

The student is working on the following problem:

Table 1. shows the number of goals that each football player striker scored between 2007 and 2011. Who is the most consistent player? Immagine a measure of "consistency" and justify why you think your method is correct.

Year	Mike Arwen	Dave Backhand	Ivan Right
2007	13	12	14
2008	12	14	10
2009	15	16	18
2010	17	15	18
2011	13	13	15

The solution of this problem is:

The concept of standard deviation is the usual way to measure consistency but it is not expected that the student knows it. Any measure proposed by the student to measure consistency is acceptable as long as it can be justified.

Example of canonical solution: computing the variance (or standard deviation) for each player: First, compute the mean:

Mean number of goals for Mike: 14

Mean number of goals for Dave: 14

Mean number of goals for Ivan: 15

Then, compute the sum of square deviations from the mean for each player.

Then devide by the number of data points to get the variance:

Variance for Mike:  $12/5 = 3.2$

Variance for Dave:  $10/5 = 2$

Variance for Ivan:  $44/5 = 8.8$

So according to the variance, Dave is the most consistent player.



Hello! I am your tutor. Can you walk me through your solution?

hey! tbh i'm kinda lost with this problem. we're supposed to find the "most consistent" player but i'm not really sure what consistency means in this context. like, i get that it means how often they are steady in performance, but no idea how to measure that with math. any tips on where to start?



Absolutely, let's tackle this step by step. Think about what it means to be consistent in the context of scoring goals. If a player is consistent, how would their number of goals scored change from year to year?

oh ok so if a player's consistent, their number of goals would not change much from year to year, right? they'd score kinda the same amount each year. so we'd look for a small difference in their scores over the years? how do we do that with numbers tho?



Right, you're on the right path! If a player's performance is consistent, the number of goals they score each year should be close to their average number of goals per year. How would you calculate the average number of goals for each player?

oh, i know how to do averages! you add up all the numbers and then divide by how many numbers there are. so for mike arwen, we'd add up all his goals from 2007 to 2011 and then divide by 5, since there are 5 years. lemme do that real quick...



for mike arwen it's  $(13 + 12 + 15 + 17 + 13)/5 = 70/5 = 14$ . so his average is 14 goals per year. should i do the same for dave backhand and ivan right?



Exactly, you've computed the average correctly for Mike Arwen. Yes, please go ahead and calculate the average number of goals per year for Dave Backhand and Ivan Right using the same method.

cool, will do that now.



for dave backhand its  $(12 + 14 + 16 + 15 + 13)/5 = 70/5 = 14$

.

and for ivan right its  $(14 + 10 + 18 + 18 + 15)/5 = 75/5 = 15$ .

so dave's average is also 14, and ivan's is 15. what do i do next?



Great work on finding the averages! Now, how might you measure the amount that each player's number of goals deviates from their average in each year? This will help you assess their consistency. Think about comparing each year's goals to the player's average you just calculated.

ohhhh, so do you mean i should look at how far each player's goals in a year are from their average? like, if mike's average is 14, and he scored 13 in one year, then that's a difference of 1 from the average, right?



i guess i need to do that for all the years and players? and then what, add those differences up?



You're on the right track with looking at the differences from the average! But rather than just adding them up, consider what would happen if a player scored both above and below their average in different years. Adding the differences directly might not give the true picture of their consistency. Is there a way you could take into account the distance from the average without cancelling out above and below average performances?

hmm, i see what you mean. if we just add them the above and below stuff could cancel out. so i guess we need to make the differences all positive? could we like, square the differences so they're all positive and then add them up? that way, the negatives won't cancel the positives.



Yes, that's an excellent approach! By squaring the differences, you ensure that they are all positive. This method is actually the beginning step in calculating variance, which is a statistical measure of consistency. Would you like to try squaring the differences from the average for each player and year, and then sum those up?

sure! i can do that. so for mike arwen, i'll subtract each year's goals from the average and then square the differences. lemme write down the steps...



mike's differences squared would be:

$$(13 - 14)^2 = (-1)^2 = 1$$

$$(12 - 14)^2 = (-2)^2 = 4$$

$$(15 - 14)^2 = 1^2 = 1$$

$$(17 - 14)^2 = 3^2 = 9$$

$$(13 - 14)^2 = (-1)^2 = 1$$

now i'll add them up:

$$1 + 4 + 1 + 9 + 1 = 16$$

should i do this for dave and ivan too?



Yes, that's the perfect approach. Now do the same for Dave Backhand and Ivan Right, and let's see what those sums of squared differences look like.

got it, i'll do dave backhand now:



dave's differences squared would be:

$$(12 - 14)^2 = (-2)^2 = 4$$

$$(14 - 14)^2 = 0^2 = 0$$

$$(16 - 14)^2 = 2^2 = 4$$

$$(15 - 14)^2 = 1^2 = 1$$

$$(13 - 14)^2 = (-1)^2 = 1$$

adding those up:

$$4 + 0 + 4 + 1 + 1 = 10$$

and now for ivan right:

ivan's differences squared would be:

$$(14 - 15)^2 = (-1)^2 = 1$$

$$(10 - 15)^2 = (-5)^2 = 25$$

$$(18 - 15)^2 = 3^2 = 9$$

$$(18 - 15)^2 = 3^2 = 9$$

$$(15 - 15)^2 = 0^2 = 0$$

adding those up:

$$1 + 25 + 9 + 9 + 0 = 44$$

so mike has a sum of squared differences of 16, dave has 10, and ivan has 44. what does this mean?

You're doing a fantastic job! The sums of the squared differences that you've computed reflect how much each player's annual goal count deviates from their respective average. A lower sum indicates more consistency, as their performance varies less from year to year.

Looking at the sums of squared differences you've calculated, can you infer which player has been the most consistent over the past 5 years according to this measure?

thanks! since we're looking for the player with the smallest deviation from their average, dave backhand is the most consistent with the lowest sum of squared differences, which is 10. so dave should get the "most consistent" player award according to this measure.



Exactly! You've arrived at a logical conclusion based on the measure of consistency you've calculated. Dave Backhand has the smallest sum of squared differences, suggesting that his performance varied the least from his average, and thus he's the most consistent player according to this metric. Well done on your analysis! Is there anything else you would like to discuss or any other questions you have about this problem?

nope, i think i got it now, thanks! just to make sure, we found out who's most consistent by looking at how the scores deviate from the average, right? and we squared those to avoid negatives. this stuff is kinda cool, didn't know you could do that with math. i should be good with this problem now, thanks!



Absolutely right! You've summarized the process perfectly. It's fascinating how mathematical concepts like averages and variance can provide insights in various scenarios, including sports statistics. I'm glad you found it interesting, and I'm happy to have helped you. If you ever have more questions or another topic you're curious about, feel free to reach out. Happy studying and good luck with your math explorations!

thanks a lot! have a great day! goodbye!



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