

The student is working on the following problem:

Let ABC be a right-angled triangle at A with AC=6cm and BC=12cm.

Determine $d(C, AB)$ the distance from C to the line (AB) and

show that $d(B, AC)$, the distance from B to the line (AC) is equal to $6 * \sqrt{3}$

The solution of this problem is:

ABC is a right-angled triangle at A so the hypotenuse is BC and sides AB and AC are perpendicular. First, $d(C, AB) = AC = 6\text{cm}$ (orthogonal projection).

Second, by the Pythagorean theorem, $BC^2 = AB^2 + AC^2$ so $AB^2 = BC^2 - AC^2$.

We compute $AB^2 = BC^2 - AC^2 = 12^2 - 6^2 = 144 - 36 = 108$ Since (AB) is perpendicular to (AC), A is the orthogonal projection of B on (AC), so $d(B, AC) = AB$. Thus, $d(B, AC) = AB = \sqrt{108} = \sqrt{6^2 * 3} = 6 * \sqrt{3}$



Hello! I am your tutor. Can you walk me through your solution?

Hey! So, I'm kinda stuck. I know that to find the distance from a point to a line I need to use some sort of perpendicular distance thing, but I'm not sure where to start. The triangle is right-angled at A so I guess that's useful somehow. Do I use Pythagoras to find AB? Then what?



Great! You're thinking along the right lines by considering the right angle at A and the Pythagorean theorem. The distance from a point to a line is indeed the shortest, or perpendicular, distance. Since you have a right-angled triangle and point C lies on the hypotenuse, let's consider the properties of perpendicular lines in this context.

How might the fact that ABC is right-angled at A help you determine the distance from C to line AB? What is the relationship between the side AC and the line AB?

Oh, I see what you're hinting at, I think. Since ABC is right-angled at A, that means AC is already perpendicular to AB. So, the distance from C to line AB would just be the length of AC, wouldn't it? That means $d(C, AB)$ is 6cm, right?





Exactly! You've got it. The side AC is the perpendicular distance from point C to line AB, so you have correctly identified $d(C, AB)$ as 6 cm.

Now, let's look at the distance from B to line (AC). How can you relate the sides of the triangle to find the length of AB, which you've correctly identified as the next step using the Pythagorean theorem? Remember that BC is the hypotenuse of the right-angled triangle.

Oh, so if AC is one side and it's 6 cm, and BC is the hypotenuse and it's 12 cm, then I can use the Pythagorean theorem to find AB. The Pythagorean theorem is $a^2 + b^2 = c^2$ where c is the hypotenuse. So I gotta solve $6^2 + AB^2 = 12^2$. This means:



$$36 + AB^2 = 144$$

Subtract 36 from both sides to get:

$$AB^2 = 108$$

So AB would be the square root of 108, which is:

$$AB = \sqrt{108}$$

$\sqrt{108}$ is $\sqrt{36 \times 3}$, which is $6\sqrt{3}$. So, $AB = 6\sqrt{3}$. But this is the length of AB, not $d(B, AC)$... now I'm stuck again. How do I find $d(B, AC)$? It must be the same idea with the perpendicular line cuz it's a right triangle.



That's a perfect application of the Pythagorean theorem to find AB. Now you have $AB = 6\sqrt{3}$ cm. For the distance from point B to line AC, think about the relationship between B and side AC. Given that angle A is a right angle, where would the perpendicular distance from B to side AC lie? What does this imply about the distance from B to line AC in the context of a right-angled triangle?

Oh, I think I get it now. Since angle A is a right angle, that means the line segment BA is already perpendicular to AC. This would mean that the distance from B to line AC is just the length of BA, which we found to be $6\sqrt{3}$ cm. So $d(B, AC)$ is $6\sqrt{3}$ cm too. Is that right?





Yes, that is correct! Since ABC is a right-angled triangle at A, side AB is perpendicular to side AC, which makes AB the distance from point B to the line (AC). Therefore, you've correctly determined that $d(B, AC) = AB = 6\sqrt{3}$ cm. Well done!

Awesome! That makes sense now. So I've found both distances, $d(C, AB) = 6$ cm and $d(B, AC) = 6\sqrt{3}$ cm. Thanks for helping me work it out! Was there anything else we needed to do with this problem?



You're welcome! It looks like you've solved the problem completely. You found both $d(C, AB)$, which is the perpendicular distance from point C to line AB, and $d(B, AC)$, the perpendicular distance from point B to line AC in the context of a right-angled triangle.

If you've understood how and why these distances are found this way, then you've accomplished the task. Just ensure you're comfortable with the concepts and could apply them to similar problems in the future. If you have any more questions or another topic you'd like to cover, feel free to ask!

Got it! I think I understand it now. The perpendicular distances really just end up being sides of the triangle when it's right-angled. I'll practice with some more problems like this to make sure I've got it down. No more questions for now. Thanks a bunch for your help! Goodbye!



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