4. Perform the basic cycle of reciprocation and division as often as required to reduce the last remainder function to zero.

Since each division reduces the degree of the numerator by one, we should expect the number of partial quotients to be simply related to the degrees of the numerator and denominator of the given function. The relationship may be determined by examination of the fractions

$$\frac{n}{n}, \frac{n}{n-1}, \frac{n-1}{n-1}, \ldots, \frac{2}{1}, \frac{1}{1}, \frac{1}{0}, \frac{0}{0} \qquad \blacktriangleright \text{(5-88)}$$

which, respectively, represent the degrees of the numerator and denominator of the original function and the successive reciprocated remainder functions. The above assumes that the original function does not have a pole at the origin; when it does, the 0/0 fraction of (5-88) is not present since the last partial quotient is not a constant but a linear term.

When $n = 2$, (5-88) contains five fractions, and thus there are five partial quotients; when $n = 3$, there are seven partial quotients. If there is a pole at the origin, then it is clear that there are only four partial quotients for $n = 2$ and six for $n = 3$. In general, we see that the number of partial quotients is equal to one more than the number of *internal* critical frequencies. It should be clear that the assumption of equal degrees in numerator and denominator results in no loss of generality, since for unequal degrees the first fraction of (5-88) is omitted, thus reducing the number of fractions by unity, and the number of internal critical frequencies is also reduced by unity.

This number of partial quotients is the *minimum* number of data necessary to specify the function; for example, for $H(s + a)/(s + b)$, the *three* parameters H, a, and b must be specified, whereas for $H(s + a)/s$, only H and a need be given.

The above discussion is summarized as:

THEOREM 5-13. The number of partial quotients obtained in a continued-fraction expansion of F_{RCZ} about infinity is equal to one more than the number of internal critical frequencies. This is the minimum number of data necessary to specify the function completely.

The division step of the cycle may be interpreted in terms of the properties of the real part of $F_{RCZ}(j\omega)$ and $F_{RCY}(j\omega)$. Since only a nonzero real constant value equal to $F_{RCZ}(\infty)$ is removed from F_{RCZ}, the real part of the remainder function is still nonnegative for all ω, as it must be for an F_{RCZ} function. Removal of a pole at infinity from F_{RCY} does not affect its real part since the real part of this pole is zero for real frequencies. However, if we attempted to remove a constant value equal to $F_{RCY}(\infty)$ from F_{RCY}, the real part of the remainder would *not* be nonnegative for all ω and consequently the remainder function would not be of the F_{RCY} form.

Now suppose that the original function to be expanded is of the F_{RCY} class. In accordance with the above discussion we *cannot* remove its constant value at infinity from F_{RCY}. Therefore *only* if F_{RCY} possesses a pole at infinity do we carry out the division of the first cycle; no operation is performed in this cycle for an F_{RCY} with no pole at infinity. Then the remainder function is reciprocated to yield an F_{RCZ} function from which a constant is removed. The succeeding cycles now proceed as for an original F_{RCZ} function.

It should be clear that *the step in the first cycle is the critical one.* If the polynomials are written in the correct order and the operation in the first cycle is performed correctly, then the remaining cycles follow automatically. The reader should now refer back to Example 5-2*a*, *c*, and *d* and note why negative coefficients were obtained in these expansions.