

**The student is working on the following problem:**

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

**The solution of this problem is:**

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A:  $1,646,000 / 50,000 = 32.92 \rightarrow 32$  seats
- B:  $6,936,000 / 50,000 = 138.72 \rightarrow 138$  seats
- C:  $154,000 / 50,000 = 3.08 \rightarrow 3$  seats
- D:  $2,091,000 / 50,000 = 41.82 \rightarrow 41$  seats
- E:  $685,000 / 50,000 = 13.70 \rightarrow 13$  seats
- F:  $988,000 / 50,000 = 19.76 \rightarrow 19$  seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82
- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Hey! Tbh I'm not really sure where to start with this one. How do I figure out how many seats to give each state?





That's quite alright; it can be a complex issue to tackle. When faced with a situation like this, we aim for fairness in representation. One typical approach is proportional representation, where each state gets a share of the seats that corresponds with its share of the overall population.

Could you share your intuition about how to distribute the seats in relation to the populations of the states? If you were to express that mathematically, what might it look like?

Okay, I think I kinda get what you mean. So like if a state has more people, it should get more seats, right? I guess I could add up all the populations to see the total, and then find out what percentage of the total population each state has? Then maybe give out the seats based on that?



Yes, that sounds like a solid line of thinking! Why don't you try expressing that mathematically? What might be your first step in this process?

Alright, my first step then would be to add up all the populations of the states to get the total population of the country. Let me do that:



Total population = Population of state A + Population of state B + Population of state C + Population of state D + Population of state E + Population of state F

Total population = 1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000

Gonna calculate that now...



Fantastic. Let me know what total you come up with.

Okay, after adding them all up:



Total population = 1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000

Total population = 12,500,000

That should be the total population of the country. Is that right?



Yes, now you have the total population. What do you think your next step should be, considering you have 250 seats to distribute based on this population?

The next step is to figure out the fair share of seats for each state. I guess for that, I'd calculate the percentage of the total population each state has and then apply that percentage to the 250 seats.



So for any state, I would do:

Percentage of total population for a state = (Population of the state / Total population) \* 100

Then, to find the number of seats for the state, I would do:

Seats for the state = (Percentage of total population for the state) \* (Total number of seats/100)

Does that sound right?



You're on the right path with looking at the percentages and relating them to the seats. Indeed, each state's population percentage is important. Now, try to consider the role of the "standard divisor" in your calculations and what it might represent in this context. How could this help you move forward with your process?

Oh, okay. I think the "standard divisor" would be the ratio of the total population to the total number of seats. So that would be like how many people one seat represents on average.



I would calculate the standard divisor like this:

Standard Divisor = Total population / Total number of seats

So let's calculate that:

Standard Divisor = 12,500,000 / 250

Do the math...



Great idea. Once you've done the calculation for the standard divisor, think about how this number can be used to figure out an initial allocation of seats for each state. Can you explain your understanding of the next steps after finding the standard divisor?

Sure, so after I get the standard divisor, I would use that number to divide the population of each state. That will give me an initial idea of how many seats each state should get based on its population.



So for each state, it would be:

Initial seat allocation for a state = Population of the state / Standard Divisor

This should give me a starting point. Then I'll have to adjust those numbers since they probably won't come out to be nice whole numbers, and I can't have a fraction of a seat.

Let me first calculate the Standard Divisor.

So:

Standard Divisor = 12,500,000 / 250  
Standard Divisor = 50,000

This means, on average, one seat represents 50,000 people. So now I'll use this to calculate the initial seat allocation for each state. Here I go...



You're making excellent progress with those calculations. As you work through this, think about the nature of whole numbers in relation to legislative seats and how you might handle any fractions that result from this division. Once you have your initial seat allocations, how might you deal with any seats that remain unassigned? Try to articulate these thoughts and your subsequent steps.

Alright, let's do this. I'll calculate the initial seat allocation.



For state A:

Initial seats for A = Population of A / Standard Divisor  
Initial seats for A =  $1,646,000 / 50,000$   
Initial seats for A = 32.92... which rounds to 33 seats

I'll repeat this for the other states, but I already know I shouldn't just round these numbers normally because if I did, the total might not add up to 250 seats. I remember something about rounding down to the nearest whole number first, then dealing with the extra seats by assigning them to the states with the largest fractions leftover?

Let's calculate the initial allocations for each state first, and then I can worry about the rounding and leftover seats. Give me a sec...



Absolutely, take your time to calculate those initial seat allocations for each state. How will you proceed after finding all the initial allocations?

Okay, I'll calculate the initial seat allocations for each state first.



For state B:

Initial seats for B = Population of B / Standard Divisor  
Initial seats for B =  $6,936,000 / 50,000$   
Initial seats for B = 138.72

For state C:

Initial seats for C = Population of C / Standard Divisor  
Initial seats for C =  $154,000 / 50,000$   
Initial seats for C = 3.08

For state D:

Initial seats for D = Population of D / Standard Divisor  
Initial seats for D =  $2,091,000 / 50,000$   
Initial seats for D = 41.82

For state E:

Initial seats for E = Population of E / Standard Divisor  
Initial seats for E =  $685,000 / 50,000$   
Initial seats for E = 13.7

For state F:

Initial seats for F = Population of F / Standard Divisor  
Initial seats for F =  $988,000 / 50,000$   
Initial seats for F = 19.76

Before rounding these numbers, I'll add up all the initial allocations by rounding them down to the nearest whole number to avoid going over 250 seats, then see how many seats are left.

Now let me do that...



Sounds like a plan. After you've added up these initial allocations (by rounding down), it is important to determine how many seats remain unallocated. Reflect on the strategy you mentioned earlier regarding the largest remainders for these unallocated seats. What could be your approach to this part of the problem?

Alright, first I'll round down the initial allocations:



State A: 33 (rounded down from 32.92)  
State B: 138 (rounded down from 138.72)  
State C: 3 (rounded down from 3.08)  
State D: 41 (rounded down from 41.82)  
State E: 13 (rounded down from 13.7)  
State F: 19 (rounded down from 19.76)

Now I add these up to see how many seats we've allocated so far:

Total allocated seats =  $33 + 138 + 3 + 41 + 13 + 19$   
Total allocated seats = 247

So, 247 seats have been allocated, but we have 250 seats available in total. That means there are 3 seats still unallocated.

For these remaining seats, I'll assign them to the states with the largest remainders. Those would be the decimals I cut off when I rounded down the initial allocations.

So, looking back at the decimals:

State A: .92  
State B: .72  
State C: .08  
State D: .82  
State E: .7  
State F: .76

The largest remainders are for States A, D, and F.

So, I'd give an extra seat to each of these states.

Is that the correct next step?



That's a clear and logical way to distribute the remaining seats. Now that you have allocated the additional seats to the states with the largest remainders, can you see a connection between the fractions left over and how fair the representation is for each state? As you consider your entire process, also think about the overall aim of achieving proportional representation and the implications of using this method in a real-world scenario.

Yeah, by giving the extra seats to the states with the largest fractional remainders, I make sure that the states with the most people 'left over' after the initial allocation gets a little bit closer to having proportional representation.



The fractions sort of show me how much a state has been "underrepresented" when I rounded down. States with bigger remainders were closer to getting another seat than those with smaller remainders. By giving them the extra seats, I'm making the representation as fair as possible without going over the total number of seats.

So after distributing the remaining seats, the final seat allocation would look like this:

State A: 34 (33 + 1 additional seat)  
State B: 138 (same, no additional seats)  
State C: 3 (same, no additional seats)  
State D: 42 (41 + 1 additional seat)  
State E: 13 (same, no additional seats)  
State F: 20 (19 + 1 additional seat)

Total =  $34 + 138 + 3 + 42 + 13 + 20 = 250$  seats

Looks like we've got proportional representation while distributing all the seats. It's definitely an interesting process and makes me think about how important fair representation is in a democracy.

That's the final allocation then, right? Did I get that all sorted out properly?



It seems you've worked through the process thoughtfully and understood the importance of keeping the representation as proportional as possible. Great job! If you have any more questions in the future or need to tackle another problem, don't hesitate to reach out. Goodbye, and take care!

Thanks a lot for your help! I definitely learned a bunch about proportional representation. I'll hit you up if I have any more math headaches. Goodbye, and take care too!



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