The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign sits proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining sits to stats having the largest remainders.

Total population = 1,646,000 (A) + 6,936,000 (B) + 154,000 (C) + 2,091,000 (D) + 685,000 (E) + 988,000 (F) = 12,500,000

Standard divisor = Total population / Number of seats = 12,500,000 / 250 = 50,000

Initial quotas:

- A: 1,646,000 / 50,000 = 32.92 -> 32 seats
- B: 6,936,000 / 50,000 = 138.72 -> 138 seats
- C: 154,000 / 50,000 = 3.08 -> 3 seats
- D: 2,091,000 / 50,000 = 41.82 -> 41 seats
- E: 685,000 / 50,000 = 13.70 -> 13 seats
- F: 988,000 / 50,000 = 19.76 -> 19 seats

Total initial seats assigned = 32 + 138 + 3 + 41 + 13 + 19 = 246

Seats left to distribute = 250 - 246 = 4

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70

- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.

Hello! I am your tutor. Can you walk me through your solution?

Hey, so I'm not really sure how to start this problem. I think we need to give out the 250 seats based on the population of each state right? But how do I do that?

That's exactly right; we do need to divide the seats based on the population of each state. Let's start by asking this question: If we want to represent the entire country fairly, what might be the first thing we'd need to know about the country's population as a whole?

Okay, so I guess we need to know the total population of the country. That means I need to add up all the populations of the states, right? Let me do that. Give me a sec...

Take your time. Once you have that total, let me know, and we can move forward with the next step.

Alright, so I'm gonna add up the populations of all the states. Here's what I'm doing:

$$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$$

Let me calculate that...

That looks like a great approach to find the total population. Once you have that calculated, we can think about what the next reasonable step would be.

Okay, so here's the total population when I add them all up:

$$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 = 12,500,000$$

So the total population of the country is 12,500,000 people. Now what do we do?

Great job calculating the total population! Now, think about how you could use this number in combination with the number of available seats to find a baseline for how many people each seat should represent. What calculation might give us that information?

Hmm, maybe we divide the total population by the number of seats to see how many people one seat represents? So like, $12,500,000$ divided by 250 seats. Lemme do that calculation...

Sounds like you're on track with your thought process. Let me know what you find from that calculation.

When I divide $12,500,000$ by 250, I get:

$$12,500,000 \div 250 = 50,000$$

So, each seat should represent roughly $50,000$ people. Is that right?

That's correct; each seat represents 50,000 people. Now, with this new piece of information, what might be your next step in determining the seat allocation for each state?

Okay, so now I guess I divide the population of each state by 50,000 to see how many seats each one should get. Let me start with state A. Give me a moment...

$$1,646,000 \div 50,000 = 32.92$$

So...that doesn't give me a whole number of seats. What should I do about that? Just round it?

That's a very good observation about the non-whole number result. When considering fair representation, what does your intuition say about rounding here? What might be the fairest way to proceed?

Hmm, I think if I round down, the state might be underrepresented and if I round up, it might be overrepresented. Maybe we need to look at all the states and then decide how to round?

Like if some states have a decimal closer to .5 and above, we can round up, and if it's lower than .5, we can round down. But I'm not sure what to do if it's exactly .5. What do you think?

It sounds like you're thoughtfully considering how to allocate the seats fairly. Reflect on this rounding approach, and think about the possible outcomes and their implications. What needs to be achieved for the allocation to be as just and balanced as possible? Consider thinking about what would happen if you applied this rounding system to all the states and see if that aligns with the goal of proportional representation.

Okay, well, I think the main goal is to make sure the total number of seats adds up to 250 by the end since we can't have more or less. If I just round each state's number, I might end up with more or less than 250 seats when I add them all together.

I guess for now, I should try to divide each state's population by the 50,000 and see all the numbers I'm working with before deciding how to round. So, I'll do the division for all states and then think about how to deal with the decimals. Let me write this down…

That sounds like a thoughtful plan. Go ahead and calculate the initial quotas for each state using the standard divisor we determined, and write down your findings. Once we have all the initial quotas, we'll be able to look at them as a whole and think about our next steps.

Alright, here are my calculations for each state:

State A:
$1,646,000 \div 50,000 = 32.92$

State B:
$6,936,000 \div 50,000 = 138.72$

State C:
$154,000 \div 50,000 = 3.08$

State D:
$2,091,000 \div 50,000 = 41.82$

State E:
$685,000 \div 50,000 = 13.7$

State F:
$988,000 \div 50,000 = 19.76$

Do these look okay? What's the next step?

Those numbers look like a good start. Now that you have the initial quotas, what can you do to examine the whole picture and decide how to move forward with the non-whole numbers you've ended up with? Think about how these initial quotas reflect the number of seats available and the goal of fair representation.

So, if I just took the whole number part of those quotas, I would still need to make sure that the total is 250 seats. Let me add the whole numbers and see what I get:

State A: 32 seats
State B: 138 seats
State C: 3 seats
State D: 41 seats
State E: 13 seats
State F: 19 seats
Adding those together gives:

$$32 + 138 + 3 + 41 + 13 + 19 = 246 \text{ seats}$$

But we have 250 seats to fill, which means we have 4 extra seats to distribute. Since we can't split seats, I think we should give those extra seats to the states with the highest decimal parts of their quotas. Does that sound right?

It sounds like you're putting a lot of thought into how to allocate the remaining seats. Given the information you have, what do you believe would be the fairest way to distribute the remaining seats amongst the states?

I think the fairest way to allocate the remaining seats would be by giving them to the states with the highest decimal parts. This way, states that are closest to earning another seat get priority.

From the numbers I have:

- State A has a decimal part of .92

- State B has .72

- State C has .08

- State D has .82

- State E has .70

- State F has .76

So the four states with the highest decimal parts are A, D, F, and B. If I give one extra seat to each of these states, the new allocation should be:

State A: 33 seats
State B: 139 seats
State C: 3 seats
State D: 42 seats
State E: 13 seats
State F: 20 seats

Adding these up should give us the 250 seats total:

$$33 + 139 + 3 + 42 + 13 + 20 = 250$$

That should do it, I think. Does this look fair and correct?

You've done a thorough job working through this problem and considering the principles of fair representation. Well done! If you have further questions in the future, don't hesitate to reach out. Goodbye and take care!

Thanks a lot! I think I understand how to do it now. Goodbye!

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