

The student is working on the following problem:

Table 1. shows the number of goals that each football player striker scored between 2007 and 2011. Who is the most consistent player? Immagine a measure of "consistency" and justify why you think your method is correct.

| Year | Mike Arwen | Dave Backhand | Ivan Right |
|------|------------|---------------|------------|
| 2007 | 13 | 12 | 14 |
| 2008 | 12 | 14 | 10 |
| 2009 | 15 | 16 | 18 |
| 2010 | 17 | 15 | 18 |
| 2011 | 13 | 13 | 15 |

The solution of this problem is:

The concept of standard deviation is the usual way to measure consistency but it is not expected that the student knows it. Any measure proposed by the student to measure consistency is acceptable as long as it can be justified.

Example of canonical solution: computing the variance (or standard deviation) for each player: First, compute the mean:

Mean number of goals for Mike: 14

Mean number of goals for Dave: 14

Mean number of goals for Ivan: 15

Then, compute the sum of square deviations from the mean for each player.

Then devide by the number of data points to get the variance:

Variance for Mike: $12/5 = 3.2$

Variance for Dave: $10/5 = 2$

Variance for Ivan: $44/5 = 8.8$

So according to the variance, Dave is the most consistent player.



Hello! I am your tutor. Can you walk me through your solution?

Hey! Actually, I'm kinda stuck. We're supposed to design measures of consistency for these players, using their goals from the past 5 years, but I'm not really sure how to do that. We never learned about this kind of stuff in my math class. Can you help me out? What does consistency mean in math here?



Of course! Let's start with a simple question. When you think about a player being consistent, what comes to your mind? How would you describe a consistent player based on their number of goals each season?

Oh, okay. So, I guess a consistent player would be someone who scores around the same number of goals each season, not too high or too low from year to year. They would be pretty predictable, no big surprises.



That's a good insight. Given your understanding, how might you use the numbers in the table to find out which player's performance is most predictable?

Hmm, maybe we could look at the differences in goals scored from one year to the next for each player. Like, if someone scores 10 goals one year and then 20 the next, that's a big jump, but if they score 10 one year and then like 11 or 9 next, that's more consistent. So should I just take the differences between each year for each player?



That sounds like an interesting approach. What would you do after finding those differences for each year for each player?

After finding the differences for each year, I could maybe, like, add them up or get an average of the differences for each player? That way I could see, on average, how much the goals are changing each year. The player with the smallest average change might be considered the most consistent. Is that how it works?



That's an interesting approach! How do you think you would handle the issue of whether the change in goals is an increase or decrease?

Oh, so if one player scores more in one year and less in another, the positives and negatives could cancel each other out when we add them up, right? We want to look at how much they change regardless of whether it's up or down. So maybe we should take the absolute value of each change before we average them out? This way, we're just looking at the size of the change, not the direction. Does this make sense?



Absolutely, you're thinking about this in the right way. What would your next step be then, considering what you've just said?

So my next step would be to calculate the absolute differences between the goals for consecutive years for each player. Then, I'd add up those absolute differences for each player and find the average. The player with the lowest average absolute difference would be considered the most consistent. I can start doing the math for this if it sounds good?



Yes, that sounds like a valid method to measure consistency. Go ahead and do the math for this approach. If you have any questions or need guidance while you're working through the calculations, feel free to ask.

Alright, first, I'll find the differences in goals scored from year to year for each player. I'll calculate the absolute values since we said that's what matters.



For Mike Arwen:

$$\text{From 2007 to 2008: } |13 - 12| = 1$$

$$\text{From 2008 to 2009: } |12 - 15| = 3$$

$$\text{From 2009 to 2010: } |15 - 17| = 2$$

$$\text{From 2010 to 2011: } |17 - 13| = 4$$

$$\text{Now I'll add those up and find the average: } (1 + 3 + 2 + 4)/4 = 10/4 = 2.5$$

Now I'll do the same for Dave Backhand:

$$\text{From 2007 to 2008: } |12 - 14| = 2$$

$$\text{From 2008 to 2009: } |14 - 16| = 2$$

$$\text{From 2009 to 2010: } |16 - 15| = 1$$

$$\text{From 2010 to 2011: } |15 - 13| = 2$$

$$\text{Adding up and finding the average: } (2 + 2 + 1 + 2)/4 = 7/4 = 1.75$$

And then for Ivan Right:

$$\text{From 2007 to 2008: } |14 - 10| = 4$$

$$\text{From 2008 to 2009: } |10 - 18| = 8$$

$$\text{From 2009 to 2010: } |18 - 18| = 0$$

$$\text{From 2010 to 2011: } |18 - 15| = 3$$

$$\text{Adding up and finding the average: } (4 + 8 + 0 + 3)/4 = 15/4 = 3.75$$

So based on this method, Dave Backhand has the lowest average change in goals, which is 1.75. That would mean he's the most consistent player according to this measure. Did I do it right?



You've applied the method you proposed consistently across all players. Now that you have these results, how do you feel about the method? Does it capture everything about consistency or is there anything that might be missing or misleading in this approach?

Hmm, I think this method does show which player has the most consistent change from year to year, but now I'm thinking, it doesn't account for how high or low the actual numbers are to start with. Like, even if Ivan Right had bigger changes, maybe his scores were overall higher? It's possible that the highs and lows of a player could still be pretty close to each other, making them consistent as well.



Maybe we need something that looks at how spread out each player's goals are from their average number of goals? Like, not just the change from year to year, but how much they vary from their own average over the 5 years. But I'm not really sure how to calculate that. What do you think?



You're on the right track. You've mentioned looking at how spread out a player's goals are from their average number of goals. How do you think you might represent this mathematically? What could you do with each player's set of goals to find out more about this kind of consistency?

Oh right, so for each player, I could find out their average (or mean) number of goals over the 5 years. Then I guess I could look at how far each year's number of goals is from that average. So, I would calculate the difference between each year's goals and the average number of goals for each player. After that, maybe take the absolute values again – to avoid positives and negatives canceling out. And then, like before, find the average of those differences for each player.



Should I work this out with the numbers from the table? Is this what you're suggesting?



It seems you have good intuition about another possible measure of consistency. Go ahead and perform the necessary calculations with the numbers from the table, just as you described. If you encounter any issues or uncertainties, don't hesitate to ask for assistance.

Okay, I'll start by finding the average number of goals for each player over the 5 years.



For Mike Arwen:

The sum of goals: $13 + 12 + 15 + 17 + 13 = 70$.

His average goals over 5 years is $70/5 = 14$.

Now for the deviation from the average for each year,

$|13 - 14| = 1$, $|12 - 14| = 2$, $|15 - 14| = 1$, $|17 - 14| = 3$,
 $|13 - 14| = 1$.

Then find the average of these differences: $(1 + 2 + 1 + 3 + 1)/5 = 8/5 = 1.6$.

I'll do the same for Dave Backhand and Ivan Right. Give me just a sec.



Take your time. What will be your process for the other two players?



For Dave Backhand and Ivan Right, the process will be to first calculate their average goals over the five years by adding up all their goals and then dividing by 5. After that, I'll find out how much each year's goals deviate from their average, take the absolute values of those deviations, and then find the mean of those absolute deviations. Okay, here it goes:

Dave Backhand:

Sum of goals: $12 + 14 + 16 + 15 + 13 = 70$.

His average goals over 5 years is $70/5 = 14$.

Now, find the deviation from the average for each year:

$|12 - 14| = 2$, $|14 - 14| = 0$, $|16 - 14| = 2$, $|15 - 14| = 1$,
 $|13 - 14| = 1$.

The average of these differences: $(2 + 0 + 2 + 1 + 1)/5 = 6/5 = 1.2$.

Now for Ivan Right:

Sum of goals: $14 + 10 + 18 + 18 + 15 = 75$.

His average goals over 5 years is $75/5 = 15$.

Deviations from the average:

$|14 - 15| = 1$, $|10 - 15| = 5$, $|18 - 15| = 3$, $|18 - 15| = 3$,
 $|15 - 15| = 0$.

The average of these differences: $(1 + 5 + 3 + 3 + 0)/5 = 12/5 = 2.4$.

Based on this method, Dave Backhand has the lowest average deviation from his own average goals, which is 1.2. Mike Arwen is next with 1.6, and Ivan Right has the highest with 2.4. So, it looks like Dave Backhand is the most consistent when we consider how players' goals vary from their average goals over the 5 years. Does this all sound okay?



It seems you've just completed a thoughtful analysis! Could you recap what you've just done? Reflect on the process and what it tells you about the concept of consistency in performances. How do you think this approach compares with the method you used earlier?

Sure, so I did two methods to try and figure out who's the most consistent soccer player.



In the first method, I looked at how much each player's number of goals changes from one year to the next, by calculating the absolute difference between their goals in consecutive years. Dave Backhand had the lowest average year-to-year change, which made him seem the most consistent there.

In the second method, I calculated the average number of goals for each player over the 5 years. Then I figured out how far off each year's goals were from that average and looked at the average of those differences. Again, Dave Backhand had the lowest average deviation from his own average number of goals, so based on that, he's the most consistent.

Reflecting on it, the first method gives us a sense of consistency as like steadiness or regularity from year to year. The second method shows us how much a player's seasonal performance deviates from their typical or average performance, which is like another way to measure consistency.

Both are legit, but they might give different results for other situations. I guess it depends on how you want to think about consistency. Overall, I think the second method is better because it uses the average which considers all the years together, not just individual differences from year to year.

What do you think about the methods and which one should we use to decide who gets the award?



You've done great work in exploring different methods and reflecting on their implications. Remember, the beauty of mathematics is in how it allows us to capture different aspects of a concept like consistency through various approaches. You've thought critically about which method might be more informative for the decision at hand.

It's important for the organizers to decide which measure best represents their definition of consistency for the award. Both methods have merit, and there isn't one "correct" answer—it comes down to what aspect of consistency they value most.

Great job today, and keep up with this analytical thinking! If you have more questions in the future or need further clarification on different topics, don't hesitate to reach out. Goodbye, and take care!

Thanks a lot for helping me think this through! I'll keep practicing how to analyze data this way. Goodbye, and take care too!



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