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:

1.1

````
```{python}
1 + 1
```
````

2

1.2

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1/2

book

1.3

1. git wsl
2. rstudio quarto
3. https://github.com/Roku-3/rindoku_RL main rstudio

1.3.1

git

```
git pull origin HEAD             # github

git add .                        #

git commit -m "edit chapter 2"   #
                                 # -m

git push origin HEAD             # github
```

2 push

`git branch` `main` push
push

1.3.2

github

2

2.0.1

197X

3 :

/

3.1

(policy)
(reward)
(value function)
(model)

3.2

3.3

0~1 0 1 0.5

$$V(s_t) \leftarrow V(s_t) + \alpha \left[V\left(s_{t+1}\right) - V(s_t) \right]$$

$V(s)$ s α

TD ; temporal-difference learning

3.4

1.1 self play

```{python}
    '''
    import numpy as np
    import pickle
    '''
```

```
'\nimport numpy as np\nimport pickle\n'
```

```{python}
    """

    BOARD_ROWS = 3
    BOARD_COLS = 3
    BOARD_SIZE = BOARD_ROWS * BOARD_COLS

    class State:
        def __init__(self):
            self.data = np.zeros((BOARD_ROWS, BOARD_COLS))
            self.winner = None
            self.hashVal = None
            self.end = None
```

```python
def getHash(self):
    if self.hashVal is None:
        self.hashVal = 0
        for i in self.data.reshape(BOARD_ROWS * BOARD_COLS):
            if i == -1:
                i = 2
            self.hashVal = self.hashVal * 3 + i
    return int(self.hashVal)

def isEnd(self):
    if self.end is not None:
        return self.end
    results = []
    for i in range(0, BOARD_ROWS):
        results.append(np.sum(self.data[i, :]))
    for i in range(0, BOARD_COLS):
        results.append(np.sum(self.data[:, i]))

    results.append(0)
    for i in range(0, BOARD_ROWS):
        results[-1] += self.data[i, i]
    results.append(0)
    for i in range(0, BOARD_ROWS):
        results[-1] += self.data[i, BOARD_ROWS - 1 - i]

    for result in results:
        if result == 3:
            self.winner = 1
            self.end = True
            return self.end
        if result == -3:
            self.winner = -1
            self.end = True
            return self.end

    sum = np.sum(np.abs(self.data))
    if sum == BOARD_ROWS * BOARD_COLS:
        self.winner = 0
        self.end = True
        return self.end

    self.end = False
    return self.end
```

```
    def nextState(self, i, j, symbol):
        newState = State()
        newState.data = np.copy(self.data)
        newState.data[i, j] = symbol
        return newState

    # print board
    def show(self):
        for i in range(0, BOARD_ROWS):
            print('-------------')
            out = '| '
            for j in range(0, BOARD_COLS):
                if self.data[i, j] == 1:
                    token = '*'
                if self.data[i, j] == 0:
                    token = '0'
                if self.data[i, j] == -1:
                    token = 'x'
                out += token + ' | '
            print(out)
        print('-------------')

def getAllStatesImpl(currentState, currentSymbol, allStates):
    for i in range(0, BOARD_ROWS):
        for j in range(0, BOARD_COLS):
            if currentState.data[i][j] == 0:
                newState = currentState.nextState(i, j, currentSymbol)
                newHash = newState.getHash()
                if newHash not in allStates.keys():
                    isEnd = newState.isEnd()
                    allStates[newHash] = (newState, isEnd)
                    if not isEnd:
                        getAllStatesImpl(newState, -currentSymbol, allStates)

def getAllStates():
    currentSymbol = 1
    currentState = State()
    allStates = dict()
    allStates[currentState.getHash()] = (currentState, currentState.isEnd())
    getAllStatesImpl(currentState, currentSymbol, allStates)
    return allStates
```

```python
allStates = getAllStates()

class Judger:
    def __init__(self, player1, player2, feedback=True):
        self.p1 = player1
        self.p2 = player2
        self.feedback = feedback
        self.currentPlayer = None
        self.p1Symbol = 1
        self.p2Symbol = -1
        self.p1.setSymbol(self.p1Symbol)
        self.p2.setSymbol(self.p2Symbol)
        self.currentState = State()
        self.allStates = allStates

    def giveReward(self):
        if self.currentState.winner == self.p1Symbol:
            self.p1.feedReward(1)
            self.p2.feedReward(0)
        elif self.currentState.winner == self.p2Symbol:
            self.p1.feedReward(0)
            self.p2.feedReward(1)
        else:
            self.p1.feedReward(0.1)
            self.p2.feedReward(0.5)

    def feedCurrentState(self):
        self.p1.feedState(self.currentState)
        self.p2.feedState(self.currentState)

    def reset(self):
        self.p1.reset()
        self.p2.reset()
        self.currentState = State()
        self.currentPlayer = None

    def play(self, show=False):
        self.reset()
        self.feedCurrentState()
        while True:
            if self.currentPlayer == self.p1:
                self.currentPlayer = self.p2
            else:
```

```
                self.currentPlayer = self.p1
            if show:
                self.currentState.show()
            [i, j, symbol] = self.currentPlayer.takeAction()
            self.currentState = self.currentState.nextState(i, j, symbol)
            hashValue = self.currentState.getHash()
            self.currentState, isEnd = self.allStates[hashValue]
            self.feedCurrentState()
            if isEnd:
                if self.feedback:
                    self.giveReward()
                return self.currentState.winner

# AI player
class Player:
    def __init__(self, stepSize = 0.1, exploreRate=0.1):
        self.allStates = allStates
        self.estimations = dict()
        self.stepSize = stepSize
        self.exploreRate = exploreRate
        self.states = []

    def reset(self):
        self.states = []

    def setSymbol(self, symbol):
        self.symbol = symbol
        for hash in self.allStates.keys():
            (state, isEnd) = self.allStates[hash]
            if isEnd:
                if state.winner == self.symbol:
                    self.estimations[hash] = 1.0
                else:
                    self.estimations[hash] = 0
            else:
                self.estimations[hash] = 0.5

    def feedState(self, state):
        self.states.append(state)

    def feedReward(self, reward):
        if len(self.states) == 0:
            return
```

```
        self.states = [state.getHash() for state in self.states]
        target = reward
        for latestState in reversed(self.states):
            value = self.estimations[latestState] + self.stepSize * (target - self.estim
            self.estimations[latestState] = value
            target = value
        self.states = []

    def takeAction(self):
        state = self.states[-1]
        nextStates = []
        nextPositions = []
        for i in range(BOARD_ROWS):
            for j in range(BOARD_COLS):
                if state.data[i, j] == 0:
                    nextPositions.append([i, j])
                    nextStates.append(state.nextState(i, j, self.symbol).getHash())
        if np.random.binomial(1, self.exploreRate):
            np.random.shuffle(nextPositions)
            self.states = []
            action = nextPositions[0]
            action.append(self.symbol)
            return action

        values = []
        for hash, pos in zip(nextStates, nextPositions):
            values.append((self.estimations[hash], pos))
        np.random.shuffle(values)
        values.sort(key=lambda x: x[0], reverse=True)
        action = values[0][1]
        action.append(self.symbol)
        return action

    def savePolicy(self):
        fw = open('optimal_policy_' + str(self.symbol), 'wb')
        pickle.dump(self.estimations, fw)
        fw.close()

    def loadPolicy(self):
        fr = open('optimal_policy_' + str(self.symbol),'rb')
        self.estimations = pickle.load(fr)
        fr.close()
```

```
# | 1 | 2 | 3 |
# | 4 | 5 | 6 |
# | 7 | 8 | 9 |
class HumanPlayer:
    def __init__(self, stepSize = 0.1, exploreRate=0.1):
        self.symbol = None
        self.currentState = None
        return
    def reset(self):
        return
    def setSymbol(self, symbol):
        self.symbol = symbol
        return
    def feedState(self, state):
        self.currentState = state
        return
    def feedReward(self, reward):
        return
    def takeAction(self):
        data = int(input("Input your position:"))
        data -= 1
        i = data // int(BOARD_COLS)
        j = data % BOARD_COLS
        if self.currentState.data[i, j] != 0:
            return self.takeAction()
        return (i, j, self.symbol)

def train(epochs=20000):
    player1 = Player()
    player2 = Player()
    judger = Judger(player1, player2)
    player1Win = 0.0
    player2Win = 0.0
    for i in range(0, epochs):
        print("Epoch", i)
        winner = judger.play()
        if winner == 1:
            player1Win += 1
        if winner == -1:
            player2Win += 1
        judger.reset()
    print(player1Win / epochs)
    print(player2Win / epochs)
```

````
        player1.savePolicy()
        player2.savePolicy()

    def compete(turns=500):
        player1 = Player(exploreRate=0)
        player2 = Player(exploreRate=0)
        judger = Judger(player1, player2, False)
        player1.loadPolicy()
        player2.loadPolicy()
        player1Win = 0.0
        player2Win = 0.0
        for i in range(0, turns):
            print("Epoch", i)
            winner = judger.play()
            if winner == 1:
                player1Win += 1
            if winner == -1:
                player2Win += 1
            judger.reset()
        print(player1Win / turns)
        print(player2Win / turns)

    def play():
        while True:
            player1 = Player(exploreRate=0)
            player2 = HumanPlayer()
            judger = Judger(player1, player2, False)
            player1.loadPolicy()
            winner = judger.play(True)
            if winner == player2.symbol:
                print("Win!")
            elif winner == player1.symbol:
                print("Lose!")
            else:
                print("Tie!")

    train()
    compete()
    play()
    """
```
````

```
'\n\nBOARD_ROWS = 3\nBOARD_COLS = 3\nBOARD_SIZE = BOARD_ROWS * BOARD_COLS\n\nclass State:\n
```
```

4

4.1 k

k 1 k t A_t R_t a X_a a

$$q_*(a) := \mathbb{E}[X_a]$$

t a $Q_t(a)$ $q_*(a)$

4.2

$$Q_t(a) := \frac{t\ a}{t\ a} = \frac{\sum_{i=1}^{t-1} R_i\, \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

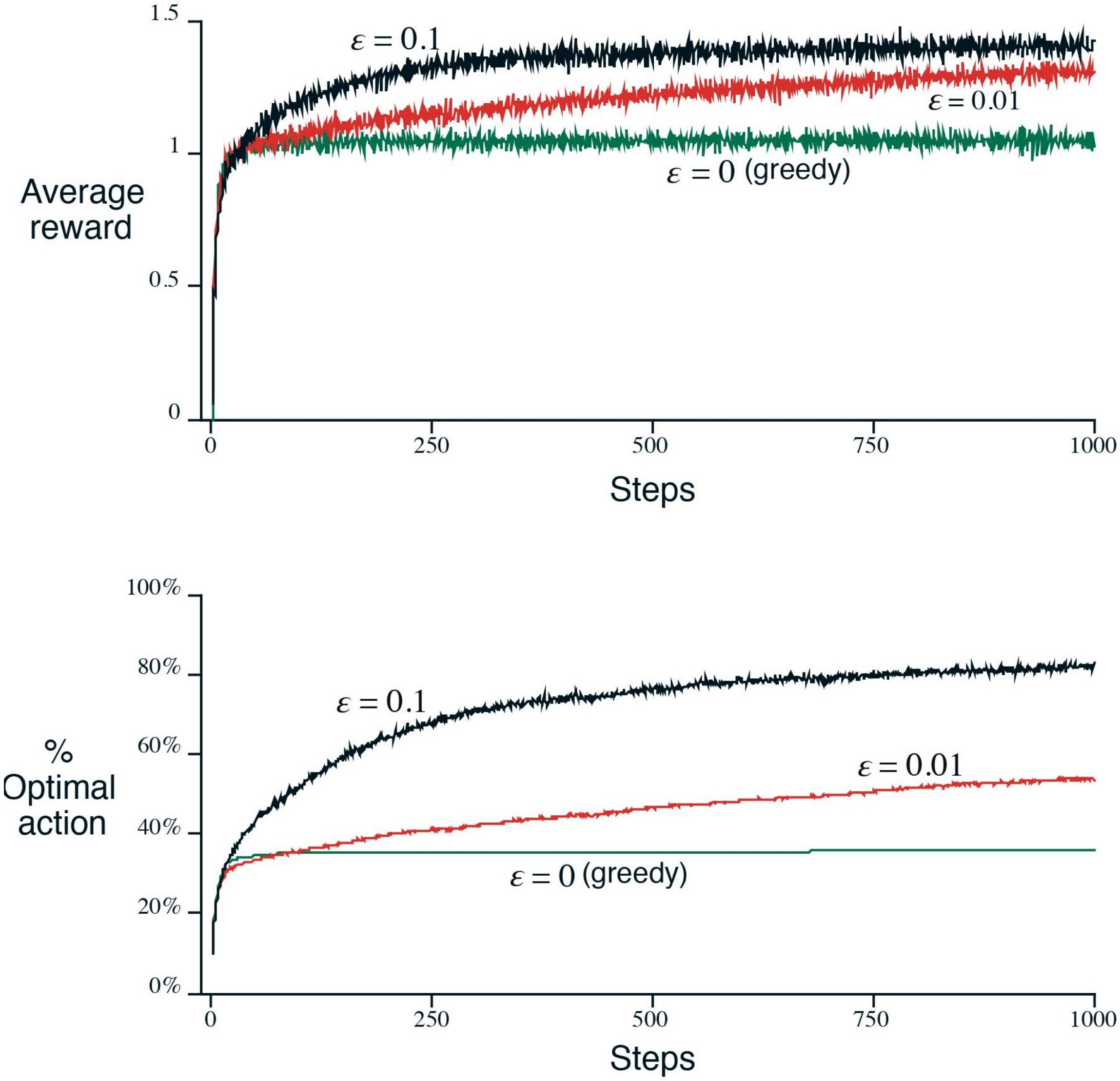
$\mathbb{1}$ 1 0

$$A_t = \arg\max_a Q_t(a)$$

$\arg\max_a Q_t(a)$ a ϵ ϵ-

4.3 10

2000 k k=10 $q_*(a)$ $(N(0,1))$ X_a $N(q_*(a),1)$ $Q_1(a)$ 0 1000 1 2000 ϵ- 2000



$\epsilon = 0.1$ $\epsilon = 0.01$ $\epsilon = 0.01$

4.4

1 n Q_n

$$Q_n := \frac{R_1 + R_2 + \quad + R_n}{n-1}$$

R_n Q_n

$$Q_{n+1} = \frac{1}{n}\sum_{i=1}^{n} R_i = \frac{1}{n}(R_n + \sum_{i=1}^{n-1} R_i) = \frac{1}{n}(R_n + (n-1)Q_n) = Q_n + \frac{1}{n}(R_n - Q_n)$$

Q_n n

$$NewEstimate \leftarrow OldEstimate + StepSize\ [Target - OldEstimate]$$

$[Target{-}OldEstimate]$ $Target($ n $)$ $StepSize$ $1/n$ α $\alpha_t(a)$

ϵ- k

4.5

(X_a) , $\alpha \in (0, 1]$

$$Q_{n+1} := Q_n + \alpha[R_n - Q_n]$$

Q_{n+1}

$$Q_{n+1} = Q_n{+}\alpha[R_n{-}Q_n] = \alpha R_n{+}(1{-}\alpha)Q_n = \alpha R_n{+}(1{-}\alpha)[\alpha R_{n-1}{+}(1{-}\alpha)Q_{n-1}] = \alpha R_n{+}(1{-}\alpha)\alpha R_{n-1}{+}(1{-}\alpha)^2 Q$$

$(1{-}a)^n{+}\sum_{i-1}^{n}\alpha(1{-}\alpha)^{n-i} = 1$ Q_{n+1} Q_1 $1{-}\alpha$
α a n $\alpha_n(a)$ 1

$$\sum_{n=1}^{\infty}\alpha_n(a) = \infty \quad \sum_{n=1}^{\infty}\alpha_n^2(a) < \infty$$

````
```{python}
print("Hello Python!")
```
````

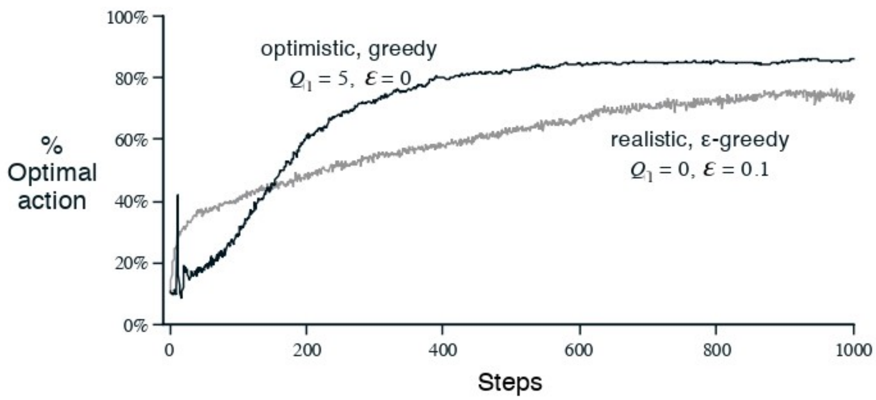
```
Hello Python!
```

4.6

$Q_1(a)$

10 $Q_1(a)$ 0 +5 $q_*(a)$ $q_*(a)$ +5 2.87×10^{-7}

10 $Q_1(a)$ +5 $Q_1(a)$ 0 ϵ-

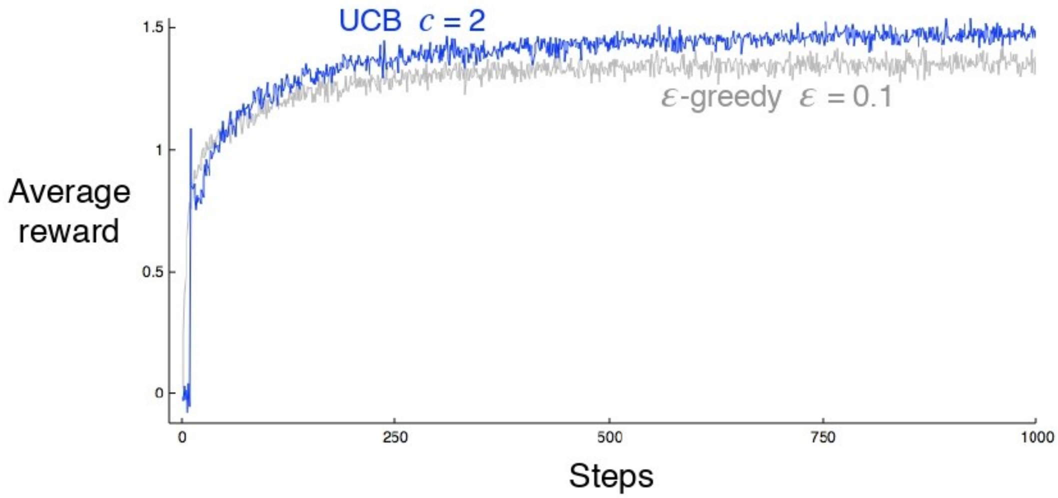


4.7

ϵ-

$$A_t := \arg\max_a \left(Q_t(a) + c\sqrt{\frac{\log_e t}{N_t(a)}} \right)$$

$N_t(a)$ t a $c > 0$ **UCB** UCB 10



UCB ϵ-

4.8

a $H_t(a)$

$$\Pr\{A_t = a\} := \pi_t(a) := \frac{e^{H_t(a)}}{\sum_{b=1}^{k} e^{H_t(b)}}$$

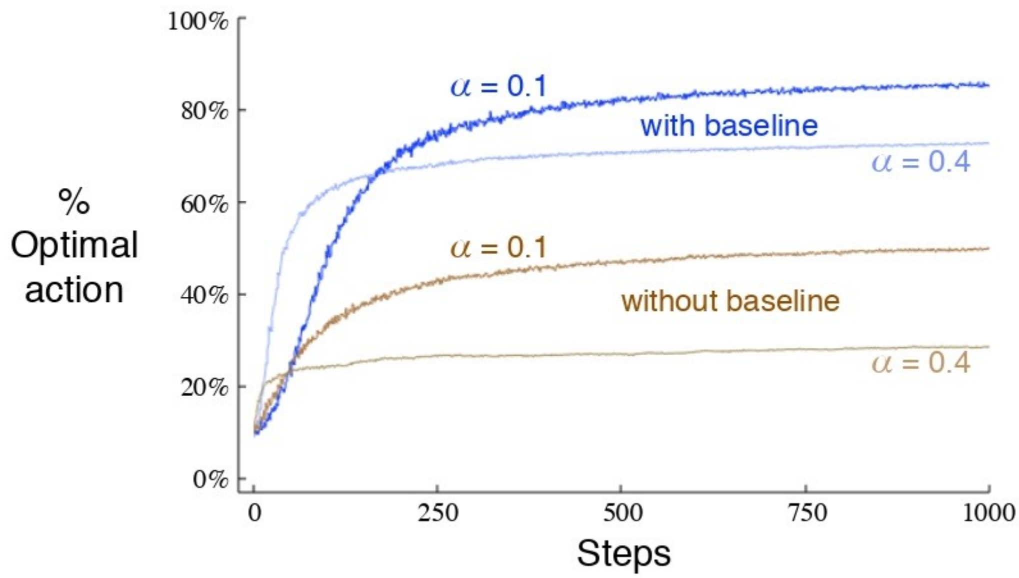
t a $\pi_t(a)$

A_t R_t

$$H_{t+1}(A_t) = H_t(A_t) + \alpha(R_t - \overline{R}_t)(1 - \pi_t(A_t)), H_{t+1}(a) := H_t(A_t) + \alpha(R_t - \overline{R}_t)\pi(a) \quad (a \neq A_t)$$

$\overline{R}_t$ t

k $q_*(a)$ +4 1 $\overline{R}_t$=0

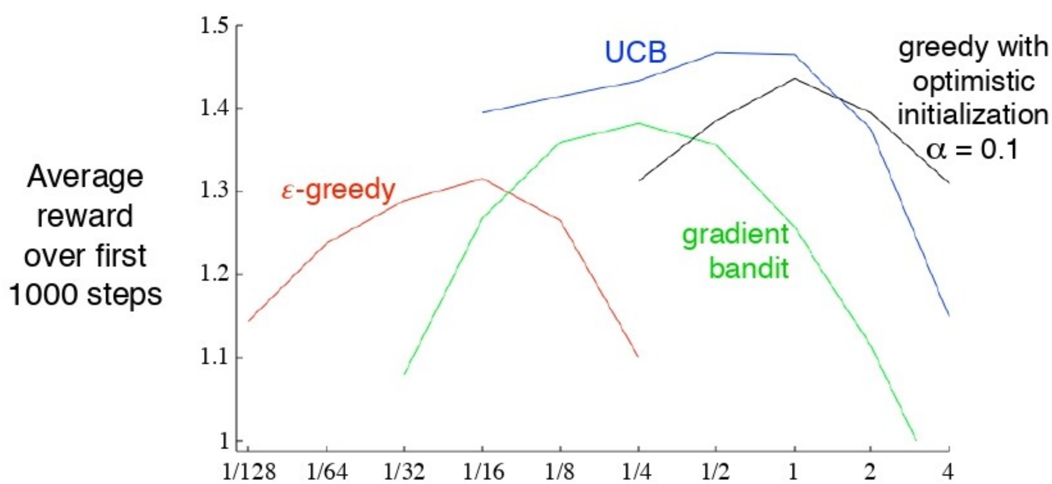


4.9

k

4.10

k x $(\epsilon\ \alpha\ c\ \ Q_0)$



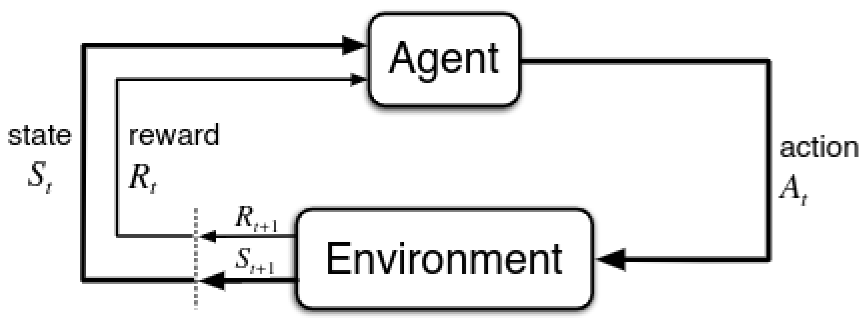
5

a $q_*(a)$

MDP s $q_*(s,a)$ $v_*(s)$

5.1

t S A 1 R_{t+1} S_{t+1}



$$p\left(s', r \mid s, a\right) = \Pr\left\{S_{t+1} = s', R_{t+1} = r \mid S_t = s, A_t = a\right\}.$$

$$r(s,a) = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p\left(s', r \mid s, a\right),$$

$$p\left(s' \mid s, a\right) = \Pr\left\{S_{t+1} = s' \mid S_t = s, A_t = a\right\} = \sum_{r \in \mathcal{R}} p\left(s', r \mid s, a\right),$$

$$r\left(s, a, s'\right) = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a, S_{t+1} = s'\right] = \frac{\sum_{r \in \mathcal{R}} r p\left(s', r \mid s, a\right)}{p\left(s' \mid s, a\right)}.$$

5.2

5.3

5.3.1

G_t T

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots + R_T$$

5.3.2

$T = \infty$

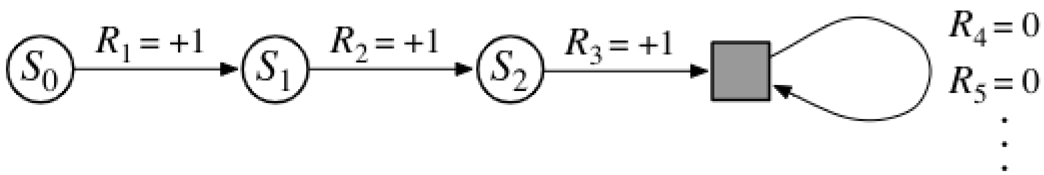
γ

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$G_t \doteq R_{t+1} + \gamma G_{t+1}$$

5.4

T 0



$T = \infty$ $\gamma = 1$

$$G_t = \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1}$$

5.5