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1

:

1.1

```
```python
1 + 1
```
```

2

1.2

| |
|-------|
| |
| |
| 11/30 |
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book

1.3

1. git wsl
2. [rstudio quarto](#)
3. https://github.com/Roku-3/rindoku_RL main rstudio

1.3.1

```
git
git pull origin HEAD          # github
git add .                     #
git commit -m "edit chapter 2" #
                               # -m
git push origin HEAD          # github
2                               push
git branch main               push
push
```

1.3.2

github

2

2.0.1

197X

3 :

/

3.1

(policy)
(reward)
(value function)
(model)

3.2

3.3

0~1 0 1 0.5

$$V(s_t) \leftarrow V(s_t) + \alpha [V(s_{t+1}) - V(s_t)]$$

TD ; temporal-difference learning

3.4

1.1 self play

```

```{python}
'''
 import numpy as np
 import pickle
'''

'''
'\nimport numpy as np\nimport pickle\n'

```{python}
'''

BOARD_ROWS = 3
BOARD_COLS = 3
BOARD_SIZE = BOARD_ROWS * BOARD_COLS

class State:
    def __init__(self):
        self.data = np.zeros((BOARD_ROWS, BOARD_COLS))
        self.winner = None
        self.hashVal = None
        self.end = None

```

```

def getHash(self):
    if self.hashVal is None:
        self.hashVal = 0
        for i in self.data.reshape(BOARD_ROWS * BOARD_COLS):
            if i == -1:
                i = 2
            self.hashVal = self.hashVal * 3 + i
        return int(self.hashVal)

def isEnd(self):
    if self.end is not None:
        return self.end
    results = []
    for i in range(0, BOARD_ROWS):
        results.append(np.sum(self.data[i, :]))
    for i in range(0, BOARD_COLS):
        results.append(np.sum(self.data[:, i]))

    results.append(0)
    for i in range(0, BOARD_ROWS):
        results[-1] += self.data[i, i]
    results.append(0)
    for i in range(0, BOARD_ROWS):
        results[-1] += self.data[i, BOARD_ROWS - 1 - i]

    for result in results:
        if result == 3:
            self.winner = 1
            self.end = True
            return self.end
        if result == -3:
            self.winner = -1
            self.end = True
            return self.end

    sum = np.sum(np.abs(self.data))
    if sum == BOARD_ROWS * BOARD_COLS:
        self.winner = 0
        self.end = True
        return self.end

    self.end = False
    return self.end

```



```

def nextState(self, i, j, symbol):
    newState = State()
    newState.data = np.copy(self.data)
    newState.data[i, j] = symbol
    return newState

# print board
def show(self):
    for i in range(0, BOARD_ROWS):
        print('-----')
        out = '| '
        for j in range(0, BOARD_COLS):
            if self.data[i, j] == 1:
                token = '*'
            if self.data[i, j] == 0:
                token = '0'
            if self.data[i, j] == -1:
                token = 'x'
            out += token + ' | '
        print(out)
    print('-----')

def getAllStatesImpl(currentState, currentSymbol, allStates):
    for i in range(0, BOARD_ROWS):
        for j in range(0, BOARD_COLS):
            if currentState.data[i][j] == 0:
                newState = currentState.nextState(i, j, currentSymbol)
                newHash = newState.getHash()
                if newHash not in allStates.keys():
                    isEnd = newState.isEnd()
                    allStates[newHash] = (newState, isEnd)
                    if not isEnd:
                        getAllStatesImpl(newState, -currentSymbol, allStates)

def getAllStates():
    currentSymbol = 1
    currentState = State()
    allStates = dict()
    allStates[currentState.getHash()] = (currentState, currentState.isEnd())
    getAllStatesImpl(currentState, currentSymbol, allStates)
    return allStates

```

```

allStates = getAllStates()

class Judge:
    def __init__(self, player1, player2, feedback=True):
        self.p1 = player1
        self.p2 = player2
        self.feedback = feedback
        self.currentPlayer = None
        self.p1Symbol = 1
        self.p2Symbol = -1
        self.p1.setSymbol(self.p1Symbol)
        self.p2.setSymbol(self.p2Symbol)
        self.currentState = State()
        self.allStates = allStates

    def giveReward(self):
        if self.currentState.winner == self.p1Symbol:
            self.p1.feedReward(1)
            self.p2.feedReward(0)
        elif self.currentState.winner == self.p2Symbol:
            self.p1.feedReward(0)
            self.p2.feedReward(1)
        else:
            self.p1.feedReward(0.1)
            self.p2.feedReward(0.5)

    def feedCurrentState(self):
        self.p1.feedState(self.currentState)
        self.p2.feedState(self.currentState)

    def reset(self):
        self.p1.reset()
        self.p2.reset()
        self.currentState = State()
        self.currentPlayer = None

    def play(self, show=False):
        self.reset()
        self.feedCurrentState()
        while True:
            if self.currentPlayer == self.p1:
                self.currentPlayer = self.p2
            else:

```

```

        self.currentPlayer = self.p1
    if show:
        self.currentState.show()
    [i, j, symbol] = self.currentPlayer.takeAction()
    self.currentState = self.currentState.nextState(i, j, symbol)
    hashValue = self.currentState.getHash()
    self.currentState, isEnd = self.allStates[hashValue]
    self.feedCurrentState()
    if isEnd:
        if self.feedback:
            self.giveReward()
        return self.currentState.winner

# AI player
class Player:
    def __init__(self, stepSize = 0.1, exploreRate=0.1):
        self.allStates = allStates
        self.estimations = dict()
        self.stepSize = stepSize
        self.exploreRate = exploreRate
        self.states = []

    def reset(self):
        self.states = []

    def setSymbol(self, symbol):
        self.symbol = symbol
        for hash in self.allStates.keys():
            (state, isEnd) = self.allStates[hash]
            if isEnd:
                if state.winner == self.symbol:
                    self.estimations[hash] = 1.0
                else:
                    self.estimations[hash] = 0
            else:
                self.estimations[hash] = 0.5

    def feedState(self, state):
        self.states.append(state)

    def feedReward(self, reward):
        if len(self.states) == 0:
            return

```

```

        self.states = [state.getHash() for state in self.states]
        target = reward
        for latestState in reversed(self.states):
            value = self.estimateds[latestState] + self.stepSize * (target - self.estimateds[latestState])
            self.estimateds[latestState] = value
            target = value
        self.states = []

def takeAction(self):
    state = self.states[-1]
    nextStates = []
    nextPositions = []
    for i in range(BOARD_ROWS):
        for j in range(BOARD_COLS):
            if state.data[i, j] == 0:
                nextPositions.append([i, j])
                nextStates.append(state.nextState(i, j, self.symbol).getHash())
    if np.random.binomial(1, self.exploreRate):
        np.random.shuffle(nextPositions)
        self.states = []
        action = nextPositions[0]
        action.append(self.symbol)
        return action

    values = []
    for hash, pos in zip(nextStates, nextPositions):
        values.append((self.estimateds[hash], pos))
    np.random.shuffle(values)
    values.sort(key=lambda x: x[0], reverse=True)
    action = values[0][1]
    action.append(self.symbol)
    return action

def savePolicy(self):
    fw = open('optimal_policy_' + str(self.symbol), 'wb')
    pickle.dump(self.estimateds, fw)
    fw.close()

def loadPolicy(self):
    fr = open('optimal_policy_' + str(self.symbol), 'rb')
    self.estimateds = pickle.load(fr)
    fr.close()

```

```

# | 1 | 2 | 3 |
# | 4 | 5 | 6 |
# | 7 | 8 | 9 |
class HumanPlayer:
    def __init__(self, stepSize = 0.1, exploreRate=0.1):
        self.symbol = None
        self.currentState = None
        return
    def reset(self):
        return
    def setSymbol(self, symbol):
        self.symbol = symbol
        return
    def feedState(self, state):
        self.currentState = state
        return
    def feedReward(self, reward):
        return
    def takeAction(self):
        data = int(input("Input your position:"))
        data -= 1
        i = data // int(BOARD_COLS)
        j = data % BOARD_COLS
        if self.currentState.data[i, j] != 0:
            return self.takeAction()
        return (i, j, self.symbol)

def train(epochs=20000):
    player1 = Player()
    player2 = Player()
    judger = Judger(player1, player2)
    player1Win = 0.0
    player2Win = 0.0
    for i in range(0, epochs):
        print("Epoch", i)
        winner = judger.play()
        if winner == 1:
            player1Win += 1
        if winner == -1:
            player2Win += 1
        judger.reset()
    print(player1Win / epochs)
    print(player2Win / epochs)

```

```

        player1.savePolicy()
        player2.savePolicy()

def compete(turns=500):
    player1 = Player(exploreRate=0)
    player2 = Player(exploreRate=0)
    judger = Judger(player1, player2, False)
    player1.loadPolicy()
    player2.loadPolicy()
    player1Win = 0.0
    player2Win = 0.0
    for i in range(0, turns):
        print("Epoch", i)
        winner = judger.play()
        if winner == 1:
            player1Win += 1
        if winner == -1:
            player2Win += 1
        judger.reset()
    print(player1Win / turns)
    print(player2Win / turns)

def play():
    while True:
        player1 = Player(exploreRate=0)
        player2 = HumanPlayer()
        judger = Judger(player1, player2, False)
        player1.loadPolicy()
        winner = judger.play(True)
        if winner == player2.symbol:
            print("Win!")
        elif winner == player1.symbol:
            print("Lose!")
        else:
            print("Tie!")

    train()
    compete()
    play()
    """
...

'\n\nBOARD_ROWS = 3\nBOARD_COLS = 3\nBOARD_SIZE = BOARD_ROWS * BOARD_COLS\n\nclass State:\n

```

4

4.1 k

$$k \qquad 1 \qquad \qquad \qquad k \qquad \qquad \text{t} \qquad A_t \qquad R_t \quad a \qquad X_a \quad a$$

$$q_*(a) := \mathbb{E}[X_a]$$

$$\text{t} \quad a \qquad Q_t(a) \qquad q_*(a)$$

4.2

$$Q_t(a) := \frac{t \ a}{t \ a} = \frac{\sum_{i=1}^{t-1} R_i \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

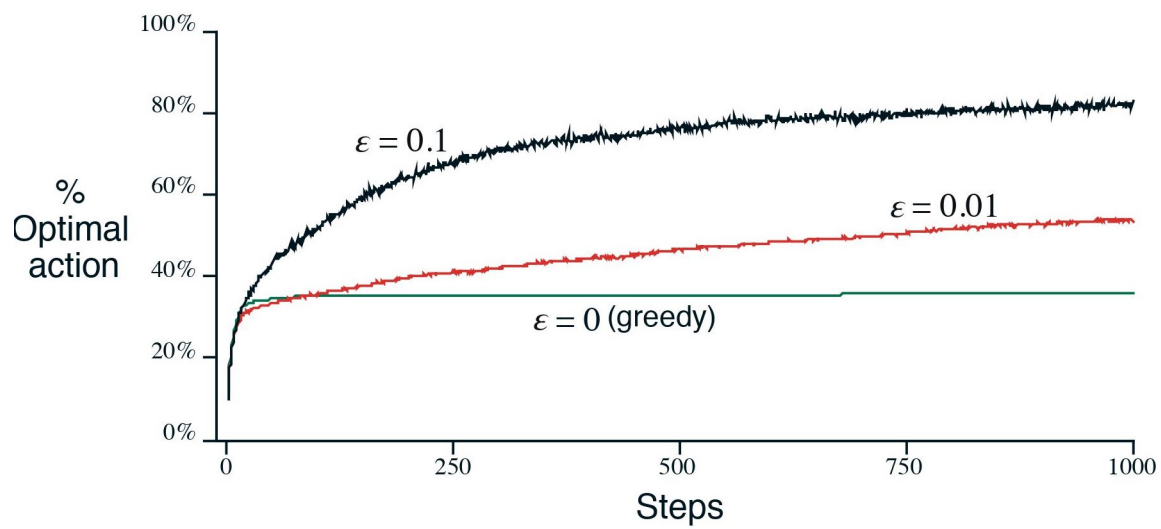
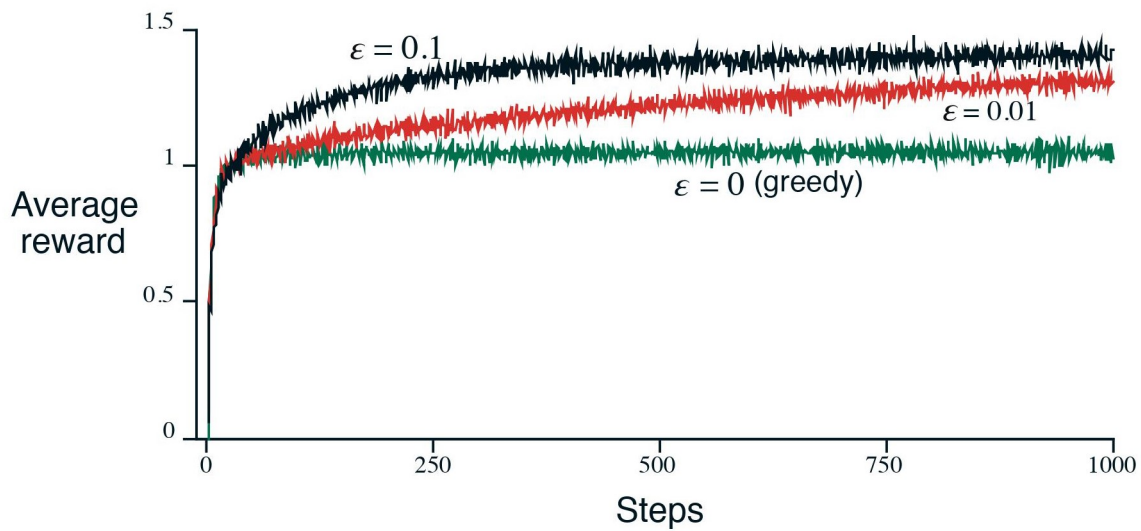
$$\mathbb{1} \qquad 1 \quad 0$$

$$A_t = \arg \max_a Q_t(a)$$

$$\arg \max_a Q_t(a) \qquad a \qquad \qquad \qquad \epsilon \qquad \qquad \epsilon-$$

4.3 10

$$\begin{array}{llllllllll} 2000 \text{ k} & \text{k=10} & q_*(a) & (N(0,1)) & X_a & N(q_*(a),1) & Q_1(a) & 0 & 1000 & 1 \\ 2000 & \epsilon- & 2000 & & & & & & & \end{array}$$



$\epsilon = 0.1$ $\epsilon = 0.01$

$\epsilon = 0.01$

4.4

1

n Q_n

$$Q_n := \frac{R_1 + R_2 + \dots + R_n}{n - 1}$$

R_n Q_n

$$Q_{n+1} = \frac{1}{n} \sum_{i=1}^n R_i = \frac{1}{n} (R_n + \sum_{i=1}^{n-1} R_i) = \frac{1}{n} (R_n + (n-1)Q_n) = Q_n + \frac{1}{n} (R_n - Q_n)$$

$$Q_n \, n$$

$$NewEstimate \leftarrow OldEstimate + StepSize \, [Target - OldEstimate]$$

$$\begin{aligned} [Target-OldEstimate] & \qquad \qquad \qquad Target(\, \, n \, \,) \qquad \qquad \qquad StepSize \, 1/n \qquad \qquad \qquad \alpha \, \alpha_t(a) \\ \epsilon - & \qquad \qquad \qquad k \end{aligned}$$

4.5

$$(\, \, X_a \,) \qquad \qquad \qquad , \qquad \qquad \qquad \alpha \in (0,1]$$

$$Q_{n+1} := Q_n + \alpha[R_n - Q_n]$$

$$Q_{n+1}$$

$$Q_{n+1} = Q_n + \alpha[R_n - Q_n] = \alpha R_n + (1 - \alpha)Q_n = \alpha R_n + (1 - \alpha)[\alpha R_{n-1} + (1 - \alpha)Q_{n-1}] = \alpha R_n + (1 - \alpha)\alpha R_{n-1} + (1 - \alpha)^2 Q_{n-1}$$

$$(1 - a)^n + \sum_{i=1}^n \alpha (1 - \alpha)^{n-i} = 1 \quad Q_{n+1} \qquad Q_1 \qquad 1 - \alpha$$

$$\alpha \qquad \qquad \qquad a \, n \qquad \qquad \qquad \alpha_n(a) \qquad \qquad \qquad 1$$

$$\sum_{n=1}^\infty \alpha_n(a) = \infty \quad \sum_{n=1}^\infty \alpha_n^2(a) < \infty$$

```

{python}
print("Hello Python!")

```

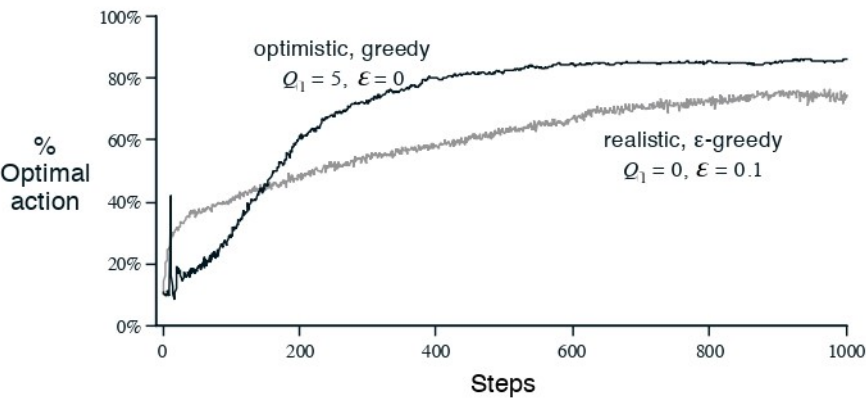
Hello Python!

4.6

$Q_1(a)$

$10 \qquad Q_1(a) \ 0 \ +5 \ q_*(a) \qquad q_*(a) +5 \qquad 2.87 \times 10^{-7}$

$10 \qquad Q_1(a) +5 \qquad Q_1(a) \ 0 \ \epsilon-$



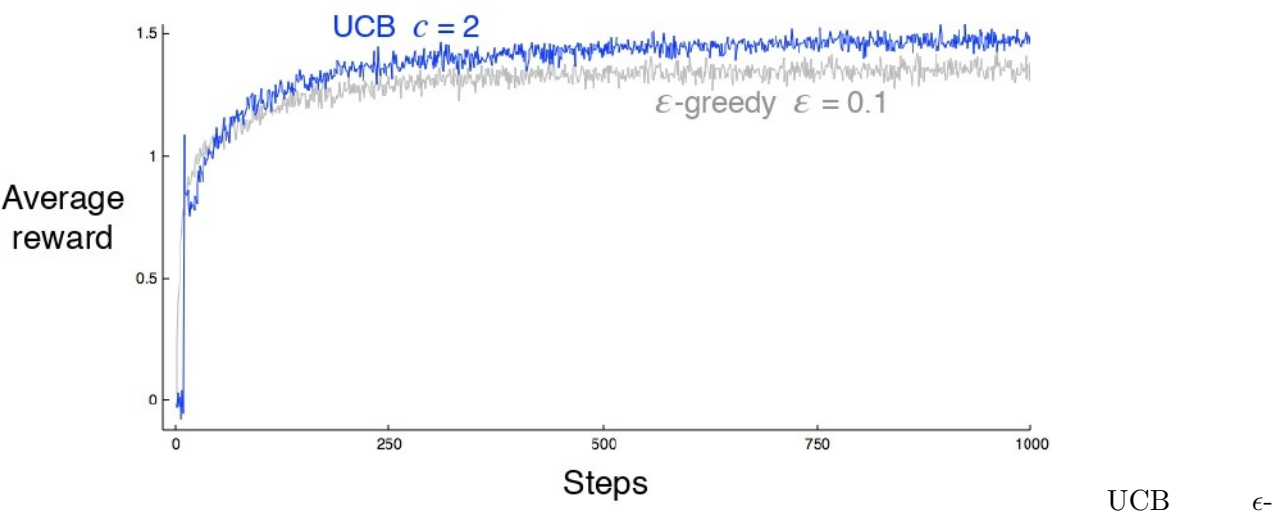
4.7

$\epsilon-$

$$A_t := \arg \max_a \left(Q_t(a) + c \sqrt{\frac{\log_e t}{N_t(a)}} \right)$$

$N_t(a) \ t \ a \ c > 0$

UCB **UCB** 10



4.8

$a \qquad H_t(a)$

$$\Pr\{A_t = a\} := \pi_t(a) := \frac{e^{H_t(a)}}{\sum_{b=1}^k e^{H_t(b)}}$$

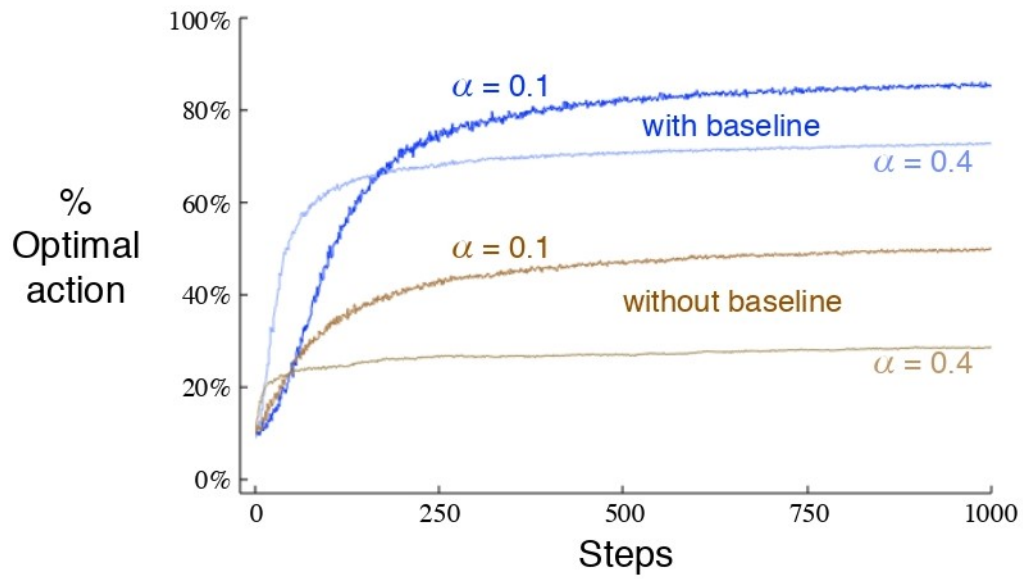
$t \quad a \quad \pi_t(a)$

$A_t \quad R_t$

$H_{t+1}(A_t) = H_t(A_t) + \alpha(R_t - \overline{R}_t)(1 - \pi_t(A_t)), H_{t+1}(a) := H_t(A_t) + \alpha(R_t - \overline{R}_t)\pi(a) \quad (a \neq A_t)$

$\overline{R}_t \qquad \text{t}$

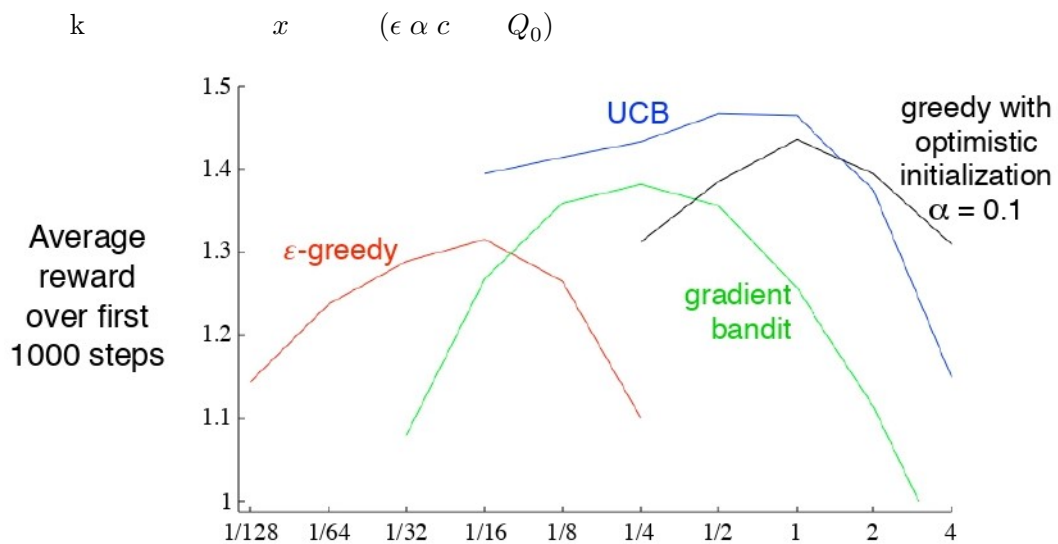
k $q_*(a) \quad +4 \quad 1 \qquad \overline{R}_t=0$



4.9

k

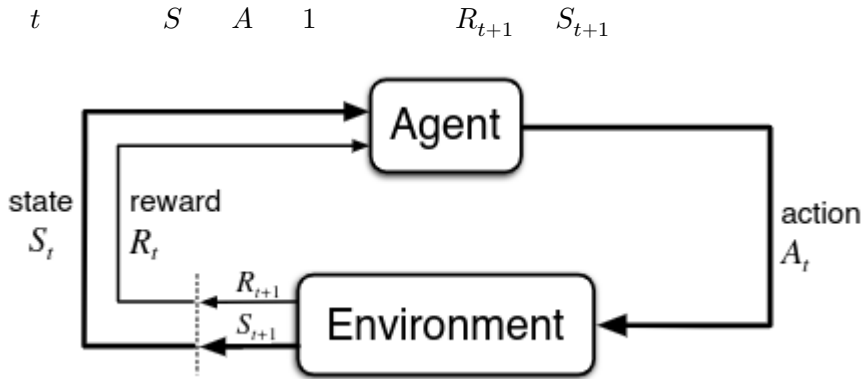
4.10



5

$$\begin{array}{ccccc} a & q_*(a) & & & \\ \text{MDP} & s & q_*(s, a) & & v_*(s) \end{array}$$

5.1



$$p(s', r | s, a) = \Pr \{S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a\}.$$

$$r(s, a) = \mathbb{E}[R_{t+1} | S_t = s, A_t = a] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p(s', r | s, a),$$

$$p(s' | s, a) = \Pr \{S_{t+1} = s' | S_t = s, A_t = a\} = \sum_{r \in \mathcal{R}} p(s', r | s, a),$$

$$r(s, a, s') = \mathbb{E}[R_{t+1} | S_t = s, A_t = a, S_{t+1} = s'] = \frac{\sum_{r \in \mathcal{R}} r p(s', r | s, a)}{p(s' | s, a)}.$$

5.2

5.3

5.3.1

$G_t \qquad T$

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots + R_T$$

5.3.2

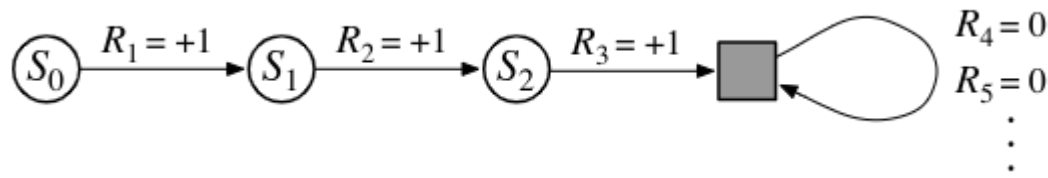
$T = \infty$

γ

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

5.4

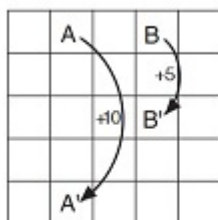
$T \quad 0$



$T = \infty \qquad \gamma = 1$

$$G_t = \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1}$$

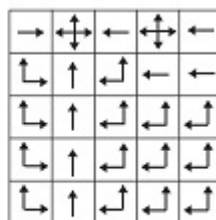
5.5



Gridworld

| | | | | |
|------|------|------|------|------|
| 22.0 | 24.4 | 22.0 | 19.4 | 17.5 |
| 19.8 | 22.0 | 19.8 | 17.8 | 16.0 |
| 17.8 | 19.8 | 17.8 | 16.0 | 14.4 |
| 16.0 | 17.8 | 16.0 | 14.4 | 13.0 |
| 14.4 | 16.0 | 14.4 | 13.0 | 11.7 |

V_*



π_*