

Let $A = \{a_1, a_2, \dots, a_k\}$ be the set of intervals selected by the Greedy-EFT algorithm, where each a_i is chosen based on earliest finish time and is non-overlapping with all previously selected intervals.

We prove by induction on k , the number of intervals selected by the algorithm, that A is of maximum possible size.

Base case ($k = 1$): The algorithm selects the interval with the earliest finish time. Any other interval must finish later or overlap, so we cannot choose more than one compatible interval starting from time 0. Hence, A is optimal in this case.

Inductive step: Assume that the algorithm produces an optimal solution for $k - 1$ intervals. When selecting the k -th interval, say a_k , the algorithm has already chosen $\{a_1, \dots, a_{k-1}\}$ and selects the next interval a_k with the earliest finish time that does not cause any conflicts with previous selections.

Suppose there exists an optimal solution $O = \{o_1, \dots, o_k\}$ of size k that selects a different set of intervals from the greedy solution $A = \{a_1, \dots, a_k\}$.

By the nature of the algorithm, a_k is the earliest finishing interval available that does not overlap with a_1 through a_{k-1} . Therefore, its finish time satisfies $f(a_k) \leq f(o_k)$.

This means a_k leaves at least as much room for future intervals as o_k would have, ensuring that the greedy schedule can fit k compatible intervals just like O . Hence, A is also of size k and optimal.

Therefore, by induction, the greedy algorithm always produces an optimal solution.