Fermi-Dirac function:

The equation $f(E)$ is called the Fermi-Dirac probability function and specifies the fraction of all states at energy E (electron volts) occupied under the conditions of thermal equilibrium.

From quantum statistics.

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}} \quad (1)$$

where K = Boltzmann constant eV/°K

T = Temp. in °K.

E_F = Fermi level or characteristic energy for the crystal, eV.

Note: The Fermi level represents the energy state with 50% probability of being filled if no forbidden band exists.

If in equation (1) we put $E = E_F$

then $f(E) = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$ for any value of temperature

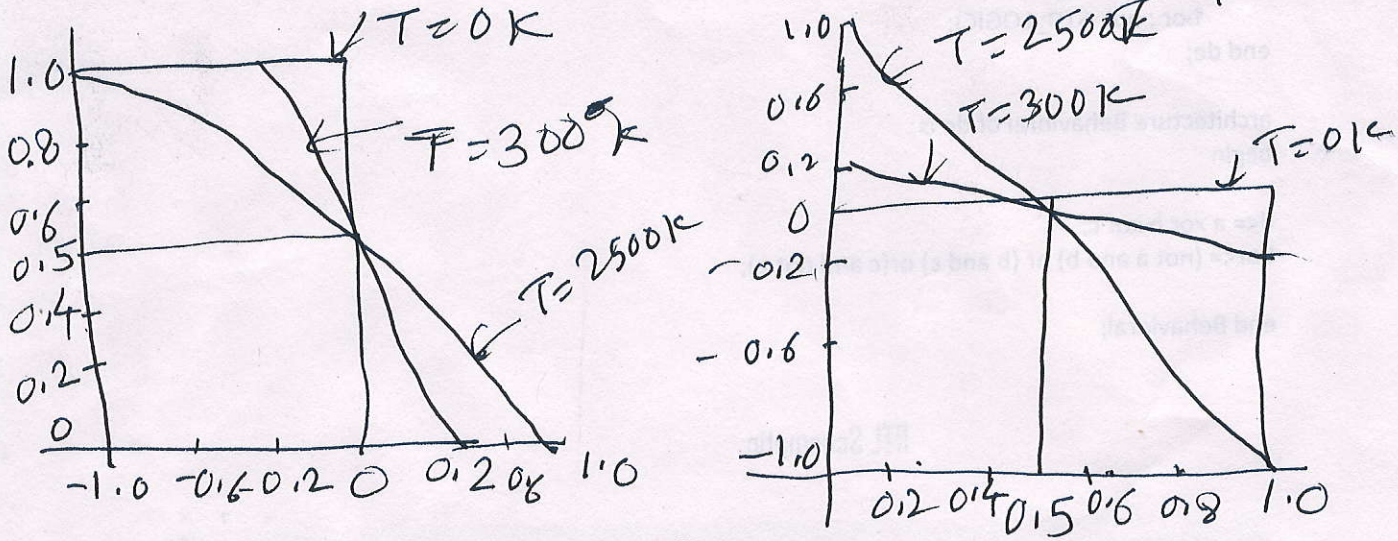
Fermi-Dirac function $f(E)$ at $T = 0$ K.

There are 2 possibilities

(1) $E < E_F$: Exponential term in equation (1) becomes zero and hence $f(E) = 1$. We conclude that all the quantum levels with energies less than E_F will be occupied at absolute zero.

(ii) $E > E_F$: exponential term of equation (1) becomes infinite and hence $f(E) = 0$. So there is no probability of finding an occupied quantum state of energy greater than E_F at absolute zero.

Refer to the 2 diagrams, in them x and y axes are interchanged.



(a) $f(E)$ versus $E - E_F$

Note: Classically all the particles should have zero energy at 0K. But in the figure electrons actually have energies extending from 0 to E_F at absolute zero. This is due to Pauli Exclusion principle which states than no 2 electrons may have the same set of quantum numbers i.e. all the electrons cannot have same the same energy at 0K (Thus absolute zero is the condition of lowest possible energy and not the zero energy)

As the temp. is raised more thermal energy is stored in electrons and their average kinetic energy is increased. Now some of the electrons acquire energy greater than E_F.

Carrier concentrations n & p in intrinsic semiconductor.

(i) $n = N_C e^{-(E_C - E_F)/KT}$

where $N_C = 2\left(\frac{2\pi m_n KT}{h^2}\right)^{3/2}$

(ii) $p = N_V e^{-(E_F - E_V)/KT}$.

$N_V = 2\left(\frac{2\pi m_p KT}{h^2}\right)^{3/2}$

In intrinsic semiconductor $n = p$.

$\therefore N_C e^{-(E_C - E_F)/KT} = N_V e^{-(E_F - E_V)/KT}$

Taking log of both sides

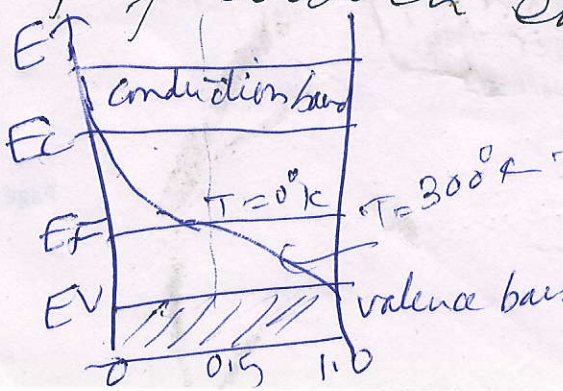
$\ln \frac{N_C}{N_V} = \frac{E_C + E_V - 2E_F}{KT}$

$\therefore E_F = \frac{E_C + E_V}{2} - \frac{KT}{2} \ln \frac{N_C}{N_V}$

If $m_p = m_n$ = effective masses of holes & electrons are same then

$E_F = \frac{E_C + E_V}{2}$

$\therefore$ Hence Fermi level lies in the center of forbidden energy band.



FERMI LEVEL ~~FOR~~ IN EXTRINSIC SEMICONDUCTORS

(1) FERMI LEVEL IN P TYPE SEMICONDUCTOR

It is assumed that

(i) The density of acceptor atoms far exceeds the density of ~~donor~~ donor atoms

(ii) Every acceptor atom has accepted one electron from the valence band

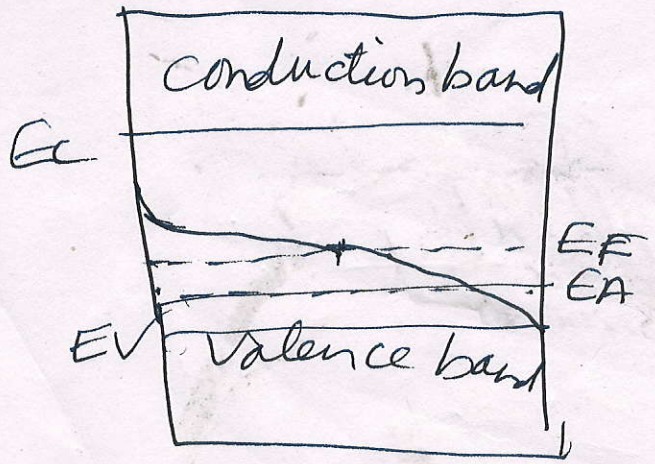
(iii) The density of electrons in the conduction band is much smaller than the density of holes in the valence band.

$n = N_A$

$n = N_A = N_V e^{-(E_F - E_V)/KT}$

solving:

$$E_F = E_V + KT \log\left(\frac{N_V}{N_A}\right)$$



Fermi level in p type semiconductor.

FERMI LEVEL IN N TYPE SEMICONDUCTOR

It is assumed that

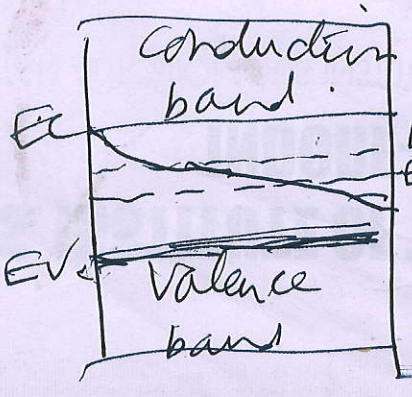
(i) The density of donor atoms far exceeds the density of acceptor atoms.

(ii) Every donor atom has donated one electon.

(iii) The density of electons in the conduction band is much ~~smaller~~ larger than the density of holes in the valence band.

Now that the first N_D states in the conduction band will be filled. Due to these filled states it becomes difficult for the valence band electons to bridge the energy gap due to thermal agitation.

Therefore the fermi level must move closer to the conduction band to indicate that many energy states in that band are filled by the donor ~~atoms~~ electons and fewer holes exist in the conduction band



$$n = N_D = N_D N_c e^{-(E_c - E_F)/kT}$$

$$n = N_D$$

$$= N_C e^{-(E_C - E_F)/KT}$$

Taking logarithm.

$$\log N_D = \log N_C - (E_C - E_F)/KT$$

Solving

$$E_F = E_C - KT \log\left(\frac{N_C}{N_D}\right).$$

Equation (19.21) may be obtained theoretically as follows: Since I_{th} is the current for zero retarding voltage, the current obtained when the barrier height is increased by E_r is determined from the right-hand side of Eq. (19.20) by changing E_W to $E_W + E_r$. Hence

$$I = SA_0T^2e^{-(E_W+E_r)/kT} = I_{th}e^{-E_r/kT} \tag{19.23}$$

where use was made of Eq. (19.20). From Eqs (1.7) and (19.22)

$$\frac{E_r}{kT} = \frac{qV_r}{1.60\times10^{-19}kT} = \frac{V_r}{V_T} \tag{19.24}$$

Hence Eq. (19.21) follows from Eq. (19.23).

From Eq. (19.21) it follows that only about 1.4 percent of the electrons from tungsten at 2700°K have energies in excess of 1 eV. If the emitter is an oxide-coated cathode operating at 1000°K, a similar calculation shown that only 0.001 percent of the electrons have an energy greater than 1 eV. These numerical values indicate that most of the electrons from a heated filament are emitted with extremely small initial speeds. A statistical analysis[2, 3] shows that the average energy of an escaping electron is $2kT$, and hence, at operating temperatures of 2700 and 1000°K, the average energies of the emitted electrons are 0.47 and 0.17 eV, respectively.

Example For an average energy of 0.26 eV of an emitted electron from a heated cathode, estimate its average velocity.

Solution An emitted electron will have only kinetic energy at the point of emission. Therefore we can write

$$\text{Kinetic energy} = \frac{1}{2}mv_{av}^2$$

where m is the mass of a free electron, and v_{av} is its average velocity. Hence

$$0.26 \text{ eV} = 0.26\times1.6\times10^{-19} \text{ J} = \frac{1}{2}\cdot9.1\times10^{-31}\cdot v_{av}^2$$

so that

$$v_{av} = 3.02\times10^5 \text{ m/s}$$

19.5 Carrier Concentrations in an Intrinsic Semiconductor

To calculate the conductivity of a semiconductor from Eq. (2.17) it is necessary to know the concentration of free electrons n and the concentration of holes p. From Eqs (19.3) and (19.4), with E in electron volts,

$$dn = N(E)f(E)\,dE \tag{19.25}$$

where dn represents the number of conduction electrons per cubic meter whose energies lie between E and $E + dE$. The density of states $N(E)$ is derived in Sec. 19.3 on the assumption that the bottom of the conduction band is at zero potential. In a semiconductor the lowest energy in the conduction band is E_C, and hence Eq. (19.5) must be generalized as follows:

$$N(E) = \gamma(E - E_C)^{\frac{1}{2}} \quad \text{for } E > E_C \tag{19.26}$$

The Fermi function $f(E)$ is given by Eq. (19.3), namely,

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \tag{19.27}$$

$$p = \int_{-\infty}^{E_V} \gamma (E_V - E)^{\frac{1}{2}} e^{-(E_F - E)/kT}\, dE \tag{19.34}$$

This integral, which represents the area under the bottom curve in Fig. 19.8*c*, evaluates to

$$p = N_V\, e^{-(E_F - E_V)/kT} \tag{19.35}$$

where N_V is given by Eq. (19.31), with m_n replaced by m_p, the effective mars of a hole.

The Fermi Level in an Intrinsic Semiconductor It is important to note that Eqs (19.30) and (19.35) apply to both intrinsic and extrinsic or impure semiconductors. In the case of intrinsic material the subscript i will be added to n and p. Since the crystal must be electrically neutral,

$$n_i = p_i \tag{19.36}$$

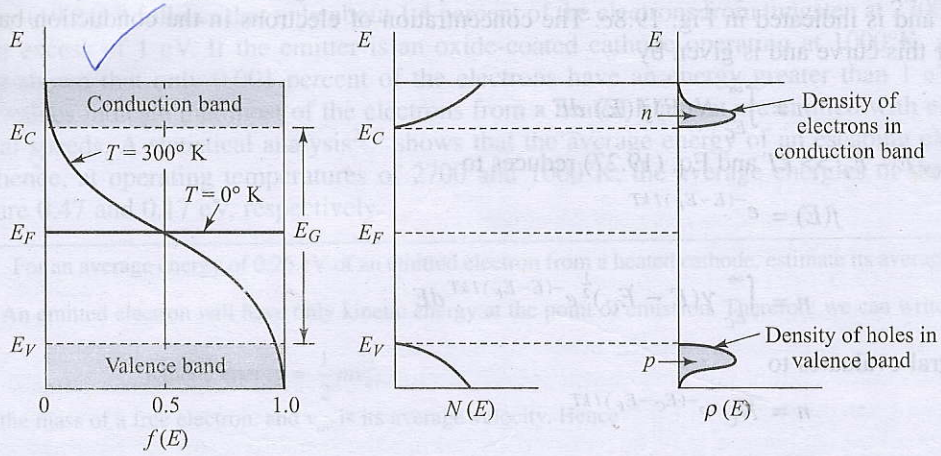


Fig. 19.8 *(a) Energy-band diagram for an intrinsic semiconductor. The Fermi-Dirac probability function f(E) is drawn at 0°K and at room temperature. (b) The density of states N(E) in each band. (c) The density of carriers ρ(E) = N(E)f(E) for electrons and N(E)[1 – f(E)] for holes at room temperature. (Not drawn to scale.)*

and we have from Eqs (19.30) and (19.35)

$$N_C\, e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

Taking the logarithm of both sides, we obtain

$$\ln \frac{N_C}{N_V} = \frac{E_C + E_V - 2E_F}{kT}$$

Hence

$$E_F = \frac{E_C + E_V}{2} - \frac{kT}{2} \ln \frac{N_C}{N_V} \tag{19.37}$$

If the effective masses of a hole and a free electron are the same, $N_C = N_V$, and Eq. (19.37) yields

$$E_F = \frac{E_C + E_V}{2} \tag{19.38}$$

Hence the Fermi level lies in the center of the forbidden energy band, as shown in Fig. 19.8.

Example The effective mass of electrons in silicon is $1.1m$, and that of holes is $0.56m$, where m is the free electr mass. Determine the displacement of the intrinsic Fermi level from the middle of the forbidden band gap at room temperatu

Solution From Eqs (19.37) and (19.41),

$$E_F = \frac{E_C + E_V}{2} - \frac{kT}{2} \ln\left(\frac{m_n}{m_p}\right)$$

$$= \frac{E_C + E_V}{2} - \frac{0.026}{2} \ln\left(\frac{m_n}{m_p}\right)$$

$$= \frac{E_C + E_V}{2} - 0.0088 \text{ eV}$$

Given that the forbidden gap of silicon is 1.1 eV at room temperature, the displacement of 8.8 meV from the middle o the band gap is very small, and in most cases is neglected.

19.6 Fermi Level in a Semiconductor Having Impurities

From Eqs (2.16) and (2.17) it is seen that the electrical characteristics of a semiconductor material depend on the concentration of free electrons and holes. The expressions for n and p are given by Eqs (19.30) and (19.35), respectively, and these are valid for both intrinsic semiconductors and semiconductors with impurities. The only parameter in Eqs (19.30) and (19.35) which changes with impurities is the Fermi level E_F. In order to see how E_F depends on temperature and impurity concentration, we recall that, in the case of no impurities (an intrinsic semiconductor), E_F lies in the middle of the energy gap, indicating equal concentrations of free electrons and holes. If a donor-type impurity is added to the crystal, then, at a given temperature and assuming all donor atoms are ionized, the first N_D states in the conduction band will be filled. Hence it will be more difficult for the electrons from the valence band to bridge the energy gap by thermal agitation. Consequently, the number of electron-hole pairs thermally generated for that temperature will be reduced. Since the Fermi level is a measure of the probability of occupancy of the allowed energy states, it is clear that E_F must move closer to the conduction band to indicate that many of the energy states in that band are filled by the donor electrons, and fewer holes exist in the valence band. This situation is pictured in Fig. 19.9*a* for an n-type material. The same kind of argument leads to the conclusion that E_F must move from the center of the forbidden gap closer to the valence band for a p-type material, as indicated in Fig. 19.9*b*. If for a given concentration of impurities the temperature of, say, the n-type material increases, more electron-hole pairs will be formed, and since all donor atoms are ionized, it is possible that the concentration of thermally generated electrons in the conduction band

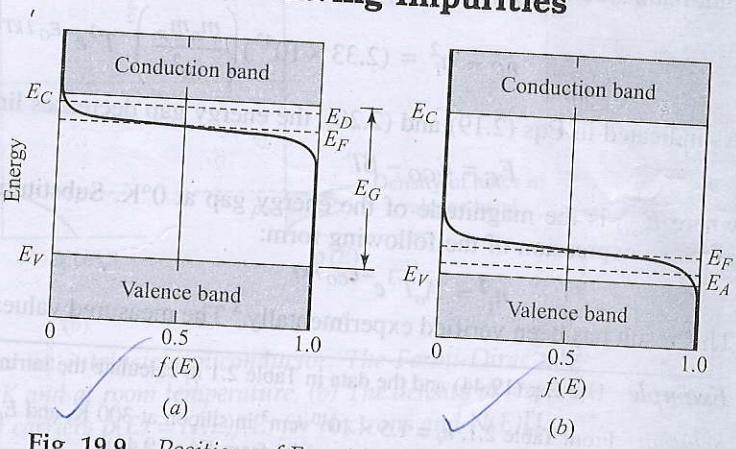


Fig. 19.9 *Positions of Fermi level in (a) n-type and (b) p-type semiconductors.*