Historical side note, Regression to Mediocrity

Regression to the mean

Brian Caffo, Jeff Leek, Roger Peng PhD
Johns Hopkins Bloomberg School of Public Health

A historically famous idea, Regression to the Mean

- Why is it that the children of tall parents tend to be tall, but not as tall as their parents?
- Why do children of short parents tend to be short, but not as short as their parents?
- Why do parents of very short children, tend to be short, but not a short as their child? And the same with parents of very tall children?
- Why do the best performing athletes this year tend to do a little worse the following?

Regression to the mean

- These phenomena are all examples of so-called regression to the mean
- Invented by Francis Galton in the paper "Regression towvards mediocrity in hereditary stature" The Journal of the Anthropological Institute of Great Britain and Ireland , Vol. 15, (1886).
- Think of it this way, imagine if you simulated pairs of random normals
  - The largest first ones would be the largest by chance, and the probability that there are smaller for the second simulation is high.
  - In other words $P(Y < x|X = x)$ gets bigger as x heads into the very large values.
  - Similarly $P(Y > x|X = x)$ gets bigger as x heads to very small values.
- Think of the regression line as the intrisic part.
  - Unless $\mathrm{Cor}(Y, X) = 1$ the intrinsic part isn't perfect

Regression to the mean

- Suppose that we normalize X (child's height) and Y (parent's height) so that they both have mean 0 and variance 1.
- Then, recall, our regression line passes through $(0, 0)$ (the mean of the X and Y).
- If the slope of the regression line is $Cor(Y, X)$, regardless of which variable is the outcome (recall, both standard deviations are 1).
- Notice if X is the outcome and you create a plot where X is the horizontal axis, the slope of the least squares line that you plot is $1/Cor(Y, X)$.

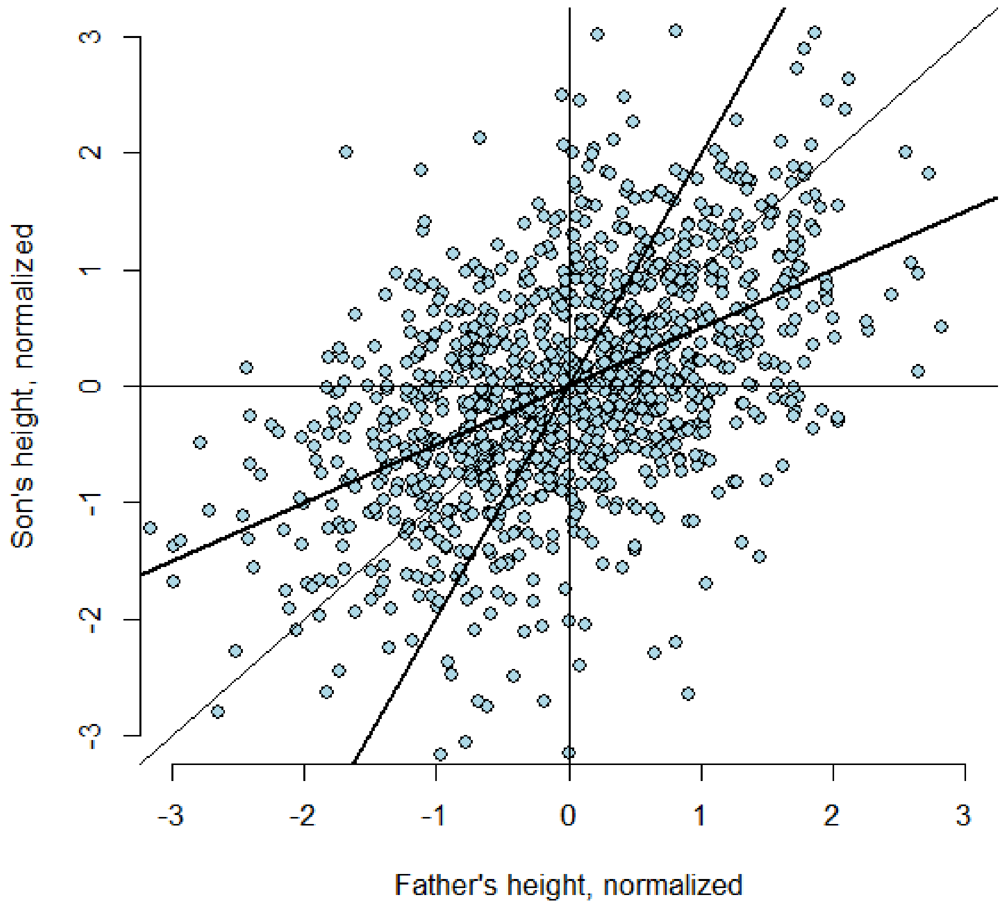
Normalizing the data and setting plotting parameters

```
library(UsingR)
data(father.son)
y <- (father.son$sheight - mean(father.son$sheight)) / sd(father.son$sheight)
x <- (father.son$fheight - mean(father.son$fheight)) / sd(father.son$fheight)
rho <- cor(x, y)
myPlot <- function(x, y) {
  plot(x, y,
       xlab = "Father's height, normalized",
       ylab = "Son's height, normalized",
       xlim = c(-3, 3), ylim = c(-3, 3),
       bg = "lightblue", col = "black", cex = 1.1, pch = 21,
       frame = FALSE)
}
```

Plot the data, code

```
myPlot(x, y)
abline(0, 1) # if there were perfect correlation
abline(0, rho, lwd = 2) # father predicts son
abline(0, 1 / rho, lwd = 2) # son predicts father, son on vertical axis
abline(h = 0); abline(v = 0) # reference lines for no relathionship
```

Plot the data, results



Discussion

- If you had to predict a son's normalized height, it would be $\mathrm{Cor}(Y, X) * X_i$
- If you had to predict a father's normalized height, it would be $\mathrm{Cor}(Y, X) * Y_i$
- Multiplication by this correlation shrinks toward 0 (regression toward the mean)
- If the correlation is 1 there is no regression to the mean (if father's height perfectly determine's child's height and vice versa)
- Note, regression to the mean has been thought about quite a bit and generalized