R Reference Card

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Getting help

Most R functions have online documentation.

help(topic) documentation on topic

?topic id.

help.search("topic") search the help system

apropos ("topic") the names of all objects in the search list matching the regular expression "topic"

help.start() start the HTML version of help

str(a) display the internal *str*ucture of an R object

summary (a) gives a "summary" of a, usually a statistical summary but it is generic meaning it has different operations for different classes of a

1s() show objects in the search path; specify pat="pat" to search on a

ls.str() str() for each variable in the search path

dir() show files in the current directory

methods (a) shows S3 methods of a

methods(class=class(a)) lists all the methods to handle objects of

Input and output

load () load the datasets written with save

data(x) loads specified data sets

library(x) load add-on packages

read.table(file) reads a file in table format and creates a data frame from it; the default separator sep="" is any whitespace; use header=TRUE to read the first line as a header of column names; use as.is=TRUE to prevent character vectors from being converted to factors; use comment.char="" to prevent "#" from being interpreted as a comment; use skip=n to skip n lines before reading data; see the help for options on row naming, NA treatment, and others

read.csv("filename", header=TRUE) id. but with defaults set for reading comma-delimited files

read.delim("filename", header=TRUE) id. but with defaults set for reading tab-delimited files

read.fwf(file,widths,header=FALSE,sep="",as.is=FALSE) cbind(...) id. by columns read a table of fixed width formatted data into a 'data.frame'; widths is an integer vector, giving the widths of the fixed-width fields

save(file,...) saves the specified objects (...) in the XDR platformindependent binary format

save.image(file) saves all objects

cat(..., file="", sep=" ") prints the arguments after coercing to character; sep is the character separator between arguments

print(a, ...) prints its arguments; generic, meaning it can have different methods for different objects

format (x,...) format an R object for pretty printing

write.table(x,file="",row.names=TRUE,col.names=TRUE, x[x > 3 & x < 5]sep=" ") prints x after converting to a data frame; if quote is TRUE, x[x %in% c("a", "and", "the")] elements in the given set

character or factor columns are surrounded by quotes ("); sep is the Indexing lists field separator; eol is the end-of-line separator; na is the string for missing values; use col.names=NA to add a blank column header to get the column headers aligned correctly for spreadsheet input

sink(file) output to file, until sink()

Most of the I/O functions have a file argument. This can often be a character string naming a file or a connection. file="" means the standard input or output. Connections can include files, pipes, zipped files, and R variables. On windows, the file connection can also be used with description = "clipboard". To read a table copied from Excel, use

x <- read.delim("clipboard")

To write a table to the clipboard for Excel, use

write.table(x, "clipboard", sep="\t", col.names=NA)

For database interaction, see packages RODBC, DBI, RMySQL, RPgSQL, and ROracle. See packages XML, hdf5, netCDF for reading other file formats.

Data creation

c(...) generic function to combine arguments with the default forming a vector; with recursive=TRUE descends through lists combining all elements into one vector

from: to generates a sequence; ":" has operator priority; 1:4 + 1 is "2,3,4,5" seq(from, to) generates a sequence by= specifies increment; length= specifies desired length

seq(along=x) generates 1, 2, ..., length(along); useful for for

rep(x,times) replicate x times; use each= to repeat "each" element of x each times: rep(c(1,2,3),2) is 1 2 3 1 2 3: length(x) number of elements in x rep(c(1,2,3),each=2) is 1 1 2 2 3 3

data.frame(...) create a data frame of the named or unnamed arguments: data.frame(v=1:4,ch=c("a","B","c","d"),n=10): shorter vectors are recycled to the length of the longest

list(...) create a list of the named or unnamed arguments; list (a=c(1,2),b="hi",c=3i);

array(x,dim=) array with data x; specify dimensions like dim=c (3, 4, 2); elements of x recycle if x is not long enough

matrix(x,nrow=,ncol=) matrix; elements of x recycle

factor(x,levels=) encodes a vector x as a factor

gl(n,k,length=n*k,labels=1:n) generate levels (factors) by specifying the pattern of their levels; k is the number of levels, and n is the number of replications

expand.grid() a data frame from all combinations of the supplied vectors or factors

rbind(...) combine arguments by rows for matrices, data frames, and

Slicing and extracting data

Indexing vectors

```
nth element
x[n]
                                  all but the nth element
x[-n]
                                  first n elements
x[1:n]
x[-(1:n)]
                                  elements from n+1 to the end
                                  specific elements
x[c(1,4,2)]
x["name"]
                                  element named "name"
                                  all elements greater than 3
x[x > 3]
                                  all elements between 3 and 5
```

```
list with elements n
x[n]
             nth element of the list
x[[n]]
x[["name"]] element of the list named "name"
x$name
             id.
Indexing matrices
            element at row i, column j
x[i,j]
x[i,]
            row i
x[,i]
            column i
x[,c(1,3)] columns 1 and 3
x["name",] row named "name"
Indexing data frames (matrix indexing plus the following)
x[["name"]] column named "name"
x$name
             id.
```

Variable conversion

```
as.array(x), as.data.frame(x), as.numeric(x),
     as.logical(x), as.complex(x), as.character(x),
      ... convert type; for a complete list, use methods (as)
```

Variable information

```
is.na(x), is.null(x), is.array(x), is.data.frame(x),
      is.numeric(x), is.complex(x), is.character(x),
      ... test for type; for a complete list, use methods (is)
```

dim(x) Retrieve or set the dimension of an object; dim(x) < -c(3,2)

dimnames (x) Retrieve or set the dimension names of an object

nrow(x) number of rows; NROW(x) is the same but treats a vector as a onerow matrix

ncol(x) and NCOL(x) id. for columns

class(x) get or set the class of x; class(x) <- "myclass"</pre>

unclass(x) remove the class attribute of x

attr(x, which) get or set the attribute which of x

attributes (obj) get or set the list of attributes of obj

Data selection and manipulation

which.max(x) returns the index of the greatest element of x which.min(x) returns the index of the smallest element of x

rev(x) reverses the elements of x

sort(x) sorts the elements of x in increasing order; to sort in decreasing order: rev(sort(x))

cut (x, breaks) divides x into intervals (factors); breaks is the number of cut intervals or a vector of cut points

match(x, y) returns a vector of the same length than x with the elements of x which are in y (NA otherwise)

which (x == a) returns a vector of the indices of x if the comparison operation is true (TRUE), in this example the values of i for which x[i] == a (the argument of this function must be a variable of mode logi-

choose (n, k) computes the combinations of k events among n repetitions = n!/[(n-k)!k!]

na.omit(x) suppresses the observations with missing data (NA) (suppresses the corresponding line if x is a matrix or a data frame)

na.fail(x) returns an error message if x contains at least one NA

unique(x) if x is a vector or a data frame, returns a similar object but with fft(x) Fast Fourier Transform of an array the duplicate elements suppressed table(x) returns a table with the numbers of the differents values of x filter(x, filter) applies linear filtering to a univariate time series or (typically for integers or factors) **subset(x, ...)** returns a selection of x with respect to criteria (..., typically comparisons: x\$V1 < 10); if x is a data frame, the option select gives the variables to be kept or dropped using a minus sign sample(x, size) resample randomly and without replacement size elements in the vector x, the option replace = TRUE allows to resample with replacement prop.table(x,margin=) table entries as fraction of marginal table sin, cos, tan, asin, acos, atan, atan2, log, log10, exp max(x) maximum of the elements of x min(x) minimum of the elements of x range(x) id. then c (min(x), max(x)) **sum(x)** sum of the elements of x diff(x) lagged and iterated differences of vector x prod(x) product of the elements of x mean (x) mean of the elements of x median(x) median of the elements of x quantile (x, probs=) sample quantiles corresponding to the given probabilities (defaults to 0,.25,.5,.75,1) weighted.mean(x, w) mean of x with weights w rank (x) ranks of the elements of x var(x) or cov(x) variance of the elements of x (calculated on n-1); if x is a matrix or a data frame, the variance-covariance matrix is calculated **sd(x)** standard deviation of x cor(x) correlation matrix of x if it is a matrix or a data frame (1 if x is a vector) var(x, y) or cov(x, y) covariance between x and y, or between the columns of x and those of y if they are matrices or data frames cor(x, y) linear correlation between x and y, or correlation matrix if they are matrices or data frames round(x, n) rounds the elements of x to n decimals log(x, base) computes the logarithm of x with base base scale(x) if x is a matrix, centers and reduces the data; to center only use the option center=FALSE, to reduce only scale=FALSE (by default center=TRUE, scale=TRUE) **pmin**(x,y,...) a vector which *i*th element is the minimum of x[i], y[i],...

pmax(x,y,...) id. for the maximum **cumsum(x)** a vector which *i*th element is the sum from x[1] to x[i]cumprod(x) id. for the product cummin(x) id. for the minimum cummax(x) id. for the maximum union(x,y),intersect(x,y),setdiff(x,y),setequal(x,y) is.element(el,set) "set" functions Re(x) real part of a complex number Im(x) imaginary part Mod(x) modulus; abs(x) is the same Arg(x) angle in radians of the complex number Conj(x) complex conjugate convolve(x,y) compute the several kinds of convolutions of two sequences

mvfft(x) FFT of each column of a matrix to each series separately of a multivariate time series Many math functions have a logical parameter na.rm=FALSE to specify missing data (NA) removal. **Matrices** t(x) transpose diag(x) diagonal %*% matrix multiplication **solve(a,b)** solves a %*% x = b for x solve(a) matrix inverse of a rowsum(x) sum of rows for a matrix-like object; rowSums(x) is a faster version colsum(x), colsums(x) id. for columns rowMeans (x) fast version of row means colMeans(x) id. for columns Advanced data processing apply(X, INDEX, FUN=) a vector or array or list of values obtained by applying a function FUN to margins (INDEX) of X lapply (X, FUN) apply FUN to each element of the list X tapply (X, INDEX, FUN=) apply FUN to each cell of a ragged array given by X with indexes INDEX by (data, INDEX, FUN) apply FUN to data frame data subsetted by INDEX merge (a,b) merge two data frames by common columns or row names xtabs(a b,data=x) a contingency table from cross-classifying factors aggregate(x,by,FUN) splits the data frame x into subsets, computes summary statistics for each, and returns the result in a convenient form; by is a list of grouping elements, each as long as the variables frame or list into a single column unstack(x, ...) inverse of stack()

stack(x, ...) transform data available as separate columns in a data

reshape(x, ...) reshapes a data frame between 'wide' format with repeated measurements in separate columns of the same record and 'long' format with the repeated measurements in separate records;

use (direction="wide") or (direction="long")

Strings

paste(...) concatenate vectors after converting to character; sep= is the string to separate terms (a single space is the default); collapse= is an optional string to separate "collapsed" results

substr(x, start, stop) substrings in a character vector; can also assign, as substr(x, start, stop) <- value</pre>

strsplit(x,split) split x according to the substring split grep(pattern, x) searches for matches to pattern within x; see ?regex

gsub(pattern, replacement, x) replacement of matches determined by regular expression matching sub() is the same but only replaces the first occurrence.

tolower(x) convert to lowercase toupper(x) convert to uppercase

match (x, table) a vector of the positions of first matches for the elements of x among table

x %in% table id. but returns a logical vector

pmatch(x,table) partial matches for the elements of x among table

nchar(x) number of characters

Dates and Times

The class Date has dates without times. POSIXct has dates and times, including time zones. Comparisons (e.g. >), seq(), and difftime() are useful. Date also allows + and -. ?DateTimeClasses gives more information. See also package chron.

as.Date(s) and as.POSIXct(s) convert to the respective class; format (dt) converts to a string representation. The default string format is "2001-02-21". These accept a second argument to specify a format for conversion. Some common formats are:

%a, %A Abbreviated and full weekday name.

%b, %B Abbreviated and full month name.

%d Day of the month (01–31).

%H Hours (00–23).

%I Hours (01–12).

% † Day of year (001–366).

%m Month (01-12).

%M Minute (00-59).

%p AM/PM indicator.

%S Second as decimal number (00-61).

%U Week (00-53); the first Sunday as day 1 of week 1.

%w Weekday (0-6, Sunday is 0).

%₩ Week (00–53); the first Monday as day 1 of week 1.

%y Year without century (00-99). Don't use.

%Y Year with century.

%z (output only.) Offset from Greenwich; -0800 is 8 hours west of.

\{\}Z (output only.) Time zone as a character string (empty if not available).

Where leading zeros are shown they will be used on output but are optional on input. See ?strftime.

Plotting

plot(x) plot of the values of x (on the y-axis) ordered on the x-axis

plot(x, y) bivariate plot of x (on the x-axis) and y (on the y-axis)

hist(x) histogram of the frequencies of x

barplot(x) histogram of the values of x; use horiz=FALSE for horizontal

dotchart (x) if x is a data frame, plots a Cleveland dot plot (stacked plots line-by-line and column-by-column)

pie(x) circular pie-chart

boxplot(x) "box-and-whiskers" plot

sunflowerplot(x, y) id. than plot() but the points with similar coordinates are drawn as flowers which petal number represents the num-

stripplot(x) plot of the values of x on a line (an alternative to boxplot() for small sample sizes)

coplot (x~y | z) bivariate plot of x and y for each value or interval of values of z

interaction.plot (f1, f2, y) if f1 and f2 are factors, plots the means of v (on the v-axis) with respect to the values of f1 (on the x-axis) and of f2 (different curves); the option fun allows to choose the summary statistic of y (by default fun=mean)

- matplot(x,y) bivariate plot of the first column of x vs. the first one of y, mtext(text, side=3, line=0, ...) adds text given by text in lty controls the type of lines, can be an integer or string (1: "solid", the second one of x vs. the second one of y, etc.
- fourfoldplot(x) visualizes, with quarters of circles, the association between two dichotomous variables for different populations (x must be an array with dim=c(2, 2, k), or a matrix with dim=c(2, 2) if
- assocplot(x) Cohen-Friendly graph showing the deviations from independence of rows and columns in a two dimensional contingency ta-
- mosaicplot(x) 'mosaic' graph of the residuals from a log-linear regression of a contingency table
- pairs (x) if x is a matrix or a data frame, draws all possible bivariate plots between the columns of x
- **plot.ts(x)** if x is an object of class "ts", plot of x with respect to time, x may be multivariate but the series must have the same frequency and dates
- ts.plot(x) id. but if x is multivariate the series may have different dates and must have the same frequency
- **qqnorm(x)** quantiles of x with respect to the values expected under a normal law
- qqplot(x, y) quantiles of y with respect to the quantiles of x
- contour (x, y, z) contour plot (data are interpolated to draw the curves), x and y must be vectors and z must be a matrix so that dim(z) = c(length(x), length(y)) (x and y may be omitted)
- filled.contour(x, y, z) id. but the areas between the contours are coloured, and a legend of the colours is drawn as well
- image(x, y, z) id. but with colours (actual data are plotted)
- persp(x, y, z) id. but in perspective (actual data are plotted)
- stars(x) if x is a matrix or a data frame, draws a graph with segments or a star where each row of x is represented by a star and the columns are the lengths of the segments
- symbols (x, y, ...) draws, at the coordinates given by x and y, symbols (circles, squares, rectangles, stars, thermometres or "boxplots") which sizes, colours ... are specified by supplementary arguments
- termplot(mod.obj) plot of the (partial) effects of a regression model (mod.obi)

The following parameters are common to many plotting functions:

add=FALSE if TRUE superposes the plot on the previous one (if it exists)

axes=TRUE if FALSE does not draw the axes and the box

- type="p" specifies the type of plot, "p": points, "1": lines, "b": points connected by lines, "o": id. but the lines are over the points, "h": vertical lines, "s": steps, the data are represented by the top of the vertical lines, "S": id. but the data are represented by the bottom of the vertical lines
- **xlim=**, **ylim=** specifies the lower and upper limits of the axes, for example with xlim=c(1, 10) or xlim=range(x)
- xlab=, ylab= annotates the axes, must be variables of mode character main = main title, must be a variable of mode character

sub= sub-title (written in a smaller font)

Low-level plotting commands

- points(x, y) adds points (the option type= can be used)
- lines(x, y) id. but with lines
- text(x, y, labels, ...) adds text given by labels at coordinates (x,y); a typical use is: plot (x, y, type="n"); text (x, y, names)

- the margin specified by side (see axis() below); line specifies the line from the plotting area
- segments (x0, y0, x1, y1) draws lines from points (x0,y0) to points (x1,y1)
- arrows (x0, y0, x1, y1, angle= 30, code=2) id. with arrows at points (x0,y0) if code=2, at points (x1,y1) if code=1, or both if code=3; angle controls the angle from the shaft of the arrow to the edge of the arrow head
- abline (a,b) draws a line of slope b and intercept a
- abline(h=y) draws a horizontal line at ordinate y
- abline(v=x) draws a vertical line at abcissa x
- abline(lm.obj) draws the regression line given by lm.obj
- rect(x1, y1, x2, y2) draws a rectangle which left, right, bottom, and top limits are x1, x2, v1, and v2, respectively
- polygon (x, y) draws a polygon linking the points with coordinates given by x and y
- **legend** (x, y, legend) adds the legend at the point (x,y) with the symbols given by legend
- title() adds a title and optionally a sub-title
- axis(side, vect) adds an axis at the bottom (side=1), on the left (2), at the top (3), or on the right (4); vect (optional) gives the abcissa (or ordinates) where tick-marks are drawn
- **rug(x)** draws the data x on the x-axis as small vertical lines
- **locator(n, type="n", ...)** returns the coordinates (x,y) after the user has clicked n times on the plot with the mouse; also draws symbols (type="p") or lines (type="l") with respect to optional graphic parameters (...); by default nothing is drawn (type="n")

Graphical parameters

These can be set globally with **par(...)**; many can be passed as parameters to plotting commands.

- adj controls text justification (0 left-justified, 0.5 centred, 1 right-justified) **bg** specifies the colour of the background (ex. : bg="red", bg="blue", ...
- the list of the 657 available colours is displayed with colors ())
- **bty** controls the type of box drawn around the plot, allowed values are: "o". "1", "7", "c", "u" ou "]" (the box looks like the corresponding character); if bty="n" the box is not drawn
- cex a value controlling the size of texts and symbols with respect to the default; the following parameters have the same control for numbers on the axes, cex.axis, the axis labels, cex.lab, the title, cex.main, and the sub-title, cex. sub
- col controls the color of symbols and lines; use color names: "red", "blue' see colors() or as "#RRGGBB"; see rgb(), hsv(), gray(), and rainbow(); as for cex there are: col.axis, col.lab, col.main, col.sub
- **font** an integer which controls the style of text (1: normal, 2: italics, 3: bold, 4: bold italics); as for cex there are: font.axis, font.lab, font.main,font.sub
- las an integer which controls the orientation of the axis labels (0: parallel to the axes, 1: horizontal, 2: perpendicular to the axes, 3: vertical)

- 2: "dashed", 3: "dotted", 4: "dotdash", 5: "longdash", 6: "twodash", or a string of up to eight characters (between "0" and "9") which specifies alternatively the length, in points or pixels, of the drawn elements and the blanks, for example 1ty="44" will have the same effect than 1tv=2
- **1wd** a numeric which controls the width of lines, default 1
- mar a vector of 4 numeric values which control the space between the axes and the border of the graph of the form c(bottom, left, top, right), the default values are c (5.1, 4.1, 4.1, 2.1)
- mfcol a vector of the form c(nr,nc) which partitions the graphic window as a matrix of nr lines and nc columns, the plots are then drawn in columns
- **mfrow** id. but the plots are drawn by row
- **pch** controls the type of symbol, either an integer between 1 and 25, or any single character within ""
- 1 2 △ 3 + 4 × 5 ◇ 6 ▽ 7 図 8 ★ 9 ⊕ 10 ⊕ 11 ☆ 12 ⊞ 13 図 14 △ 15 16 ● 17 ▲ 18 ◆ 19 ● 20 ● 21 ○ 22 □ 23 ♦ 24 △ 25 ▽ * * . XX a a ??
- **ps** an integer which controls the size in points of texts and symbols
- **pty** a character which specifies the type of the plotting region, "s": square, "m": maximal
- tck a value which specifies the length of tick-marks on the axes as a fraction of the smallest of the width or height of the plot; if tck=1 a grid is
- tcl a value which specifies the length of tick-marks on the axes as a fraction of the height of a line of text (by default tcl=-0.5)
- **xaxt** if xaxt="n" the x-axis is set but not drawn (useful in conjonction with axis(side=1, ...))
- yaxt if yaxt="n" the y-axis is set but not drawn (useful in conjonction with axis(side=2, ...))

Lattice (Trellis) graphics

xyplot(y~x) bivariate plots (with many functionalities)

barchart (y~x) histogram of the values of y with respect to those of x

dotplot(y~x) Cleveland dot plot (stacked plots line-by-line and columnby-column)

densityplot(~x) density functions plot

histogram(~x) histogram of the frequencies of x

bwplot(y~x) "box-and-whiskers" plot

qqmath(~x) quantiles of x with respect to the values expected under a theoretical distribution

- stripplot(y~x) single dimension plot, x must be numeric, y may be a
- qq(y~x) quantiles to compare two distributions, x must be numeric, y may be numeric, character, or factor but must have two 'levels'
- **splom(~x)** matrix of bivariate plots
- parallel(~x) parallel coordinates plot
- **levelplot** $(z^x y | g1 * g2)$ coloured plot of the values of z at the coordinates given by x and y (x, y) and z are all of the same length)

wireframe(z~x*y|q1*q2) 3d surface plot

cloud(z~x*y|g1*g2) 3d scatter plot

Sannolikhetslära

1.1 Några diskreta fördelningar

• Binomialfördelning

$$X \sim Bin(n, p)$$

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \dots, n$$

$$E[X] = np, \quad \text{var}(X) = np(1-p), \qquad G_X(s) = (1+p(s-1))^n$$

• Poissonfördelning

$$X \sim Po(\mu)$$

$$p_X(k) = \frac{\mu^k}{k!} e^{-\mu}, \qquad k = 0, 1, 2, \dots$$

$$E[X] = \mu, \qquad \text{var}(X) = \mu, \qquad G_X(s) = e^{(s-1)\mu}$$

Hypergeometriskfördelning

$$p_X(k) = \frac{\binom{Np}{k} \binom{N(1-p)}{n-k}}{\binom{N}{n}}, \quad k = 0, 1, \dots, n$$

$$E[X] = np, \quad \text{var}(X) = \frac{N-n}{N-1} np(1-p)$$

• Geometrisk fördelning

$$X \sim Ge(p)$$

$$p_X(k) = (1-p)^k p, \qquad k = 0, 1, 2, \dots$$

$$E[X] = \frac{1-p}{p}, \qquad \text{var}(X) = \frac{1-p}{p^2}, \qquad G_X(s) = \frac{p}{1-(1-p)s}$$

För första-gången-fördelning

$$X \sim Ffg(p)$$

$$p_X(k) = (1-p)^{k-1}p, \qquad k = 1, 2, 3 \dots$$

$$E[X] = \frac{1}{p}, \qquad var(X) = \frac{1-p}{p^2}, \qquad G_X(s) = \frac{sp}{1-(1-p)s}$$

• Negativ Binomialfördelning

$$X \sim NB(r, p)$$

$$p_X(k) = \binom{k+r-1}{r-1} p^r (1-p)^{k-r} \qquad k = 0, 1, 2, \dots$$

$$E[X] = \frac{r(1-p)}{p}, \qquad \text{var}(X) = \frac{r(1-p)}{p}, \qquad G_X(s) = \left(\frac{p}{1-(1-p)s}\right)^r$$

TAMS65

1.2 Några kontinuerliga fördelningar

• Likformig (rektangulär) fördelning på intervallet (a,b)

$$X \sim U(a,b)$$

$$f_X(x) = \frac{1}{b-a}, \qquad a \le x \le b$$

$$\text{E}[X] = \frac{a+b}{2}, \qquad \text{var}(X) = \frac{(b-a)^2}{12}, \qquad M_X(p) = \frac{e^{pb} - e^{pa}}{p(b-a)}$$

• Exponentialfördelning

$$X \sim Exp(\lambda),$$

där λ betecknar intensiteten. Ibland används väntevärdet $\mu = \frac{1}{\lambda}$ som parameter.

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x \ge 0$$

$$\mathrm{E}[X]=\frac{1}{\lambda}, \qquad \mathrm{var}(X)=\frac{1}{\lambda^2}, \qquad M_X(p)=\frac{\lambda}{\lambda-p}, \qquad \lambda>p.$$
 • Normalfördelning
$$X\sim N(\mu,\sigma)$$

$$X \sim N(\mu, \sigma)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad -\infty < x < +\infty$$

$$\mathrm{E}[X] = \mu, \qquad \mathrm{var}(X) = \sigma^2, \qquad M_X(p) = \exp\left\{p\mu + \frac{p^2}{2}\sigma^2\right\}$$

• γ²-fördelning

$$Y \sim \chi^2(n)$$

Uppkomst: Om X_1, \ldots, X_n är oberoende, var och en N(0,1), gäller att $Y = X_1^2 + \cdots + X_n^2$ $\ldots + X_n^2$ får en χ^2 fördelning med n frihetsgrader.

$$f_Y(x) = \frac{x^{(n/2)-1}e^{-x/2}}{2^{(n/2)}\Gamma(n/2)}, \quad x \ge 0,$$

där $\Gamma(\cdot)$ är gammafunktionen

$$\Gamma(c) = \int_0^\infty x^{c-1} e^{-x} dx, \quad \text{där } c > 0.$$

$$E[Y] = n,$$
 $var(Y) = 2n,$ $M_Y(p) = \frac{1}{(1-p)^{n/2}},$ $p < 1$