

The student is working on the following problem:

Table 1. shows the number of goals that each football player striker scored between 2007 and 2011. Who is the most consistent player? Imagine a measure of "consistency" and justify why you think your method is correct.

Year	Mike Arwen	Dave Backhand	Ivan Right
2007	13	12	14
2008	12	14	10
2009	15	16	18
2010	17	15	18
2011	13	13	15

The solution of this problem is:

The concept of standard deviation is the usual way to measure consistency but it is not expected that the student knows it. Any measure proposed by the student to measure consistency is acceptable as long as it can be justified.

Example of canonical solution: computing the variance (or standard deviation) for each player: First, compute the mean:

Mean number of goals for Mike: 14

Mean number of goals for Dave: 14

Mean number of goals for Ivan: 15

Then, compute the sum of square deviations from the mean for each player.

Then divide by the number of data points to get the variance:

Variance for Mike: $12/5 = 3.2$

Variance for Dave: $10/5 = 2$

Variance for Ivan: $44/5 = 8.8$

So according to the variance, Dave is the most consistent player.



Hello! I am your tutor. Can you walk me through your solution?

Hey! So, I'm kinda confused about the problem. I get that we need to figure out who's the most consistent player based on their scores, but I don't really know what 'consistency' means in terms of math or how to calculate it with these numbers. Can you help me out? What's a good place to start?



It's great that you're looking to understand 'consistency' in a mathematical sense! Think about what consistency means to you in this context and how you might express that with the numbers you've been given. Share your thoughts on what a 'consistent' scorer would look like in terms of goals scored each year.

Oh, okay, if I think about consistency, a player who is consistent would have scores that don't change much from year to year. Like, they would score almost the same number of goals each season, not one year a lot and another year just a few.



So, from the numbers in the table, we could look for the player whose goals each year are closest to each other? I guess we could subtract the smallest number of goals from the highest for each player and see who has the smallest difference? What do you think?



That's an insightful starting point! You're considering the range – the difference between the highest and lowest scores for each player. Can you try applying your idea to the data in the table and explain how this might reflect consistency?

Sure! Let's check the range for each player's scores from 2007 to 2011:



For Mike Arwen:

Highest is 17 goals and lowest is 12 goals.

So, the range would be $17 - 12 = 5$.

For Dave Backhand:

Highest is 16 goals and lowest is 12 goals.

So, the range would be $16 - 12 = 4$.

For Ivan Right:

Highest is 18 goals and lowest is 10 goals.

So, the range would be $18 - 10 = 8$.

Based on the range, Dave Backhand has the smallest range of 4, which would mean he's the most consistent according to this measure. Mike Arwen's range is 5, and Ivan Right's is quite a bit bigger at 8. Does that sound like a good way to measure consistency?



That's a coherent approach using the range as a measure of consistency. Now that you have explored the range, can you consider other ways to incorporate all of the data points provided for each player to measure consistency? How might you account for each year's performance in relation to the others?

Hmm, okay... another way might be to look at the average number of goals scored by each player over the 5 years, and see how much each year's score deviates from that average. So, if a player's score is really close to the average every year, that could mean they're consistent.



Let me calculate the average (I think it's also called the 'mean') of each player's goals first. Here's how to do it:

For Mike Arwen:

Add up all the goals and then divide by 5 (since we have 5 years of data).

$$\text{Average} = (13 + 12 + 15 + 17 + 13)/5 = 70/5 = 14.$$

For Dave Backhand:

$$\text{Average} = (12 + 14 + 16 + 15 + 13)/5 = 70/5 = 14.$$

For Ivan Right:

$$\text{Average} = (14 + 10 + 18 + 18 + 15)/5 = 75/5 = 15.$$

Now that I have the averages, maybe I can subtract each year's number of goals from the average and see how much it varies? What should I do next to find out who's the most consistent?



You're navigating through this quite well. Given that you have the average number of goals for each player, how might you express mathematically the idea of calculating the variation of each year's goals from the average?

Oh, okay... To express mathematically how much each year's goals vary from the average, we could do something like taking the difference between each year's number of goals and the player's average, then maybe adding those differences up for each player? But wait, some differences would be negative and some positive... Hmm, we can't have that canceling out.



Maybe we square those differences so they are all positive, is that right? Then add them up to get a total that reflects how much the scores changed from the average for each player. That could show us consistency, maybe? How does it sound?



Yes, you're on the right track with the notion of squaring the differences to eliminate negative numbers. After squaring these differences, what will your next step be? Consider how you could use the total sum of these squared differences to represent each player's consistency.

Got it! So we take the differences, square them, and then add them up to get a total for each player. This total will tell us something about their consistency. The smaller the total, the more consistent the player is, right?



So what we're talking about is actually variance, isn't it? Even though I don't know that term very well, it sounds like it fits the concept. Like, variance would measure how much the values vary from the mean.

Okay, let's calculate this for each player.

For Mike Arwen:

We already have the average, which is 14.

So we take each score, subtract 14, square the result, and then sum these up.

Variance for Mike Arwen: $[(13 - 14)^2 + (12 - 14)^2 + (15 - 14)^2 + (17 - 14)^2 + (13 - 14)^2] = [(-1)^2 + (-2)^2 + (1)^2 + (3)^2 + (-1)^2] = [1 + 4 + 1 + 9 + 1] = 16.$

Should I do the same for the other two players? Is this what we should be doing?



Yes, you're employing the method correctly for Mike Arwen. Keep in mind to complete the process of calculating variance, after summing the squared differences, you'll want to divide by the number of data points.

Could you continue this process for the other two players, and then evaluate who is the most consistent based on your variance calculations?

After finishing your calculations, think about what this number is telling you in the context of consistency among players.

Okay! Let's do the variance calculation for Dave Backhand and Ivan Right.



For Dave Backhand:

His average is also 14.

Variance for Dave Backhand: $[(12 - 14)^2 + (14 - 14)^2 + (16 - 14)^2 + (15 - 14)^2 + (13 - 14)^2] = [(-2)^2 + (0)^2 + (2)^2 + (1)^2 + (-1)^2] = [4 + 0 + 4 + 1 + 1] = 10$.

Before we divide by the number of data points, I'll calculate for Ivan Right.

For Ivan Right:

His average is 15.

Variance for Ivan Right: $[(14 - 15)^2 + (10 - 15)^2 + (18 - 15)^2 + (18 - 15)^2 + (15 - 15)^2] = [(-1)^2 + (-5)^2 + (3)^2 + (3)^2 + (0)^2] = [1 + 25 + 9 + 9 + 0] = 44$.

Okay, now let's divide these by the number of data points, which is 5 (for the 5 years).

Mike Arwen's Variance: $\frac{16}{5} = 3.2$.

Dave Backhand's Variance: $\frac{10}{5} = 2$.

Ivan Right's Variance: $\frac{44}{5} = 8.8$.

Looking at these calculated variances, Dave Backhand has the smallest variance, which means he has the lowest variation in his goal scores around the average, suggesting he's the most consistent according to this measure. Mike Arwen would be next, and Ivan Right has the largest variance, indicating he's the least consistent.

So this is telling us that Dave Backhand maintains goal-scoring closer to his average each year when compared to Mike Arwen and Ivan Right, hence Dave is the most consistent. Is that a good way to explain it? 😊



Exactly, you've done an excellent job working through the calculations and interpreting the results. Your understanding of the concept of variance and how it relates to consistency is right on point. Great work today! If you have any more questions in the future, don't hesitate to ask. Goodbye and take care!

Thanks a lot for your help! I actually understood it better now. I don't have any more questions for now, but I'll definitely reach out if I need help again. Goodbye!



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