UNIVERSITY OF BUEA

FET LEVEL 1

Course Title: CEF 201 Analysis

TUTORIAL SHEET N°3
ON
LIMITS - CONTINUITY

Exercice I (Limits at a point)

find when ~~the~~ it exists, the following limit:

(a) $\lim\limits_{x \to 2} \frac{2}{\sqrt{3x-5}}$ (b) $\lim\limits_{x \to 0}(2 - \frac{1}{x^2})$ (c) $\lim\limits_{x \to 1} 3x^2 + \frac{1}{\sqrt{1-x}}$

(d) $\lim\limits_{x \to 4} \frac{\sqrt{x}-2}{x-4}$ (e) $\lim\limits_{x \to 0} \frac{|x|}{x}$ (f) $\lim\limits_{x \to 0}(\sqrt{x} - 1 + \frac{1}{x})$

(g) $\lim\limits_{x \to 3} -\frac{1}{x-3}$ (h) $\lim\limits_{x \to -4}(3x^2 + 2x^3)$

Exercice II: (Limits at infinity) (Same assignment as ex. 1)

(a) $\lim\limits_{x \to +\infty}(-3x^2 + 7x - x^3)$ (b) $\lim\limits_{x \to -\infty}(\frac{2x^2+3x+4}{3x^2+5})$ (c) $\lim\limits_{x \to +\infty}(2 - \frac{1}{x})^3$

(d) $\lim\limits_{x \to +\infty}(\sqrt{x+1} - \sqrt{x})$ (e) $\lim\limits_{x \to -\infty}(2x+1)^3 - 10x^2$ (f) $\lim\limits_{x \to +\infty}(\sqrt{x^2+x+1} - \sqrt{x})$

Exercice III: The function f is defined by $f(x) = \frac{6x-25}{2x-8}$

a) Find the domain of f.

b) Determine two real numbers a and b such that $f(x) = a + \frac{b}{2x-8}$

c) Determine all the limits of f at the boundaries of its domain. Deduce the equations to the eventual asymptote to the graph of f.

Exercice IV: Let $f(x) = \frac{\sqrt{x-5}}{x}$

1) Prove that for all $x \geq 5$, $0 \leq f(x) \leq \frac{1}{\sqrt{x}}$.

2) Deduce the value of $\lim\limits_{x \to +\infty} f(x)$.

3) Use similar arguments as in question 1 and 2 to find $\lim\limits_{x \to +\infty} \frac{x^2 + \sin x}{2x^2+1}$

Exercice V: For $f(x) = \begin{cases} 1 + \frac{1}{x-3} & \text{if } x < 3 \\ 1 - \frac{1}{x-3} & \text{if } x > 3 \end{cases}$

Does f admit a limit at $x = 3$.

Exercice VI: For each of the following functions, give the domain, the limits at the boundaries of the domain and all possible equations interpreting the behaviour of the graph of the function:

(a) $Q(x) = \frac{x^2+2x+3}{x+1}$ (b) $f(x) = 2x - 1 + \frac{1}{x-3}$

c) $f(x) = \frac{\sqrt{x+2} - 2}{x-2}$

Exercice VII: study the continuity of f at $x = a$ in each of the following cases:

(a) $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ 9x^2 + x + 1 & \text{if } x \geq 0 \end{cases}$ $a = 0$ (b) $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) + 3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ $a = 0$

(c) $f(x) = \begin{cases} \frac{1}{(x+3)^3} & \text{if } x \leq -1 \\ 2 - x & \text{if } -1 < x \leq 1 \\ \frac{3}{x+2} & \text{if } x > 1 \end{cases}$ at $a = -1$ and at $a = 1$

Exercice VIII: Using the (ε, δ) definition show that:

a) $\lim_{x \to -1} 3x^2 + 2x - 4 = -3$ b) $\lim_{x \to 2} \frac{-x+3}{2x+1} = \frac{2}{5}$

c) $f: x \mapsto \sqrt{x}$ is continuous at $x = 1$

d) $f: x \mapsto \frac{1}{x^2+1}$ is continuous at $x = 5$