

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

Total population = 1,646,000 (A) + 6,936,000 (B) + 154,000 (C) + 2,091,000 (D) + 685,000 (E) + 988,000 (F)
= 12,500,000

Standard divisor = Total population / Number of seats = 12,500,000 / 250 = 50,000

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

Total initial seats assigned = 32 + 138 + 3 + 41 + 13 + 19 = 246

Seats left to distribute = 250 - 246 = 4

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82
- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Hey! Honestly, I'm kinda lost with this... 🤔 I think we need to divide the seats based on the population of each state? But I have no idea how to actually do it. Can you help me out?



Of course! You're on the right track considering that seats should be distributed based on the population of each state to ensure fair representation. To begin, we need to determine the total population of the country. Can you calculate the total population by adding up the population of each state?

Sure, I'll add those up. Okay, so the total population is:

$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$.

Lemme calculate that... It adds up to:

$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 = 12,500,000$

So, the total population of the country is 12,500,000. What do I do next?



Great job calculating the total population! Next, you'll want to determine how many people, on average, each seat should represent. This is done to ensure that each seat in the legislature is representing a similar number of people.

Try calculating the "standard divisor" by dividing the total population by the total number of available seats. Can you do that?

Alright, I need to divide the total population by the number of available seats. There are 250 seats, so:

$12,500,000 \div 250 = 50,000$.

So I think each seat should represent 50,000 people. Is that correct?



Exactly right! Each seat represents 50,000 people on average, which is your standard divisor. Now that you have this number, the next step is to determine the initial allocation of seats to each state.

We begin by dividing each state's population by the standard divisor. The result gives you the initial whole number of seats for each state, without considering any remainders yet. This is often referred to as the initial quota. Could you go ahead and calculate the initial quotas for each state by dividing their populations by the standard divisor? Remember to round down to the nearest whole number for now.

Okay, let's calculate it:



For State A:

$1,646,000 \div 50,000 = 32.92$, so State A gets initially 32 seats.

For State B:

$6,936,000 \div 50,000 = 138.72$, so State B gets initially 138 seats.

For State C:

$154,000 \div 50,000 = 3.08$, so State C gets initially 3 seats.

For State D:

$2,091,000 \div 50,000 = 41.82$, so State D gets initially 41 seats.

For State E:

$685,000 \div 50,000 = 13.7$, so State E gets initially 13 seats.

For State F:

$988,000 \div 50,000 = 19.76$, so State F gets initially 19 seats.

Adding up all the initial quotas, we have:

$32 + 138 + 3 + 41 + 13 + 19 = 246$ seats. We've allocated 246 out of 250 seats, so we have 4 seats left to assign. What's next?



Well done! You've correctly determined the initial quotas for all states and realized there are 4 surplus seats remaining to be distributed. The next step involves handling these surplus seats.

We should distribute the remaining seats to the states with the largest remainders from their initial quotas. The remainders are the decimal parts you calculated earlier but didn't use yet. First, identify the remainders for each state. Then, can you tell me which states have the largest remainders? These will get the remaining seats, one for each of the highest remainder values until all surplus seats are allocated.

Got it! Here are the remainders I found when calculating the quotas:



State A: 0.92
State B: 0.72
State C: 0.08
State D: 0.82
State E: 0.70
State F: 0.76

Now, let's rank these remainders from highest to lowest to see which states get the extra seats:

1. State A: 0.92
2. State D: 0.82
3. State F: 0.76
4. State B: 0.72
5. State E: 0.70
6. State C: 0.08

The four states with the largest remainders are State A, State D, State F, and State B. So they each get one of the remaining 4 seats. Here's the updated seat allocation:

State A: $32 + 1 = 33$ seats
State B: $138 + 1 = 139$ seats
State C: 3 seats
State D: $41 + 1 = 42$ seats
State E: 13 seats
State F: $19 + 1 = 20$ seats

That gives us a total of 250 seats allocated. Did I do that right?



That's correct! You've successfully distributed the surplus seats based on the largest remainders. Every state now has a number of seats that is as close to proportional with its population as possible given the constraints. Can you summarize the final seat allocation you've just calculated to ensure we have a clear record of the outcome?

Sure! Here's the final seat allocation for the legislative body:



State A: 33 seats
State B: 139 seats
State C: 3 seats
State D: 42 seats
State E: 13 seats
State F: 20 seats

And adding them all up to double check, we get:

$$33 + 139 + 3 + 42 + 13 + 20 = 250 \text{ seats total.}$$

Everything adds up! Each state has a fair representation in the legislative body based on its population. Cool, I think I get it now! 🤗



Fantastic! You've clearly understood the method and executed it perfectly. You've allocated the legislative seats fairly according to the population of each state, using the method of largest remainders. Remember, the method you've used ensures that each representative is responsible for roughly the same number of people, which is a core principle of proportional representation.

If you have any more questions or if there's another topic you'd like to explore, feel free to ask. Great work today!

Thank you so much! I think I'm good for now. This was a really interesting problem, and I learned a lot about proportional representation. I'll reach out if I have more questions in the future. Bye!



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