ASTR 5550: HW4

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```
# Libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import random
from scipy.stats import chi2
import os,sys
import seaborn as sns
sns.set_style('whitegrid')

# import helper script file
## change working directory
os.chdir("C:/Users/rokka/GH-repos/GitHubPages/Code-Reference-Notebook/CU-Boulder/AstroPhys/HW

## import my own code
import hw_helper_func2 as hf   # this is my own code I made (for probability/distribution fu
```

(**JK note:** To view the code with the functions I made myself to (hopefully) help with all assignments click here)

1. Combining Poisson Distributions

Given two Poisson distributions:

$$P(x, \mu_A) = \frac{\mu_A^x}{x!}e^{-\mu_A} \text{ and } P(x, \mu_B) = \frac{\mu_B^x}{x!}e^{-\mu_B}$$

Show that they combine to a Poisson distribution:

$$P(x, \mu_C) \text{ where } \mu_C = \mu_A + \mu_B$$

Hint: For any given integer x, the one must sum all possibilities of $P(i, \mu_A)P(x - i, \mu_B)$.

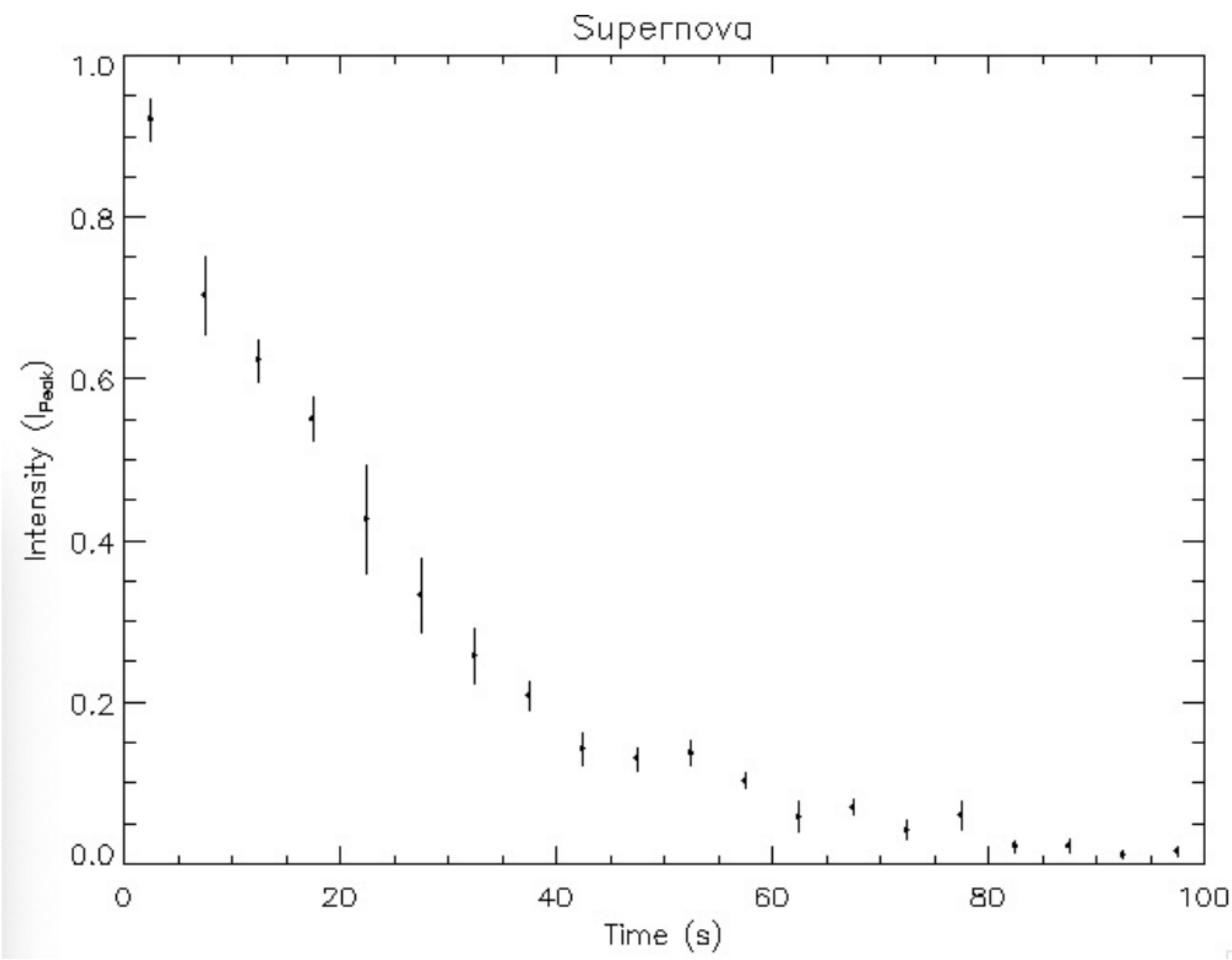
```
Didn't finish yet
```

2. Supernova Light Curve

After a supernova reaches its maximum brightness, the light curve exponentially decays as do the radioactive materials. The decay time can tell us its type. Examine the light curve below.

$I = [0.921, 0.704, 0.623, 0.550, 0.426, 0.332, 0.258, 0.208, 0.143, 0.130, 0.137, 0.103, 0.058, 0.070, 0.042, 0.060, 0.022,$

$\sigma = [0.026, 0.048, 0.026, 0.027, 0.068, 0.046, 0.034, 0.017, 0.020, 0.014, 0.015, 0.009, 0.019, 0.010, 0.012, 0.018, 0.007,$



```python
I = np.array([0.921, 0.704, 0.623, 0.550, 0.426, 0.332, 0.258, 0.208, 0.143, 0.130,
              0.137, 0.103, 0.058, 0.070, 0.042, 0.060, 0.022, 0.022, 0.011, 0.015])
sigma =  np.array([0.026, 0.048, 0.026, 0.027, 0.068, 0.046, 0.034, 0.017, 0.020, 0.014,
                   0.015, 0.009, 0.019, 0.010, 0.012, 0.018, 0.007, 0.008, 0.005, 0.005])
```

Part (a)

Assuming that σ represents a 1-sigma Gaussian uncertainty, find the most likely parameters under the hypothesis that the intensity undergoes an exponential decay:

$$I = I_0 e^{-t/\tau}$$

Here, τ is the decay time. As one can see, I_0 should be nearly unity but, for this problem, do not fix $I_0 = 1$. Calculate the uncertainty in τ. Plot the observations and the fit.

Hint: One way is to perform a linear fit to $ln(I)$. Be careful how you treat the uncertainty σ; Taylor expand $ln(I \pm \sigma)$ to calculate the uncertainties of $ln(I)$.

$$I = I_o e^{-t/\tau}$$

$$\Rightarrow \ln(I) = \ln(I_o e^{-t/\tau})$$

$$= \ln(I_o) + \ln(e^{-t/\tau})$$

$$= \ln(I_o) + (-t/\tau)$$

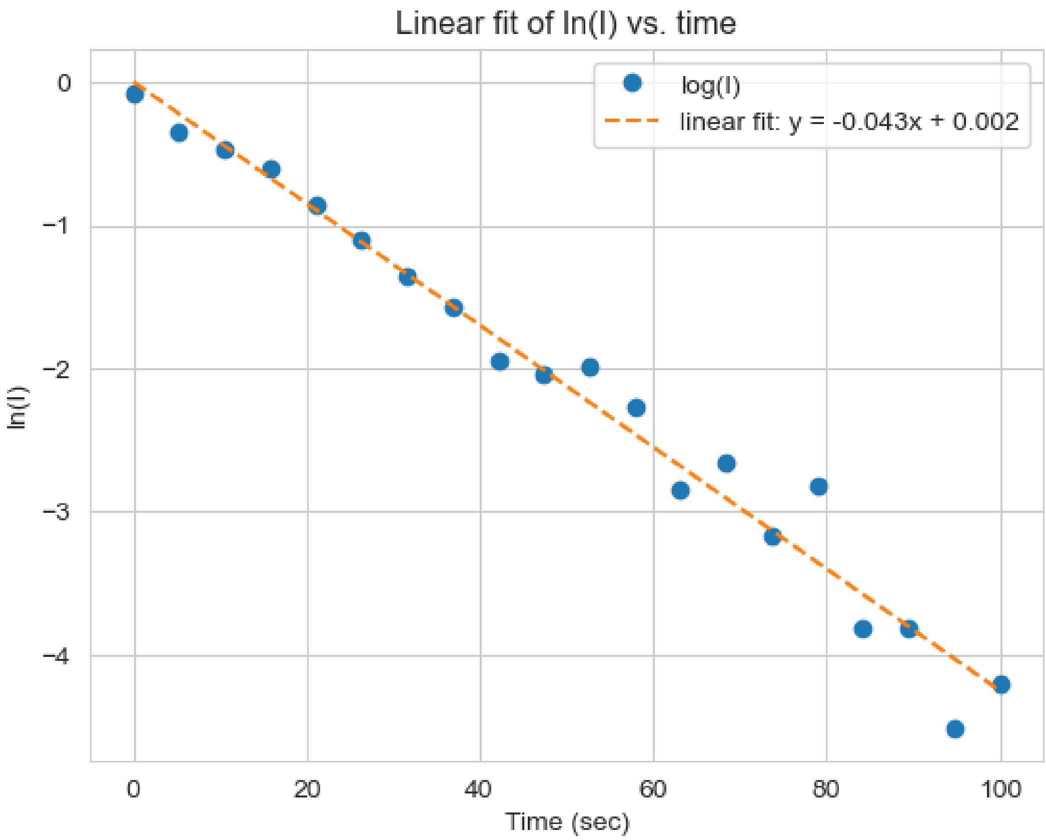
$$\ln(I) = -\left(\frac{1}{\tau}\right) t + \ln(I_o)$$

$$y = (m)(x) + b$$

$$\Rightarrow \boxed{\tau = \frac{-1}{\text{Slope}}} \quad \& \quad \boxed{\begin{array}{l}\ln(I_o) = \text{intercept} \\ \Rightarrow I_o = e^{\text{intercept}}\end{array}}$$

Calculating uncertainty in tau not finished yet

```
# Linear fit of ln(I)
## x-axis (time)
t = np.linspace(0,100,len(I))            # the x-axis (time) appears to go from 0 to 100 sec i
## linear fit
coef = np.polyfit(t,np.log(I),deg=1)     # obtain coefficients for a linear fit ("polyfit" of
lin_fit = np.poly1d(coef)                # linear fit function

# plot linear fit
plt.plot(t,np.log(I),'o',label = 'log(I)')
plt.plot(t,lin_fit(t),'--',label='linear fit: y = {0:.3f}x + {1:.3f}'.format(coef[0],coef[1])
plt.title("Linear fit of ln(I) vs. time")
plt.xlabel('Time (sec)')
plt.ylabel('ln(I)')
plt.legend()
```



```
print("slope of linear fit:",coef[0])
print("intercept of linear fit:",coef[1])
```

```
slope of linear fit: -0.04250665756665193
intercept of linear fit: 0.0021199482368504264
```

```
print("tau:", -1/coef[0])
print("I0:",np.exp(coef[1]))
```

```
tau: 23.52572649194929
I0: 1.002122196915861
```

Part (b)

Calculate χ^2_ν and compare it to the expected PDF/CDF of χ^2_ν. Plot your results. Is the hypothesis justified? What is the probability for χ^2_ν to be above the calculated value?

Jasmine's note-to-self

χ^2_N ("traditional"; unbinned, non-reduced)

$$
\begin{aligned}
\chi^2_N &= \sum_{i=1}^{N} \frac{(x_i - \mu')^2}{\sigma^2} \\
&\approx \sum_{i=1}^{N} \frac{(x_i - \mu')^2}{\mu'} \\
&\approx \sum_{i=1}^{N} \frac{(x_i - \mu')^2}{\sigma_i^2}
\end{aligned}
$$

Where:

- $\sigma^2 \equiv$ parent variance
- $\mu' \equiv$ expected variance
- $\sigma_i^2 \equiv$ variance of an individual measurement

Generalizing, we can write:

$$\chi^2_\nu = \sum_{i=1}^{N} \frac{(y_i - y(x_i))^2}{\sigma_i^2} \qquad\qquad \rightarrow \nu = N - m$$

$$\chi^2_R = \frac{1}{\nu}\chi^2_\nu \qquad\qquad \rightarrow 1$$

```
# hypothesized formula for intensity
def hyp_function_I(t):
    """hypothesis function for intensity (I) undergoing exponential decay
    I = I_0*exp(-t/tau)
    """
    tau = -1/coef[0]            # use tau calculated from linear fit in part (a)
    I0 = np.exp(coef[1])        # use I_0 calculated from linear fit in part (a)
    return I0*np.exp(-t/tau)    # I_0*e^(-t/tau)
```

If I use

$$\chi^2_\nu = \sum_{i=1}^{N} \frac{(y_i - y(x_i))^2}{\sigma_i^2}$$

Where:

- $y_i = i^{th}$ element in I (measured intensity)
- $y(x_i) =$ calculated I_i using the hypothesized formula ($I_i = I_0 e^{-t_i/\tau}$)
- $\sigma_i = i^{th}$ element in the given list of σ (variance in measurement)

```
# calculate I from hypothesis function
calculated_I = hyp_function_I(t=t)
```

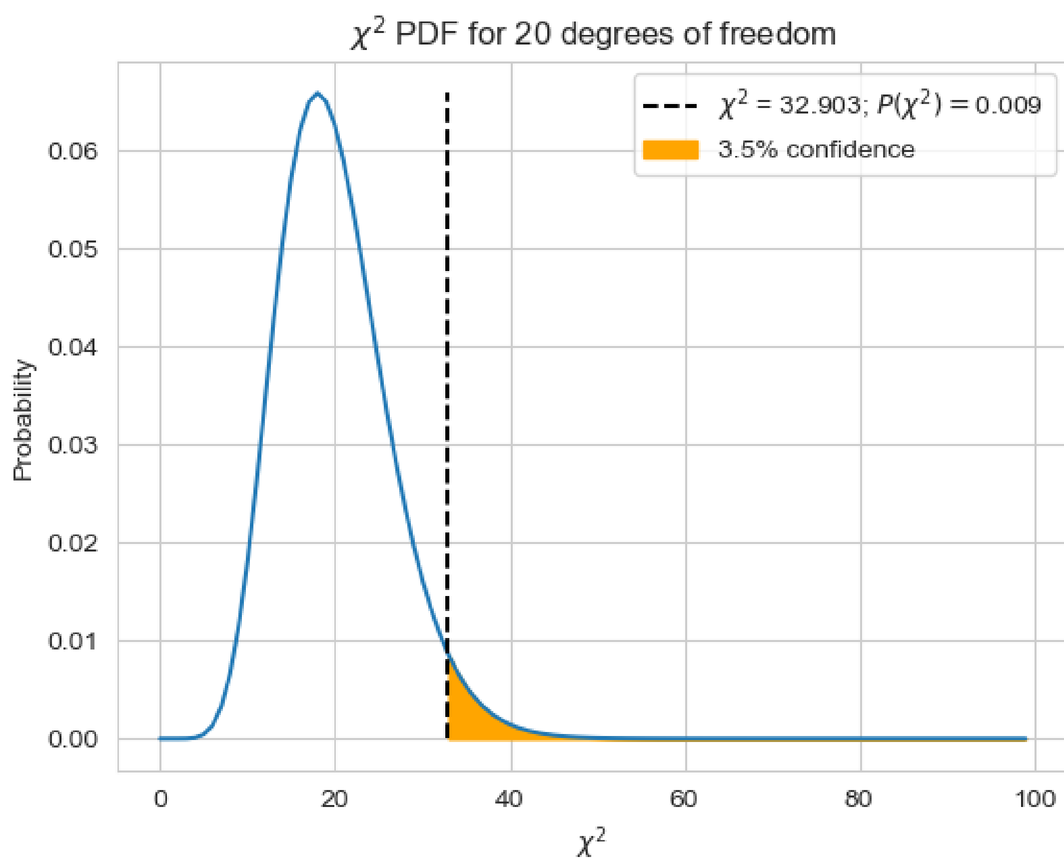
```
q2b = hf.chi_squared(x=I,parameters=1,sigma=sigma)
q2b.calculate_chi2(x=I,mu_prime=calculated_I,sigma_2=sigma**2,set_to_object=True)
print('chi-squared ("traditional"; non-reduced) = ',q2b.cs)
```

```
chi-squared ("traditional"; non-reduced) =  32.90276036256053
```

```
df = 20
confidence = q2b.calculate_chi2_confidence(cs=q2b.cs,df=df,set_to_object=True )
print("For chi-squared={0:.3f} with {1:.0f} deg. of freedom:".format(q2b.cs,df))
print("Probability:", chi2.pdf(x=q2b.cs,df=df))
print("Confidence:", confidence)
```

```
For chi-squared=32.903 with 20 deg. of freedom:
Probability: 0.00871534107424103
Confidence: 0.03457894779339932
```

```
q2b.plot_chi2_pdf(df=20)
```

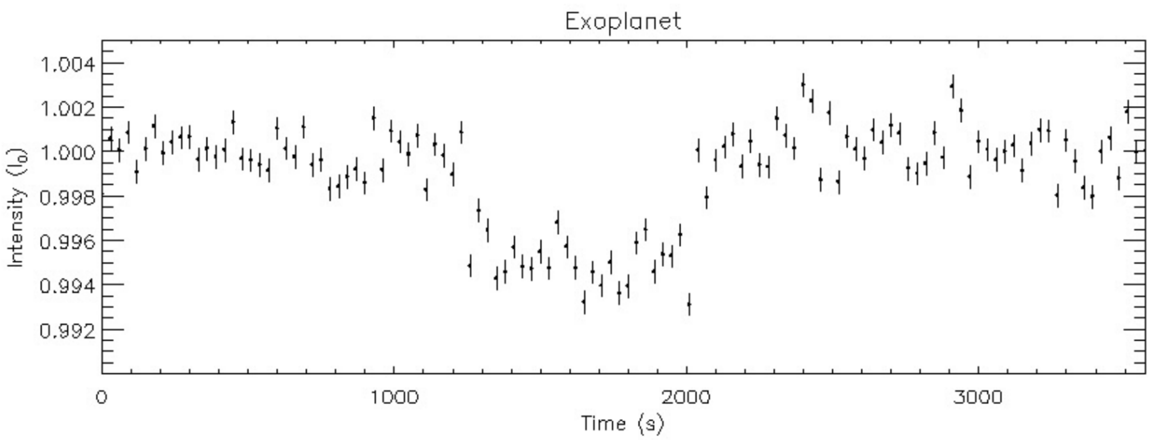


3. Extra-Solar Planet

The Kepler mission used the transit method in which one examines a time series of a star's intensity for a negative excursion. Under this method, the parent distribution of a star's intensity can be well established. In this example, the star's intensity is measured at a 30 sec cadence and found to be $I_0 + 0.001 I_0$ (1-sigma) with a Gaussian parent distribution.

Finding a transit often involves several steps. The first step is to identify intervals that may have a transiting planet. One way is to examine one hour (120-point) stretches (sliding every half an hour, 60 points) for a non-constant distribution.

Read in the text file, `HW4_data_A`, from Canvas. It contains 120 points of intensity in units of I_0, one every 30 seconds. Create a corresponding time array going from 0 to 3570 seconds. Assume the uncertainty in time is negligible.




```
# load data
data_array = np.loadtxt('hw4/HW4_data.txt')
data_array
```

```
array([0.998088, 1.00058 , 1.00007 , 1.00085 , 0.999086, 1.00013 ,
       1.00114 , 0.999925, 1.00042 , 1.00063 , 1.00064 , 0.999636,
       1.00013 , 0.999742, 1.00007 , 1.00132 , 0.999664, 0.999608,
       0.999392, 0.999136, 1.00104 , 1.00014 , 0.99976 , 1.0011  ,
       0.999397, 0.99961 , 0.998294, 0.998431, 0.998846, 0.99919 ,
       0.998581, 1.00151 , 0.999167, 1.00092 , 1.00044 , 0.999879,
       1.00071 , 0.998279, 1.00031 , 0.999818, 0.998973, 1.00086 ,
       0.994858, 0.997352, 0.99644 , 0.994283, 0.994575, 0.995698,
       0.994824, 0.994713, 0.995482, 0.994736, 0.996792, 0.995706,
       0.994752, 0.993211, 0.994558, 0.993956, 0.995004, 0.993613,
```

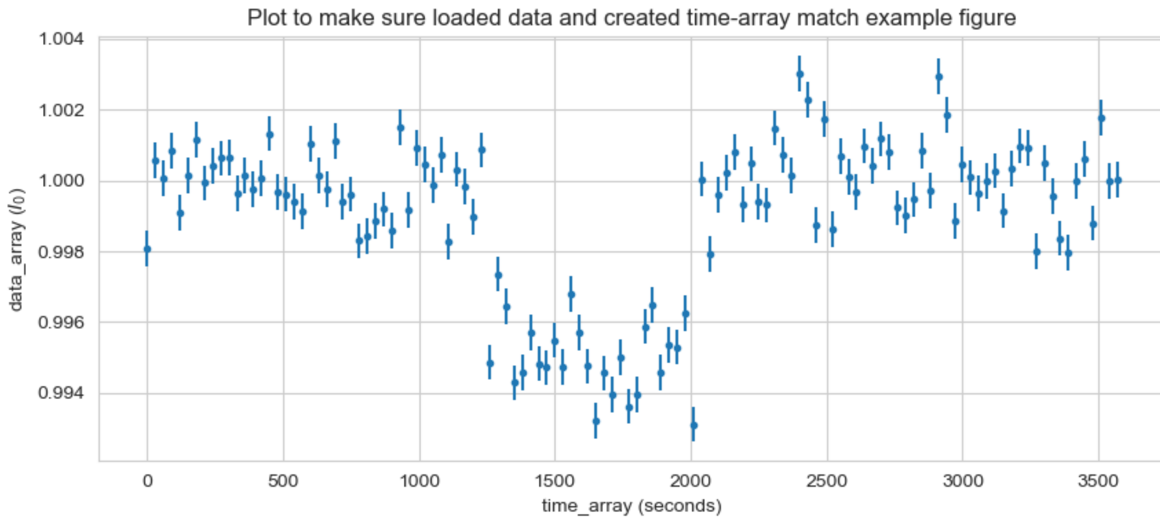
```
0.993948, 0.995872, 0.996478, 0.994574, 0.995362, 0.99529 ,
0.996243, 0.993112, 1.00004 , 0.99792 , 0.999602, 1.00021 ,
1.00079 , 0.999311, 1.00047 , 0.999404, 0.999304, 1.00148 ,
1.00072 , 1.00014 , 1.00301 , 1.00228 , 0.998727, 1.00173 ,
0.998629, 1.00067 , 1.00009 , 0.999671, 1.00097 , 1.0004  ,
1.00117 , 1.00081 , 0.999229, 0.998998, 0.999459, 1.00084 ,
0.999723, 1.00293 , 1.00184 , 0.998846, 1.00044 , 1.00008 ,
0.999624, 0.999991, 1.00027 , 0.99914 , 1.00034 , 1.00097 ,
1.00092 , 0.998008, 1.00049 , 0.999557, 0.998361, 0.997969,
0.999991, 1.00061 , 0.998795, 1.00178 , 0.999991, 1.00003 ])
```

```
# create time-array
t0 = 0          # start of time array
tf = 3570       # end of time array
dt = 30         # timestep (30 seconds)

time_array = np.arange(t0,tf+dt,dt)
print("length of time_array:",len(time_array))
```

```
length of time_array: 120
```

```
# plot (with error bars)
plt.figure(figsize=(10,4))
plt.errorbar(time_array,data_array,yerr=0.0005*data_array,fmt='.')
plt.xlabel('time_array (seconds)')
plt.ylabel('data_array ($I_0$)')
plt.title("Plot to make sure loaded data and created time-array match example figure")
```

```
Text(0.5, 1.0, 'Plot to make sure loaded data and created time-array match example figure')
```



Part (a)

Start by eliminating the possibility that the negative excursion is a random fluctuation. Plot the PDF of the expected χ^2_ν under the hypothesis that the intensity is constant. Calculate χ^2_ν and compare to show that this event is **not** consistent with a constant intensity. What is the mean of the intensity (I_μ) and the uncertainty of the mean ($\sigma_{I\mu}$)? Is I_μ less than 1 by more than the $\sigma_{I\mu}$?

Hint: σ of the parent distribution is known ($0.001 I_0$)

```
print("mean of intensity:",np.mean(data_array))
```

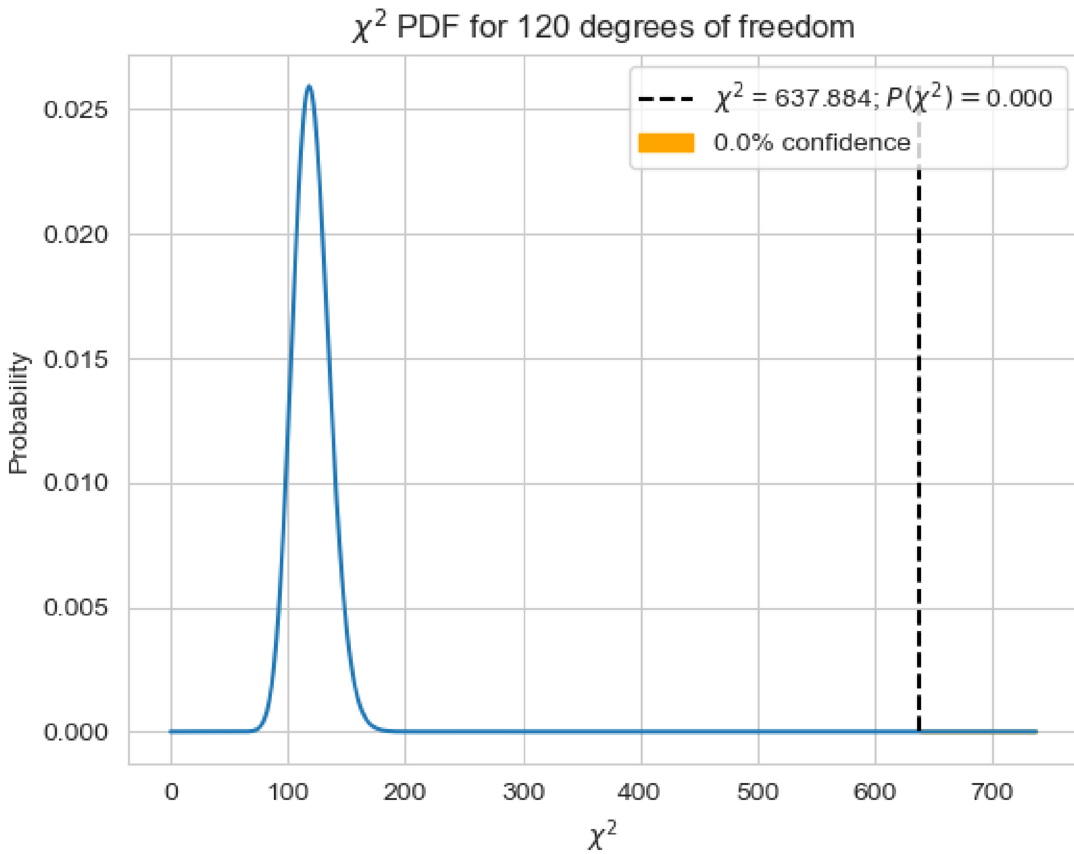
```
mean of intensity: 0.9989476166666668
```

```
# uncertainty of mean
# (come back to this later)
```

```
cs_constant = hf.chi_squared(x=data_array,sigma=0.001*data_array)
cs_constant.calculate_chi2(x=data_array,mu_prime=np.mean(data_array),sigma_2=(0.001*data_arra
```

```
637.8838224539259
```

```
cs_constant.plot_chi2_pdf(cs=cs_constant.cs,df=len(data_array))
```



Part (b)

Now that the interval is identified as significant and negative, let's examine and fit the negative excursion. Keeping it simple, use a three-parameter $(I_0, t_{start}, t_{end})$ fit:

$$I = \begin{cases} I_0 - \Delta I & t_{start} \leq t \leq t_{end} \\ I_0 & \text{otherwise} \end{cases}$$

Do a least-squares fit with a method of choose. My method is to guess t_{start} and t_{end} then calculate χ^2_ν along with ΔI. Increment t_{start} and t_{end} and recalculate ΔI until χ^2_ν is minimum. Plot the data (with error bars if you can) and overplot your fit. What are I_0, t_{start}, and t_{end}?

```
# TODO: Have a way to "automate" finding lowest chi-squared for given time-intervals to try
class exo_partb:
    def __init__(self,t_start,t_end,I0=1,data=data_array,time=time_array,sigma=None):
```

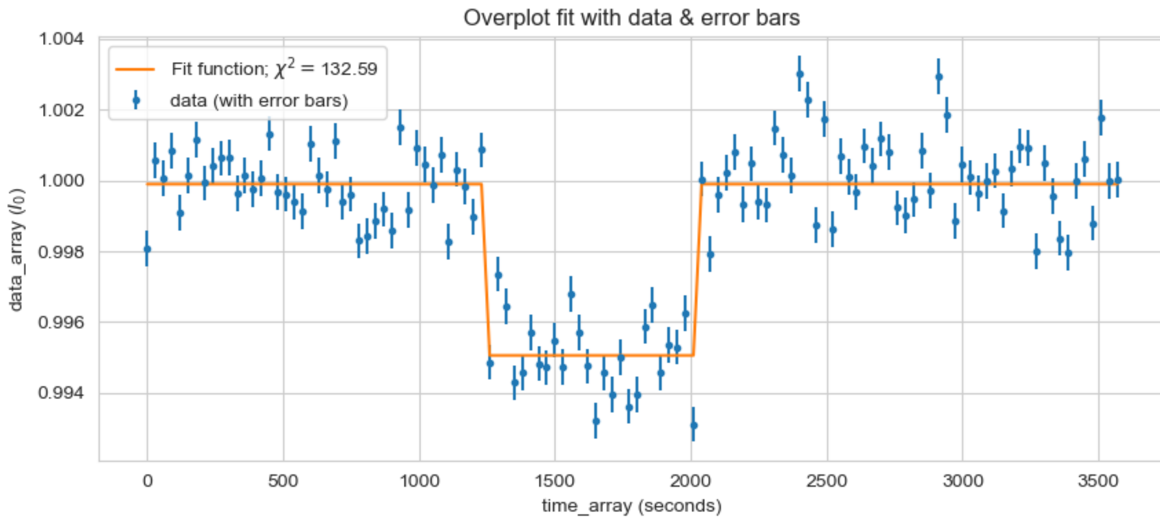
```
        self.data_array = data
        self.time_array = time
        self.t_start = t_start
        self.t_end = t_end
        self.I0 = I0

        if sigma is None:
            self.sigma = 0.001*self.data_array
        else:
            self.sigma = sigma

    def fit_udu(self,t_start=None, t_end=None, I0=None,
                set_to_object=True,
                calculate_chi_squared = True,
                sigma=None):
        """Fit 'up-down-up'
        """
        if t_start is None:
            t_start = self.t_start
        if t_end is None:
            t_end = self.t_end
        if I0 is None:
            I0 = self.I0

        delta_I = I0 - np.mean(self.data_array[(self.time_array>=t_start) & (self.time_array
        fit = []
        for t in time_array:
            if t >= t_start and t <= t_end:
                fit.append(I0 - delta_I)
            else:
                fit.append(I0)

        fit = np.array(fit)
        if set_to_object:
            self.fit = fit

        if calculate_chi_squared:
            if sigma is None:
                sigma = self.sigma
            self.chi2 = hf.nonreduced_chi2(x=self.data_array,mu_prime=fit,sigma_2=sigma**2)
            self.reduced_chi2 = hf.reduced_chi2(x=self.data_array,mu_prime=fit,sigma_2=sigma
                                                parameters=3,verbose=False)
```

```
            print("For fit with parameters, t_start={0:.0f}, t_end={1:.0f}, & I0={2:.3f}:".fo
            print("(non-reduced) chi-squared:",self.chi2)
            print("reduced chi-squared:",self.reduced_chi2)
            return fit

        return fit

    def plot_data_eb(self,title="Plot Data with Error bars"):
        """Plot data with error bars"""
        plt.errorbar(self.time_array,self.data_array,yerr=self.sigma/2,fmt='.',label='data (w
        plt.xlabel('time_array (seconds)')
        plt.ylabel('data_array ($I_0$)')
        plt.title(title)

    def overplot_fit_to_data(self,fit=None,title="Overplot fit with data & error bars"):
        if fit is None:
            fit = self.fit_udu(t_start=self.t_start,t_end=self.t_end,I0=self.I0)

        self.plot_data_eb(title=title)
        plt.plot(self.time_array,fit,label='Fit function; $\chi^2 =${0:.2f}'.format(self.chi2
```

```
t_start = 1250
t_end = 2010

q3b = exo_partb(t_start=t_start,t_end=t_end,I0=np.mean(data_array[time_array<t_start]))
```

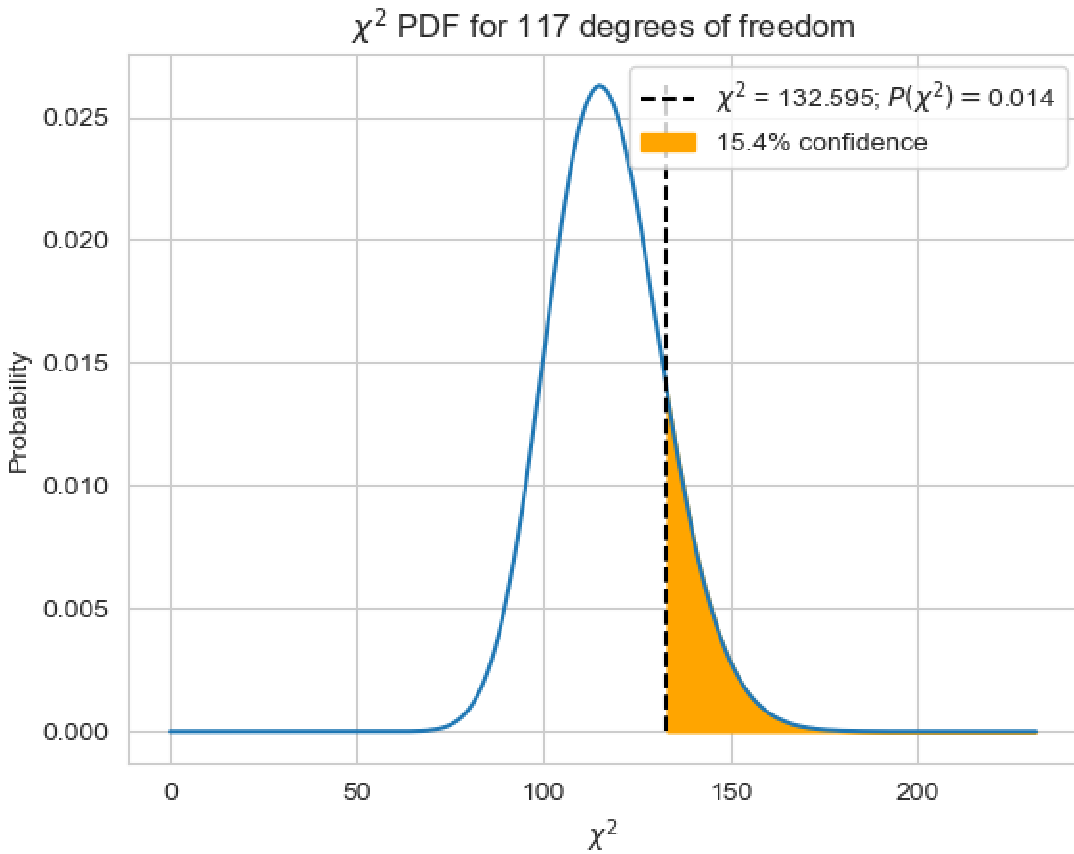
```
# plot (with error bars)
plt.figure(figsize=(10,4))
q3b.overplot_fit_to_data()
plt.legend()
```

```
For fit with parameters, t_start=1250, t_end=2010, & I0=1.000:
(non-reduced) chi-squared: 132.59462317601458
reduced chi-squared: 1.133287377572774
```




```
print("Not asked in the problem, but Jasmine was just curious")
c3b = hf.chi_squared(x=data_array,parameters=3,sigma=0.001*data_array)
c3b.plot_chi2_pdf(cs=q3b.chi2)
```

```
Not asked in the problem, but Jasmine was just curious
```



Part (c)

Estimate the uncertainties of I_0, t_{start}, and t_{end}. Explain how you arrive at your values.

Hint: The uncertainty of ΔI is straight-forward. Recall that you can calculate σ_I, but $\partial t/\partial I$ can only be estimated. Can one have an uncertainty in time that is less than δt (30 seconds)?

```
# not finished yet
```

4. Kolmogorov-Smirnov Test

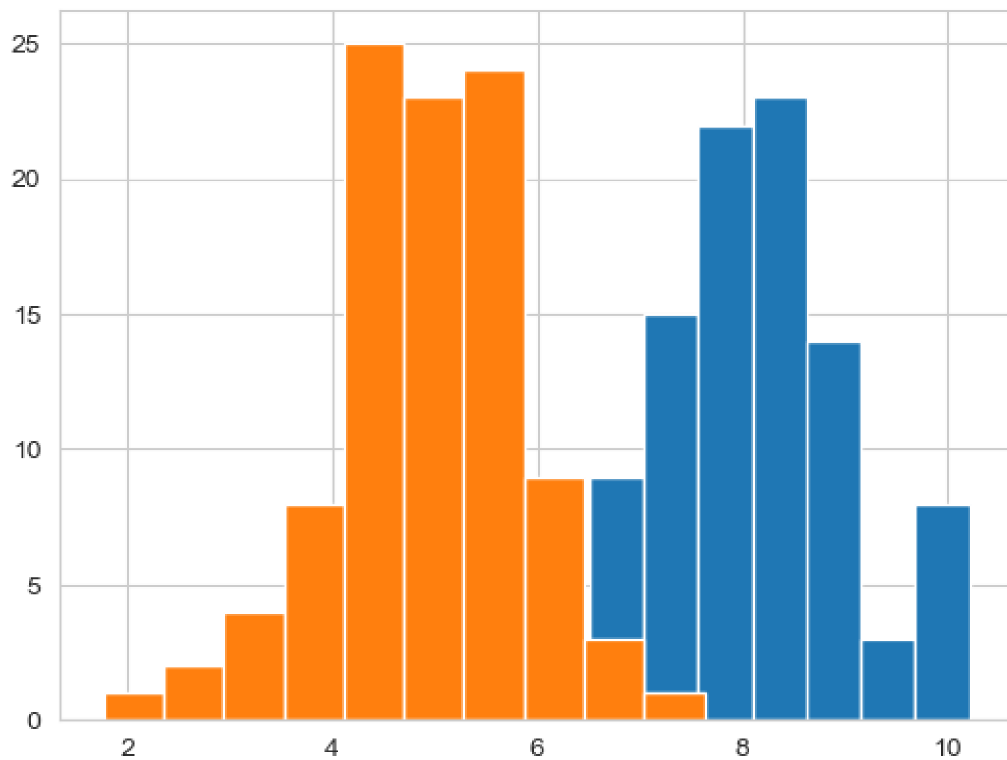
Using a random number generator, create two distributions:

$$f_1(x) = P(x, \mu_1, n); \mu_1 = 8, n = 100$$

$$f_2(x) = P(x, \mu_2, n); \mu_2 = 5, n = 100$$

```
n_points = 100

# distribution 1
mu1 = 8
fx1 = np.random.normal(mu1, size=n_points)

# distribution 2
mu2 = 5
fx2 = np.random.normal(mu2,size=n_points)
```

```
# histogram of two distributions
plt.hist(fx1)
plt.hist(fx2)
plt.show()
```



Part (a)

Calculated and plot the two CDFs for $n = 100$. Compare the two distributions using the Kolmogorov-Smirnov Test with $\alpha = 0.1$. The more exact formula for the threshold is:

$$D > \sqrt{-\frac{1}{2}\ln\left(\frac{\alpha}{2}\right)}\sqrt{\frac{n+m}{nm}}; n, m \text{ are number of points}$$

```
# not finished yet
```

Part (b)

Repeat the test several (5 to 10) times recreating the distributions. Do f_1 and f_2 consistently pass or fail the test?

```
# not finished yet
```

Part (c)

Repeat the test for higher n, say 1000 (for both f_1 and f_2) several times. Does the test at $n = 1000$ reveal that the two distributions are not from the same parent? What does this exercise tell us about the Kolmogorov-Smirnov Test?

```
# not finished yet
```