Guide* to answers for written examination in TSBB06 Multi-dimensional signal analysis, 2015-04-08

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PART I

Exercise 1 The homogeneous coordinates of the two points are given as

$$\mathbf{y}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{y}_2 = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

The dual homogeneous coordinates of the intersecting line are

$$\mathbf{l} = \mathbf{y}_1 \times \mathbf{y}_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -10 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

The correctness of this result can be checked: $\mathbf{y}_1 \cdot \mathbf{l} = \mathbf{y}_2 \cdot \mathbf{l} = 0$.

Exercise 2 Some examples:

- Two images taken with a common camera center of the same scene, for example by rotating a camera around its center between two images. The coordinates of corresponding points in the two images are related by a homography.
- Two images taken from different viewpoints (with different camera centers) of a planar surface. Again, the coordinates of corresponding points in the two images are related by a homography.
- A single image of a planar surface, e.g., the ground plane in a scene. The mapping to/from image coordinates to a coordinate system in the plane is given by a homography.
- If an image is projected onto a slanted surface, e.g., a PC projector that is not perpendicular to the screen, the projected image should be transformed by a homography to look as it shold.

Exercise 3 See the IREG compendium, section 9.2.1.

Exercise 4 The Plücker coordinates of the line that intersects $\mathbf{x}_1$ and $\mathbf{x}_2$ is given as

$$\mathbf{L} = \mathbf{x}_1 \mathbf{x}_2^T - \mathbf{x}_2 \mathbf{x}_1^T$$

and the point of intersection between this line and the plane $\mathbf{p}$ is given as $\mathbf{x}_0 = \mathbf{L}\,\mathbf{p}$.

*This guide is not an authoritative description of how answers to the questions must be given in order to pass the exam.

Since the two points are distinct, it follows that the two vectors $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^4$ are linearly independent. This means that $\mathbf{x}_0 = \mathbf{0}$ exactly when $\mathbf{x}_1 \cdot \mathbf{p} = \mathbf{x}_2 \cdot \mathbf{p} = 0$, i.e., when both points $\mathbf{x}_1, \mathbf{x}_2$ lie in the plane $\mathbf{p}$.

Exercise 5 The projections of the two 3D points into the camera image are given as

$$\mathbf{y}_1 \sim \mathbf{C}\,\mathbf{x}_1 = \begin{pmatrix} -3 \\ -2 \\ -4 \end{pmatrix}, \quad \mathbf{y}_2 \sim \mathbf{C}\,\mathbf{x}_2 = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}.$$

This means that the 3D line $\mathbf{L}$ which intersects $\mathbf{x}_1$ and $\mathbf{x}_2$ must also intersect the camera center. The camera center $\mathbf{n}$ satisfies $\mathbf{C}\,\mathbf{n} = \mathbf{0}$ and is given as

$$\mathbf{n} = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}.$$

The fact that $\mathbf{L}$ intersects the camera center is verified by the observation that $\mathbf{x}_1 = 2\,\mathbf{n} - \mathbf{x}_2$, i.e., it is possible to write the homogeneous coordinates of either of the three points as a linear combination of the other two.

PART II

Exercise 6 The inhomogeneous method is based on setting one element in $\mathbf{z}$ equal to 1. For example, we can set the last element to 1, and denote the remaining elements as $\bar{\mathbf{z}}$. The homogeneous equation then becomes:

$$\mathbf{0} = \mathbf{A}\,\mathbf{z} = \begin{pmatrix} \mathbf{A}_0 & \mathbf{b} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{z}} \\ 1 \end{pmatrix} = \mathbf{A}_0 \bar{\mathbf{z}} + \mathbf{b},$$

where $\mathbf{b}$ is the rightmost column of $\mathbf{A}$, and $\mathbf{A}_0$ are the remaining columns of $\mathbf{A}$. This is an inhomogeneous equation in the unknown $\bar{\mathbf{z}}$, and has a solution:

$$\bar{\mathbf{z}} = -(\mathbf{A}_0^\top \mathbf{A}_0)^{-1} \mathbf{A}_0^\top \mathbf{b} = -\mathbf{A}_0^+ \mathbf{b}.$$

Exercise 7 See the IREG compendium, section 11.2.2.

Exercise 8 See the IREG compendium, section 11.3.

Exercise 9 See the IREG compendium, section B.4.

Exercise 10 See the IREG compendium, section 13.2.

PART III

Exercise 11 See lecture 2A, slides 26-27.

Exercise 12 See lecture 2F, slide 23.

Exercise 13 See lecture 2C, slide 13.

Exercise 14 The definition of dual coordinates is made in lecture, slide 25. The transformation from dual to proper coordinates is discussed in the same lecture, slide 36.

Exercise 15 The algebraic relation between a basis and its dual basis is presented in lecture 2B, slide 3. How this can be used for computing coordinates is discussed in the same lecture, slide 4.

PART IV

Exercise 16 See lecture 2C, slide 28.

Exercise 17 See lecture 2f, slides 31-37.

Exercise 18 See computer exercise D.

Exercise 19 Applications for PCA are discussed in lecture 2F, slide 26. The problem formulation in PCA is presented in the same lecture, slide 8.

Exercise 20 The relation between the analyzing wavelet functions and the corresponding mother wavelet is discussed in lecture 2F, slide 39. A comparison between the wavelet transform and the Fourier transform is presented in the same lecture, slide 41.