Information page for written examinations at Linköping University

Examination date	2015-01-13
Room	G35, G36
Time	14-18
Course code	TSBB06
Exam code	TEN2
Course name **Exam name**	Multi-dimensional Signal Analysis
Department	Dept of EE (ISY)
Number of questions in the examination	20
Teacher responsible/contact person during the exam time	Klas Nordberg
Contact number during the exam time	0739-037628
Visit to the examination room approx.	Around 15pm & 17pm
Name and contact details of the course examiner	Klas Nordberg 013-281634 klas@isy.liu.se
Equipment permitted	Calculator
Other important information	
Which type of paper should be used, cross-ruled or lined	Cross-ruled rules
Number of exams in the bag	

Guide

The written examination consists of 4 parts, one part for each of the four course aims in the curriculum.

- Part I: Geometry
- Part II: Estimation
- Part III: Linear signal representation
- Part IV: Signal processing applications

Each part consists of 3 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of more detailed explanations or simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: two parts must have at least 3p and two parts must have at least 4p, and there must be 2 B-type exercises passed with full 2p.

To pass with grade 4: two parts must have at least 4p and two parts must have at least 5p, and there must be 4 B-type exercises passed with full 2p.

To pass with grade 5: all parts must have at least 5p, and there must be 6 B-type exercises passed with full 2p.

The answers to the A-exercises should preferably be given in the blank spaces of this examination thesis, below the questions. Use additional sheets if necessary, with no more than one exercise per sheet

Write your anonymous examination ID (AID) at the top of the pages in this examination thesis and any sheet appended to the examination thesis.

Good luck!
Klas Nordberg

PART I: GEOMETRY

Exercise 1 (A, 1p) A rotation matrix $\mathbf{R}$ is an element of the matrix group $SO(3)$. What are the algebraic constraints that $\mathbf{R}$ necessary must satisfy in order to assure that $\mathbf{R} \in SO(3)$?

Exercise 2 (A, 1p) The 2D points $\mathbf{y}_1$ and $\mathbf{y}_2$ have the same distance to a line $\mathbf{l}$, but their respective *signed distances* to $\mathbf{l}$ have different signs. What is the geometric interpretation of this observation?

Exercise 3 (A, 1p) Two 3D planes have dual homogeneous coordinates $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively. How can we determine if they are parallel based on their homogeneous coordinates?

Exercise 4 (B, 2p) Give at least two examples where computations based on homogeneous coordinates becomes simpler, or can lead to more general results, compared to using Cartesian coordinates.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 5 (B, 2p) Two 2D lines are parallel and two other lines are not parallel. Show that an affine 2D transformation preserves these properties.

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PART II: ESTIMATION

Exercise 6 (A, 1p) You have a set of corresponding 3D points, before and after a rigid transformation. You want to estimate the rigid transformation, although both point sets contain some amount of noise. How must the point sets be modified in order to allow the orthogonal Procrustes algorithm to give the rotation part of the rigid transformation?

Exercise 7 (A, 1p) What is the result of applying a direct linear transformation (DLT) to the relation $\mathbf{y}'_k \sim \mathbf{H}\,\mathbf{y}_k, k = 1, \ldots, N$?

Exercise 8 (A, 1p) A 3D plane $\mathbf{p}$ is determined from the positions of three 3D points, using the homogeneous method. Explain in what way $\mathbf{p}$ is different if you make the estimation based on Hartley-normalized coordinates or without this normalization, and why.

Exercise 9 (B, 2p) Explain why methods based on minimization of geometric errors are the preferred approach in geometric estimation. Explain why algebraic minimization techniques are still relevant in that case.

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Exercise 10 (B, 2p) The homogeneous method is used for minimizing an algebraic error that is derived from some measurements that are perturbed by noise. The result can then be classified into three characteristic cases: (A) there is one unique solution, (B) there are multiple solutions, and (C) there is no solution to the estimation problem. For each of the three cases, characterize the corresponding data matrix in terms of its SVD profile.

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PART III: LINEAR SIGNAL REPRESENTATIONS

Exercise 11 (A, 1p) A set of N vectors $\mathbf{b}_k \in V, k = 1, \ldots, N$, form a frame for V if they satisfy the *frame condition*. How it this condition formulated?

Exercise 12 (A, 1p) Convolution between two discrete signals, f and g, can be interpreted as producing a linear combination between a set of vectors and a set of scalars. Explain how we can arrive at this idea.

Exercise 13 (A, 1p) Give an example of a scalar product that can be used for one-variable functions, e.g., in L^2.

Exercise 14 (B, 2p) The vectors $\mathbf{b}_1 = (1, -1)$ and $\mathbf{b}_2 = (3, -1)$ form a basis for $\mathbb{R}^2$. Use the scalar product defined by $\mathbf{G}_0 = \mathbf{I}$, and determine the corresponding dual basis vectors $\tilde{\mathbf{b}}_1$ and $\tilde{\mathbf{b}}_2$. Show that the $\tilde{\mathbf{b}}_1$ and $\tilde{\mathbf{b}}_2$ are in a dual relation with $\mathbf{b}_1$ and $\mathbf{b}_2$.

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Exercise 15 (B, 2p) Let $V = \mathbb{R}^n$ be a vector space with a scalar product defined by the matrix $\mathbf{G}_0$. $U \subset V$ is a subspace represented by the basis matrix $\mathbf{B}$. Let $\mathbf{v} \in V$, and describe how to determine $\mathbf{v}_1$, the orthogonal production of $\mathbf{v}$ onto U. Describe also how to determine $\mathbf{v}_0$, the orthogonal complement of $\mathbf{v}_1$, and demonstrate that $\mathbf{v}_0$ is indeed orthogonal to any $\mathbf{u} \in U$.

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PART IV: SIGNAL PROCESSING APPLICATIONS

Exercise 16 (A, 1p) Signal processing based on conversion from analog to digital representations, and vice versa, can use over-sampling to reduce the amount of noise that is introduced by such processing. Describe at least one such noise source, that appears in a practical application.

Exercise 17 (A, 1p) Normalized convolution can be used to process a discrete signal that has missing data/samples. Describe an application that is likely to generate missing or uncertain samples in a signal.

Exercise 18 (A, 1p) If you apply PCA to blocks of natural audio or image signals, the first principal component can often be characterized in a simple way. How?

Exercise 19 (B, 2p) Based on a mother wavelet, $\psi(t)$, how do you define the continuous wavelet transform (CWT) of the function $f(t)$? Describe the variables of the transform function in a clear way. The discrete wavelet transform (DWT) is a sampled version of the CWT. In which points is the CWT sampled to give the DWT? Give a mathematical expression of the sample points, or illustrate in a figure.

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Exercise 20 (B, 2p) In filter optimization, the residual error of the resulting filter in general decreases when more coefficients are added to the filter. This, however, is not always he case. Describe a situation when a filter coefficient $f[x]$ does not contribute to reducing the residual error, and explain why this is so.

WRITE YOUR ANSWER ON A SEPARATE SHEET