

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Hey! So umm, I'm kinda lost with this. I think that I need to give a number of seats based on like how big the population is for each state but I'm not sure how to do that exactly. Any tips on how to start?



Certainly! Think about how you might begin by considering all the states together. What might be the first step in understanding how to distribute the seats fairly in relation to the populations of the states?

Okay, I think the first step is to find the total population of the country by adding all the populations of the states together. So let me do that real quick. Adding up $1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$. Lemme get my calculator...



That sounds like a good plan! Once you have the total population, think about what that number can tell you in terms of seats per individual. How could you use that to figure out a fair distribution?

Alright, so the total population would be $1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 = 12,500,000$.



Next, I think I need to find out how many people each seat represents by dividing the total population by the number of seats, which is 250. So let me do that division next. Here goes: $12,500,000 \div 250 = 50,000$. So like, 1 seat represents 50,000 people, right?



That's correct! Each seat represents 50,000 people in your calculation. What will you do with that information? Consider how it relates to the population of each state.

Oh...so I think I need to divide the population of each state by 50,000 to find out how many seats each state should get. So, like for state A, it would be $1,646,000 \div 50,000$. I'll calculate that next. Give me a sec...



That's right; you're on a good track. After doing that for state A, remember to do the same for all other states as well. How do you think you should handle any fractions or decimals that result from these divisions?

Alright, so for state A it's $1,646,000 \div 50,000$ which gives us 32.92.



For any decimals, I guess I should round to the nearest whole number cause you cannot have a part of a seat. But if I do that for each state, won't I end up with possibly more or less than 250 seats total? What should I do about that?



Could you perhaps consider what options you might have for dealing with the decimals? How might you ensure that the total doesn't exceed or fall short of 250?

Hmm, maybe like if I round down for all the states to get rid of the decimals, right? Then I can add up all those seats and see if I hit 250 or not. If I have extra seats left over, I guess I can give them to the states with the largest remainders from the rounding. I'm gonna calculate the rounded down number of seats for each state first. Here goes:



For state A: $1,646,000 \div 50,000$ gives 32 seats (ignoring the .92 for now).

I'll do the same for the other states and keep track of the decimals to see which states have the largest ones. Lemme calculate that really quick...



That sounds like a solid plan. Once you've calculated the initial number of seats for each state by rounding down, you can then assess how many seats are left to distribute and decide how to allocate them according to the remainders. How do you think this process helps to ensure each state receives fair representation? After completing the calculations, can you summarize your approach and findings?

Okay so rounding down the seats for each state I get:



State A: $1,646,000 \div 50,000$ gives 32 seats.

State B: $6,936,000 \div 50,000$ gives 138 seats.

State C: $154,000 \div 50,000$ gives 3 seats.

State D: $2,091,000 \div 50,000$ gives 41 seats.

State E: $685,000 \div 50,000$ gives 13 seats.

State F: $988,000 \div 50,000$ gives 19 seats.

Adding these up: $32+138+3+41+13+19 = 246$ seats.

This leaves us 4 seats short of 250. So, now I would look at the decimals I ignored and give the extra seats to the states with the largest remainders. This ensures each state gets as fair a representation as possible.

I guess what I found is, you divide the state populations by the number of people each seat should represent, here it's 50,000, round down, and distribute any leftover seats by looking at the remainders. Lemme check those remainders now.



Your methodology seems very well thought out. After assigning the initial seats by rounding down, you addressed the leftover seats with a clear and systematic approach – looking at the remainders. This is a good illustration of how mathematical methods can be applied to real-world problems, such as ensuring fair representation in a legislative body. Can you recapitulate in your own words the steps you followed and what this method achieves in the broader context of proportional representation? Once you do that, you'll have a solid understanding of the concept.

Sure! Here's a recap of what I did:



1. First, I found the total population of the country by adding up the populations of each state.
2. Then, I figured out how many people each of the 250 legislative seats should represent by dividing the total population by 250.
3. Next, I divided the population of each state by the number of people per seat to see how many seats they'd get if everything was perfect.
4. Since I can't divide seats into parts, I rounded down to get a whole number of seats for each state.
5. After giving out the seats by rounding down, I ended up with fewer than 250 seats in total.
6. To fix this, I looked at the decimal parts I'd chopped off when rounding down and gave the leftovers to the states with the biggest remainders – till all 250 seats were fairly distributed.

This method ensures that the representation is proportional: states with bigger populations get more seats, although it's not perfect due to rounding. But then, by dealing with the remainders, we make it as fair as possible given the whole number of seats we have. With this, every state gets a number of seats that closely reflects its population size in the country's legislative body.



You've grasped the concept very well! It's been a pleasure to explore this problem with you. If you have any more questions in the future or need further clarification, feel free to ask. Goodbye and good luck with your studies!

Thanks so much for your help! I definitely understand it better now. Goodbye and I'll for sure reach out if I get stuck on more math stuff in the future. Bye!



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