Guide* to answers for written examination in TSBB06 Multi-dimensional signal analysis, 2014-01-15

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PART I

Exercise 1 See the IREG compendium, eq (9.1)

Exercise 2 See the IREG compendium, eq (2.36)

Exercise 3 Two alternatives: (I) From the dual homogeneous coordinates of the two planes:

$$\mathbf{p}_1 \sim \begin{pmatrix} \hat{\mathbf{p}}_1 \\ -\Delta_1 \end{pmatrix} \mathbf{p}_2 \sim \begin{pmatrix} \hat{\mathbf{p}}_2 \\ -\Delta_2 \end{pmatrix},$$

where $\hat{\mathbf{p}}_1$ and $\hat{\mathbf{p}}_2$ are normal vectors of the respective plane, it follows that the two planes are parallel if, $\hat{\mathbf{p}}_1 = \hat{\mathbf{p}}_2$, i.e., if the first three elements of $\mathbf{p}_1$ and $\mathbf{p}_2$ are parallel. (II) form the dual Plücker coordinates of the 3D line that lies in the intersection of $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$\tilde{\mathbf{L}} = \mathbf{p}_1\mathbf{p}_2^\top - \mathbf{p}_2\mathbf{p}_1^\top$$

If the two planes are parallel $\tilde{\mathbf{L}}$ must be a line at infinity, i.e., its fourth column (or row) must be zero.

Exercise 4 Some examples:

- The point of intersection between two lines, or the line that intersects two points, can be computer by a simple cross product.
- In general, simple geometric relations have a simple algebraic correspondence when applied to homogenous representations.
- After proper normalization, also geometric distances can easily be obtained from homogeneous representations.
- A large set of transformations, not just linear transformations on Cartesian coordinates, are represented as linear transformations on the homogeneous coordinates.
- Points, lines, and planes at infinity can be introduced in a consistent way.
- Both points and lines obtain a compact and consistent (connected by dual transformations) representation, which both can be estimated from measurements based on simple algebraic methods.
- Homogeneous representation of lines works also for vertical lines.

Exercise 5 See the IREG compendium, section 3.5.1.

*This guide is not an authoritative description of how answers to the questions must be given in order to pass the exam.

PART II

Exercise 6 It must be translated so that the centroid is located at the origin (barycentric coordinates). See the IREG compendium, Algorithm 16 in section 13.2.

Exercise 7 See the IREG compendium, eq (11.11).

Exercise 8 In general, the plane is uniquely determined from the three points, i.e., it passes through the points without any residual error. Therefore, it does not matter if we transform the problem to another coordinate system, the estimated plane will be the same.

Exercise 9 For motivation of minimization of geometric errors and the connection to algebraic minimization see, e.g., section 10.3 in the IREG compendium.

Exercise 10 The three cases are

A The data matrix has one unique singular value that is close to zero.

B The data matrix has multiple singular values that are close to zero.

C There data matrix has no singular value that is close to zero.

PART III

Exercise 11 See lecture 2F, page 8.

Exercise 12 See lecture 2A, pages 26 − 27.

Exercise 13 See lecture 2A, page 12. More general forms of scalar products appear for example in lecture 2D, page 17.

Exercise 14 The basis matrix $\mathbf{B}$ and the metric tensor $\mathbf{G}$ are given as:

$$\mathbf{B} = \begin{pmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}, \quad \mathbf{G} = \mathbf{B}^\top \underbrace{\mathbf{G}_0}_{=\mathbf{I}} \mathbf{B} = \mathbf{B}^\top \mathbf{B} = \begin{pmatrix} 2 & 4 \\ 4 & 10 \end{pmatrix}$$

The dual basis matrix $\tilde{\mathbf{B}}$ is computed as

$$\tilde{\mathbf{B}} = \begin{pmatrix} \tilde{\mathbf{b}}_1 & \tilde{\mathbf{b}}_2 \end{pmatrix} = \mathbf{B}\,\mathbf{G}^{-1} = \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 10 & -4 \\ -4 & 2 \end{pmatrix} \frac{1}{4} = \begin{pmatrix} -1 & 1 \\ -3 & 1 \end{pmatrix} \frac{1}{2}$$

The dual basis vectors should be in a dual relation to the original basis vectors:

$$\mathbf{B}^\top \underbrace{\mathbf{G}_0}_{=\mathbf{I}} \tilde{\mathbf{B}} = \mathbf{B}^\top \mathbf{G}_0 \tilde{\mathbf{B}} = \begin{pmatrix} 1 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -3 & 1 \end{pmatrix} \frac{1}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}, \quad \text{OK!}$$

Exercise 15 See lecture 2C, pages 11 − 12.

PART IV

Exercise 16 See lecture 2F, page 30.

Exercise 17 See lecture 2C, page 34.

Exercise 18 It consists of a DC component, i.e., constant sample values.

Exercise 19 See lecture 2F page 46, and lecture 2H, pages $47 - 48$.

Exercise 20 Each filter coefficient $f[k]$ has a corresponding basis function $b_k(u) = e^{-iuk}$ in the frequency domain. If this function is orthogonal to the ideal frequency function $F_I(u)$ then the optimal value of the coefficient is zero. This means that the coefficient does not contribute to minimizing the cost function ϵ.