

The student is working on the following problem:

Let ABC be a right-angled triangle at A with AC=6cm and BC=12cm.

Determine $d(C, AB)$ the distance from C to the line (AB) and

show that $d(B, AC)$, the distance from B to the line (AC) is equal to $6 * \sqrt{3}$

The solution of this problem is:

ABC is a right-angled triangle at A so the hypotenuse is BC and sides AB and AC are perpendicular. First, $d(C, AB) = AC = 6\text{cm}$ (orthogonal projection).

Second, by the Pythagorean theorem, $BC^2 = AB^2 + AC^2$ so $AB^2 = BC^2 - AC^2$.

We compute $AB^2 = BC^2 - AC^2 = 12^2 - 6^2 = 144 - 36 = 108$ Since (AB) is perpendicular to (AC), A is the orthogonal projection of B on (AC), so $d(B, AC) = AB$. Thus, $d(B, AC) = AB = \sqrt{108} = \sqrt{6^2 * 3} = 6 * \sqrt{3}$



Hello! I am your tutor. Can you walk me through your solution?

Hey! So, the problem is about a right triangle ABC where AC is 6 cm and BC is 12 cm. I'm not sure where to start to find the distance from C to the line AB. Can you help me with that?



Sure! Consider the properties of a right-angled triangle. Can you identify which line in the triangle is perpendicular to the line AB?

Oh, since it's a right triangle with a right angle at A, the side AC would be perpendicular to the line AB. That means side AC is the perpendicular distance from C to the line AB, right? So, $d(C, AB)$ is just AC which is 6 cm?



That's an interesting observation you've made! Can you explain why the fact that AC is perpendicular to AB means that the distance from C to the line AB is the length of AC? How is distance defined in this context?

The distance from a point to a line is the shortest distance between them, which is along the perpendicular. Since AC is already perpendicular to AB and C is a point on AC, then the length of AC is the shortest distance from C to the line AB. So, $d(C, AB)$ is equal to the length of AC which is 6 cm.





Excellent reasoning! Now, can you tackle the second part of the problem regarding the distance from B to the line AC using the information you've just discussed? What might be a good starting point?



Alright, for the second part, we need to find $d(B, AC)$, which is the distance from point B to the line AC. For this, we probably need to find the length of the perpendicular from B to AC. Since ABC is a right triangle, angle C is 90 degrees, and that means AB is the hypotenuse.

Because we have two sides of the right triangle (AC and BC), I can use the Pythagorean theorem to find AB. So, let's do that first:

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 6^2 + 12^2$$

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

$$AB = \sqrt{180}$$

$$AB = \sqrt{36 \times 5}$$

$$AB = 6\sqrt{5}$$

Now I need to use this to find $d(B, AC)$. Hmm... I'm not exactly sure how to find that perpendicular distance. What should I do next?



You're on the right track with using the Pythagorean theorem, but I noticed a small discrepancy in your calculations. Let's go back to the step where you apply the theorem. You correctly stated:

$$AB^2 = AC^2 + BC^2$$

You then substituted AC and BC with 6 cm and 12 cm respectively. Can you try performing those calculations again? Remember to look closely at each step for arithmetic accuracy.

Oops, my bad! Let's try it again.



$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 6^2 + 12^2$$

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

I made a mistake here. It should be:

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

The correct calculation should be:

$$AB^2 = 36 + 144$$

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

Wait, that seems wrong again. Let's recalculating that correctly:

$$AB = \sqrt{36 + 144}$$

$$AB = \sqrt{180}$$

Oh, I made a mistake in factoring it out. The correct factorization should be:

$$AB = \sqrt{9 \cdot 20}$$

$$AB = 3\sqrt{20}$$

$$AB = 3\sqrt{4 \cdot 5}$$

$$AB = 3 \cdot 2\sqrt{5}$$

$$AB = 6\sqrt{5}$$

Now we have the correct length for AB. How should we proceed to find d(B,AC)?



It looks like there might still be a small mix-up. Let's carefully revisit that Pythagorean theorem calculation one more time to ensure we have it right. Take each step carefully:

$$AB^2 = AC^2 + BC^2$$

Substitute in the lengths you know:

$$AB^2 = 6^2 + 12^2$$

Can you recalculate that? What is 6^2 and what is 12^2 ? After you find those, add them together and find the square root to get AB.

Okay, let's go through the calculation one more time, carefully.



$$AB^2 = AC^2 + BC^2$$

Substituting the lengths:

$$AB^2 = 6^2 + 12^2$$

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

Now, to find AB we take the square root of 180:

$$AB = \sqrt{180}$$

$$AB = \sqrt{36 \cdot 5}$$

$$AB = 6\sqrt{5}$$

Now that we have the length of AB, we want to find the distance from B to the line AC. This distance would be represented by a line segment from B that is perpendicular to AC. In a right-angled triangle, the altitude to the hypotenuse (which is what we're describing here) also creates two smaller right-angled triangles that are similar to the original triangle. So, we can relate sides through these similar triangles to find the length. However, in this case, there's a quicker way to find the distance from B to AC, which makes use of the area of the triangle.

We can calculate the area of the triangle in two ways: $\frac{1}{2} \cdot \text{base} \cdot \text{height}$ and also by applying Heron's formula.

Since we know the base and height as AC and BC respectively, let's calculate the area using that method:

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

$$\text{Area} = \frac{1}{2} \cdot AC \cdot BC$$

$$\text{Area} = \frac{1}{2} \cdot 6 \text{ cm} \cdot 12 \text{ cm}$$

$$\text{Area} = 36 \text{ cm}^2$$

Now we can use this area to find $d(B, AC)$, keeping in mind that the altitude (or the perpendicular distance from a point to the opposite side) will also form the base of another expression for the area of the triangle:

$$\text{Area} = \frac{1}{2} \cdot AB \cdot d(B, AC)$$

Plugging in the area we found and the length of AB:

$$36 \text{ cm}^2 = \frac{1}{2} \cdot 6\sqrt{5} \text{ cm} \cdot d(B, AC)$$

Now we can solve for $d(B, AC)$:

Now we can solve for $d(B, AC)$.

$$d(B, AC) = 72 \text{ cm}^2 / (6\sqrt{5} \text{ cm})$$

$$d(B, AC) = 12\sqrt{5} \text{ cm}^2 / \sqrt{5}$$

$$d(B, AC) = 12 \text{ cm}$$

Okay, there it is. That's not what I expected,



You're doing well with the process and using the formula for the area of a triangle. However, I would encourage you to re-examine your calculations, especially the last step where you're finding $d(B, AC)$.

Perhaps you should consider the simplification process once more, and the properties of square roots when you're dividing by a square root. What does simplifying that expression give you?

You're right, I made a mistake simplifying the expression for $d(B, AC)$. Let's try that again.



We have:

$$\text{Area} = 36 \text{ cm}^2 = 1/2 * AB * d(B, AC)$$

Filling in the values we have:

$$36 \text{ cm}^2 = 1/2 * 6\sqrt{5} \text{ cm} * d(B, AC)$$

To solve for $d(B, AC)$, we'll multiply both sides by 2 and then divide by $6\sqrt{5} \text{ cm}$:

$$d(B, AC) = 72 \text{ cm}^2 / (6\sqrt{5} \text{ cm})$$

$$d(B, AC) = (72 / 6) \text{ cm}^2 / \sqrt{5}$$

$$d(B, AC) = 12 \text{ cm}^2 / \sqrt{5}$$

$$d(B, AC) = 12 / \sqrt{5} \text{ cm}$$

Now to rationalize the denominator:

$$d(B, AC) = (12 / \sqrt{5}) * (\sqrt{5} / \sqrt{5}) \text{ cm}$$

$$d(B, AC) = 12\sqrt{5} / 5 \text{ cm}$$

Now I can see my previous calculation wasn't simplified correctly. This is the distance from B to the line AC.



That was a thoughtful calculation. Now that you have an expression for $d(B, AC)$, it might be helpful to go back to the earlier part of the problem where you calculated the length of AB . Look at what you found for AB again, and think about how that length compares to the expression you just found for $d(B, AC)$. Is there anything about the way AB was calculated that could relate to what you're trying to find here?

You're right, the way AB was calculated might help. AB was calculated to be $6\sqrt{5}$ cm.



I have the distance from B to AC, which I found to be $d(B, AC) = \frac{12\sqrt{5}}{5}$ cm.

Looking at these two expressions, I see AB is the hypotenuse of the right triangle, and $d(B, AC)$ is a height (altitude) drawn to the hypotenuse. By geometric properties of a right triangle, the product of the lengths of the two altitudes is equal to the square of the length of the altitude from the right angle, which in our case is AC, with a length of 6 cm.

$$\text{So, } d(B, AC) \times d(C, AB) = AC^2$$

Let's substitute in the known values:

$$\frac{12\sqrt{5}}{5} \times 6 = 36$$

Simplifying the expression on the left:

$$\frac{12\sqrt{5} \times 6}{5} = 36$$

$$\frac{72\sqrt{5}}{5} = 36$$

Now we can simplify the fraction by dividing both sides by 12:

$$\frac{6\sqrt{5}}{5} = 3$$

Then multiply both sides by 5:

$$6\sqrt{5} = 15$$

Looks like I went wrong somewhere again. Gonk! Let's try one more time:

$$\frac{12\sqrt{5} \times 6}{5} = 36$$

$$12\sqrt{5} \times \frac{6}{5} = 36$$

$$\frac{72\sqrt{5}}{5} = 36$$

Now, let's find the value for $\sqrt{5}$:

$$\sqrt{5} = \frac{36 \times 5}{72}$$

$$\sqrt{5} = \frac{180}{72}$$

$$\sqrt{5} = \frac{5}{2}$$

Which does not make sense since $\sqrt{5}$ is irrational. I see now that I've made an error in assuming that the product of the distances is equal to AC^2 . That property actually applies to the segments of the hypotenuse created by the altitude.

The actual relationship is $d(B, AC) * AB = AC^2 = 6^2$



That's a great effort in trying to piece together the relationship between the parts of the triangle. Let's pause for a moment and think carefully about the relationships in a right triangle with an altitude drawn from the right angle vertex.

You mentioned that the product of the distances is related to the length of one of the sides squared. Could you reflect on the properties of the right triangle you have and the significance of the altitude in relation to the hypotenuse and the other two sides? What specific relationship do the segments of the hypotenuse have with the altitude, and how might that relate to the area of the triangle? Take your time and consider these relationships carefully.

Okay, I might have been mixing up the geometric relationships. In a right-angled triangle, if an altitude is drawn to the hypotenuse, it creates two smaller right-angled triangles. Each of those smaller triangles is similar to the original triangle, and also to each other.



Now, if we look at the original triangle ABC and the altitude from B to AC, which we'll call h, this altitude creates two right triangles: ABC and ABD, where D is the foot of the altitude on AC.

For these two triangles, the following relationships exist:

- The altitude h is the geometric mean of the segments it creates on the hypotenuse (let's call the segments p and q, where p is from A to D and q is from D to C).
- The altitude h also relates to the sides of the triangle in that the altitude h squared will be equal to the product of the segments of the hypotenuse it creates, that is, $h^2 = pq$.
- The hypotenuse AB of the larger right-angled triangle is the geometric mean of the whole side it subtends (AC in our case) and the whole side opposing it.

Now we know AC = 6 cm (our h, as it's perpendicular to AB), and we are looking for d(B, AC) which would be h in the triangle ABD where the hypotenuse is AB.

Since the triangle is right-angled at A, the altitude from B to AC can be considered the height when AB is the base. We can use the area of the triangle to link $d(B, AC)$ to the other sides, with the area given by $\frac{1}{2} * \text{base} * \text{height}$.

The area, A of the triangle ABC would be:

$$A = \frac{1}{2} * AC * BC$$

For our triangle with sides:

AC = 6 cm (height)

BC = 12 cm (base)

The area is then:

$$A = \frac{1}{2} * 6 \text{ cm} * 12 \text{ cm} = 36 \text{ cm}^2$$

This area can also be expressed in terms of the hypotenuse AB and the altitude from B (our unknown distance d(B, AC)):

$$A = \frac{1}{2} * AB * d(B, AC)$$

Equating the two expressions for the area, we have:
 36 cm



That's quite a thorough analysis using the properties of similar triangles and the altitude's relationship to the hypotenuse. It's important to carefully use those relationships to avoid any potential mix-ups.

Now, could you try to write down mathematically how the area of triangle ABC (which is right-angled at A) can be expressed using both sets of sides (AC and BC, and then AB and your to-be-determined altitude from B)?

Once you have those equations set up, think about how you can manipulate them to solve for the distance from B to the line AC. Remember to consider the area you've already calculated, the length of AB you've determined, and how these relate to the altitude from B to line AC.

Please write down what you come up with, and let's review that together.

Sure, let's start by writing down mathematically how to express the area of triangle ABC with both sets of sides.



First, using sides AC and BC, where AC is the height:

$$A = \frac{1}{2} \cdot AC \cdot BC = \frac{1}{2} \cdot 6 \cdot 12 = 36 \text{ cm}^2$$

Next, using the hypotenuse AB (which we found to be $6\sqrt{5}$ cm) and the altitude from B ($d(B, AC)$):

$$A = \frac{1}{2} \cdot AB \cdot d(B, AC) = \frac{1}{2} \cdot 6\sqrt{5} \cdot d(B, AC)$$

Equating the two expressions for the area gives us:

$$36 \text{ cm}^2 = \frac{1}{2} \cdot 6\sqrt{5} \cdot d(B, AC)$$

Solving for $d(B, AC)$:

$$d(B, AC) = \frac{36 \cdot 2}{6\sqrt{5}}$$

$$d(B, AC) = \frac{72}{6\sqrt{5}}$$

$$d(B, AC) = \frac{12}{\sqrt{5}}$$

To rationalise the denominator, we multiply by $\frac{\sqrt{5}}{\sqrt{5}}$:

$$d(B, AC) = \frac{12\sqrt{5}}{5}$$

Now we have rationalised the expression for the distance from B to the line AC as $d(B, AC) = \frac{12\sqrt{5}}{5}$.

Reverse checking with the area using AB and $d(B, AC)$:

$$A = \frac{1}{2} \cdot AB \cdot d(B, AC) = \frac{1}{2} \cdot 6\sqrt{5} \cdot \frac{12\sqrt{5}}{5} = \frac{1}{2} \cdot \frac{72 \cdot 5}{5} = 36 \text{ cm}^2$$

The area is consistent, so we've verified that indeed $d(B, AC) = \frac{12\sqrt{5}}{5}$.



Great job on solving for $d(B, AC)$ and rationalizing the denominator! Now, let's go back and review each aspect of your final expression for $d(B, AC)$, making sure that it makes sense with what you know about the triangle and the lengths of its sides. Does everything check out?

Yes, let's review.



The final expression for $d(B, AC)$ is $d(B, AC) = \frac{12\sqrt{5}}{5}$.

Let's check this against what we know about right triangles and their properties:

1. The triangle ABC is a right-angled triangle at A.
2. AC and BC are perpendicular, making them the height and base for calculating the area with respect to the hypotenuse AB.
3. The distance from a vertex to the line containing the opposite side (in a right-angled triangle) is calculated using the altitude from the vertex to the hypotenuse, which is also the corresponding height when the hypotenuse is considered the base of the triangle.
4. $d(B, AC)$ was found by using the area of the triangle calculated in two different ways, first with base and height as AC and BC and then with base and height as AB and the altitude we wanted to find.

Since the expression for the triangle's area matches using both sets of measurements, the calculated $d(B, AC)$ seems to be correct.

Additionally, we should recall that according to the properties of the geometric mean in right triangles:

- The altitude to the hypotenuse ($d(B, AC)$ in our triangle) is the geometric mean of the projections of the two legs onto the hypotenuse. This altitude divides the hypotenuse into two segments, which are not relevant to our current calculation, but they are related to the complete sides of the triangle.

Since I already correctly used the triangle's area to connect AB and $d(B, AC)$, and since the numbers match, the expression $\frac{12\sqrt{5}}{5}$ seems valid as the distance from B to the line AC. It confirms the formula relating the area, hypotenuse, and an altitude in a right triangle, where the area was calculated to be 36 cm^2 , and the hypotenuse AB was found to be $6\sqrt{5} \text{ cm}$.

Thus, everything checks out with the final expression for $d(B, AC)$.



You've carefully considered the properties of right triangles and the relationships between the sides and the area. Now, take a moment to reflect on the whole problem again, from the calculation of AB 's length to determining $d(B, AC)$. Review each step and the result you obtained to ensure everything is consistent and correct.

Sure, let's review the entire problem step by step:



1. We have a right-angled triangle ABC with a right angle at A, $AC = 6$ cm, and $BC = 12$ cm.

2. We calculated the length of AB, the hypotenuse, using the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 6^2 + 12^2$$

$$AB^2 = 36 + 144$$

$$AB^2 = 180$$

$$AB = \sqrt{180}$$

We factored 180 into 36×5 to simplify the square root:

$$AB = \sqrt{36 \times 5}$$

$$AB = 6\sqrt{5} \text{ cm}$$

3. We used the fact that side AC is an altitude from C to AB to determine that $d(C, AB) = AC = 6$ cm.

4. Next, we needed to find $d(B, AC)$, so we considered the area of triangle ABC was constant and could be expressed in two ways. We had the area using the base and height (BC and AC):

$$\text{Area } A = \frac{1}{2} \cdot AC \cdot BC = \frac{1}{2} \cdot 6 \cdot 12 = 36 \text{ cm}^2$$

5. Then we expressed the area using AB as the base and $d(B, AC)$ as the height:

$$\text{Area } A = \frac{1}{2} \cdot AB \cdot d(B, AC)$$

Thus:

$$36 = \frac{1}{2} \cdot 6\sqrt{5} \cdot d(B, AC)$$

6. We solved for $d(B, AC)$:

$$d(B, AC) = \frac{72}{6\sqrt{5}}$$

To simplify it, we rationalized the denominator:

$$d(B, AC) = 12/\sqrt{5} = 12\sqrt{5}/5 \text{ cm}$$

Each step seems logical and consistent. We found the distance from C to AB is 6 cm, and we've shown that the distance from B to AC is $\frac{12\sqrt{5}}{5}$ cm. The calculation for AB's length was verified with the Pythagorean theorem and all intermediate steps appear to be correct.

The only remaining part that I initially said incorrectly in the beginning of



Indeed, you've clearly outlined the steps of your solution. It's important that we reflect on the first assertion you made regarding the triangle and what was provided in the original statement of the problem. Take a close look at your final result for the value of $d(B, AC)$, considering the properties of a right-angled triangle at A and the length of AB that you calculated using the Pythagorean theorem. Does your result for $d(B, AC)$ logically fit with the lengths of the sides of the triangle and their relationship in a right-angled triangle? What can you infer about the distances in a right-angled triangle concerning the sides and the altitude?

Reflecting on the problem again:



1. We know that ABC is a right-angled triangle at A.
2. AC is the altitude of triangle ABC relative to base AB, making $AC = d(C, AB) = 6$ cm.
3. We calculated that $AB = 6\sqrt{5}$ cm using the Pythagorean theorem.
4. We are asserting that the altitude from B to line AC ($d(B, AC)$) is $\frac{12\sqrt{5}}{5}$ cm.

Considering the relationships in a right-angled triangle, especially concerning the altitude from vertex to the hypotenuse, we can note the following:

- The altitude to the hypotenuse of a right-angled triangle (triangle ABC, in this case) creates two smaller triangles (triangle ABD and triangle BDC) that are similar to each other and also to the original triangle (triangle ABC).
- The altitude ($d(B, AC)$) from B to AC creates a right-angled triangle ABD with AB as the hypotenuse and AD as one of its legs.
- The length of the altitude ($d(B, AC)$) also has a relationship with the segments it divides on the hypotenuse (AB) -- it is the geometric mean of these two segments.
- Moreover, the altitude ($d(B, AC)$) from the vertex (B) of the right angle (ABC) onto the hypotenuse (AB) is the geometric mean of the projections of the sides onto the hypotenuse.

Therefore, if the sides AC and BC of right-angled triangle ABC are 6 cm and 12 cm respectively, and AB is $6\sqrt{5}$ cm, then $d(B, AC)$ can also be considered in terms of the geometric mean of the two segments it divides on AB. However, it is not straightforward to determine these segments without further information from the problem.

What we can reliably use is the uniformity of the area calculated by the triangle of sides AC and BC with AB as the hypotenuse and altitude

the two pairs of sides (altitude-height with AC and BC, and altitude-height with AB and $d(B, AC)$). This has enabled us to determine that $\frac{12\sqrt{5}}{5}$ cm is indeed the altitude from B to AC.

To see if it logically fits with the rest of the triangle, it might help to express the altitude as $d(B, AC) = \frac{12\sqrt{5}}{5} = 2.4\sqrt{5}$



You've made a strong effort so far in understanding the relationships between different parts of the right triangle. Now, focus on what you have determined for $d(B, AC)$ and compare it to the lengths of the sides AC and BC. Given that triangle ABC is a right triangle and that the sides AC and BC are integers, what might you expect about the simplicity or complexity of the distance from B to the straight line AC? Is there a simpler form that $d(B, AC)$ could be expressed in, considering the properties of the triangle you've been working with?

Assuming you have correctly found that AB is $6\sqrt{5}$ cm and you have used the area of the triangle to find the altitude from B to line AC, think about whether the result of $d(B, AC)$ fits with the pattern of the sides' lengths being integer values. Consider simplifying your result or rechecking your calculation against the properties of right triangles you noted earlier.

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