

The student is working on the following problem:

Let ABC be a right-angled triangle at A with AC=6cm and BC=12cm.

Determine $d(C, AB)$ the distance from C to the line (AB) and

show that $d(B, AC)$, the distance from B to the line (AC) is equal to $6 * \sqrt{3}$

The solution of this problem is:

ABC is a right-angled triangle at A so the hypotenuse is BC and sides AB and AC are perpendicular. First, $d(C, AB) = AC = 6\text{cm}$ (orthogonal projection).

Second, by the Pythagorean theorem, $BC^2 = AB^2 + AC^2$ so $AB^2 = BC^2 - AC^2$.

We compute $AB^2 = BC^2 - AC^2 = 12^2 - 6^2 = 144 - 36 = 108$ Since (AB) is perpendicular to (AC), A is the orthogonal projection of B on (AC), so $d(B, AC) = AB$. Thus, $d(B, AC) = AB = \sqrt{108} = \sqrt{6^2 * 3} = 6 * \sqrt{3}$



Hello! I am your tutor. Can you walk me through your solution?

um, so i guess I'm a bit confused about where to start but i think first we need to find the hypotenuse AB of the triangle ABC?



so since AC is 6 cm and BC is 12 cm and it's a right angled triangle, we can use Pythagoras theorem to find AB.

It should be like $AB^2 = AC^2 + BC^2$

So plugging in the values it's like $AB^2 = 6^2 + 12^2 = 36 + 144 = 180$

Which means $AB = \sqrt{180}$, so after simplifying that it would be $AB = 6\sqrt{5}$

Is this right so far?



Very good start on attempting to use the Pythagorean theorem – you've got the right idea there! However, take another look at the formula and the values you're plugging in. What is the relationship between AC, BC, and AB in the context of the Pythagorean theorem for a right-angled triangle? Remember, one of these sides is the hypotenuse. Can you identify which one it is and revisit your equation accordingly?

oh oops, my bad. Since it's a right-angled triangle at A, BC should be the hypotenuse. So the equation should be $BC^2 = AC^2 + AB^2$



So it is $BC^2 = 6^2 + AB^2$

We know BC is 12 cm so $12^2 = 6^2 + AB^2$ that means $144 = 36 + AB^2$

So $AB^2 = 144 - 36 = 108$ which means $AB = \sqrt{108}$ and I know that could be simplified to

$AB = 6\sqrt{3} \times 2$ which is same as $AB = 12\sqrt{3}$ cm. I hope this one is correct?



Yes, you've rearranged the equation correctly to solve for AB . Now, let's go back and simplify $\sqrt{108}$ properly. You're on the right track with simplification. How can you factor 108 to make it easier to take the square root? Look for a perfect square factor within 108. What do you get when you break it down?

Okay, I think I jumped the gun there. We have $AB^2 = 108$, so $AB = \sqrt{108}$.



To break it down 108 can be factored into 36×3 since 36 is a perfect square. So it would look like:

$$AB = \sqrt{36 \times 3} = \sqrt{36} \times \sqrt{3} = 6 \times \sqrt{3}$$

So simplifying I got,

$AB = 6\sqrt{3}$ cm. Alright, this should be the correct simplification now.



Exactly, you've now correctly simplified the square root. Great job on working through that step! Can we recap what you've discovered so far and then think about the distances from points to lines in the context of this triangle? How might we relate what we have found to finding the distance from C to line AB, and from B to line AC?

Sure! So we found that in the right-angled triangle ABC, with right angle at A, AC is 6 cm, BC is 12 cm, and AB is $6\sqrt{3}$ cm.



Now for finding the distance $d(C, AB)$, that's the shortest distance from point C to the line AB, which is the perpendicular distance. Since triangle ABC is right-angled at A, we already have this distance – it's just the length AC, which is 6 cm.

For $d(B, AC)$, the distance from B to line AC, it would be another perpendicular dropped from point B to line AC. Since we're dropping a perpendicular from B to AC, this will create another right-angled triangle.

To find $d(B, AC)$, we can use the area of the triangle ABC. There are two ways to express the area of triangle: one-half of the base times height. Here, one can consider AC as the base and $d(B, AC)$ as the height or vice versa.

So area of triangle ABC can be expressed as

$$Area = \frac{1}{2} \times AC \times d(B, AC)$$

or

$$Area = \frac{1}{2} \times AB \times AC.$$

Since we know AC and AB, we should be able to calculate the area and then solve for $d(B, AC)$. Should we do this next?



It sounds like you have a strong understanding of the concepts involved and how we might proceed.

Recapping, you've skillfully found that $AB = 6\sqrt{3}$ cm and deduced that $d(C, AB) = AC = 6$ cm. You also started thinking about the area of triangle ABC for finding $d(B, AC)$. You have all the pieces you need.

Reflect on what you've learned so far. Think about how these concepts relate generally to right-angled triangles and how the properties of perpendicular lines and the Pythagorean theorem can be used in a variety of problems involving triangles and distances. Working through this process is a great way to build a deeper understanding.

I believe you're equipped to complete the problem from here. Take your time to think it through, and don't hesitate to review the steps you've taken to solve for the missing parts. If you have any questions in the future or need further clarification, feel free to ask.

Goodbye and good luck with your future studies!

Alright, thanks! I'll think about that and try to work it out. See you next time! Goodbye!



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