

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Oh hey! I'm kinda lost with this problem.



I think we need to divide the seats relative to the population of each state, right? But I'm not sure how to divide the seats fairly. How do I start? 🤔



That's a great starting point! You've identified that seats should be divided in a way that represents each state's population fairly. Now, when thinking about dividing things fairly, what comes to mind regarding the steps you could take to do this? What would you need to know or calculate first?

Hmm, I guess I would need to know like the total population of the whole country first? I could add up the populations of all the states, right? Then I would divide the total number of seats by the total population maybe? 🤔



That sounds like a plan. Go ahead and work through those calculations whenever you're ready. What do you get?

Alright! Here goes the math:



Total population would be the sum of all the states:

$$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$$

So the total population is:

$$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 = 12,500,000$$

Now, I guess I should get some sort of "rate" or "ratio" of seats to population. I'd divide the total number of seats by the total population:

$$\frac{250}{12,500,000}$$

That gives me a ratio of:

$$\frac{250}{12,500,000} = \frac{1}{50,000}$$

So, for every 50,000 people, there should be 1 seat in the legislative body.

Is this looking right so far? 🤔



Great job calculating the total population and finding the ratio of seats to population. With this information, what do you think would be the next logical step in the process of determining fair seat distribution for each state?

Thanks! So next I guess I need to figure out how many seats each state gets.



For that, I think I have to multiply the population of each state by the ratio I found?

Like for state A with a population of 1,646,000, it would be:

$$1,646,000 \times \frac{1}{50,000}$$

Let me do that calculation for state A... 🧮

It's $1646000 \div 50000$ which gives 32.92.

We can't have a fraction of a seat, so I think I have to round to the nearest whole number, right?

That means state A would get 33 seats.

Do I do this for all states now? 🤔



You're on the right track with your calculation for state A. However, think about the steps we take in rounding numbers, especially in a situation like this where we're distributing a limited number of seats. What options do we have for rounding, and is there a particular method we might need to use for this purpose?

Oh right, rounding could be tricky 'cause if we just round normally, we might end up with too many or too few seats when we add them all up. I think we're supposed to keep the total at exactly 250 seats.



Maybe we could use something like rounding down always, and then give the extra seats to the states with the largest remainders? Does that make sense? 🤔



Yes, you've identified an important consideration. Given your understanding, what would be your next step?

Okay, so my next step would be to figure out how many seats each state gets by rounding down the results, that way I make sure I don't go over 250 seats.



So for state A, when I did the multiplication I got 32.92 which would round down to 32 seats.

I think I need to do this same process for states B, C, D, E, and F. And then, I'll see how many seats we have left and give those extra seats to the states with the largest decimal parts. I'll have to keep track of the remainders.

Should I go ahead and calculate the seat numbers for all states first?
🤔



It seems like you have a solid plan. What do you think you should do next?

I'll go ahead and calculate the seat numbers for each state then! Just rounding down at first:



State B:

$$6,936,000 \times \frac{1}{50,000} = 138.72$$

So, 138 seats after rounding down.

State C:

$$154,000 \times \frac{1}{50,000} = 3.08$$

So, 3 seats after rounding down.

State D:

$$2,091,000 \times \frac{1}{50,000} = 41.82$$

So, 41 seats after rounding down.

State E:

$$685,000 \times \frac{1}{50,000} = 13.7$$

So, 13 seats after rounding down.

State F:

$$988,000 \times \frac{1}{50,000} = 19.76$$

So, 19 seats after rounding down.

Now I'll add these to make sure they don't go over 250.

$$32 + 138 + 3 + 41 + 13 + 19 = 246$$

Okay, looks like we have 246. So, we have 4 seats left 'cause we need a total of 250. Time to distribute the remaining seats to the states with the largest decimal remainders.

From my calculations the remainders were:

State A - .92

State B - .72

State C - .08

State D - .82

State E - .7

State F - .76

So the seats should go to states A, B, D, and F.

That will make the seat numbers:

State A - 33

State B - 139

State C - 3

State D - 42

State E - 13

State F - 20

This should add up to 250 now.

$$33 + 139 + 3 + 42 + 13 + 20 = 250$$

So I think that would fairly distribute the seats?

Am I doing this right? 🤖



Recap what you have done so far and consider how the method you have used ensures that the seats are allocated fairly. How does this process reflect the broader principles of proportionality and fairness in representation? Reflect on the connection between the method and the goal of the seat allocation.

Okay, recap time. So what I did first was find the total population of the new country by adding all the state populations. That was 12,500,000 people.



Next, I figured out how many people each seat in the legislative body would represent by dividing the total population by the 250 seats. That was 1 seat for every 50,000 people.

Then, I multiplied each state's population by the ratio $\frac{1}{50,000}$. I rounded down the resulting numbers to avoid going over the total 250 seats. After this, I distributed the remaining seats to the states with the largest decimal remainders to get as close as possible to their exact proportion of the population.

This method aims to be fair because it tries to give each state representation proportional to its population. It doesn't perfectly match the ratios because of the rounding, but by assigning left over seats to those with the highest decimals, it gets as close as possible.

The goal of the seat allocation is to make sure each resident of the country has roughly equal representation in the legislative body, no matter which state they're from. With the method I used, the allocation of seats reflects the principle of proportionality, which is a big part of achieving fairness in representation.

So yeah, I guess by using proportionality I tried to distribute the seats fairly according to each state's population. Does that sound right? 😊



You've recapitulated the process and highlighted its main goal very clearly. Proportionality is indeed crucial to ensuring that representation is fair and reflects the population distribution. Remember, the broader implications of such methods extend to various realms of decision-making and governance, aiming to maintain the balance between quantity and equity.

The journey you took through this problem reflects the essence of problem-solving: breaking down a complex issue into manageable steps and applying a logical process towards an equitable solution. Well done!

If you have any more questions in the future or need further guidance, feel free to reach out. Goodbye and take care! 🙌😊

Got it, thank you so much for helping me figure this out! I learned a lot about fairness and proportionality today. I'll remember to reach out if I get stuck on another problem. Goodbye! 🙌😊



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