Department of Applied Mathematics & Computational Science
Shri G. S. Institute of Technology & Science
B.E. II Year ELEX & TC / MA 27014 – Mathematics III

Assignment-1

1. Form the partial differential equations by eliminating the arbitrary constants from the following equations:

(i) $z=(x-a)^2+(y-b)^2$ (ii) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (iii) $\log\left(az-1\right)=x+ay+b.$

2. Form the partial differential equations by eliminating the arbitrary function from the following equations:

(i) $z=xy+f\left(x^2+y^2\right)$ (ii) $z=x^n f\left(\frac{y}{x}\right)$ (iii) $z=xf_1\left(x+y\right)+f_2\left(x+y\right).$

3. Form the partial differential equation from the following:
(i) $F\left(x+y+z,x^2+y^2-z^2\right)=0$ (ii) $F\left(x^2+y^2,x^2-z^2\right)=0$

(iii) $F\left(xy+z^2,x+y+z\right)=0.$

4. Solve the following partial differential equations:
(i) $(y-xz)p+(yz-x)q=(x+y)(x-y)$ (ii) $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$
(iii) $x(y^2+z)p+y(x^2+z)q=z(x^2-y^2)$ (iv) $\left(x^2-y^2-z^2\right)p+2xyq=2xz.$

5. Solve the following linear partial differential equations:
(i) $\left(D^2+DD'-6D'^2\right)z=x^2y+e^{3x+y}$ (ii) $\left(D^2-6DD'+5D'^2\right)z=xy+e^x\sinh y$

(iii) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=e^{x-y}\sin(2x+3y)$ (iv) $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial x}=e^{x+2y}+4\sin(x+y).$

6. Using the method of separation of variables, solve $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$ if $u\left(x,0\right)=4x-\frac{1}{2}x^2.$

7. Obtain the three possible solution of two dimensional heat equation
$\frac{\partial u}{\partial t}=c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right).$

8. Obtain a Fourier series to represent the function $f(x)=|x|,-\pi<x<\pi$, and hence deduce

that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots=\frac{\pi^2}{8}.$

9. If $f(x)=x+1,\text{for}\,0<x<\pi,$ find its Fourier (i) sine series (ii) cosine series.

10. Find the Fourier series representation of the function $f(x)=\sin ax,$ in (-2, 2).

11. Find the Fourier transform of the functions (i) $f(x)=e^{-\frac{x^2}{2}}$ and (ii) $f(x)=\begin{cases}x^2, & |x|<a\\ 0, & |x|>a\end{cases}$

12. Find the Fourier sine transform of the following functions

(i) $f(x) = xe^{-\frac{x^2}{2}}$ (ii) $f(x) = ae^{-\alpha x} + be^{-\beta x}$

13. Find the Fourier cosine transform of the following functions

(i) $f(x) = \dfrac{1}{1+x^2}$ (ii) $f(x) = ae^{-\alpha x} + be^{-\beta x}$.

14. Express $f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$, as a Fourier sine integral and hence evaluate

$\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin \lambda x \, d\lambda$.

15. Using Fourier integral method prove that $\int_0^\infty \frac{\sin \pi\lambda}{1-\lambda^2} \sin \lambda x d\lambda = \begin{cases} \frac{\pi}{2}\sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$.