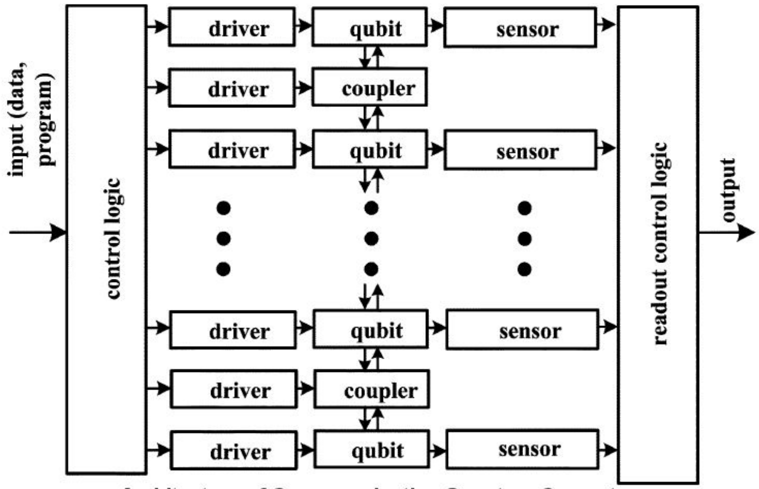

Architecture of Superconductive Quantum Computer

# An Introduction to Quantum Computing

Robert Olsthoorn

University of Florida

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# Abstract

**Quantum computing is a potentially revolutionary principle which will be continued to be researched and studied for the foreseeable future as the importance of efficiency and the limit of binary computing is approached. This paper aims to provide an overview of the field of quantum computing for individuals with a minor understanding of physics, computer science, and mathematics. An introduction to quantum computing will leave the reader with a comfortable overview of the field and insight into which topic in particular they find most interesting.**

**This paper will talk briefly about the recent history of quantum computing as well as a small subset of quantum mechanicss as it relates to quantum computations and the cornerstones which currently make quantum computing possible. It aims to establish the differences between conventional and quantum computing with a goal to speak about how certain algorithms will run more efficiently and what applications in the field this can be used for. Near the end, we will look at the current issues within the field and its future importance.**

# Contents

# 1 History

Quantum computing is a relatively new field in relation to computer science as a discipline with the informal start originating in the late 1970's and early 1980's as Richard Feynman speculated that quantum mechanics could not be effectively modeled through a classical computer. In accordance with Moore's law, the size of a silicon ship would continue to shrink until the individual elements were no larger than several atoms and would be subject to quantum effects at that scale. Feynman published an abstract model in 1982 in which he analyzed the outcome of using a quantum simulator in order to avoid the exponential slowdown which is common with classical computers. (*3*)

In 1985, David Deutsch published a paper proving that any physical process could be, in theory, effectively rendered on a quantum computer. As a result, a quantum computer, which is able to operate in an exponential time, could provide a wide array of values for heavy data crunching, modelling of complex systems, or in the general solving NP-Complete classical problems in polynomial time. [1] Deutsch proved a basic algorithm which will be worked through later in the paper.

Until 1994, the quantum computing field remained relatively unchanged until Shor was able to prove and set a method for a common NP-Hard factorization problem which could call on the benefits allowed through quantum computers, which would run in a time much shorter than what will be ever possible on classical computers. (*3*) As a field, this momentus finding was able to push the field of research for quantum computing out of the view of the select who were performing research on the project to the public eye. Shor's algorithm will be explored later in the paper as well.

[1] In computational complexity theory, a decision problem is NP-complete when it is both in NP and NP-hard. The set of NP-complete problems is often denoted by NP-C or NPC. The abbreviation NP refers to "nondeterministic polynomial time".
Although any given solution to an NP-complete problem can be verified quickly (in polynomial time), there is no known efficient way to locate a solution in the first place.

# 2 Quantum Mechanics

As a quick note, the material that will only be covered consists of a very small section of quantum mechanics encompassing finite dimenstional quantum mechanic where the vector spaces which represent the states of the dimension are finite in size.

## 2.1 Double Slit Experiment

Young's double slit experiment is one of the most foundational experiments related to the field of quantum mechanics and demonstrates the wave-particle duality of photons when conducted. Before approaching the quantum model, it is interesting to explore a classical model and the probabilities associated with it before moving on.

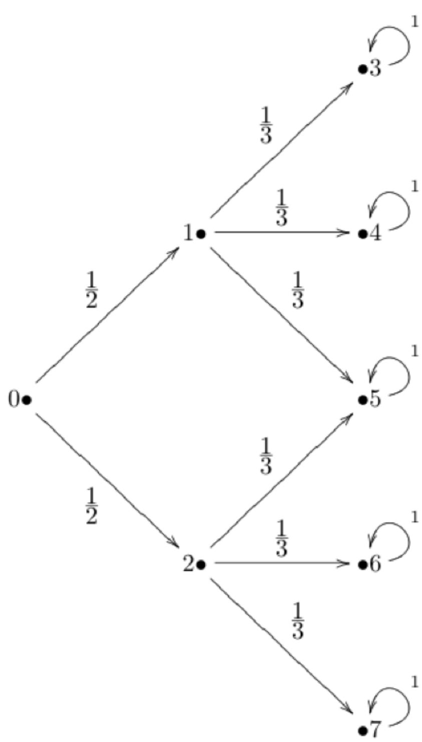


Figure 1: Corresponding graph to scenario

Pretend for a moment, that there is an experiment where there is a sharpshooter who is guaranteed to always shoot through one or the other open windows, with equal probability. Once the bullet passes through the window, it has an equal probability of hitting three targets. There is one target which is shared between both open windows.

The probability matrix associated with this scene is shown. FIGURE 2 OF ORIGINAL MATRIX

By representing the data points as a matrix, it is possible to identify the probability where the bullet might be found on the next time click by simply using matrix multiplication. (*4*). The

Figure 2: Matrix representing the progression of two time clicks

$$B \star B \quad = \quad B^2 \quad = \quad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

matrix shown above, *B*, represents the state of the system after one time click. By multiplying the matrix by itself you are able to represent the state after two time clicks.

The takeaway from this example is to show that after two time clicks the bullets will be in the state

$$B^2X = [0, 0, 0, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}]^T$$

Which means that $B^2[5, 0]$ is equivalent to $\frac{1}{3}$. Which is the two states $\frac{1}{6} + \frac{1}{6}$.

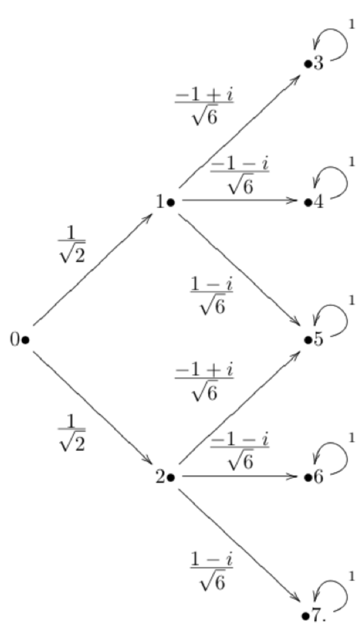


Figure 3: Corresponding quantum modulus graph to scenario

Pretend for a moment that the shooter has now been changed to a flashlight which can spread light into both of the windows with a similar setting. Once the light has passed through the windows, it again travels randomly to one of the three respective target locations. Represented in this graph is the modulus, where the modulus squared represents the probability of the specific event taking place. $\frac{1}{\sqrt{2}}^2$ is $\frac{1}{2}$ and more importantly $\left|\frac{\pm 1 \pm i}{\sqrt{6}}\right|^2 = \frac{1}{3}^2$ The above matrices represented the state of

[2]The complex number weights represented here are not to represent the actual quantum probability weights as this would require acquiring the distance of the slit spacing, the width of the individual slits. Rather the numbers given are to represent the point of quantum interference which will be highlighted later on

Figure 4: Matrix representing the state after one time click

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Figure 5: Matrix representing the state after two time clicks

$$P^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1+i}{\sqrt{12}} & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & \frac{1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ \frac{-1+i}{\sqrt{12}} & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Figure 6: Probability matrix of the modulus squared after two time clicks

$$|P^2[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ \mathbf{0} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

the experiment using the moduli of the components. In order to interpret this information in reference to the classical scenario it is helpful to consider the probability of the individual locations. This can be shown by squaring the individual components of the $P^2$ matrix.

For the most part, the probability matrix for $P^2$ is the same as the probability matrix for $B^2$; however, there is one important distinction to be made. Which is that while $B^2[5,0] = \frac{1}{3}$ in the quantum simulation $P^2[5,0] = 0$. On a mathematical basis, this is trivially written as

$$\frac{1}{\sqrt{2}}(\frac{-1+i}{\sqrt{6}} + \frac{1}{\sqrt{2}}(\frac{1-i}{\sqrt{6}} = \frac{-1+i}{\sqrt{12}} + \frac{1-i}{\sqrt{12}} = 0$$

This may seem troubling at first, but it must be remembered that photons are subject to particle interference and thus, on the shared target of the windows, the probability drops to 0. Furthermore, one may try to argue that if the experiment was carried out using only one photon, then the probability would again return to $\frac{1}{3}$, but this assumption would also be incorrect. A single photon is said to have a superposition in being in every possible position simultaneously. This is not mimicked in the classical world; however, due to the photon residing in every position at once it is prone to interference when the shared target is reached. As with most quantum phenomena, the particle is only determined to be in a certain state with a certain probability when a measurement is taken. In quantum mechanics, a measurement causes the superposition sate to collapse to a certain pure state.

This phenomena is central to the success of quantum computing, as it allows for an exponential number of comutations or simulations to be run in parallel thus becoming incredibly more efficient.

# 3 Qubit

## 3.1 Notation

## 3.2 Implementation

# 4 Applications

## 4.1 Implementation

# 5 Algorithms

## 5.1 Shor's Algorithm

## 5.2 Grover's Algorithm

## 5.3 Deutsch Algorithm

# 6 Error Correction and Measurement

# 7 Future of Quantum Computing

> However, this record of the solar nebula may have been partly erased by the complex history of the meteorite parent bodies, which includes collision-induced shock, thermal metamorphism, and aqueous alteration

```
({\it 1, 2, 5--7\/}).
```

Compiled, the last two lines of the code above, of course, would give notecalls in *Science* style:

...thermal metamorphism, and aqueous alteration (*1, 2, 5–7*).

Under the same luck, the author could set up his or her reference list as a simple enumeration,

```
{\bf References and Notes}

\begin{enumerate}
\item G. Gamow, {\it The Constitution of Atomic Nuclei
and Radioactivity\/} (Oxford Univ. Press, New York, 1931).
\item W. Heisenberg and W. Pauli, {\it Zeitschr.\ f.\
Physik\/} {\bf 56}, 1 (1929).
\end{enumerate}
```

yielding

**References and Notes**

1. G. Gamow, *The Constitution of Atomic Nuclei and Radioactivity* (Oxford Univ. Press, New York, 1931).
2. W. Heisenberg and W. Pauli, *Zeitschr. f. Physik* **56**, 1 (1929).

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  - Any additional `.sty` and `.bst` files called by the source code (though, for reasons noted earlier, we *strongly* discourage the use of such files beyond those mentioned in this document).


# References and Notes

1. Preskill, John. "Quantum Computing: Pro and Con." Diss. California Instute of Technology, 1996. Print. Covers the applications in which it will be used as well as the technical difficulties that are encountered with creating a quantum computer. Also encompasses the future of quantum computing

2. Rieffel, Eleanor, and Wolfgang Polak. "An Introduction to Quantum Computing for Non-Physicists." Diss. FX Palo Alto Laboratory, 2000. Print. Covers some algorithm efficiencies for conventional vs quantum computing and also covers basic applications of quantum computing in the field including cryptography.

3. West, Jacob. "The Quantum Computer." An Introduction to Quantum Computing. Rice University, 28 Apr. 2000. Web. 25 Oct. 2015. Provides a general purpose overview of the field of quantum computing. Includes a brief history of the field as well as current obstacles and research being done in the field.

4. Yanofsky, Noson S. "An Introduction to Quantum Computing." Diss. Department of Computer and Information Science, Brooklyn College, CUNY, 2007. Print. Presents an introduction to the mathematics behind quantum computing as well as an overview of the architecture necessary for quantum computing. This paper also presents Deutsch's Algorithm which will be spoken about and overviewed.