FACULTY OF ENGINEERING AND TECHNOLOGY

ALGEBRA (CEF 203)

TUTORIAL EXERCISES ON MATRICES AND SYSTEMS OF LINEAR EQUATIONS

1. Let $A = \begin{pmatrix} -1 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & -3 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 3 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 1 & -1 \\ 4 & 2 \end{pmatrix}$.
   Perform, where possible, each of the following operations.
   i) $2A - 3C$ ii) $AD + C$ iii) DB iv) $BC - 2D$ v)AB

2. Find a row echelon form of each of the following matrices and hence deduce the rank of each.
$$A = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -2 & -5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 2 & -1 & 3 \\ 2 & 3 & 0 & 1 & 2 \\ 1 & 3 & -2 & -2 & -1 \\ 5 & 1 & 3 & -1 & 8 \end{pmatrix}$$

3. Find the reduced row echelon form of the given matrix.
$$A = \begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 5 & 0 \\ 0 & -2 & 1 & 0 \\ 3 & 0 & 0 & 7 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & -8 & 1 \\ 2 & 1 & -1 & 1 \\ -3 & 1 & 14 & 7 \end{pmatrix}$$

4. Find the determinant of each of the following matrices.
$$A = \begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 5 & 0 \\ 7 & -3 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 2 & -4 \\ 5 & 0 & 7 & 0 \\ 1 & 6 & -2 & 3 \\ 4 & 0 & 0 & 2 \end{pmatrix}$$

5. Decide whether the given matrix is invertible, and if it does, find its inverse.
$$A = \begin{pmatrix} -2 & 5 \\ 5 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 3 & -6 \\ 3 & -7 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 & -1 & 3 \\ -2 & -3 & 4 & -5 \\ 4 & 8 & -5 & 9 \\ -1 & -4 & 0 & 5 \end{pmatrix}, D = \begin{pmatrix} 1 & 0 & 10 & \\ 1 & -1 & 1 & 3 \\ 5 & 2 & 1 & 1 \\ 2 & 0 & -3 & 9 \end{pmatrix}$$

6. Solve each of the following systems of equations using Crammer's rule.

a)
$$\begin{aligned} x - 2y &= -6 \\ 5x + y &= 5 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ -4x + 5y - 5z &= 3 \\ 2x + 3y - 6z &= 9 \end{aligned}$$

b)

7. Determine whether each of the following systems of equations has a solution. If it does, find all possible solutions.

a)
$$\begin{aligned} 2x + 3y + z &= 4 \\ x - 2z &= -4 \\ -3x + y + 2z &= -1 \end{aligned}$$

b)
$$\begin{aligned} x - y + 2z &= 3 \\ 3x + 2y - z &= 5 \\ x + 4y - 5z &= -1 \end{aligned}$$

c)
$$\begin{aligned} 3x + 5y - 2z &= 3 \\ 2x - y + 3z &= 2 \\ 7x + 3y + 4z &= 1 \end{aligned}$$

8. Show that the linear system
$$\begin{aligned} x + y - z &= a \\ 3x - y + 2z &= b \\ x - 3y + 4z &= c \end{aligned}$$
has a unique solution if and only if $2a - b + c = 0$.

9. For what real numbers α does the system
$$\begin{aligned} x + 2y + 2z &= 7 \\ 3x + 5y + z &= 6 \\ -x - 4y + \alpha z &= 20 \end{aligned}$$
have a unique solution?

10. Find the $PA = LU$ decomposition of the following matrices:
$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 4 \\ 1 & -2 & -1 \end{pmatrix}, B = \begin{pmatrix} 4 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}, C = \begin{pmatrix} 4 & 2 \\ 1 & 0 \\ -3 & 5 \end{pmatrix}$$

11. Solve the following systems of equations using the $PA = LU$ decomposition method.

a)
$$\begin{aligned} 2x_1 + x_2 - 3x_3 &= 12 \\ x_1 - x_2 + 4x_3 &= -18 \\ 2x_1 - x_3 &= 6 \end{aligned}$$

b)
$$\begin{aligned} x_1 + 5x_2 + 2x_3 &= 7 \\ -3x_1 + x_2 + 5x_3 &= 1 \\ 2x_1 - 3x_2 + x_3 &= 8 \end{aligned}$$

12. A nursery stocks three brands of lawn preparation as follows: Brand A is made of 30 percent of fertilizer, 20 percent weed killer and 50 percent of insecticide; Brand B comprises 60 percent fertilizer, 30 percent weed killer and 10 percent insecticide; Brand C comprises 90 percent of fertiliser, 10 percent of weed killer and no insecticide. By forming a systems of equations, find the number of kilograms of each Brand that can be used to produce 100 kg mixture containing 19 kg of weed killer and twice as much fertilizer as insecticide.