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Global fits to D^0 CPV parameters using an HFAG like fit

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Abstract

The new $D^0 \to K\pi$ result from LHCb provides a credibly powerful constraint on mixing parameters. This note describes a fit in the style of HFAG to combine our result with previous measurements.

Contents

1 Introduction

To fully understand the global impact of the updated WS $D^0 \to K\pi$ analysis, a combination of global results of the neutral D system is necessary. We present an HFAG-like fit for the underlying parameters $|q/p|$, ϕ, x and y using the updated 2011+2012 LHCb $D^0 \to K\pi$ results.

2 Chi-square calculation

The purpose of our fit is to combine the errors on several different measurements of the same parameters, where each measurement may have a different relation to the underlying true mixing parameters (eg measuring (x'^2, y') in place of (x, y)), and where the numbers in each measurement may be strongly correlated. To do so we construct an overall χ^2 for all the results:

$$\chi^2 \;=\; \vec{\epsilon}^T \sigma^{-1} \vec{\epsilon} \tag{1}$$

where the elements of $\vec{\epsilon}$ are given by $\epsilon_i = m_i - p_i$. Here $\vec{m}$ is the list of measured values from experiments, and $\vec{p}$ is a set of "proposed" values for the mixing parameters; we use MINUIT to vary $\vec{p}$ so as to minimise χ^2. Finally, σ is an $N \times N$ matrix where N is the number of measurements, with $\sigma_{ij} = e_i c_{ij} e_j$. Here e_i is the reported error on measurement i, and c_{ij} is the correlation coefficient between measurements i and j.

Notice that, if the measurements are uncorrelated, then σ reduces to a diagonal matrix where the elements are the squares of the measurement errors. In this case χ^2 is simply the sum $\sum\limits_i \epsilon_i^2/e_i^2$, that is, each element is the difference between a measurement and the corresponding prediction, divided by the error on the measurement, squared. In other words, if there are no correlations we recover the usual chi-square goodness-of-fit metric.

3 Fit variants

In full generality, we wish to fit for no less than seven underlying related mixing parameters:

- x and y, the normalised mass and width differences
- R_D^+ and R_D^-, the ratios of rates
- δ, the strong phase difference
- $|q/p|$ and ϕ, the magnitude and phase of the indirect CP violation.

The observed inputs, however, are not all direct measurements of these quantities. From $D^0 \to K_S\pi\pi$ we get direct measurements of x, y, $|q/p|$ and ϕ; $D^0 \to K\pi$ results also yield $R_D^\pm$ directly, although sometimes quoted as $R_D = \frac{1}{2}(R_D^+ + R_D^-)$ and $A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-}$. However, we also measure the derived parameters $x'^{2(\pm)}$, $y'^{(\pm)}$, y_{CP}, and A_Γ, defined as:

$$\begin{aligned}
x' &= x\cos\delta + y\sin\delta && (2)\\
y' &= y\cos\delta - x\sin\delta && (3)\\
x'^{\pm} &= \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (x'\cos\phi \pm y'\sin\phi) && (4)\\
y'^{\pm} &= \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (y'\cos\phi \mp x'\sin\phi) && (5)\\
2y_{CP} &= (|q/p| + |p/q|)\, y\cos\phi - (|q/p| - |p/q|)\, x\sin\phi && (6)\\
2A_\Gamma &= (|q/p| - |p/q|)\, y\cos\phi - (|q/p| + |p/q|)\, x\sin\phi && (7)\\
& && (8)
\end{aligned}$$

where the helper quantity A_M is given by

$$A_M = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}. \quad (9)$$

To calculate $\vec{\epsilon}$, then, we take in a vector of proposed mixing parameters from MINUIT, calculate the resulting observable parameters from the equations above, and subtract the actually observed numbers.

In addition to the fully-general fit allowing all these variables to float, there are some variants imposing different no-CPV constraints:

- No CP violation. In this fit we set $|q/p| = 1$, $\phi = 0$, and $R_D^+ = R_D^-$, and fit only for x, y, δ, and R_D.
- No direct CP violation. With no direct CP violation, $R_D^+ = R_D^-$; in addition, the four parameters x, y, ϕ and $|q/p|$ are related (in the limit that CPV is small) by the

constraint

$$
\begin{aligned}
|q/p| &= 1 - \frac{y}{x}\tan\phi && (10)\\
\phi &= \tan^{-1}\left(\frac{1-|q/p|^2}{1+|q/p|^2}\frac{x}{y}\right) && (11)
\end{aligned}
$$

Thus we have two variants on this fit:

2a Here we allow $|q/p|$ to float and calculate ϕ from the constraint.

2b We allow ϕ to float and calculate $|q/p|$ from the constraint.

- All CPV allowed. As A_D is quite small, the contribution of a new physics phase to ϕ is far below our current sensitivity; consequently the constraint above is a reasonable approximation. We therefore run three variants of the all-CPV-allowed scenario:

  3a All parameters float, no constraint.

  3b ϕ is calculated from $|q/p|$ as above, rather than allowed to float. R_D^+ and R_D^- are both free, as before.

  3c As in 3b, but with $|q/p|$ calculated from the constraint and ϕ allowed to float.

In addition, we do a fit not allowing direct CP violation, in which the free parameters are the underlying[1] x_{12}, y_{12}, and ϕ. These parameters are related (in the limit of no direct CP violation) to $|q/p|$, x, y and ϕ (no subscripts!) as follows:

$$
\begin{aligned}
x &= \frac{1}{\sqrt{2}}\mathrm{sg}(\cos\phi_{12})\sqrt{x_{12}^2 - y_{12}^2 + |x_{12}^2 + y_{12}^2| - 4x_{12}^2 y_{12}^2 \sin^2(\phi_{12})} && (12)\\
y &= \frac{1}{\sqrt{2}}\sqrt{y_{12}^2 - x_{12}^2 + |x_{12}^2 + y_{12}^2| - 4x_{12}^2 y_{12}^2 \sin^2(\phi_{12})} && (13)\\
|q/p| &= \left(\frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin(\phi_{12})}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin(\phi_{12})}\right)^{1/4} && (14)\\
\phi &= -\frac{1}{2}\frac{\sin(2\phi_{12})}{\cos(2\phi_{12}) + \frac{y_{12}^2}{x_{12}^2}}. && (15)
\end{aligned}
$$

Our approach in this fit is to allow MINUIT to believe that the parameters x_{12}, y_{12}, and ϕ_{12} are free, but interpret them internally as giving rise to the non-underlying parameters x, y, ϕ, and $|q/p|$, and use these values in the calculation of χ^2, as outlined for the other fits.

[1] See Kagan and Sokoloff, `http://arxiv.org/abs/0907.3917`.

4 Measurements Used

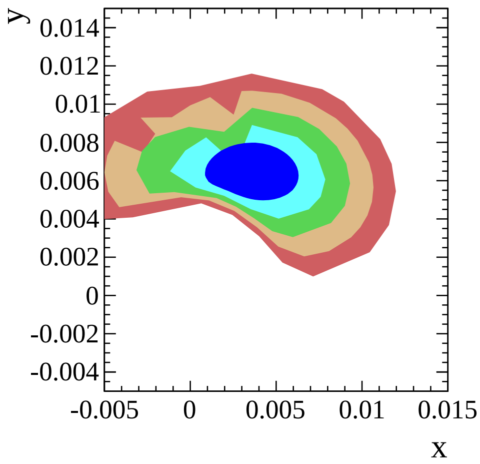
Of all of the current measurements available, only a few are resonable for certain fits. Table 1 lists all the possible measurements pertaining to fits excluding CP Violation. Table **??** corresponds to measurements allowing only direct CP violation, and Table **??** lists all measurments pertaining to both direct and indirect CP violation allowed.

Table 1: No CPV allowed inputs

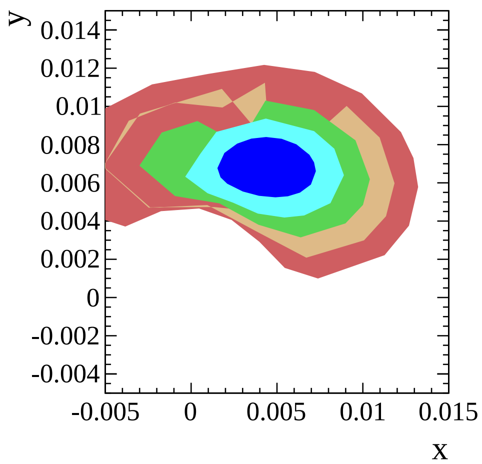
Result	Value	Correlation Coefficients
HFAG ycp	$0.00866 \pm 0.00155 \pm 0$	
LHCb $x'^2(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
HFAG $x(K^0_S\pi\pi)$	$0.00419 \pm 0.00211 \pm 0$	
HFAG $y(K^0_S\pi\pi)$	$0.00456 \pm 0.00186 \pm 0$	
CLEO $\cos(\delta)(K\pi)$	$0.81 \pm 0.2 \pm 0.06$	
CLEO $\sin(\delta)(K\pi)$	$-0.01 \pm 0.41 \pm 0.04$	
CDF R_D	$0.00304 \pm 0.00055 \pm 0$	
CDF $x'^2(K\pi)$	$-0.00012 \pm 0.00035 \pm 0$	
CDF $y'(K\pi)$	$0.0085 \pm 0.0076 \pm 0$	
Belle R_D	$0.00364 \pm 0.00017 \pm 0$	
Belle $x'^2(K\pi)$	$0.00018 \pm 0.00022 \pm 0$	
Belle $y'(K\pi)$	$0.0006 \pm 0.004 \pm 0$	
BaBar R_D	$0.00303 \pm 0.00016 \pm 0.0001$	
BaBar $x'^2(K\pi)$	$-0.00022 \pm 0.0003 \pm 0.00021$	
BaBar $y'(K\pi)$	$0.0097 \pm 0.0044 \pm 0.0031$	

5 Results

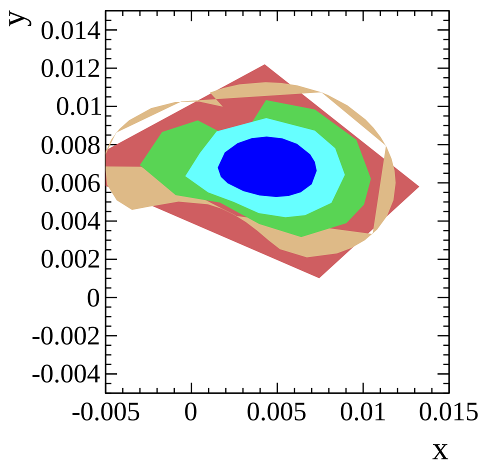
The results are split into subsections depending on the type of CP Violation allowed. Additonally, results are presented using a variety of different combinations of the available data. Figure 1 shows all variations for the no CPV allowed fits. Figure 3 shows the results for a subset of variations on All CPV allowed fits.



(a) Two dimensional error ellipses for x and y using all available measurements.



(b) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results.



(c) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF measurements.

Figure 1: Two dimensional error ellipses of x and y from fit for No CPV. Exclusion of the Belle and BaBar results drastically change the slope of the error ellipses. The differing colors represent the 1-5σ contours.

5.1 No CP Violation Allowed

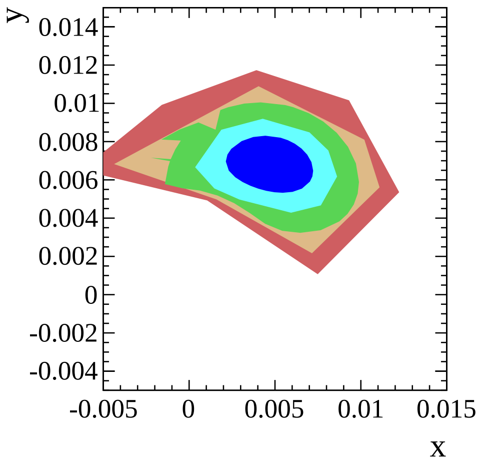
5.2 All CP Violation Allowed

6 Conclusion

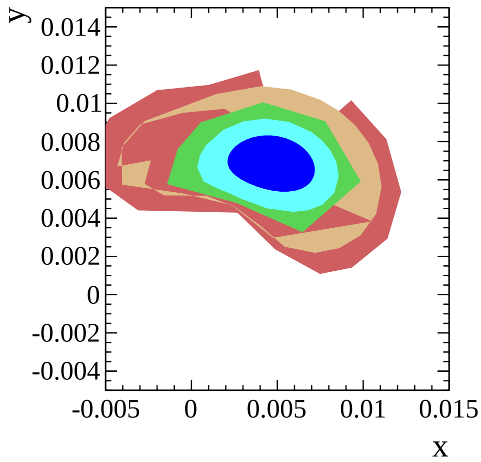
By utilizing a global, HFAG-like fit, we constrain to be $|q/p| = xxxxx \pm yyyyy$ and $\phi = zzzzzzz \pm qqqqqqqqqqqqq$, in the case of all CPV allowed. Allowing only direct CPV, $|q/p| = xxxxx \pm yyyyy$ and $\phi = zzzzzzz \pm qqqqqqqqqqqqq$. These measurements represent the most precise determination of the CP violating parameters of the netural D meson system

Table 2: All CPV allowed inputs

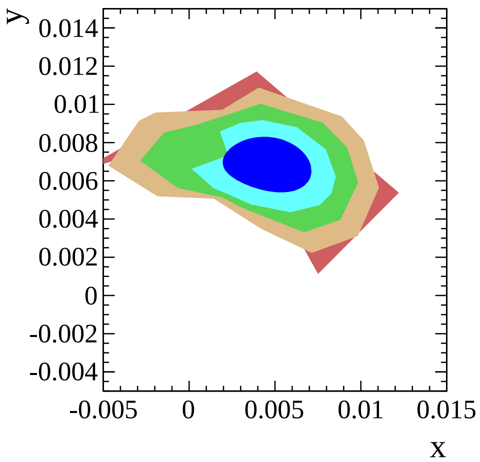
Result	Value	Correlation Coefficients
HFAG y_{CP}	$0.00866 \pm 0.00155 \pm 0$	
HFAG A_Γ	$-0.00022 \pm 0.00161 \pm 0$	
LHCb $A_\Gamma(KK)$	$-0.00035 \pm 0.00062 \pm 0.00012$	
LHCb $A_\Gamma(\pi\pi)$	$0.00033 \pm 0.00106 \pm 0.00014$	
LHCb $x'^{2+}(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'^{+}(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D^+	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
LHCb $x'^{2-}(K\pi)$	$5.5e-05 \pm 4.2e-05 \pm 2.6e-05$	
LHCb $y'^{-}(K\pi)$	$0.00481 \pm 0.00085 \pm 0.00053$	
LHCb R_D^-	$0.003568 \pm 5.8e-05 \pm 3.3e-05$	
Belle $x(K_S^0\pi\pi)$	$0.0081 \pm 0.003 \pm 0.0015$	
Belle $y(K_S^0\pi\pi)$	$0.0037 \pm 0.0025 \pm 0.0012$	
Belle $\|q/p\|$	$0.86 \pm 0.3 \pm 0.1$	
Belle ϕ	$-0.244 \pm 0.31 \pm 0.09$	
CLEO $\cos(\delta)(K\pi)$	$0.81 \pm 0.2 \pm 0.06$	
CLEO $\sin(\delta)(K\pi)$	$-0.01 \pm 0.41 \pm 0.04$	
CLEO R_D	$0.00533 \pm 0.00107 \pm 0.00045$	
CLEO $x'^{2}(K\pi)$	$0.0006 \pm 0.0023 \pm 0.0011$	
CLEO $y'(K\pi)$	$0.042 \pm 0.02 \pm 0.01$	
CDF R_D	$0.00304 \pm 0.00055 \pm 0$	
CDF $x'^{2}(K\pi)$	$-0.00012 \pm 0.00035 \pm 0$	
CDF $y'(K\pi)$	$0.0085 \pm 0.0076 \pm 0$	
Belle R_D^-	$0.0036 \pm 0.0002 \pm 0$	
Belle $x'^{2-}(K\pi)$	$6e-05 \pm 0.00034 \pm 0$	
Belle $y'^{-}(K\pi)$	$0.002 \pm 0.0054 \pm 0$	
Belle R_D^+	$0.00368 \pm 0.0002 \pm 0$	
Belle $x'^{2+}(K\pi)$	$0.00032 \pm 0.00037 \pm 0$	
Belle $y'^{+}(K\pi)$	$-0.0012 \pm 0.0058 \pm 0$	
BaBar R_D^-	$0.00303 \pm 0.0002 \pm 0.0001$	
BaBar $x'^{2-}(K\pi)$	$-0.0002 \pm 0.00041 \pm 0.00029$	
BaBar $y'^{-}(K\pi)$	$0.0096 \pm 0.0064 \pm 0.0045$	
BaBar R_D^+	$0.00303 \pm 0.0002 \pm 0.0001$	
BaBar $x'^{2+}(K\pi)$	$-0.00024 \pm 0.00043 \pm 0.0003$	
BaBar $y'^{+}(K\pi)$	$0.0098 \pm 0.0061 \pm 0.0043$	
BaBar $x(K_S^0\pi\pi)$	$0.0016 \pm 0.0023 \pm 0.0012$	
BaBar $y(K_S^0\pi\pi)$	$0.0057 \pm 0.002 \pm 0.0013$	



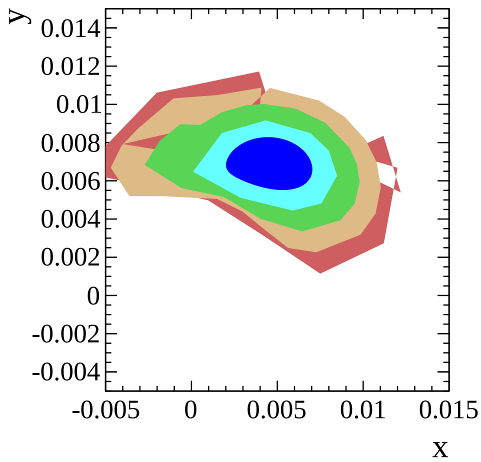
(a) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Does not include latest A_Γ result of LHCb.



(b) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Include latest A_Γ result of LHCb.

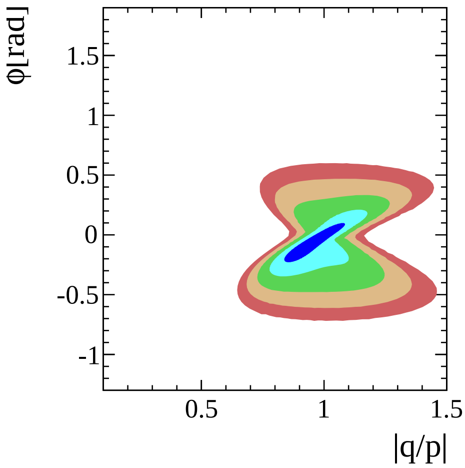


(c) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Does not include latest A_Γ result of LHCb.

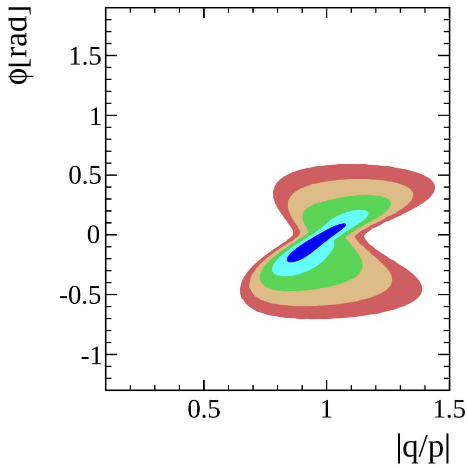


(d) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Include latest A_Γ result of LHCb.

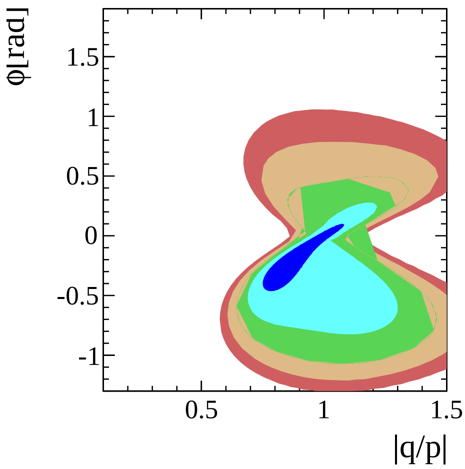
Figure 2: Two dimensional error ellipses of fit for All CPV including differing sets of data for x vs y. The biggest differences come from including the CDF result, which elongates the error ellipses. The differing colors represent the 1-5σ contours.



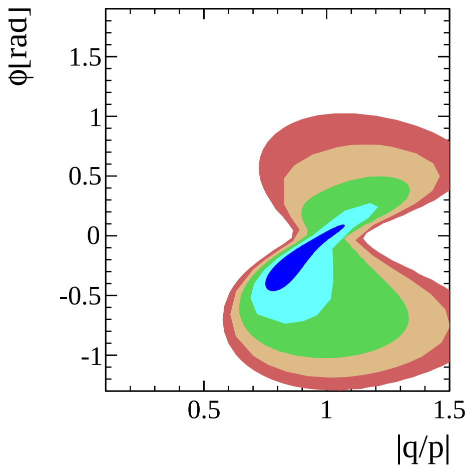
(a) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Does not include latest A_Γ result of LHCb.



(b) Two dimensional error ellipses for x and y from fit excluding Belle and BaBar $K\pi$ results. Include latest A_Γ result of LHCb.



(c) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Does not include latest A_Γ result of LHCb.



(d) Two dimensional error ellipses for x and y from fit excluding Belle, BaBar and CDF $K\pi$ results. Include latest A_Γ result of LHCb.

Figure 3: Two dimensional error ellipses of fit for All CPV including differing sets of data for ϕ vs q/p. The biggest differences come from including the CDF result, which elongates the error ellipses. The differing colors represent the 1-5σ contours.