Parallel programming
// programming

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Chapter : Scan/Prefix

Plan

1. Motivating example: Scan/Prefix is a general-purpose tool
   - See also the chapter of G. Blelloch on the LMS
2. The Scan/Prefix generic operation
   1. Version with binary tree
   2. Version with linked list
   3. Divide & conquer version

Prefix Sums
and Their Applications

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PREFIX Parallel computation

- A very useful & general pattern named also SCAN
- Almost similar pattern known as SUFFIX
- Principle:
  - A collection of values (x1,x2,….xn)
    - In an array, or a list for instance
  - Compute (y1,y2,…,yn) such that
    yk = x1 *op* x2 *op*… *op* x(k-1) *op* xk
    (in suffix case: yk=xn *op* x(n-1) *op* … *op* xk)
  - OP is a binary associative operation
  - Popular case : OP is a sum
    - *prefix sum*
- Goal: Find a parallel time = O(log n) algorithm
  - Ideally with a total work = O(n)
  - Seq computation: O(n)

```
y[0]=x[0]
for i=1 to n-1 do
        y[i]=x[i] + y[i-1]
```

Application of the Prefix, even Prefix **Sum**

Consider this array of integers, where 0 values are non-significant, and ideally, should be suppressed.

(7,0,0,9,0,1,0,0,0,3)

- Compute the new position of each *non nul* element, using the prefix sum algorithm. Application of the Prefix, even Prefix Sum

Application of the Prefix, even Prefix **Sum**

array positions [0 1 2 3 4 5 6 7 8 9]

Array: (7,0,0,9,0,$\boxed{1}$,0,0,0,3)

- Use another array= Flags: (1,0,0,1,0,$\boxed{1}$,0,0,0,1)
- Compute Prefix<u>Sum</u>(Flags): (<u>1</u>,1,1,<u>2</u>,2,$\boxed{\underline{3}}$,3,3,3,<u>4</u>)
- It corresponds to new indexes of *non nul* elements !
- Move each A' elements to its final position, starting at index 0:
- (7,9,$\boxed{1}$,3,0,1,0,0,0,3)
- NB: We just care about the red final values in the array

```
procedure line-of-sight(altitude)
  in parallel for each index i
    angle[i] ← arctan(scale × (altitude[i] - altitude[0])/ i)
  max-previous-angle ← max-prescan(angle)
  in parallel for each index i
    if (angle[i] > max-previous-angle[i])
       result[i] ← "visible"
    else
       result[i] ← not "visible"
```

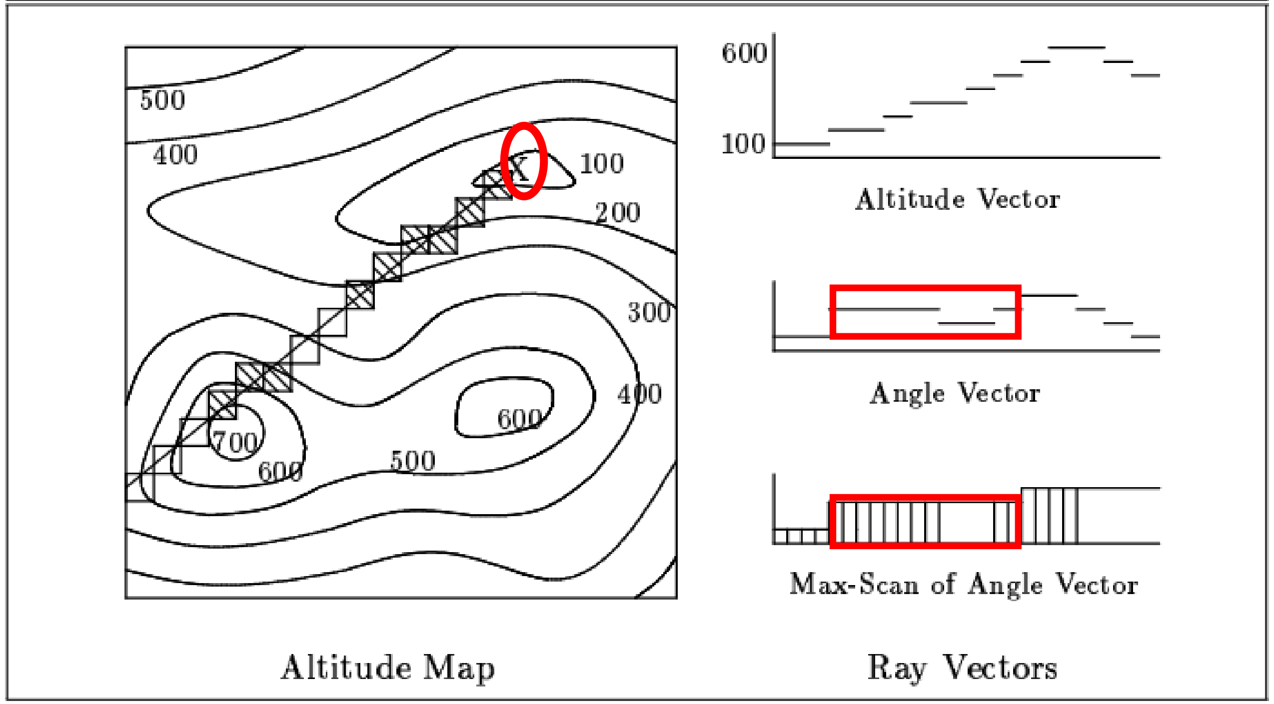


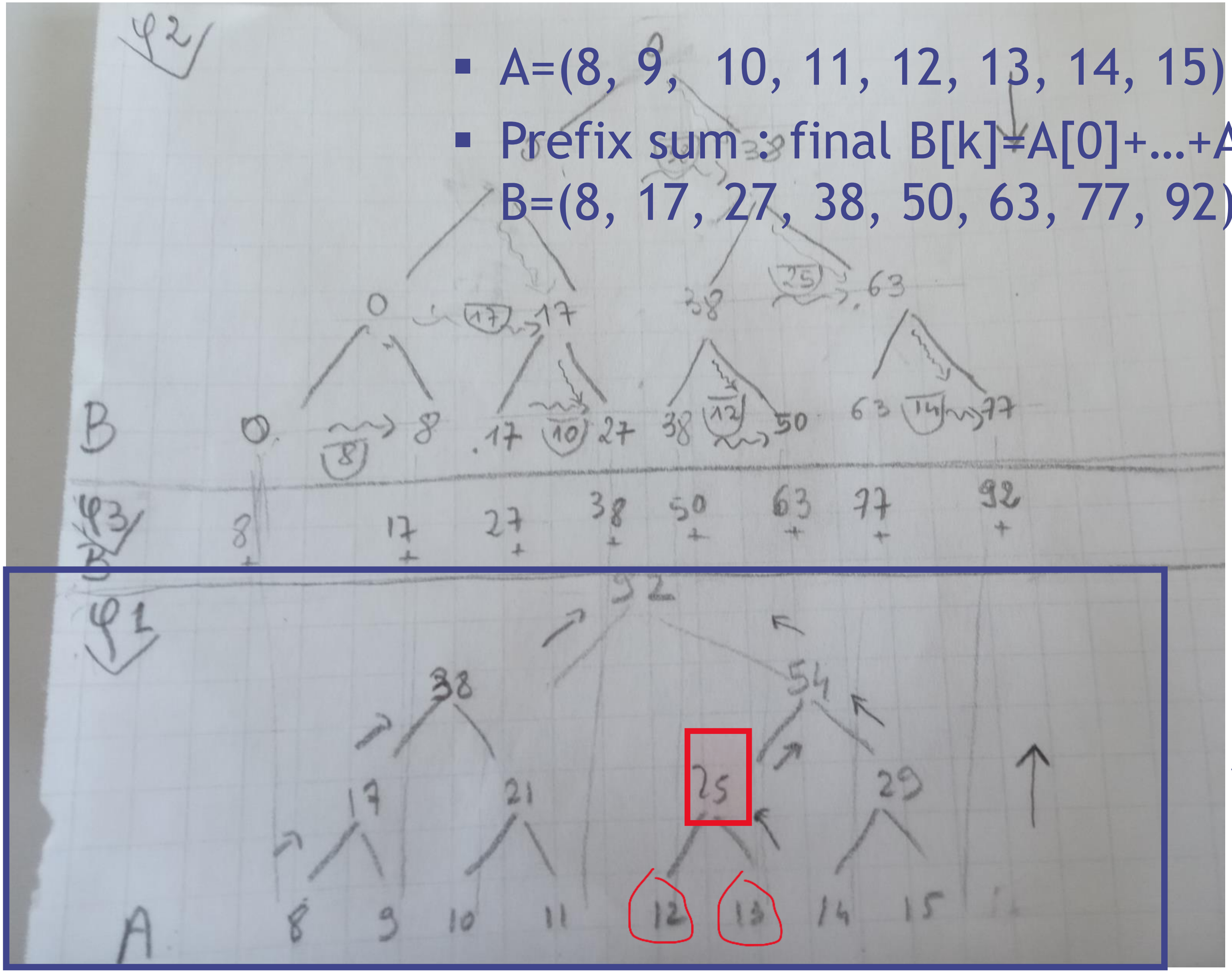
FIGURE 1.7
The line-of-sight algorithm for a single ray. The X marks the observation point. The visible points are shaded. A point on the ray is visible if no previous point has a greater angle.

Parallel prefix (v1)

- Based about a logarithmic depth binary tree traversal
- Along the recursive principle
  - OP(x1,x2,… x (n/2)) <u>accumulates on</u> (x (n/2) …, xn)
  - OP(x1,x2,… x (n/4)) <u>accumulates on</u> (x (n/4)+1 …, x(n/2))
  - OP(x (n/2)+1 …, x(3n/4) <u>accumulates on</u> (x (3n/4)+1 …, xn)

Execution of v1 algo in 3 phases: **ascend**, descend, final

- A=(8, 9, 10, 11, 12, 13, 14, 15)
- Prefix sum : final B[k]=A[0]+…+A[k] for each k
  B=(8, 17, 27, 38, 50, 63, 77, 92)



A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]
null	92	38	54	17	21	25	29

A[8]	A[9]	A[10]	A[11]	A[12]	A[13]	A[14]	A[15]
8	9	10	11	12	13	14	15

- Code EREW PRAM of Dekel et Sahni, Optimal
  - version of [Desprez], with OP=+
  - Collection size = 2^m, stored in the second half of array A. Result will be available in second half of array B
  - First halves of A & B used as intermediate storage
  - 3 phases:

1. « ascend » along the tree, from bottom to root

```
For l=m-1 to 0 do (in sequential)
     For j=2^l to 2^(l+1) -1 do_in parallel
          A[j]= A[2.j] + A[2.j +1]
      endFor
endFor
```

- Complexity: //time O(log n) = O(m) on EREW, with n/2 proc.
- Can be turned as work optimal

Execution of v1 algo in 3 phases: ascend, **descend**, final

- A=(8, 9, 10, 11, 12, 13, 14, 15)
- Prefix sum : final B[k]=A[0]+…+A[k] for each k
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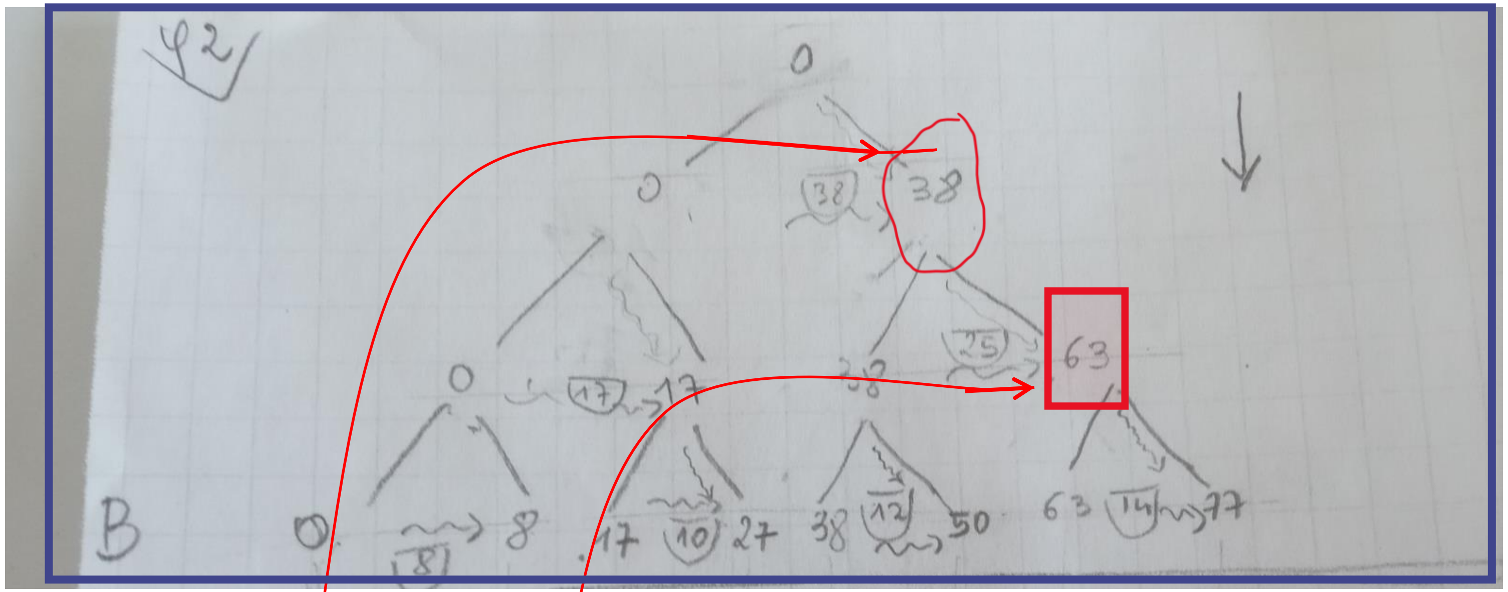
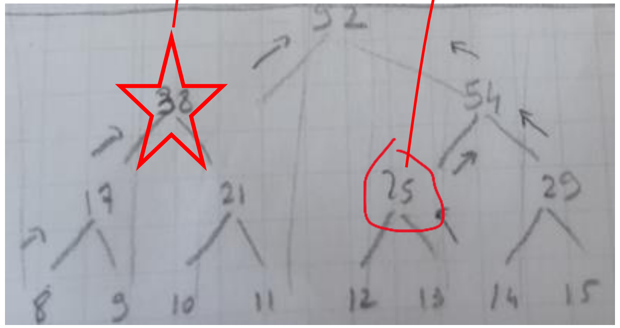


B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]
null	0	0	38	0	17	38	$\boxed{63}$

B[8]	B[9]	B[10]	B[11]	B[12]	B[13]	B[14]	B[15]
0	8	17	27	38	50	63	77



A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]
null	92	38	54	17	21	25	29

A[8]	A[9]	A[10]	A[11]	A[12]	A[13]	A[14]	A[15]
8	9	10	11	12	13	14	15

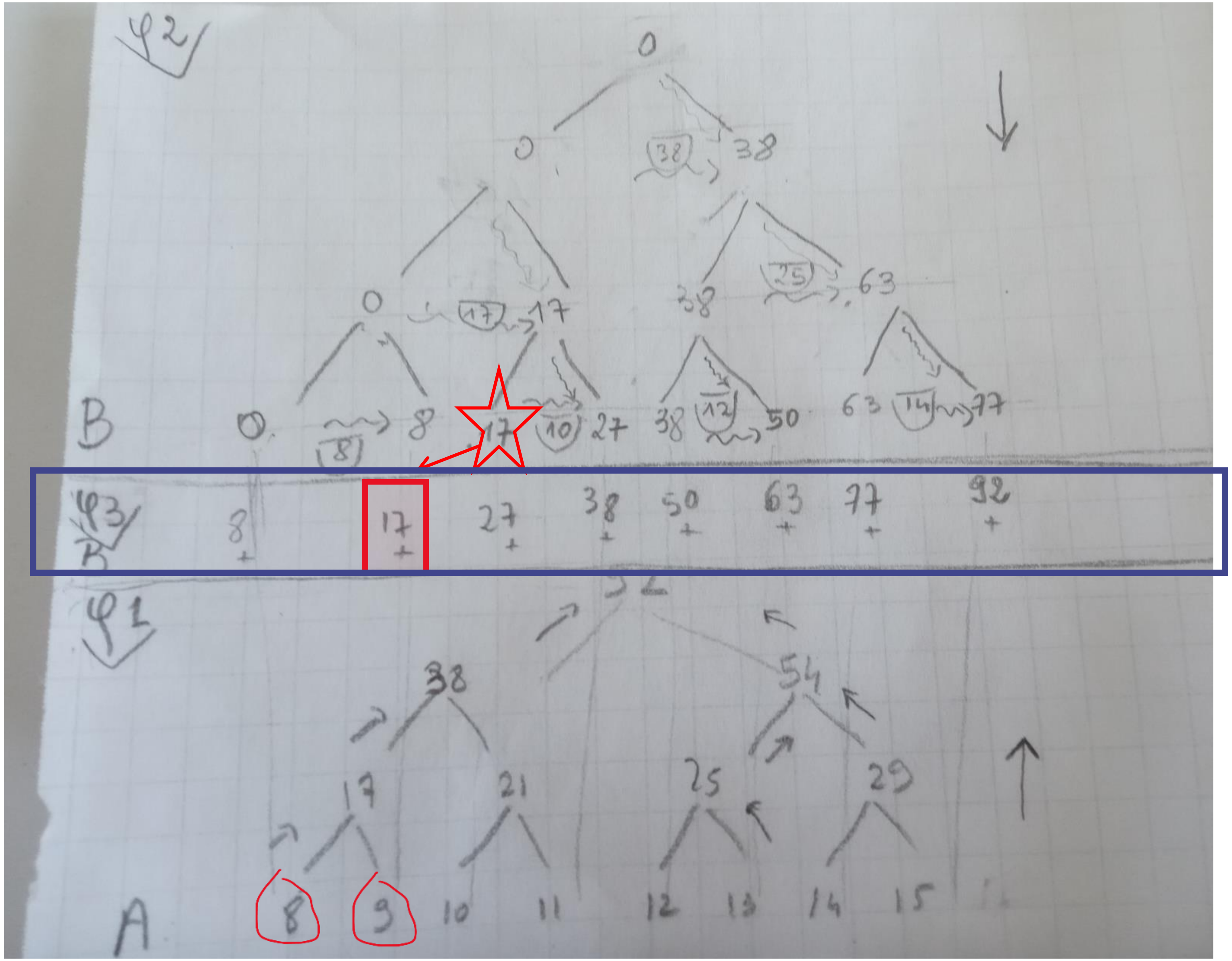
2. « descend » : for each node, accumulate (in B) value from left brother(in A) & value stored at father level (in B)

- If I'm a node at left hand side, just take the value accumulated at my father level (no left brother!)
- If I'm a node at right hand side, combine accumulated value at father and the left hand side node (=my left brother)

```
B[1]=0
For l=1 to m do (in sequential)
    For j=2^l to 2^(l+1) -1 do_in_parallel
        if EVEN(j) // j is a left hand side node,
            B[j] = B[j/2]  // j gets the value accumulated at j' father node
        if ODD(j) // j is a right hand side node
            B[j] = B[(j-1) / 2)] + A[j-1] // combine with + father & left brother values
     endFor
endFor
```

- Complexity: //time O(log n) = O(m) on EREW, with n/2 proc.
  - maxi. 2 concurrent Read access, so EREW is OK
    - eg j=8 and j=9, both access B[4] in read mode
- Can be turned as work optimal

Execution of v1 algo in 3 phases: ascend, descend, <u>**final**</u>



3. Finalize the computation of the second half of B, by adding one by one the values stored in second half of A

- In // accumulate the own value to the incomplete prefix sum value

```
For j=2^m à 2^(m+1) - 1 do in parallel
        B[j]= B[j] + A[j]
endFor
```

- Final phase can be avoided if yk=x0 *op* x1 *op* … *op* x(k-1)=> PreScan
- Complexity: //time O(1) on an EREW, with n proc.
- Can be turned as work optimal

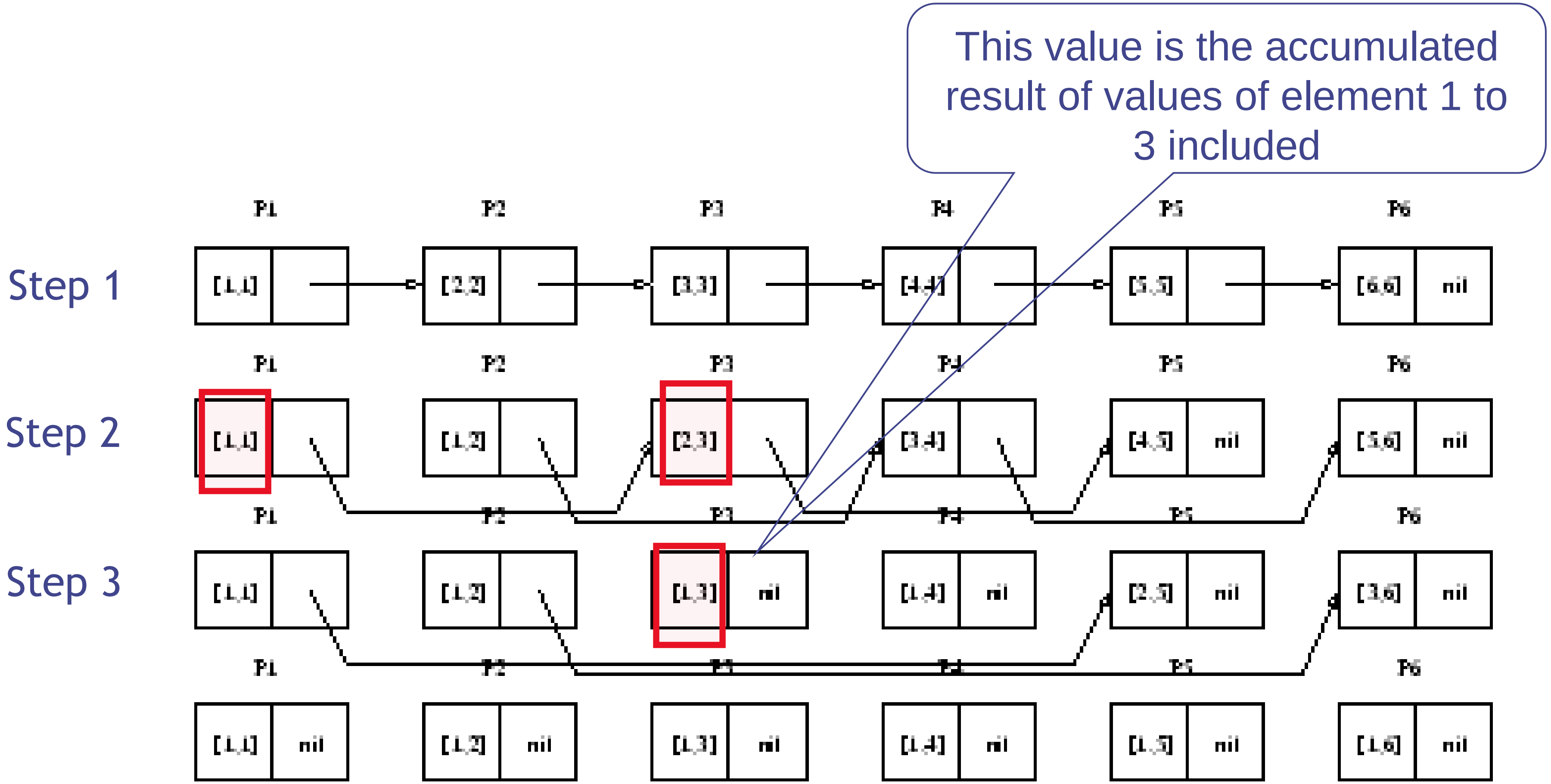
TOTAL: O(log n) with O(n) procs on an EREW PRAM,
or O(log (n / log (n)) + log(n)) = O(logn) using O(n/log n) procs

Parallel prefix (v2)

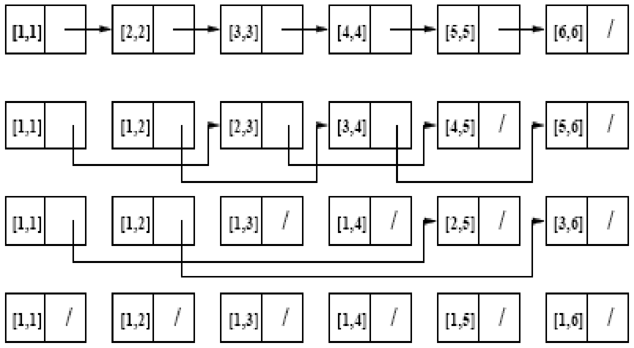
- Based on a collection : linked lisk
  - -> none of elements knows its position in the list
  - Visit the list by following pointers between elements: pointer jumping
  - Principle of recursive doubling:
    - For all proc., the distance to which information are propagated is doubling at each parallel step
    - So, the parallel time needed is in O(log of list length) so that all the information gets propagated

Principle of version 2

- Code EREW PRAM in O(logn) time, not work optimal



Code for v2



$[i, j] = x_i \otimes x_{i+1} \otimes \ldots \otimes x_j$ pour $i \leq j$

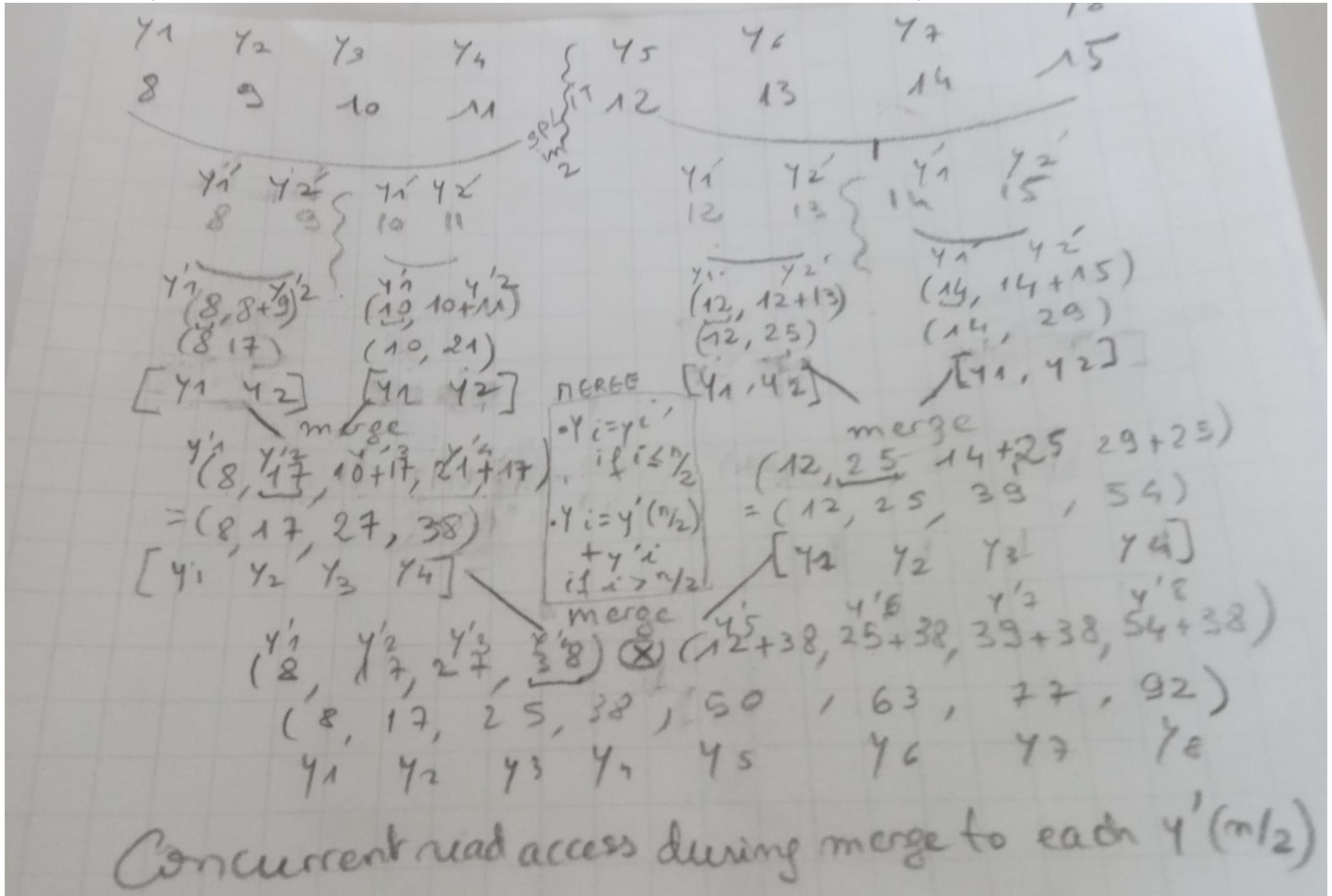
for each processor i in // **do**
    $y[i] \leftarrow x[i]$
    while ($\exists$ objet i t.q. $next[i] \neq$ NIL) **do**
        for each processor i in // **do**
            if $next[i] \neq$ NIL **then**
                $y[next[i]] \leftarrow y[i] \otimes y[next[i]]$
                $next[i] \leftarrow next[next[i]]$

Parallel prefix (v3)

- Based upon a « simple » divide & conquer and parallelized approach
- Along the recurrence principle
  - yi = y'i if i<= n/2
  - yi = y'(n/2) *OP* y'i if i> n/2

Simulation of v3 algo

- A=(8, 9, 10, 11, 12, 13, 14, 15)
- Prefix sum : final B[k]=A[1]+…+A[k] for each k
  B=(8, 17, 27, 38, 50, 63, 77, 92)

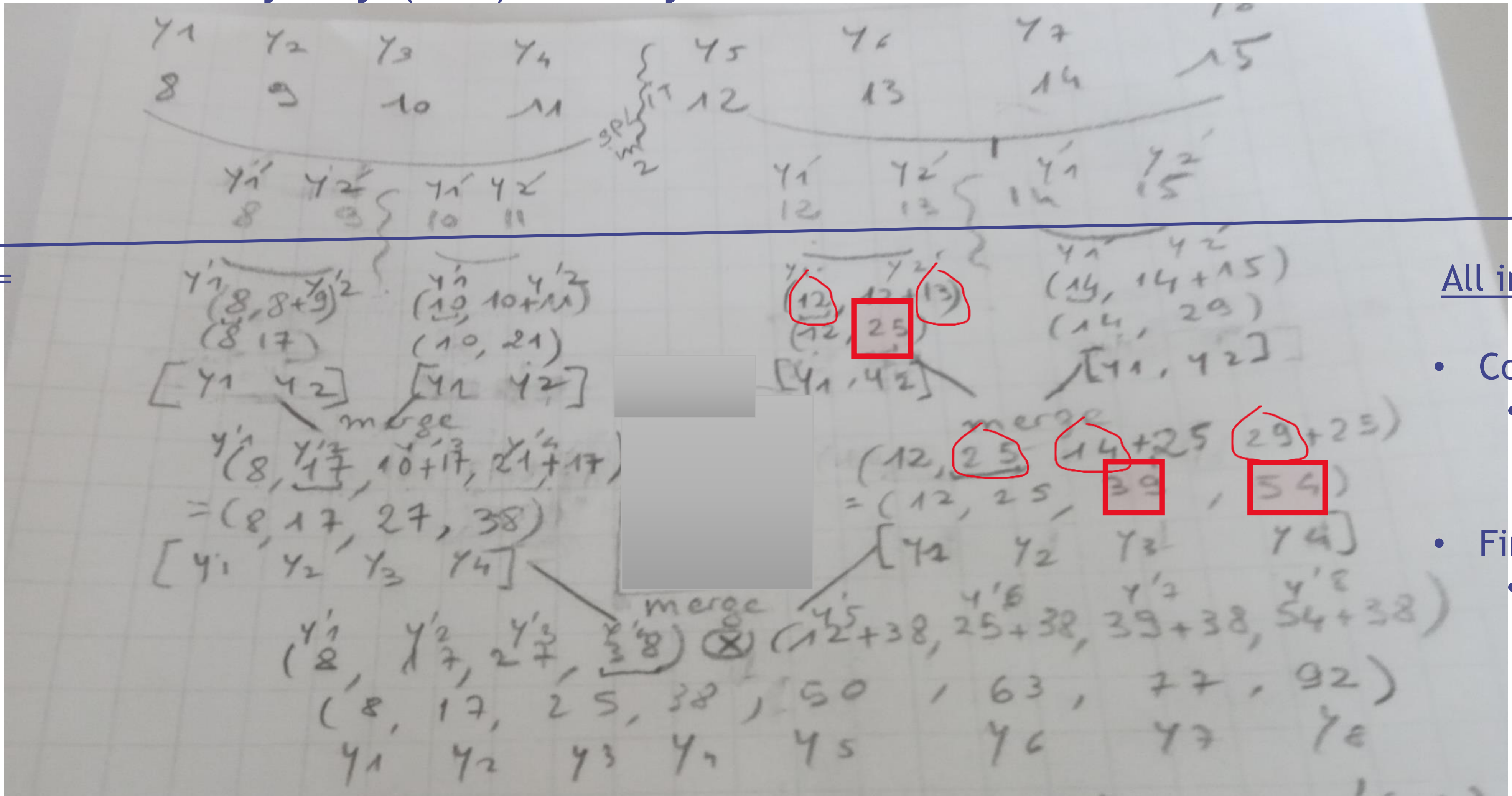


Simulation of v3 algo

- yi = y’i if i<= n/2
- yi = y’(n/2) *OP* y’i if i> n/2

DIVIDE= SPLIT phase

All in parallel



CONQUER= MERGE phase

All in parallel:

- Coarse level:
  - All sublists are merged in //
- Fine level:
  - To build each sublist of size k, k procs needed

Remarks about v3

- Requires a CREW PRAM
  - At each merging of 2 sub problems (2 sublists):
    - **Concurrent read of value y'(n/2)** by all the procs. in charge of indices > n/2
      - Duplicate in a subsidiary array of length (n/2), the value y'(n/2) as many times it needs to be read « concurrently »
      - Cf TD1

- Parallel time complexity : O(logn)
- At each parallel step, n procs. needed
  - Possible to reduce this amount by a O(logn) factor