

The student is working on the following problem:

Table 1. shows the number of goals that each football player striker scored between 2007 and 2011. Who is the most consistent player? Immagine a measure of "consistency" and justify why you think your method is correct.

Year	Mike Arwen	Dave Backhand	Ivan Right
2007	13	12	14
2008	12	14	10
2009	15	16	18
2010	17	15	18
2011	13	13	15

The solution of this problem is:

The concept of standard deviation is the usual way to measure consistency but it is not expected that the student knows it. Any measure proposed by the student to measure consistency is acceptable as long as it can be justified.

Example of canonical solution: computing the variance (or standard deviation) for each player: First, compute the mean:

Mean number of goals for Mike: 14

Mean number of goals for Dave: 14

Mean number of goals for Ivan: 15

Then, compute the sum of square deviations from the mean for each player.

Then devide by the number of data points to get the variance:

Variance for Mike: $12/5 = 3.2$

Variance for Dave: $10/5 = 2$

Variance for Ivan: $44/5 = 8.8$

So according to the variance, Dave is the most consistent player.



Hello! I am your tutor. Can you walk me through your solution?

Hey! Okay, I'm a bit lost with this problem. I think we need to measure how consistently these players score goals, right? But I don't really know any math methods for measuring "consistency". Can you help me understand what kind of methods we could use?



Absolutely, consistency involves looking at how things stay the same or change over time. For a starting point, can you think of any ways to describe how a player's goals vary from year to year? What would it mean for a player to be consistent in terms of the number of goals scored?

Sure, I think a player would be consistent if they scored about the same number of goals every year. So maybe we should look for someone whose number of goals doesn't change much across the years?



For that, could we maybe just look at the differences in the number of goals they score each year and see whose differences are smaller? Like, we could subtract each year's goals from the previous year's and add up these differences for each player?



That's a good thought process. Working along those lines, consider how you might handle the differences—whether they are positive or negative—and what this could tell you about the player's performance over the years. What might be the next step in analyzing these differences?

I see what you're saying, the differences could be positive or negative, but for consistency, we probably only care about how big the difference is, not whether it's an increase or decrease. So, I guess we could take the absolute value of the differences to make all of them positive.



Then, we could maybe average those values for each player over the 5 years to see who has the lowest average? That player should be the most consistent because their number of goals isn't changing as much from year to year. What do you think?



That's an insightful route to explore. Given the approach you've described, how do you plan on utilizing the absolute values to proceed with your analysis?

Okay, so I'll take each player's goals for each year and subtract the previous year's goals to find the difference, then take the absolute value of that difference. After I get all the differences for each player, I'll add them up and divide by 4, since we're looking at the differences across 5 years, which gives us 4 periods of change. Here's what it would look like for each player:



For Mike Arwen:

- From 2007 to 2008: $|12 - 13| = 1$
- From 2008 to 2009: $|15 - 12| = 3$
- From 2009 to 2010: $|17 - 15| = 2$
- From 2010 to 2011: $|13 - 17| = 4$
- Average difference: $(1 + 3 + 2 + 4) / 4 = 2.5$

Should I do the same for Dave Backhand and Ivan Right and then we see who has the smallest average difference?



You seem to have a solid understanding of your proposed method. What do you think should be the next step?

Great, I'll calculate the average differences for Dave Backhand and Ivan Right the same way I did for Mike Arwen. After that, we can compare the averages and see who has the smallest one. That player would be the most consistent according to this method. I'll do those calculations now.



For Dave Backhand:

- From 2007 to 2008: $|14 - 12| = 2$
- From 2008 to 2009: $|16 - 14| = 2$
- From 2009 to 2010: $|15 - 16| = 1$
- From 2010 to 2011: $|13 - 15| = 2$
- Average difference: $(2 + 2 + 1 + 2) / 4 = 1.75$

Now for Ivan Right:

- From 2007 to 2008: $|10 - 14| = 4$
- From 2008 to 2009: $|18 - 10| = 8$
- From 2009 to 2010: $|18 - 18| = 0$
- From 2010 to 2011: $|15 - 18| = 3$
- Average difference: $(4 + 8 + 0 + 3) / 4 = 3.75$

Alright, looking at the averages, Dave Backhand has the smallest one, 1.75, followed by Mike Arwen with 2.5, and then Ivan Right with 3.75. So according to this method, Dave Backhand should be the most consistent player.



You've applied your method correctly to each player. What do you think of the method you have used? Can you think of any limitations or considerations we should be aware of when interpreting the results using this approach?

Hmm, I think the method we used is pretty straightforward. It just measures how much a player's goals fluctuate from year to year. If we're talking about consistency, that seems to make sense. But I guess one limitation might be that it doesn't really tell us about how well they're playing overall, just how consistent they are. Like, a player could score very few goals but always the same amount, and they'd look really consistent.



Also, taking the absolute values treats all differences the same, no matter if it's a good or bad change from one year to the next. And since we average the differences, it's possible that one big fluctuation could kind of be hidden if the other years don't change much.

I guess another thing is, this method we used doesn't consider the actual numbers of goals scored that much, so it might be useful to include something about that. Like, maybe it's more impressive to be consistent at a higher number of goals? I'm not sure what other method we could use though. Do you have any ideas?



You've raised some interesting points about considering the overall performance along with the consistency. Reflecting on the different ways to analyze numerical data, can you think of any statistical measures that might be able to take both consistency and the magnitude of the scores into account?

Oh, right, there are things like mean and median that you learn in math, which can tell you something about the overall performance. The mean is just the average number of goals scored, so we could add all the goals up and divide by 5.



And then for consistency... maybe we could compare the goals scored each year to the mean for each player? But I'm not sure exactly how to do that. Or maybe there's some other statistical thing that can combine consistency and average performance? I'm really not sure, what do you think?



You're on the right track exploring the relationship between individual scores and some measure of central tendency, like the mean. Given those thoughts, what might be a way to use the mean to determine how far off each player's yearly goals are from it?

Okay, so for each player, we first need to calculate the mean number of goals they scored over the five years. Then, I think we can subtract the mean from each year's goals to see how far off they are from their average performance. After that, maybe we could do something like what we did with the average differences but use these differences from the mean instead?



But I remember from what we discussed before about averages, we might want to deal with the positive or negative signs... Perhaps we should take absolute values again or square the differences? Squaring would make all differences positive and also give more weight to bigger differences. Is that something like what you're thinking?



Your ideas are really developing well. Considering the points you made about squaring the differences, what effect would that have on the overall calculation of consistency?

If we square the differences between each year's goals and the mean, that would make all of the values positive, sort of like taking absolute values. But it would also make larger differences have a bigger impact since squaring a larger number makes it even larger. That sounds good because we probably want to notice more when a player has a year that's really off from their average.



So then we'd have these squared differences for each year, and adding them up would give us an idea of the overall consistency. We're kind of summing up how far away each year's performance is from the mean, with more unusual years counting for more because of the squaring. That seems like it could be like a "consistency score," maybe, where a lower score is more consistent because it means their years were closer to the mean?

Is that what you were thinking? How would we use this to figure out the most consistent player?



You've described the process quite well, illustrating the effects of squaring the differences. Given this "consistency score" concept, what do you think could be the next step to finalize the measure for each player?

Got it! So we have a sum of squared differences from the mean for each player. I think we should then divide this sum by the number of years to find the average, because we have 5 years of data and we want the consistency measure to work no matter how many years we're looking at; it would be sort of like an average squared difference from the mean for each player.



So the steps would be:

1. Find the mean goals for each player over the 5 years.
2. Calculate the squared differences from the mean for each year.
3. Add up those squared differences.
4. Divide the total by the number of data points (which is 5 in our case, not 4 because now we want to include all years).

This would give us the average of the squared differences, which could be our measure of consistency – I guess a lower number would mean more consistency. Am I getting this right? Should we calculate it for each player?



You seem to have a clear grasp of the method now. How do you feel about proceeding with those calculations for each player using the steps you've described?

I'm feeling good about it! Let's go ahead and do the calculations step by step for each player.



We'll start with calculating the mean number of goals for each player. That's the total goals scored by the player over the five years, divided by five. Here's how it breaks down for Mike Arwen:

Mike Arwen's total goals from 2007 to 2011: $13 + 12 + 15 + 17 + 13 = 70$ goals.

Mike Arwen's mean number of goals: $70 \div 5 = 14$.

Now that we have the mean for Mike Arwen, we need to calculate the squared differences from the mean for each year, then sum these up and get the average. Let's do that for Mike Arwen first, and I'll show you the results for each step:

Mike Arwen's squared differences from the mean:

- $(13 - 14)^2 = (-1)^2 = 1$
- $(12 - 14)^2 = (-2)^2 = 4$
- $(15 - 14)^2 = 1^2 = 1$
- $(17 - 14)^2 = 3^2 = 9$
- $(13 - 14)^2 = (-1)^2 = 1$

Now we add these up: $1 + 4 + 1 + 9 + 1 = 16$.

And to get the average squared difference for Mike Arwen: $16 \div 5 = 3.2$.

I'll go ahead and repeat this process for Dave Backhand and Ivan Right. Once we have the average squared differences for all three players, we can compare them!

Do you want me to go on and calculate these for Dave Backhand and Ivan Right now?



That sounds like a solid plan. What do you think should be the next step in your analysis?

Great! Next, I'll calculate the mean number of goals for Dave Backhand and Ivan Right, and then do the same thing I did for Mike Arwen: find the squared differences from the mean for each year, add them up, and calculate the average squared differences for each player.



Let's do Dave Backhand next:

Dave Backhand's total goals from 2007 to 2011: $12 + 14 + 16 + 15 + 13 = 70$ goals.

Dave Backhand's mean number of goals: $70 \div 5 = 14$.

Dave Backhand's squared differences from the mean:

- $(12 - 14)^2 = (-2)^2 = 4$
- $(14 - 14)^2 = 0^2 = 0$
- $(16 - 14)^2 = 2^2 = 4$
- $(15 - 14)^2 = 1^2 = 1$
- $(13 - 14)^2 = (-1)^2 = 1$

Total squared differences for Dave Backhand: $4 + 0 + 4 + 1 + 1 = 10$.

Average squared difference for Dave Backhand: $10 \div 5 = 2$.

Now for Ivan Right:

Ivan Right's total goals from 2007 to 2011: $14 + 10 + 18 + 18 + 15 = 75$ goals.

Ivan Right's mean number of goals: $75 \div 5 = 15$.

Ivan Right's squared differences from the mean:

- $(14 - 15)^2 = (-1)^2 = 1$
- $(10 - 15)^2 = (-5)^2 = 25$
- $(18 - 15)^2 = 3^2 = 9$

- $(18 - 15)^2 = 3^2 = 9$

- $(18 - 15)^2 = 3^2 = 9$

- $(15 - 15)^2 = 0^2 = 0$

Total squared differences for Ivan Right: $1 + 25 + 9 + 9 + 0 = 44$.

Average squared difference for Ivan Right: $44 \div 5 = 8.8$.

Now that we have the average squared differences for all three players, we can see that Dave Backhand has the smallest one with 2, followed by Mike Arwen with 3



You've done some thorough calculations. How does this mathematical approach inform us about each player's consistency? Can you summarize the results and their implications?

Sure! This mathematical approach gave us some numbers called the average squared differences for each player which tell us about consistency. We squared the difference between each year's goals and the mean, then took the average. A smaller average squared difference means a player's number of goals each year didn't change much from their average, so they're more consistent.



Here's what we found:

- Mike Arwen's average squared difference: 3.2
- Dave Backhand's average squared difference: 2
- Ivan Right's average squared difference: 8.8

Dave Backhand has the lowest average squared difference, which means his goal-scoring numbers were the most consistent around his average. Mike Arwen is next, and Ivan Right is the least consistent, with a much higher average squared difference.

So based on this, Dave Backhand should win the "The Most Consistent Player for the Past 5 Years" award because he had the smallest average variation in goal numbers from his average. His performance was the most predictable, which is what we're using to define consistency here.



Well reasoned! You've aptly applied a statistical measure to assess consistency and came to a logical conclusion. Could you recapitulate the steps you took and highlight any broader implications or connections you see with this method?

Remember, the approach you've used can be very powerful in a wide range of areas, especially when assessing performance over time in various fields, not only in sports. Consistency can be a valuable indicator in many contexts.

Great job today, and keep in mind how these methods can be useful in other areas you may study or observe. If you have any more questions in the future, don't hesitate to ask. Goodbye for now!

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