

UNIVERSITY OF PENNSYLVANIA
ESE 650: LEARNING IN ROBOTICS
SPRING 2022
[03/16] HOMEWORK 3
DUE: 04/05 TUE 11.59 PM ET

Instructions

Read the following instructions carefully before beginning to work on the homework.

- You will submit solutions typeset in \LaTeX on Gradescope (strongly encouraged). You can use `hw_template.tex` on Canvas in the “Homeworks” folder to do so. If your handwriting is *unambiguously legible*, you can submit PDF scans/tablet-created PDFs.
- Please start a new problem on a fresh page and mark all the pages corresponding to each problem. Failure to do so may result in your work not graded completely.
- Clearly indicate the name and Penn email ID of all your collaborators on your submitted solutions.
- For each problem in the homework, you should mention the total amount of time you spent on it. This helps us gauge the perceived difficulty of the problems.
- You can be informal while typesetting the solutions, e.g., if you want to draw a picture feel free to draw it on paper clearly, click a picture and include it in your solution. Do not spend undue time on typesetting solutions.
- You will see an entry of the form “HW 3” where you will upload the PDF of your solutions. You will also see entries like “HW 3 Problem 1 Code” where you will upload your solution for the respective problems. For each programming problem, you should use the appropriate `.py` template file provided on canvas, unless otherwise stated by the problem. This file should contain all the code to reproduce the results of the problem and you will upload the `.py` file to Gradescope. We will not use the autograder for this homework.
- You should include all the relevant plots in the PDF, without doing so you will not get full credit.
- **Your PDF solutions should be completely self-contained. We will run the Python file to check if your solution reproduces the results in the PDF.**

Credit The points for the problems add up to 120. You only need to solve for 100 points to get full credit, i.e., your final score will be $\min(\text{your total points}, 100)$.

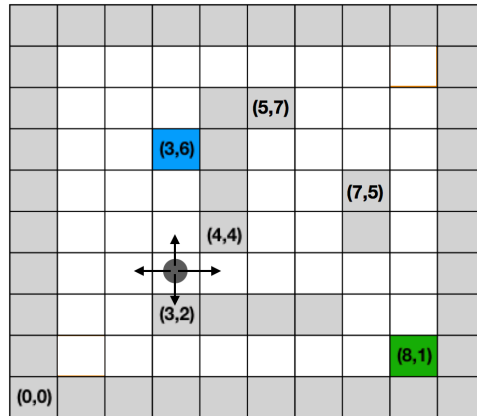
1 **Problem 1 (Policy Iteration, 40 points).** Consider the following Markov Decision
2 Process. The state-space is a 10×10 grid, cells that are obstacles are marked in gray.
3 The initial state of the robot is in blue and our desired terminal state is in green.
4 The robot gets a *reward* of 10 if it reaches the desired terminal state with a discount
5 factor of 0.9. At each non-obstacle cell, the robot can attempt to move to any of the
6 immediate neighboring cells using one of the four controls (North, East, West and
7 South). The robot cannot move diagonally. The move succeeds with probability
8 0.7 and with remainder probability 0.3 the robot can end up at some other cell as
9 follows:

$$P(\text{moves north} \mid \text{control is north}) = 0.7,$$

$$P(\text{moves west} \mid \text{control is north}) = 0.1,$$

$$P(\text{moves east} \mid \text{control is north}) = 0.1,$$

$$P(\text{does not move} \mid \text{control is north}) = 0.1.$$



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11 Similarly, if the robot desired to go east, it may end up in the cells to its north,
12 south, or stay put at the original cell with total probability 0.3 and actually move
13 to the cell east with probability 0.7. The cost pays a cost of 1 (i.e., reward is -1)
14 for each control input it takes, regardless of the outcome. If the robot ends up at a
15 state marked as an obstacle, it gets a reward of -10 for each time-step that it remains
16 inside the obstacle cell.

17 We would like to implement policy iteration to find the best trajectory for the
18 robot to go from the blue cell to the green cell.

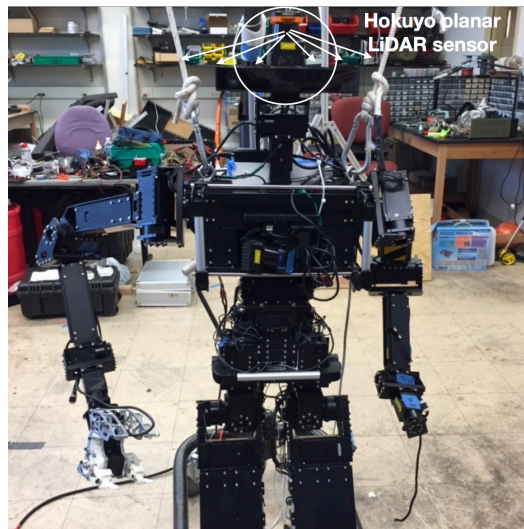
19 (a) **(10 points)** Carefully code up the above environment to run policy iteration.

20 You will need to think about how to code up the probability transition matrix
21 $\mathbb{R}^{100 \times 100} \ni T_{x,x'}(u) = P(x' \mid x, u)$, the run-time cost $q(x, u)$, and the
22 terminal cost $q_f(x)$. Policy iteration is easy to implement if you represent
23 all the above quantities as matrices and vectors. **Plot the environment to**
24 **check if it confirms to the above picture. Include this plot in your report.**

25 (b) **(15 points)** Initialize policy iteration with a feedback control $u^{(0)}(x)$ where
26 the robot always goes east, this results in a policy $\pi^{(0)} = (u^{(0)}(\cdot), u^{(0)}(\cdot), \dots)$.
27 Write the code for policy evaluation to obtain the cost-to-go from every cell

1 in the above picture for this initial policy. Plot the value function $J^{\pi^{(0)}}(x)$
 2 as a heatmap in the above picture. Include this plot in your report.
 3 (c) **(15 points)** Execute the policy iteration algorithm, you will iteratively per-
 4 form policy evaluation and policy improvement steps. For the first 4 iterations,
 5 plot the feedback control $u^{(k)}(x)$ (using arrows as shown in the lecture
 6 notes (https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.arrow.html,
 7 you can also write the control input in the cell). You should color the cell
 8 using the value function $J^{\pi^{(k)}}(x)$. Include the plots for all 4 iterations in
 9 your report.
 10 You should use a fresh python file. You will submit your own code for this
 11 problem; there is no autograder.

12 **Problem 2 (Simultaneous Localization and Mapping (SLAM) with a particle**
 13 **filter, 80 points. Do not use Google Colab to do this homework).** In this problem,
 14 we will implement mapping and localization in an indoor environment using infor-
 15 mation from an IMU and a LiDAR sensor. We have provided you data collected
 16 from a humanoid named THOR that was built at Penn and UCLA
 17 (<https://archive.darpa.mil/roboticschallenge/finalist/thor.html>). You can read more
 18 about the hardware in this paper (<https://ieeexplore.ieee.org/document/7057369>.)



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20 **Hardware setup of Thor** The humanoid has a Hokuyo LiDAR sensor ([https://hokuyo-](https://hokuyo-usa.com/products/lidar-obstacle-detection)
 21 [usa.com/products/lidar-obstacle-detection](https://hokuyo-usa.com/products/lidar-obstacle-detection) on its head (the final version of the robot
 22 had it in its chest but this is a different version); details of this are in the code (which
 23 will be explained shortly). This LiDAR is a planar LiDAR sensor and returns 1080
 24 readings at each instant, each reading being the distance of some physical object
 25 along a ray that shoots off at an angle between $(-135, 135)$ degrees with discretiza-
 26 tion of 0.25 degrees in an horizontal plane (shown as white rays in the picture).

1 We will use the position and orientation of the head of the robot to calculate the
2 orientation of the LiDAR in the body frame.

3 The second kind of observations we will use pertain to the location of the robot.
4 However, in contrast to the previous homework where we used the raw accelerometer
5 and gyroscope readings to get the orientation, we will directly use the (x, y, θ) pose
6 of the robot in the world coordinates (θ denotes yaw). These poses were created
7 presumably on the robot by running a filter on the IMU data (such estimates are
8 called odometry estimates), and just as you saw some tracking errors in the previous
9 homework, these poses will not be extremely accurate. However, we will treat them
10 conceptually the same way as we treated Vicon in the previous homework, namely
11 as a much more precise estimate of the pose of the robot that is used to check how
12 well SLAM is working.

13 **Coordinate frames** The body frame is at the top of the head (X axis pointing
14 forwards, Y axis pointing left and Z axis pointing upwards), the top of the head is at
15 a height of 1.263m from the ground. The transformation from the body frame to the
16 LiDAR frame depends upon the angle of the head (pitch) and the angle of the neck
17 (yaw) and the height of the LiDAR above the head (which is 0.15m). The world
18 coordinate frame where we want to build the map has its origin on the ground plane,
19 i.e., the origin of the body frame is at a height of 1.263m with respect to the world
20 frame at location (x, y, θ) .

21 Data and code

22 (a) **(0 points)** We have provided you 4 datasets corresponding to 4 different
23 trajectories of the robot in Towne Building at Penn. For example, dataset 0 consists
24 of two files `data/train/train_lidar0.mat` and `data/train/train_joint0.mat` which contain
25 the LiDAR readings and joint angles respectively. The functions `load_lidar_data`
26 and `load_joint_data` inside `load_data.py` read the data. You can run the function
27 `show_lidar` to see the LiDAR data. Each of the data reading functions returns
28 a data-structure where t refers to the time-stamp (in seconds) of the data, `xyth`
29 refers to (x, y, θ) pose of the LiDAR and `rpy` refers to Euler angles (roll, pitch,
30 yaw). The joint data contains a number of fields, but we are only interested in the
31 angle of the head and the neck at a particular time-stamp. You should read these
32 functions carefully and check the values returned by them. The dicts `joint_names`
33 and `joint_names_to_index` can be used to read off the data of a specific joint (we
34 only need the head and the neck).

35 (b) **(0 points)** Next look at the `slam.py` file provided to you. Read the code
36 for the class `map_t` and `slam_t` and the comments provided in the code very
37 carefully. You are in charge of filling in the missing pieces marked as `TODO`:
38 `XXXXXX`. A suggested order for studying this code is as follows: `slam_t.read_data`,
39 `slam_t.init_sensor_model`, `slam_t.init_particles`, `slam_t.rays2world`, `map_t.__init__`,
40 `map_t.grid_cell_from_xy`. Next, the file `utils.py` contains a few standard rigid-body
41 transformations that you will need. You should pay attention to the functions

1 smart_plus_2d and smart_minus_2d that will be used to code up the dynamics
2 propagation step of the particle filter.

3 (c) **(20 points, dynamics step)** Next look at main.py which has two functions
4 run_dynamics_step and run_observation_step which act as test functions to check
5 if the particle filter and occupancy grid update has been updated correctly. The
6 run_dynamics function plots the trajectory of the robot (as given by its IMU data
7 in the LiDAR data-structure). It also initializes 3 particles and plots all particles
8 at different time-steps while performing the dynamics step with a very small dy-
9 namics noise; this is a very neat way of checking if dynamics propagation in the
10 particle filter is working correctly. This function will create two plots, one for the
11 odometry trajectory and one more for the particle trajectories, both these trajec-
12 tories should match after you code up the dynamics function slam_t.dynamics_step
13 correctly. Include these plots for all datasets in your report. Briefly explain how you
14 implemented the dynamics step.

15 (d) **(20 points, observation step)** The function run_observation_step is used to
16 perform the observation step of the particle filter to get an estimate of the location
17 of the robot and updates to the occupancy grid using observations from the LiDAR.
18 First read the comments for the function slam_t.observation_step carefully.

19 We first discuss the particle filtering part.

20 (i) Compute the head and neck position for the time t . For each particle,
21 assuming that that particle is indeed the true position of the robot, project
22 the LiDAR scan `slam_t.lidar[t]['scan']` into the world coordinates using the
23 `slam_t.ray2world` function. The end points of each ray tell us which cells in
24 the map are occupied, for each particle.

25 (ii) In order to compute the updated weights of the particle, we need to know
26 the likelihood of LiDAR scans given the state (our current occupancy grid
27 in the case of SLAM). We are going to use a simple model to do so

$$\log P(\text{LiDAR scan as if the robot is at particle } p \mid m) = \sum_{ij \in O} m_{ij} \quad (1)$$

28 where O is the set of occupied cells as detected by the LiDAR scan assuming
29 the robot is at particle p and m_{ij} is our current estimate of the binarized
30 map (more on this below). In simple words, if the occupied cells as given
31 by our LiDAR match the occupied cells in the binarized map created from
32 the past observations, then we say the log-probability of particle p is large.

33 (iii) You will next implement the function `slam_t.update_weights` that takes
34 the log-probability of each particle p , its previous weights, calculates the
35 updated weights of the particles.

36 (iv) Typically, resampling step (`slam_t.stratified_resampling`) is performed only
37 if the effective number of particles (as computed in `slam_t.resample_particles`)
38 falls below a certain threshold (30% in the code). Implement resampling as
39 we discussed in the lecture notes.

1 We will now do the mapping part. We have a number of particles $p^i = (x^i, y^i, \theta^i)$
2 that together give an estimate of the distribution of the location of the robot. For
3 this homework, you will only use the particle with the largest weight to update
4 the map although typically we update the map using all particles. Our goal is
5 simple: we want to increase `map_t.log_odds` array at cells that are recorded as
6 obstacles by the LiDAR and decrease the values in all other cells. You should
7 add `slam_t.log_odds_occ` to all occupied cells and add `slam_t.log_odds_free` from
8 all cells in the map. It is also a good idea to clip the `log_odds` to like between
9 `[-slam_t.map.log_odds_max, slam_t.map.log_odds_max]` to prevent increasingly
10 large values in the `log_odds` array. The array `slam_t.map.cells` is a binarized version
11 of the map (which is used above to calculate the observation likelihood).

12 Check the `run_observation_step` function after you have implemented the obser-
13 vation step. Since the map is initialized to zero at the beginning of SLAM which
14 results in all observation log-likelihoods to be zero in (1), we need to do something
15 special for the first step. We will use the first entry in `slam_t.lidar[0]['xyth']` to
16 get an accurate pose for the robot and use its corresponding LiDAR readings to
17 initialize the occupancy grid. You can do this easily by initializing the particle filter
18 to have just one particle and simply calling the `slam_t.observation_step` as shown in
19 `main.py`.

20 Include in your report the output of the `run_observation_step` function for one
21 time-step. Briefly explain how you implemented the observation step.

22 (e) **(40 points)** You will now run the full SLAM algorithm that performs one
23 dynamics step and observation step at each iteration in the function `run_slam` in
24 `main.py`. Make sure to start SLAM only after the time when you have both LiDAR
25 scans and joint readings (the two arrays start at different times). For all 4 datasets,
26 include in your report the plots of the final binarized version of the map, the (x, y)
27 location of the particle in the particle filter with the largest weight at each time-step
28 and the odometry trajectory (x, y) (in a different color); this counts for 10 points
29 each.

30 **Some Notes** This problem is much easier and shorter than it may seem. You should
31 go through these steps carefully and in the suggested order. You should make
32 sure that the results of the previous step are correct before proceeding. The two
33 functions in `main.py` to check the dynamics and observation step are very important
34 to find bugs. **Do not use Colab to do this homework.** Debugging this is much
35 easier if you use Jupyter/IPython; using ipdb/some other IDE (say PyCharm or
36 Spyder) is invaluable to quickly code up these kinds of problems. You do not need
37 to implement more than 150 lines of code in this problem.