The continuous Bernoulli: fixing a pervasive error in variational autoencoders

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Introduction

- Variational autoencoders (VAE [KW14]) are a central tool in probabilistic modeling.
- MNIST is the most widely used testbed for VAE improvements.
- While MNIST is not binary, Bernoulli likelihoods are commonly used.
- We introduce a new distribution: the continuous Bernoulli.
- We interpret the practice as ignoring a normalizing constant.

Contributions of the paper

- Introducing and characterizing the continuous Bernoulli.
- Showing it outperforms the Bernoulli, even when the data is close to binary.
- Showcasing the importance of correctly modeling data type.

Variational autoencoders

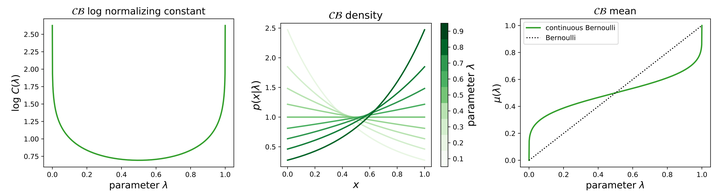
- Inference is performed on the model:
$$Z_n \sim p_0(z) \text{ and } X_n|Z_n \sim p_\theta(x|z_n)$$
by maximizing the ELBO with respect to the generative θ and approximate posterior parameters, θ and ϕ:
$$\mathcal{E}(\theta,\phi) = \sum_{n=1}^{N} E_{q_\phi}[\log p_\theta(x_n|z)] - KL(q_\phi||p_0)$$

The pervasive error in Bernoulli VAE

- In the Bernoulli case where $p_\theta(x|z) = \mathcal{B}(\lambda_\theta(z))$ the reconstruction term is given by:
$$E_{q_\phi}\Big[\sum_{d=1}^{D} x_{n,d}\log\lambda_{\theta,d}(z_n) + (1-x_{n,d})\log(1-\lambda_{\theta,d}(z_n))\Big]$$
- This loss is often used with real-valued data in $[0,1]$.

$\mathcal{CB}$: the continuous Bernoulli

- To study the effect of this practice, we introduce the $[0,1]$-supported continuous Bernoulli distribution, where $\tilde{p}(\cdot|\lambda)$ denotes the Bernoulli distribution:
$$X \sim \mathcal{CB}(\lambda) \iff p(x|\lambda) \propto \tilde{p}(x|\lambda) = \lambda^x(1-\lambda)^{1-x}$$
- The continuous Bernoulli is well defined and its pdf and expected value are given by:
$$p(x|\lambda) = C(\lambda)\lambda^x(1-\lambda)^{1-x}, \text{where } C(\lambda) = \begin{cases} \dfrac{2\tanh^{-1}(1-2\lambda)}{1-2\lambda} & \text{if } \lambda \neq 0.5 \\ 2 & \text{otherwise} \end{cases}$$
$$\mu(\lambda) := E[X] = \begin{cases} \dfrac{\lambda}{2\lambda-1} + \dfrac{1}{2\tanh^{-1}(1-2\lambda)} & \text{if } \lambda \neq 0.5 \\ 0.5 & \text{otherwise} \end{cases}$$



- We fully characterize the continuous Bernoulli distribution.
- The shape of $C(\lambda)$ reveals that ignoring it hurts performance the most near the extrema.

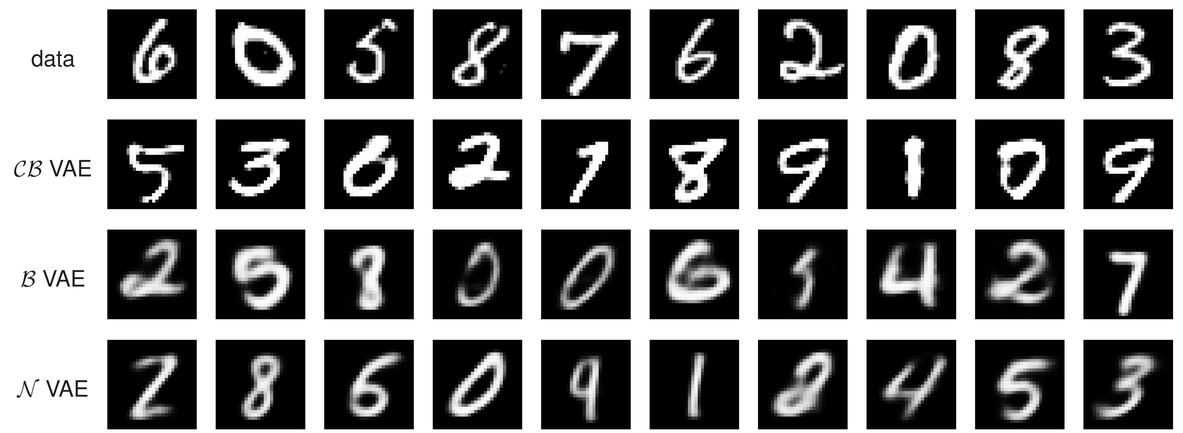
The continuous Bernoulli VAE

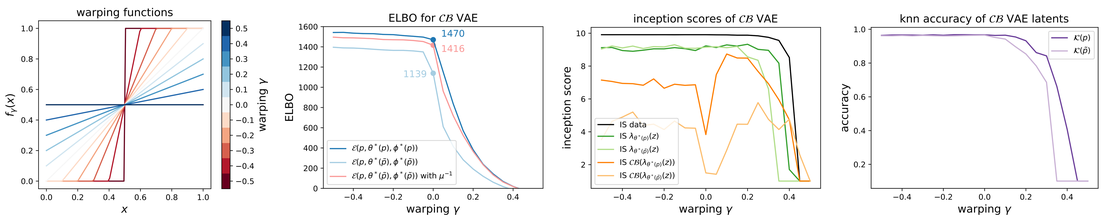
- When using a continuous Bernoulli the ELBO becomes:
$$\mathcal{E}(p,\theta,\phi) = \sum_{n=1}^{N} -KL(q_\phi||p_0) + E_{q_\phi}\Big[\sum_{d=1}^{D} x_{n,d}\log\lambda_{\theta,d}(z_n) + (1-x_{n,d})\log(1-\lambda_{\theta,d}(z_n)) + \log C(\lambda_{\theta,d}(z_n))\Big]$$
$$\mathcal{E}(\tilde{p},\theta,\phi) = \sum_{n=1}^{N} -KL(q_\phi||p_0) + E_{q_\phi}\Big[\sum_{d=1}^{D} x_{n,d}\log\lambda_{\theta,d}(z_n) + (1-x_{n,d})\log(1-\lambda_{\theta,d}(z_n))\Big]$$
- We find most explanations of using a Bernoulli likelihood lacking.
- We show that $\mathcal{E}(\tilde{p},\theta,\phi) < \mathcal{E}(p,\theta,\phi)$.

Conclusions

- Besides allowing the study of using Bernoulli likelihoods with real-valued data, the continuous Bernoulli outperforms the Bernoulli in a variety of metrics.
- Careful consideration of data type matters.

Experiments





distribution	objective	map	$\mathcal{E}(p,\theta^*,\phi^*)$	IS w/ samples	IS w/ parameters	$\mathcal{K}(\phi^*)$
$\mathcal{CB}/\mathcal{B}$	$\mathcal{E}(p,\theta,\phi)$	·	**1007**	1.15	2.31	**0.43**
	$\mathcal{E}(\tilde{p},\theta,\phi)$	μ^{-1}	916	**1.49**	**4.55**	0.42
	$\mathcal{E}(\tilde{p},\theta,\phi)$	·	475	1.41	1.39	0.42
Gaussian	$\mathcal{E}(p,\theta,\phi)$	·	**1891**	**1.86**	**3.04**	**0.42**
	$\mathcal{E}(\tilde{p},\theta,\phi)$	·	-42411	1.24	1.00	0.1
beta	$\mathcal{E}(p,\theta,\phi)$	·	**3121**	**2.98**	**4.07**	**0.47**
	$\mathcal{E}(\tilde{p},\theta,\phi)$	·	-38913	1.39	1.00	0.1

Acknowledgments and References

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[KW14] Diederik P Kingma and Max Welling, *Auto-encoding variational bayes*, International Conference on Learning Representations, 2014.

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