CADi
ROS as a Development Platform
Longitudinal Dynamics

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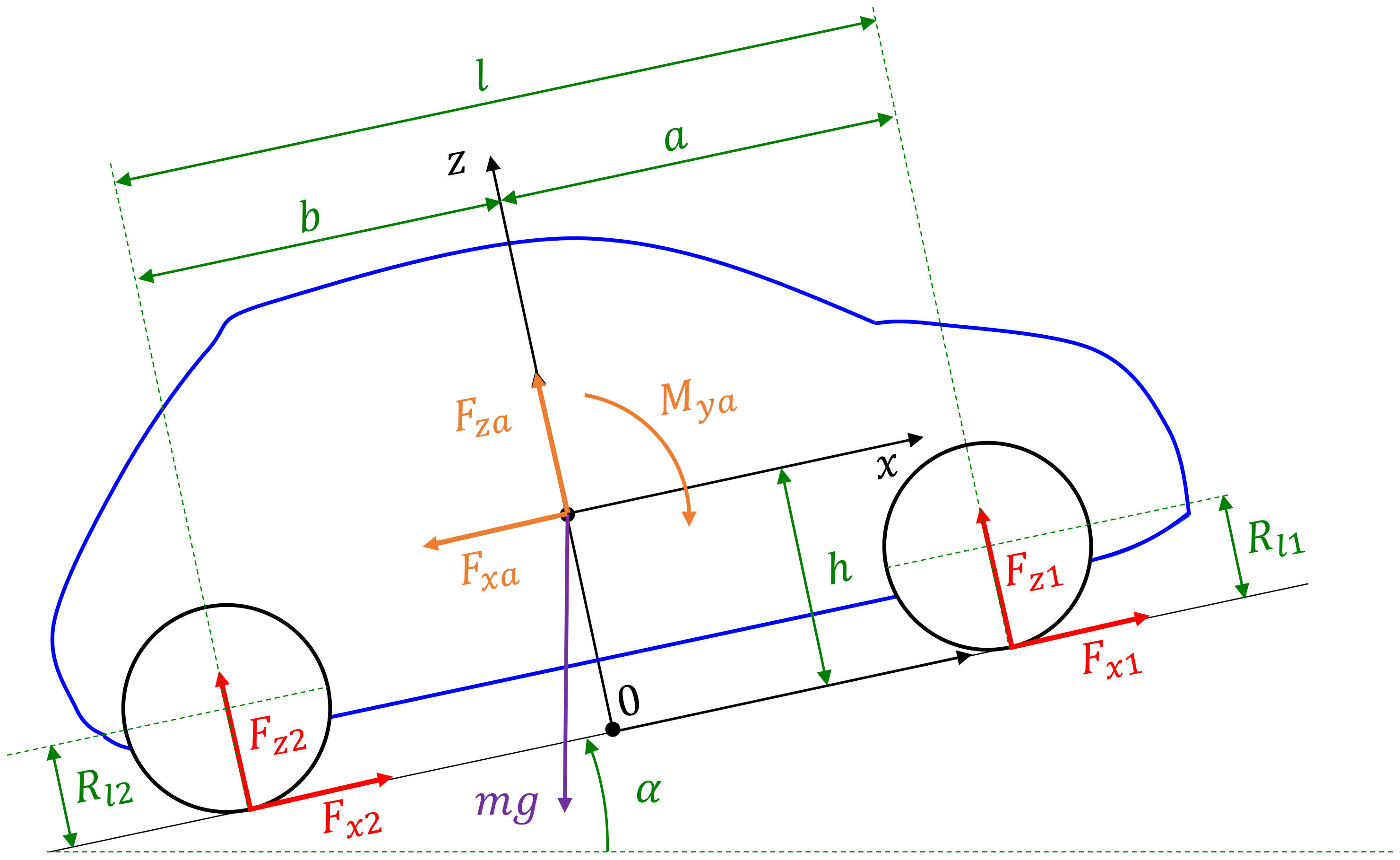
Vehicle Dynamics Study

- The vehicle dynamics problem can be decoupled into:
  - *Longitudinal dynamics*
  - *Lateral dynamics*
  - *Vertical dynamics*
  - *Roll dynamics*
- Decoupling is a key aspect as it simplifies the modeling problem.
- Exploiting this feature, we can state that vehicle drivability & handling focuses exclusively on longitudinal and lateral behavior.

Longitudinal Free Body Diagram



Considering the vehicle as a rigid body with defined geometric parameters, we have:

- Tire forces
- Aerodynamic forces & moment
- Weight

Force & Moment Equilibrium

- Force equilibrium along x:

$$F_{x1} + F_{x2} - F_{xa} - mg\sin\alpha = m\dot{V}$$

- Force equilibrium along z:

$$F_{z1} + F_{z2} + F_{za} - mg\cos\alpha = 0$$

- Moment equilibrium around 0:

$$F_{z1}a - F_{z2}b + mg\sin\alpha\, h + F_{xa}h - M_{ya} = -m\dot{V}h$$

- Solving for F_{z1} and F_{z2}:

$$F_{z1} = l^{-1} \cdot \left[mg(b\cos\alpha - h\sin\alpha) - bF_{za} - hF_{xa} + M_{ya} - m\dot{V}h\right]$$
$$F_{z2} = l^{-1} \cdot \left[mg(a\cos\alpha + h\sin\alpha) - aF_{za} + hF_{xa} - M_{ya} + m\dot{V}h\right]$$

Acceleration Limits

Assumptions

- Flat road: $\alpha = 0$
- Acceleration from zero speed:

$$V = 0 \Longrightarrow F_{xa} = F_{za} = M_{ya} = 0$$

Vertical forces

$$F_{z1} = mg\frac{b}{l} - m\dot{V}\frac{h}{l}$$

$$F_{z2} = mg\frac{a}{l} + m\dot{V}\frac{h}{l}$$

- Load transfer increases the load on the rear axle during traction, while the front axle is unloaded.

Acceleration Limits

Capsize limits

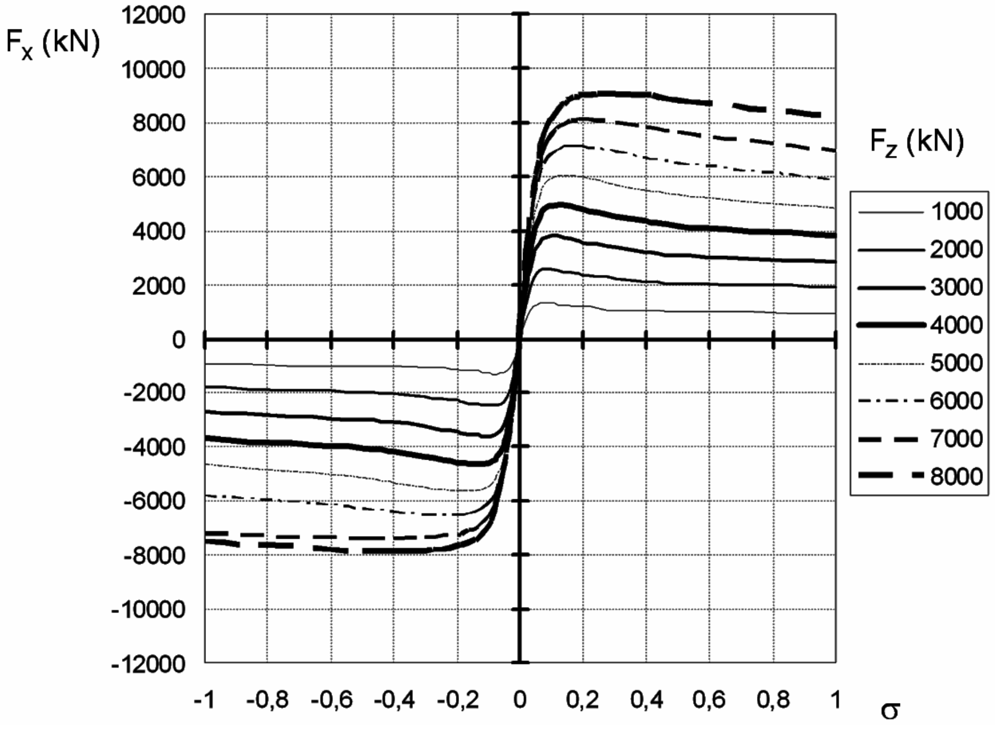
- *During traction:*

$$F_{z1} = mg\frac{b}{l} - m\dot{V}\frac{h}{l} = 0 \rightarrow \dot{V} = g\frac{b}{h}$$

- *During braking:*

$$F_{z2} = mg\frac{a}{l} + m\dot{V}\frac{h}{l} = 0 \rightarrow \dot{V} = -g\frac{a}{h}$$

Tire Behavior



- The tire characteristic is usually reported by means of F_x as a function of the slip σ

$$\sigma = \frac{\omega R_l - v_x}{v_x}$$

- This relation is highly nonlinear (Pacejka).

- However, note that there is a proportional relationship between F_x and F_z:

$$\mu_x \approx \frac{F_x}{F_z}$$

Acceleration Limits

- From the equilibrium along x, *traction is balanced by inertia*:

$$m\dot{V} = F_{x1} + F_{x2}$$

- From the equilibrium along z, *vertical forces are balanced by the weight*:

$$F_{z1} + F_{z2} = mg$$

- We define the traction force through the linear characteristic:

$$m\dot{V} = \mu_{x1}F_{z1} + \mu_{x2}F_{z2}$$

Acceleration Limits

Additional Assumptions

- *All-Wheel Drive (AWD)*: The vehicle generates longitudinal forces on all tires.

Tire limit

- Traction or braking yield the maximum friction coefficient on all tires: $\mu_{x1} = \mu_{x2} = \mu_{xmax}$

$$m\dot{V} = F_{x1} + F_{x2} = \mu_{xmax}(F_{z1} + F_{z2}) = \mu_{xmax} m g$$

$$\Downarrow$$

$$\dot{V} = \mu_{xmax} g$$

Acceleration limit allowed by the tire friction on a flat road

Aerodynamics

- **Reynolds number:** Relation among inertial and viscous forces in a fluid (air).

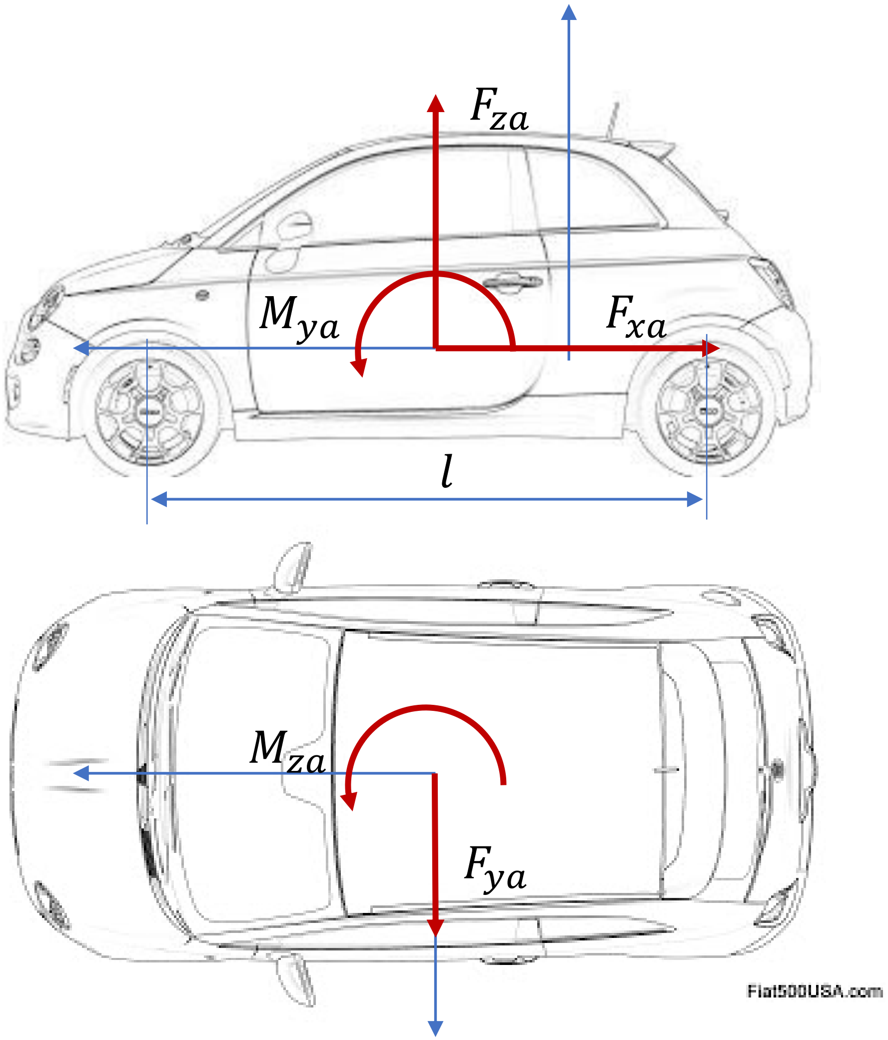
$$\mathrm{Re} = \frac{Vl}{\nu_{air}}$$

Kinematic viscosity of the air

- For a vehicle moving at speeds higher than 20-30 km/h, $\mathrm{Re} > 3 \cdot 10^6$

- In this range the aerodynamic forces are expressed as function of constant aerodynamic coefficients.

Aerodynamic Forces and Moments



$$F_{xa} = \frac{1}{2}\rho V^2 S C_x$$

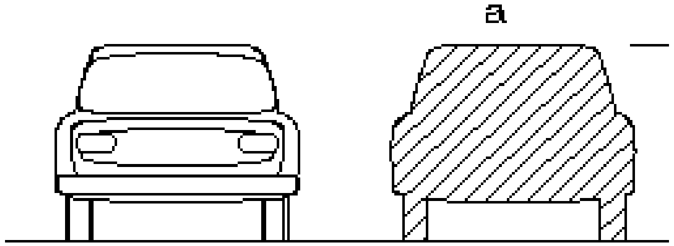
$$F_{ya} = \frac{1}{2}\rho V^2 S C_y$$

$$F_{za} = \frac{1}{2}\rho V^2 S C_z$$

$$M_{xa} = \frac{1}{2}\rho V^2 S l C_{Mx}$$

$$M_{ya} = \frac{1}{2}\rho V^2 S l C_{My}$$

$$M_{za} = \frac{1}{2}\rho V^2 S l C_{Mz}$$



S is projection area

Traction Power (Constant Speed)

- We consider the power needed to balance aerodynamic and weight forces when the vehicle moves at constant speed.
- We neglect the power dissipated in the powertrain and the wheels.
- From equilibrium along x:

$$F_{x1} + F_{x2} = F_{xa} + mg \sin \alpha$$

$$P_n = P_{tire} = V(F_{x1} + F_{x2}) = \frac{1}{2}\rho V^3 C_x S + mgV \sin \alpha$$

The power to move the vehicle comes from the tire forces

Maximum Speed (AWD)

- Assume that all four tires develop the same longitudinal friction coefficient:

$$\mu_{x1} = \mu_{x2} = \mu_{xmax}$$

$$P_{tire} = V(F_{x1} + F_{x2}) = V\mu_{xmax}(F_{z1} + F_{z2})$$

- From equilibrium along z:

$$F_{z1} + F_{z2} = mg\cos\alpha - F_{za}$$

$$P_{tire} = V\mu_{xmax}\left(mg\cos\alpha - \frac{1}{2}\rho V^2 C_z S\right)$$

Maximum Speed (AWD)

- We equate both power terms and solve for V

$$V = \sqrt{\frac{mg\cos\alpha(\mu_{xmax} - i)}{\frac{1}{2}\rho C_x S(1 + \mu_{xmax} e_a)}}$$

Where:

- $i = \tan\alpha$ is the road grade
- $e_a = \frac{C_z}{C_x}$ is the aerodynamic efficiency

Maximum Road Grade (AWD)

- Assume the same longitudinal friction coefficient on all tires:

$$\mu_{x1} = \mu_{x2} = \mu_{xmax}$$

- At low speed, the aerodynamic forces can be neglected, hence:

$$P_{tire} = mgV \sin\alpha = V\mu_{xmax}mg\cos\alpha$$

$$i = \tan\alpha = \mu_{xmax}$$

Power Balance

$$\eta_t P_m - P_n - P_{rr} = \frac{\mathrm{d}E_k}{\mathrm{d}t}$$

Rate of change of total kinetic energy

Powertrain

Road grade, aerodynamics

Rolling resistance

$$E_k = \frac{1}{2} m V^2 + \frac{1}{2} \sum_i J_i \omega_i^2 = \frac{1}{2} m_a V^2$$

$$m_a = m + \frac{J_r}{R^2} + \frac{J_p}{(R\tau_p)^2} + \frac{J_e}{(R\tau_p\tau_g)^2}$$

$$m_a \dot{V} = \frac{\eta_t P_m - P_n - P_{rr}}{V}$$

Rolling radius R
Final transmission ratio τ_p
Gearbox ratio τ_g
Wheel inertia J_r
Propeller shaft inertia J_p
Engine inertia J_e

Dynamic Simulation

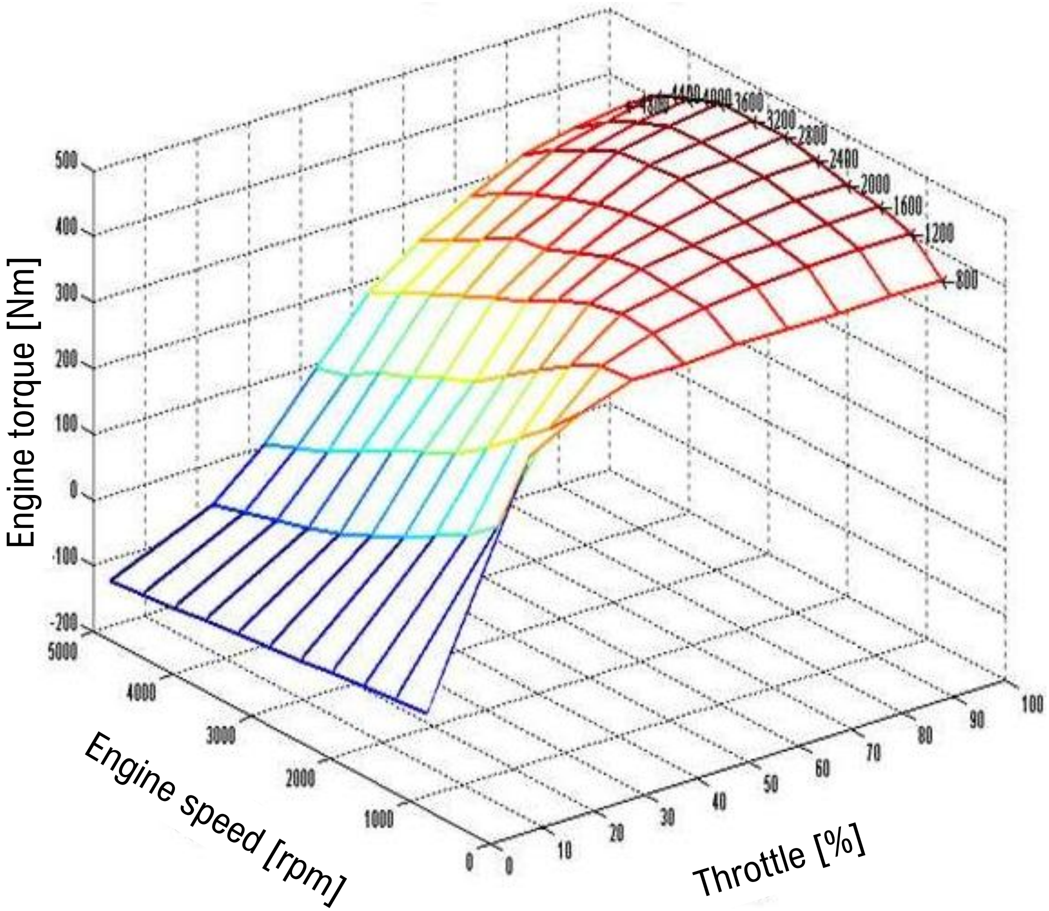
$$m_a\dot{V} = \frac{\eta_t P_m - P_n - P_{rr}}{V} = \eta_t \frac{P_m}{R\tau_p\tau_g\omega_m} - \frac{\rho V^3 C_x S/2 + mgV\sin\alpha}{V} - \frac{f(F_{z1}, F_{z2}, V)V}{V}$$

$$m_a\dot{V} = \underbrace{\eta_t \frac{T_m}{R\tau_p\tau_g}}_{\text{Powertrain}} - \underbrace{\frac{1}{2}\rho V^2 C_x S}_{\text{Aerodynamics}} - \underbrace{mg\sin\alpha}_{\text{Road grade}} - \underbrace{f(F_{z1}, F_{z2}, V)}_{\text{Rolling resistance}}$$

Given

$$F_{z1} = l^{-1} \cdot \left[mg(b\cos\alpha - h\sin\alpha) - bF_{za} - hF_{xa} + M_{ya} - m\dot{V}h\right]$$
$$F_{z2} = l^{-1} \cdot \left[mg(a\cos\alpha + h\sin\alpha) - aF_{za} + hF_{xa} - M_{ya} + m\dot{V}h\right]$$

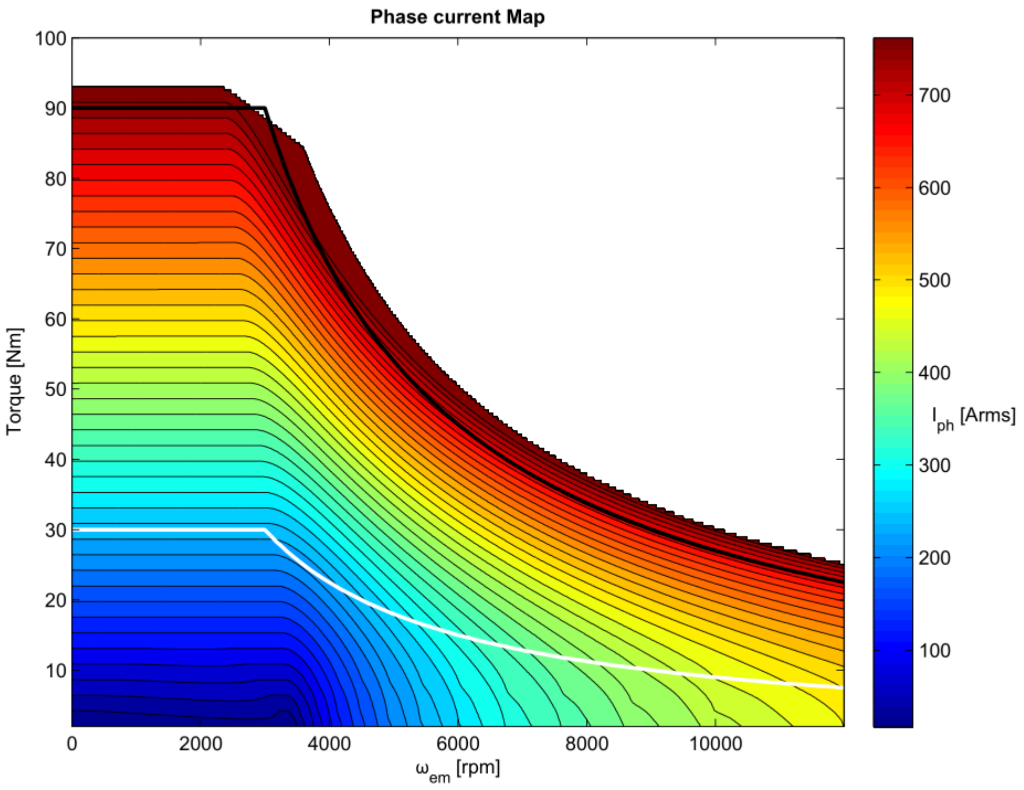
From Throttle to Torque



Internal Combustion Engine (ICE)

- Nonlinear torque characteristic depends on throttle and engine speed.
- Gear-shifting hardware and strategy are needed.
- Very low efficiency (up to 35%).
- In the case of autonomous implementation, throttle must be electrified.

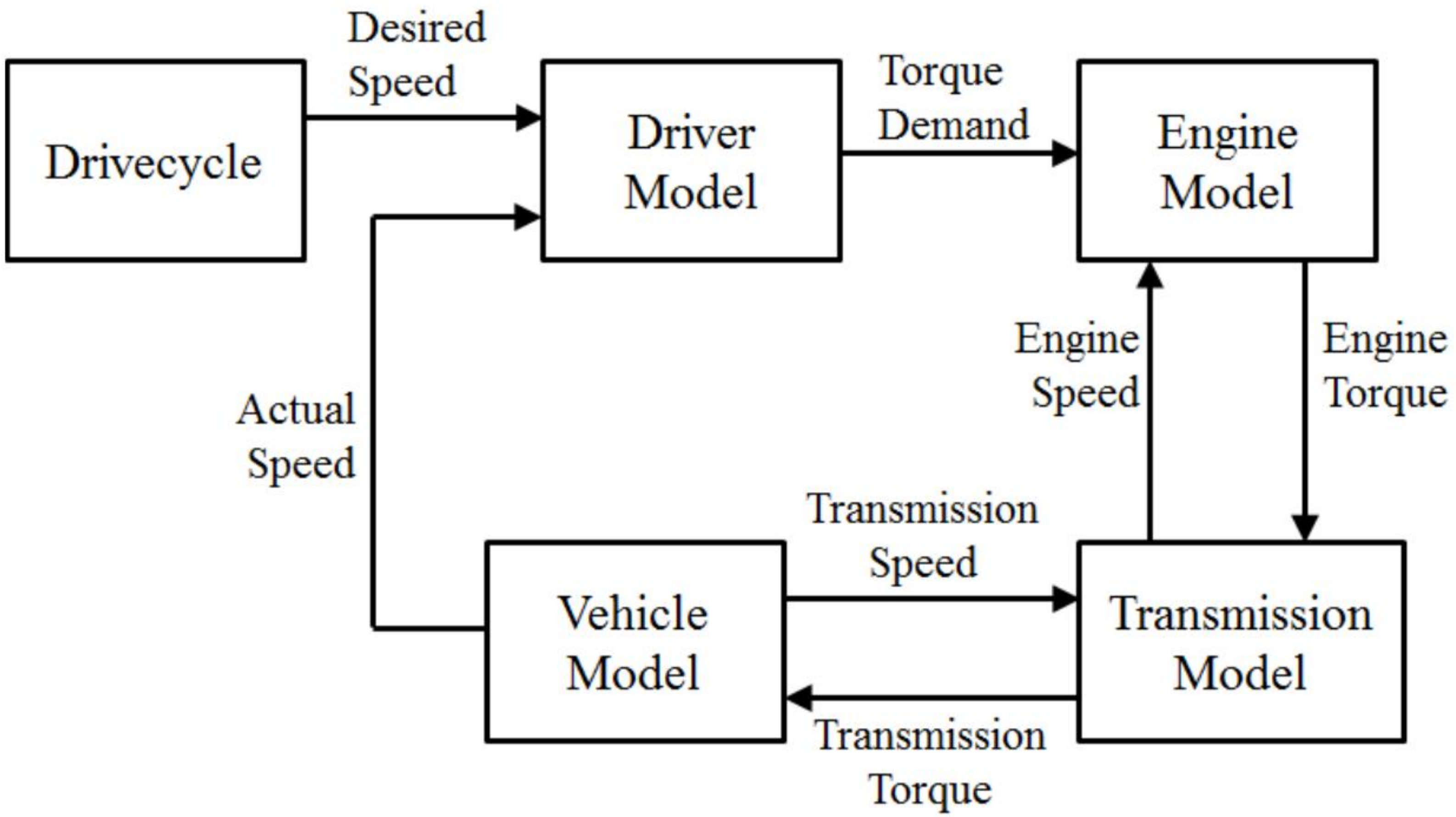
From Throttle to Torque



Electric Machine

- The torque characteristic depends on the current and speed.
- A portion of this behavior is clearly linear and independent of the speed. This portion is at stall.
- Four-quadrant operation is nearly symmetric.
- No gear-shifting required.
- High efficiency (up to 95%).
- Throttle is directly given by the torque request.

Forward Dynamic Simulation



Example

Longitudinal Dynamic Model and Control in Python

Parameters

Tires

- Wheel radius (both wheels) $R_l = 16 \text{ in}$
- Assume perfect tire grip

Aerodynamics

- Air density $\rho = 1.275 \text{ kg/m}^3$
- Aerodynamic coefficient $C_x = 0.4$
- Vehicle projection area $S = 2.2 \text{ m}^2$

Rolling resistance

- Assume a purely viscous rolling resistance $R_x \cong c_r V$, where $c_r = 0.01$.
- This allows neglecting the calculation of the vertical forces.

Road grade

- Road slope angle $\alpha = 0$
- Gravity constant $g = 9.81 \text{ m/s}^2$

Powertrain

- Vehicle mass $m = 1800 \text{ kg}$
- Total equivalent inertia $m_a = 3000 \text{ kg}$
- Gearbox ratio $\tau_g = 0.3$ (fixed)
- Final transmission ratio $\tau_p = 0.1$ (fixed)
- Max. torque request $T_{max} = \pm 100 \text{ Nm}$
- Total powertrain efficiency $\eta_t = 0.8$
- Input torque $T_{in} = 0$

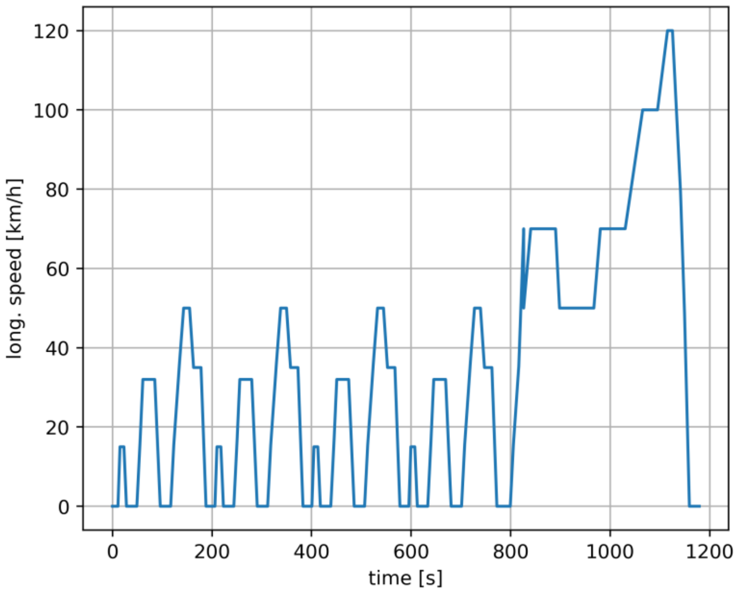
Euler discretization

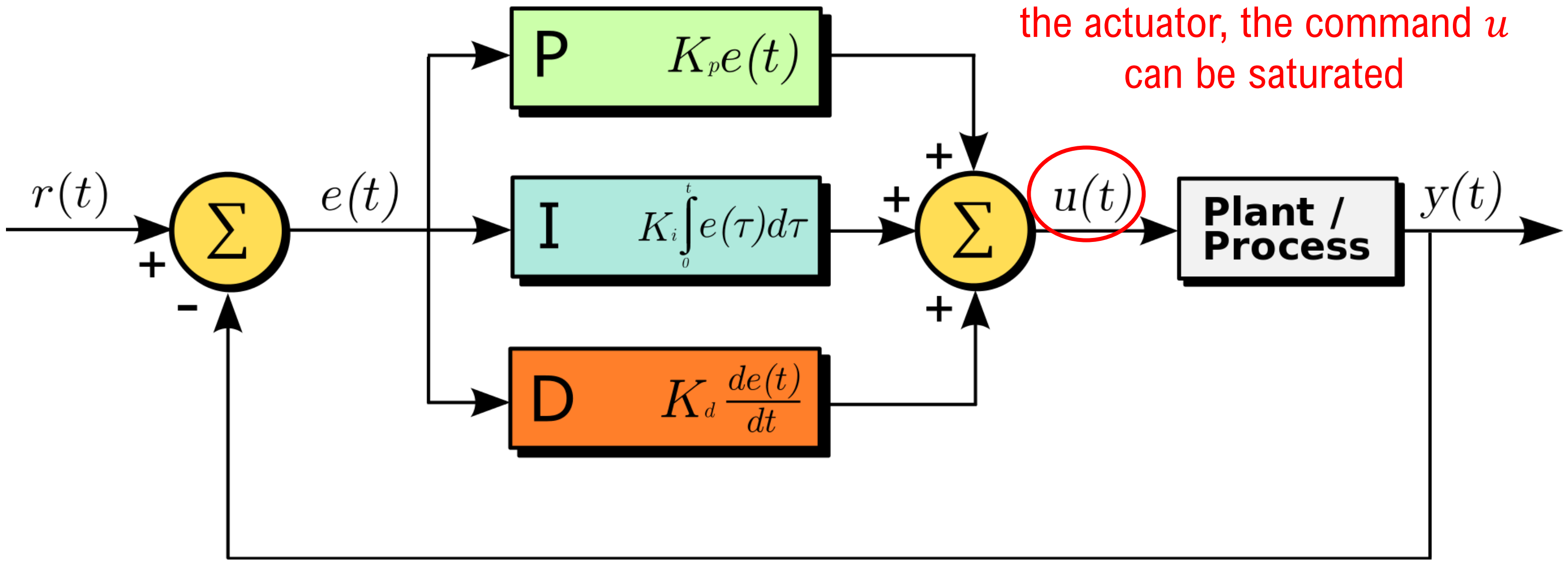
$$\frac{x_k - x_{k-1}}{\Delta t} \cong \dot{x} = f(x,u)$$

$$x_k = x_{k-1} + \Delta t f(x,u)$$

Driver Model / Control

Desired speed profile
New European Driving Cycle





Due to intrinsic limitations of the actuator, the command u can be saturated