Global fits to D^0 CPV parameters using an HFAG like fit

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Abstract

The new $D^0 \to K\pi$ result from LHCb provides a credibly powerful constraint on mixing parameters. This note describes a fit in the style of HFAG to combine our result with previous measurements.

Contents

1 Introduction

To fully understand the global impact of the updated WS $D^0 \to K\pi$ analysis, a combination of global results of the netural D system is necessary. We present an HFAG like fit for the underlying parameters $|q/p|$, ϕ, x and y utilizing the updated 2011+2012 LHCb $D^0 \to K\pi$ results.

2 Chi-square calculation

The purpose of our fit is to combine the errors on several different measurements of the same parameters, where each measurement may have a different relation to the underlying true mixing parameters (eg measuring (x'^2, y') in place of (x, y)), and where the numbers in each measurement may be strongly correlated. To do so we construct an overall χ^2 for all the results:

$$\chi^2 = \vec{\epsilon}^T \sigma^{-1} \vec{\epsilon} \tag{1}$$

where the elements of $\vec{\epsilon}$ are given by $\epsilon_i = m_i - p_i$. Here $\vec{m}$ is the list of measured values from experiments, and $\vec{p}$ is a set of "proposed" values for the mixing parameters; we use MINUIT to vary $\vec{p}$ so as to minimise χ^2. Finally, σ is an $N \times N$ matrix where N is the number of measurements, with $\sigma_{ij} = e_i c_{ij} e_j$. Here e_i is the reported error on measurement i, and c_{ij} is the correlation coefficient between measurements i and j.

Notice that, if the measurements are uncorrelated, then σ reduces to a diagonal matrix where the elements are the squares of the measurement errors. In this case χ^2 is simply the sum $\sum_i \epsilon_i^2/e_i^2$, that is, each element is the difference between a measurement and the corresponding prediction, divided by the error on the measurement, squared. In other words, if there are no correlations we recover the usual chi-square goodness-of-fit metric.

3 Fit variants

In full generality, we wish to fit for no less than seven underlying related mixing parameters:

- x and y, the normalised mass and width differences

- R_D^+ and R_D^-, the ratios of rates

- δ, the strong phase difference

- $|q/p|$ and ϕ, the magnitude and phase of the indirect CP violation.

The observed inputs, however, are not all direct measurements of these quantities. From $D^0 \to K_S \pi \pi$ we get direct measurements of x, y, $|q/p|$ and ϕ; $D^0 \to K\pi$ results also yield $R_D^\pm$ directly, although sometimes quoted as $R_D = \frac{1}{2}(R_D^+ + R_D^-)$ and $A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-}$. However, we also measure the derived parameters $x'^{2(\pm)}$, $y'^{(\pm)}$, y_{CP}, and A_Γ, defined as:

$$x' = x \cos \delta + y \sin \delta \tag{2}$$
$$y' = y \cos \delta - x \sin \delta \tag{3}$$
$$x'^\pm = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (x' \cos \phi \pm y' \sin \phi) \tag{4}$$
$$y'^\pm = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (y' \cos \phi \mp x' \sin \phi) \tag{5}$$
$$2y_{CP} = (|q/p| + |p/q|)\, y \cos \phi - (|q/p| - |p/q|)\, x \sin \phi \tag{6}$$
$$2A_\Gamma = (|q/p| - |p/q|)\, y \cos \phi - (|q/p| + |p/q|)\, x \sin \phi \tag{7}$$
$$\tag{8}$$

where the helper quantity A_M is given by

$$A_M = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}. \tag{9}$$

To calculate $\vec{\epsilon}$, then, we take in a vector of proposed mixing parameters from MINUIT, calculate the resulting observable parameters from the equations above, and subtract the actually observed numbers.

In addition to the fully-general fit allowing all these variables to float, there are some variants imposing different no-CPV constraints:

- No CP violation. In this fit we set $|q/p| = 1$, $\phi = 0$, and $R_D^+ = R_D^-$, and fit only for x, y, δ, and R_D.

- No direct CP violation. With no direct CP violation, $R_D^+ = R_D^-$; in addition, the four parameters x, y, ϕ and $|q/p|$ are related (in the limit that CPV is small) by the

constraint

$$|q/p| \;=\; 1 - \frac{y}{x}\tan\phi \tag{10}$$

$$\phi \;=\; \frac{x}{y}\tan^{-1}\left(\frac{1-|q/p|^2}{1+|q/p|^2}\right) \tag{11}$$

Thus we have two variants on this fit:

2a Here we allow $|q/p|$ to float and calculate ϕ from the constraint.

2b We allow ϕ to float and calculate $|q/p|$ from the constraint.

- All CPV allowed. As A_D is quite small, the contribution of a new physics phase to ϕ is far below our current sensitivity; consequently the constraint above is a reasonable approximation. We therefore run three variants of the all-CPV-allowed scenario:

3a All parameters float, no constraint.

3b ϕ is calculated from $|q/p|$ as above, rather than allowed to float. R_D^+ and R_D^- are both free, as before.

3c As in 3b, but with $|q/p|$ calculated from the constraint and ϕ allowed to float.

In addition, we do a fit not allowing direct CP violation, in which the free parameters are the underlying[1]

4 Measurements Used

Of all of the current measurements available, only a few are resonable for certain fits. Table 1 lists all the possible measurements pertaining to fits excluding CP Violation. Table **??** corresponds to measurements allowing only direct CP violation, and Table **??** lists all measurments pertaining to both direct and indirect CP violation allowed.

5 Results

The results are split into subsections depending on the type of CP Violation allowed. Additonally, results are presented using a variety of different combinations of the available data. Figure 1 shows all variations for the no CPV allowed fits. Figure 3 shows the results for a subset of variations on All CPV allowed fits.

Table 1: No CPV allowed inputs

| Result	Value	Correlation Coefficients
HFAG ycp	$0.00866 \pm 0.00155 \pm 0$	
LHCb $x'^2(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
HFAG $x(K_S^0\pi\pi)$	$0.00419 \pm 0.00211 \pm 0$	
HFAG $y(K_S^0\pi\pi)$	$0.00456 \pm 0.00186 \pm 0$	
CLEO $\cos(\delta)(K\pi)$	$0.81 \pm 0.2 \pm 0.06$	
CLEO $\sin(\delta)(K\pi)$	$-0.01 \pm 0.41 \pm 0.04$	
CDF R_D	$0.00304 \pm 0.00055 \pm 0$	
CDF $x'^2(K\pi)$	$-0.00012 \pm 0.00035 \pm 0$	
CDF $y'(K\pi)$	$0.0085 \pm 0.0076 \pm 0$	
Belle R_D	$0.00364 \pm 0.00017 \pm 0$	
Belle $x'^2(K\pi)$	$0.00018 \pm 0.00022 \pm 0$	
Belle $y'(K\pi)$	$0.0006 \pm 0.004 \pm 0$	
BaBar R_D	$0.00303 \pm 0.00016 \pm 0.0001$	
BaBar $x'^2(K\pi)$	$-0.00022 \pm 0.0003 \pm 0.00021$	
BaBar $y'(K\pi)$	$0.0097 \pm 0.0044 \pm 0.0031$	

5.1 No CP Violation Allowed

5.2 All CP Violation Allowed

6 Conclusion

By utilizing a global, HFAG-like fit, we constrain to be $|q/p| = xxxxx \pm yyyyy$ and $\phi = zzzzzzz \pm qqqqqqqqqqqqq$, in the case of all CPV allowed. Allowing only direct CPV, $|q/p| = xxxxx \pm yyyyy$ and $\phi = zzzzzzz \pm qqqqqqqqqqqqq$. These measurements represent the most precise determination of the CP violating parameters of the netural D meson system
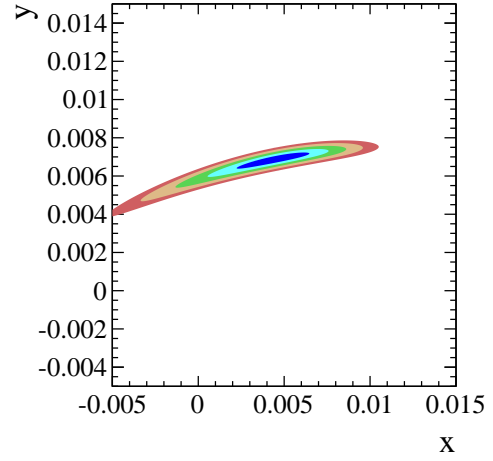
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[1]See Kagan and Sokoloff, http://arxiv.org/abs/0907.3917.
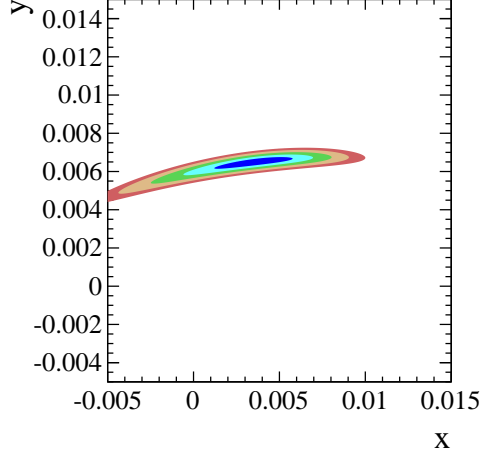
(a) Two dimensional error ellipses for x
and y using all available measurements.
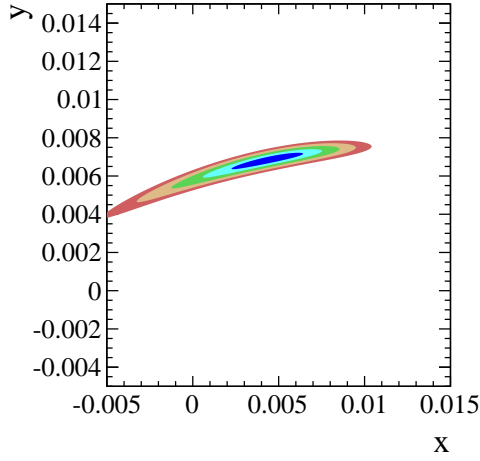
(b) Two dimensional error ellipses for
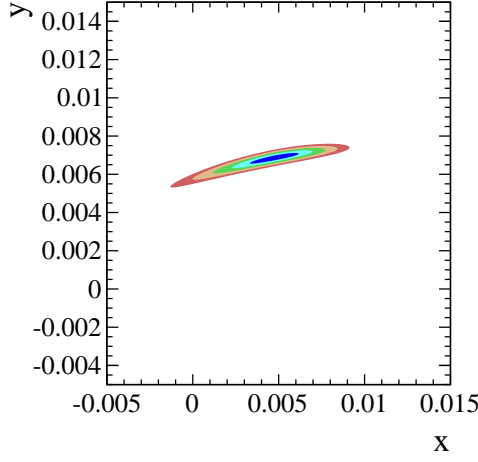x and y from fit excluding Belle and
BaBar $K\pi$ results.

(c) Two dimensional error ellipses for x
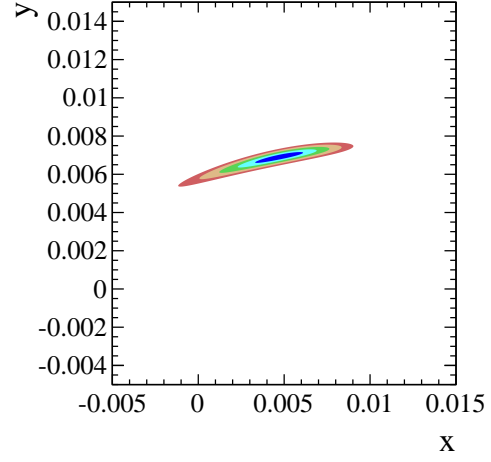and y from fit excluding Belle, BaBar
and CDF measurements.

Figure 1: Two dimensional error ellipses of x and y from fit for No CPV. Exclusion of the
Belle and BaBar results drastically change the slope of the error ellipses. The differing
colors represent the 1-5σ contours.

Table 2: All CPV allowed inputs

| Result	Value	Correlation Coefficients
HFAG y_{CP}	$0.00866 \pm 0.00155 \pm 0$	
HFAG A_Γ	$-0.00022 \pm 0.00161 \pm 0$	
LHCb $A_\Gamma(KK)$	$-0.00035 \pm 0.00062 \pm 0.00012$	
LHCb $A_\Gamma(\pi\pi)$	$0.00033 \pm 0.00106 \pm 0.00014$	
LHCb $x'^{2+}(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'^+(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D^+	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
LHCb $x'^{2-}(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'^-(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D^-	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
Belle $x(K_S^0\pi\pi)$	$0.0081 \pm 0.003 \pm 0.0015$	
Belle $y(K_S^0\pi\pi)$	$0.0037 \pm 0.0025 \pm 0.0012$	
Belle $|q/p|$	$0.86 \pm 0.3 \pm 0.1$	
Belle ϕ	$-0.244 \pm 0.31 \pm 0.09$	
CLEO $\cos(\delta)(K\pi)$	$0.81 \pm 0.2 \pm 0.06$	
CLEO $\sin(\delta)(K\pi)$	$-0.01 \pm 0.41 \pm 0.04$	
CLEO R_D	$0.00533 \pm 0.00107 \pm 0.00045$	
CLEO $x'^2(K\pi)$	$0.0006 \pm 0.0023 \pm 0.0011$	
CLEO $y'(K\pi)$	$0.042 \pm 0.02 \pm 0.01$	
CDF R_D	$0.00304 \pm 0.00055 \pm 0$	
CDF $x'^2(K\pi)$	$-0.00012 \pm 0.00035 \pm 0$	
CDF $y'(K\pi)$	$0.0085 \pm 0.0076 \pm 0$	
Belle R_D^-	$0.0036 \pm 0.0002 \pm 0$	
Belle $x'^{2-}(K\pi)$	$6e-05 \pm 0.00034 \pm 0$	
Belle $y'^-(K\pi)$	$0.002 \pm 0.0054 \pm 0$	
Belle R_D^+	$0.00368 \pm 0.0002 \pm 0$	
Belle $x'^{2+}(K\pi)$	$0.00032 \pm 0.00037 \pm 0$	
Belle $y'^+(K\pi)$	$-0.0012 \pm 0.0058 \pm 0$	
BaBar R_D^-	$0.00303 \pm 0.0002 \pm 0.0001$	
BaBar $x'^{2-}(K\pi)$	$-0.0002 \pm 0.00041 \pm 0.00029$	
BaBar $y'^-(K\pi)$	$0.0096 \pm 0.0064 \pm 0.0045$	
BaBar R_D^+	$0.00303 \pm 0.0002 \pm 0.0001$	
BaBar $x'^{2+}(K\pi)$	$-0.00024 \pm 0.00043 \pm 0.0003$	
BaBar $y'^+(K\pi)$	$0.0098 \pm 0.0061 \pm 0.0043$	
BaBar $x(K_S^0\pi\pi)$	$0.0016 \pm 0.0023 \pm 0.0012$	
BaBar $y(K_S^0\pi\pi)$	$0.0057 \pm 0.002 \pm 0.0013$	

(a) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Does not include latest A_Γ result of LHCb.

(b) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Include latest A_Γ result of LHCb.
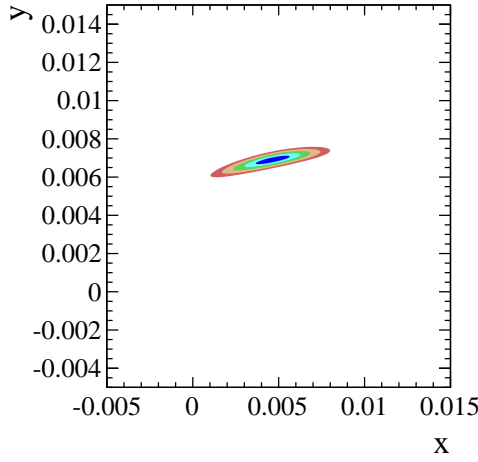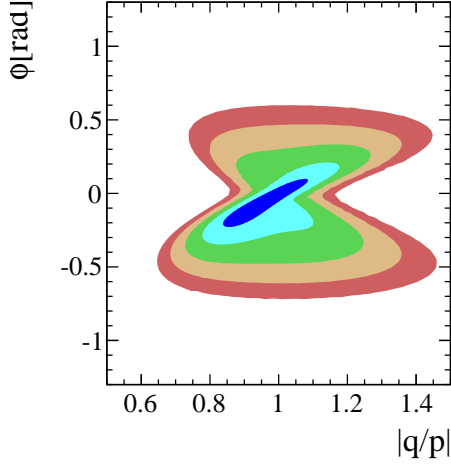
(c) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Does not include latest A_Γ result of LHCb.
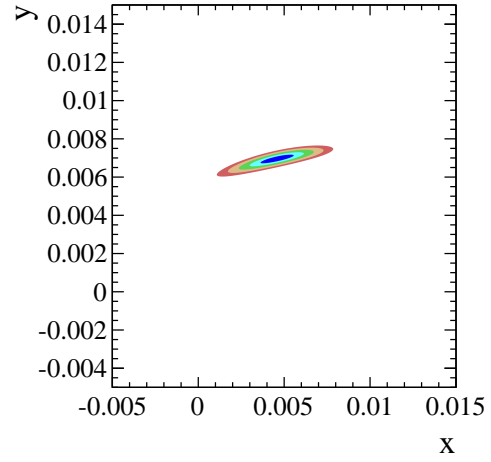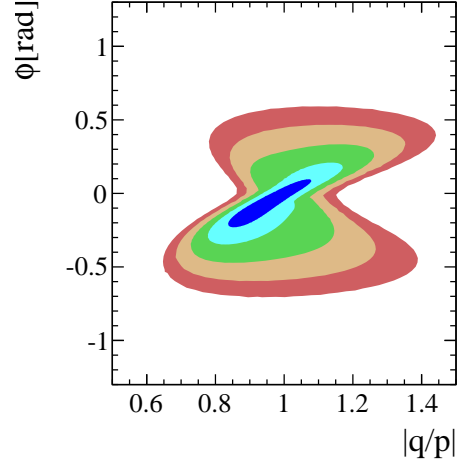
(d) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Include latest A_Γ result of LHCb.
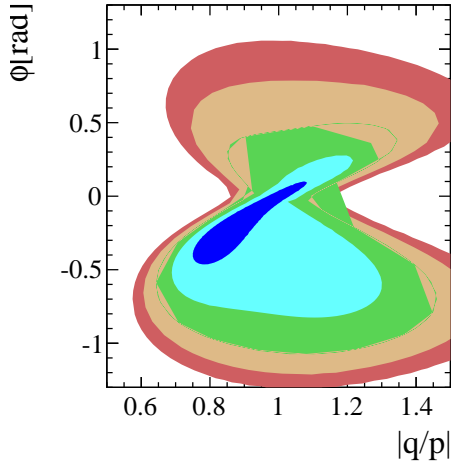
Figure 2: Two dimensional error ellipses of fit for All CPV including differing sets of data for x vs y. The biggest differences come from including the CDF result, which elongates the error ellipses. The differing colors represent the 1-5σ contours.
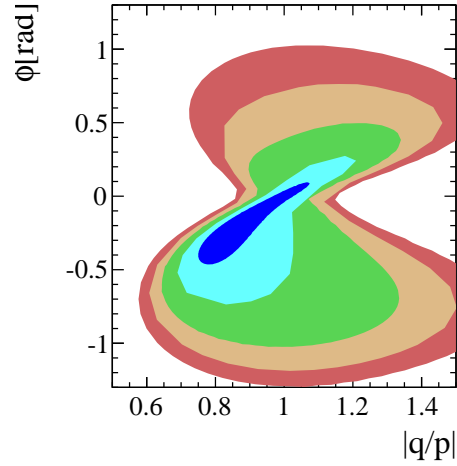
(a) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Does not include latest A_Γ result of LHCb.

(b) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Include latest A_Γ result of LHCb.

(c) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Does not include latest A_Γ result of LHCb.

(d) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Include latest A_Γ result of LHCb.

Figure 3: Two dimensional error ellipses of fit for All CPV including differing sets of data for ϕ vs q/p. The biggest differences come from including the CDF result, which elongates the error ellipses. The differing colors represent the 1-5σ contours.