Proof of Optimality: Greedy Algorithm for Interval Scheduling (EFT)

Let $A = \{a_1, a_2, \dots, a_k\}$ be the set of intervals selected by the Greedy-EFT algorithm, which always chooses the next interval with the earliest finish time that does not overlap with previously selected intervals.

Assume, for the sake of contradiction, that there exists another set of non-overlapping intervals $O = \{o_1, o_2, \dots, o_m\}$ such that $m > k$, i.e., O contains strictly more intervals than the greedy solution A.

The key property of the greedy algorithm is that at each step, it selects the interval that:

- Has the earliest finish time among all remaining intervals, and
- Does not overlap with the previously selected interval.

This choice ensures that the algorithm leaves the most remaining time available for scheduling additional intervals.

Now, compare the sequences A and O. For each index $i \in \{1, 2, \dots, k\}$, the i-th interval a_i selected by the greedy algorithm finishes no later than the i-th interval o_i in the optimal set O, i.e.,

$$f(a_i) \leq f(o_i)$$

since the greedy algorithm always picks the earliest finishing compatible interval.

As a result, after each greedy selection a_i, at least as much time remains for selecting future intervals as would remain after selecting o_i. Thus, the greedy algorithm cannot block off any more time than the optimal solution does at any step.

But this leads to a contradiction: if O contains more intervals than A, then the greedy algorithm must have missed at least one opportunity to add a non-overlapping interval — which contradicts its strategy of always choosing the interval that maximizes future flexibility.

Therefore, our assumption is false, and the greedy algorithm produces an optimal solution.