

**The student is working on the following problem:**

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

**The solution of this problem is:**

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A:  $1,646,000 / 50,000 = 32.92 \rightarrow 32$  seats
- B:  $6,936,000 / 50,000 = 138.72 \rightarrow 138$  seats
- C:  $154,000 / 50,000 = 3.08 \rightarrow 3$  seats
- D:  $2,091,000 / 50,000 = 41.82 \rightarrow 41$  seats
- E:  $685,000 / 50,000 = 13.70 \rightarrow 13$  seats
- F:  $988,000 / 50,000 = 19.76 \rightarrow 19$  seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

oh hey! honestly, I'm kinda lost on where to even start with this. I think I need to figure out how to give the states seats fairly based on their population sizes, but I'm not sure what math to use for that... any hints?



Absolutely, let's start at the beginning. First off, when we want to distribute seats proportionally based on population, we need to have a sense of the whole picture. Can you tell me what you think the first step should be when you are starting to solve a problem like this? Think about what information you would need to consider all the states fairly.

Ok, I guess the first step is probably to figure out the total population of the whole country by adding up all the populations of the states? That way we can work out the proportion of each state to the total population.



That's an excellent starting point! Go ahead and calculate the total population of the country by adding up the populations of each state. What number do you get?

Ok, so if I add them up it should look like this:



State A: 1, 646, 000

State B: 6, 936, 000

State C: 154, 000

State D: 2, 091, 000

State E: 685, 000

State F: 988, 000

So the total population would be:

$$1, 646, 000 + 6, 936, 000 + 154, 000 + 2, 091, 000 + 685, 000 + 988, 000 = 12, 500, 000$$

I hope that's right.



You've done that perfectly! With the total population known, the next step is to find out how many people should, on average, be represented by one seat. This is sometimes called the "standard divisor". How would you calculate the number of people per seat?

I guess I would take the total population and then divide it by the number of seats available. So that should be 12, 500, 000 divided by 250 seats, right? Lemme do the math...



Yes, that's the right approach. Once you've done the calculation, you'll have the average population that one seat represents. What do you find when you divide the total population by the number of available seats?

Ok so I divide the total population, 12, 500, 000 by the number of seats, which is 250. So it's like:



$$12, 500, 000 \div 250 = 50, 000$$

So it looks like one seat should represent 50, 000 people. Did I get that right?



Yes, that's correct! Each seat represents 50,000 people. Now that you have the standard divisor, you can start to figure out the initial number of seats each state should get. How might you do that? Consider using the population of each state and the standard divisor to find the initial allocation of seats.

Ok, so I take the population of each state and divide it by 50,000, since one seat represents 50,000 people. Then I round to the nearest whole number to get the initial seat allocation for each state. Should I go ahead and do that for all the states?



You're on the right track, but let's make a small correction. When allocating seats, initially, you should round down to the nearest whole number, not to the nearest whole number. This means if a state has a quota of 32.92 seats, for example, it should initially receive 32 seats, not 33.

Could you try calculating the initial number of seats for one state to start with, rounding down to the whole number? How about you begin with state A?

Got it, we round down to the nearest whole number. So for state A, the population is 1,646,000. Dividing that by 50,000 should give the initial seats. Let me calculate...



$$1,646,000 \div 50,000 = 32.92.$$

Rounding down that would give state A 32 seats to start with. Should I do the same for the rest of the states now?



Yes, that's perfectly done for state A. Now go ahead and repeat the same process for the remaining states. Remember to only take the whole number part of your result for each one. If you could calculate those and tell me what you get, that would be great.

Alright, I'll calculate it for each state and round down for each.



State B:  $6,936,000 \div 50,000 = 138.72$  so, rounding down, State B gets 138 seats.

State C:  $154,000 \div 50,000 = 3.08$  so, rounding down, State C gets 3 seats.

State D:  $2,091,000 \div 50,000 = 41.82$  so, rounding down, State D gets 41 seats.

State E:  $685,000 \div 50,000 = 13.7$  so, rounding down, State E gets 13 seats.

State F:  $988,000 \div 50,000 = 19.76$  so, rounding down, State F gets 19 seats.

Now that I have all these numbers, what should I do next?



Well done with those calculations! The next thing to do is to add up all of these initial seat allocations to see how many of the 250 seats have been allocated. Can you calculate the total number of seats allocated so far? If there are any seats left, we will need to distribute them accordingly. What is the sum of the initial allocations for all the states?

Ok, I'll add up the seats allocated for each state:



State A: 32 seats

State B: 138 seats

State C: 3 seats

State D: 41 seats

State E: 13 seats

State F: 19 seats

Adding these together gives:

$$32 + 138 + 3 + 41 + 13 + 19 = 246$$

So it looks like we've allocated 246 seats so far. That means there are 4 seats left since we have to allocate a total of 250 seats. What's next?



You've got it! There are 4 seats remaining to be allocated. We now should distribute these surplus seats to the states with the largest remainders from their initial quota calculations. Can you identify the remainders for each state from your previous calculations? Once you have those, you can determine which states will receive the extra seats.

Sure, the remainders are the fractions we left out when we rounded down for each state. Let me list the remainders:



State A: 32.92 means the remainder is 0.92.

State B: 138.72 means the remainder is 0.72.

State C: 3.08 means the remainder is 0.08.

State D: 41.82 means the remainder is 0.82.

State E: 13.7 means the remainder is 0.7.

State F: 19.76 means the remainder is 0.76.

So the four largest remainders are for States A (0.92), D (0.82), F (0.76), and B (0.72). That means these states would get the four remaining seats. Did I do that right?

Exactly right! States A, D, F, and B should each receive one of the remaining seats. Now, can you update the initial allocation for these states with the additional seats and tell me the new totals for each state?

Sure, adding one more seat to the initial allocations for states A, D, F, and B would give us:



State A:  $32 + 1 = 33$  seats

State B:  $138 + 1 = 139$  seats

State D:  $41 + 1 = 42$  seats

State F:  $19 + 1 = 20$  seats

And States C and E stay the same since they don't get any of the extra seats:

State C: 3 seats

State E: 13 seats

So now we have the final seat allocations for each state.

Indeed, you have correctly updated the seat allocations. To ensure everything sums up correctly, can you provide the final check by adding all the seats together? This way, we can confirm that all 250 seats have been appropriately distributed among the states.

Of course! Here's the final count of seats for each state:



State A: 33 seats  
State B: 139 seats  
State C: 3 seats  
State D: 42 seats  
State E: 13 seats  
State F: 20 seats

I'll add these up to make sure it sums to 250 seats:

$$33 + 139 + 3 + 42 + 13 + 20 = 250$$

Yep, it all adds up to 250 seats. Looks like we did it right!

Fantastic! Your calculations are correct, and the seats have been allocated fairly based on the method of largest remainders. This method ensured proportional representation of each state according to its population within the legislative body. Well done! If you have any more questions or another topic you'd like to tackle, feel free to ask.

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