Information page for written examinations at Linköping University

Examination date	2015-04-08
Room (1)	TER3
Time	8-12
Course code	TSBB06
Exam code	TEN2
Course name **Exam name**	Multidimensional Signal Analysis (Multidimensionell signalanalys) Written Examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	013-281634
Visit to the examination room approximately	9 and 11 am
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator
Other important information	Use cross-ruled paper
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

PART I: GEOMETRY

Exercise 1 (A, 1p) Two 2D points have Cartesian coordinates $(2,2)$ and $(2,-3)$, respectively. What are the dual homogenous coordinates of the line that intersects the two points?

Exercise 2 (A, 1p) Give a practical example of when a homography describes the transformation between the points in two 2D planes.

Exercise 3 (A, 1p) Given an angle α and normalized vector $\hat{\mathbf{n}}$, how can you determine the rotation matrix $\mathbf{R}$ of the rotation around $\hat{\mathbf{n}}$ by the angle α?

Exercise 4 (B, 2p) $\mathbf{x}_1$ and $\mathbf{x}_2$ are the homogeneous coordinates of two distinct 3D points, and $\mathbf{p}$ is the dual homogeneous coordinates of a 3D plane. Give a simple expression for the homogeneous coordinates $\mathbf{x}_0$ of the 3D point lying where the line through points $\mathbf{x}_1$ and $\mathbf{x}_2$ intersects the plane $\mathbf{p}$. Also: describe the degenerate case when the result is $\mathbf{x}_0 = \mathbf{0}$ (= zero vector).

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) A camera matrix and two 3D points are given as

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 2 \end{pmatrix}, \qquad \mathbf{x}_1 = \begin{pmatrix} -3 \\ -3 \\ 0 \\ 1 \end{pmatrix}, \qquad \mathbf{x}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}.$$

In general, the image of a 3D line projected by the camera is a 2D line. The image of the line that intersects $\mathbf{x}_1$ and $\mathbf{x}_2$, however, projects to a single point in the image plane. Explain this result: how are the two 3D points configured to produce this result?

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART II: ESTIMATION

Exercise 6 (A, 1p) A homogeneous linear equation $\mathbf{A}\mathbf{z} = \mathbf{0}$ can also be solved as an ordinary inhomogeneous linear equation. Describe how to do this.

Exercise 7 (A, 1p) Given a set of 2D points, how is Hartley-normalization done on this data set?

Exercise 8 (A, 1p) A pinhole camera can be represented by the 3×4 matrix $\mathbf{C}$ that maps homogeneous 3D coordinates $\mathbf{x}$ to homogeneous image coordinates according to: $\mathbf{y} \sim \mathbf{C}\,\mathbf{x}$. If $\mathbf{x}$ and $\mathbf{y}$ are known, how can this expression be transformed into a linear constraint on $\mathbf{C}$?

Exercise 9 (B, 2p) Let $\mathbf{A}$ be an $N \times M$ real matrix and consider the singular value decomposition (SVD) of $\mathbf{A}$. Characterize the result of this operation: how many matrices are produced, what properties do they have, and how are they related to $\mathbf{A}$?

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 10 (B, 2p) Two sets of 3D points are related by a *rigid transformation*. Assuming that we know correspondences between points in the two sets how can we estimate the rigid transformation?

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) Convolution between two discrete signals, f and g, can be interpreted as producing a linear combination between a set of vectors and a set of scalars. Explain how we can arrive at this idea.

Exercise 12 (A, 1p) $\mathbf{B}$ is matrix that, in its columns, holds the vectors of a frame in a vector space V. Any vector $\mathbf{x} \in V$ can be written as a linear combination of the frame vectors: $\mathbf{x} = \mathbf{B}\,\mathbf{c}$ for some coefficient vector $\mathbf{c}$. For a given $\mathbf{c}$, describe the complete set of coefficient vectors that can reconstruct $\mathbf{x}$ from $\mathbf{B}$.

Exercise 13 (A, 1p) U is a subspace of V and $\mathbf{B}$ hold a basis of U is its columns. $\mathbf{G}_0$ defines the scalar product in V. For a general $\mathbf{v} \in V$, how do you compute the orthogonal projection of $\mathbf{v}$ onto U?

Exercise 14 (B, 2p) How do you transform the coordinates $\mathbf{c}$ of a vector $\mathbf{v}$ relative to some basis $\mathbf{B}$ into dual coordinates $\tilde{\mathbf{c}}$? Explain the different quantities or variables that you are using in your solution. Also: explain what dual coordinates are.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 15 (B, 2p) $B = \{\mathbf{b}_k, k = 1, \dots, n\}$ is a basis of some vector space V. The corresponding dual basis is $\tilde{B} = \{\tilde{\mathbf{b}}_k, k = 1, \dots, n\}$. What algebraic relation should be satisfied between the two bases (1p)? Show that this relation allows us to compute coordinates of any $\mathbf{v} \in V$ relative the basis B in terms of scalar products with the dual basis $\tilde{B}$ (1p).

WRITE YOUR ANSWER ON A SEPARATE SHEET

PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) Normalized convolution allows you to compute an expansion of the local signal, e.g., in terms of a polynomial basis. What interpretation can be made of the corresponding coordinates in this case?

Exercise 17 (A, 1p) The sampling theorem states that the sampling frequency f_S only needs to satisfy $f_S \geq 2\, f_N$, where f_N is the Nyquist frequency of the signal. Why can it be of interest to use f_S that is significantly larger than $2\, f_N$?

Exercise 18 (A, 1p) In filter optimization, why is it not always the case that the optimization error decreases when we add a coefficient to the filter?

Exercise 19 (B, 2p) What is the primary application of Principal Component Analysis (PCA)? What problem is solved in PCA: what is known and what is unknown?

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 20 (B, 2p) The wavelet transform analyzes a function in terms of functions derived from a *mother wavelet* $\psi(t)$. How are the analyzing functions related to $\psi(t)$? How is this different compared to the Fourier transform?

WRITE YOUR ANSWER ON A SEPARATE SHEET