

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

hey! um so, i'm not really sure where to start. i think i need to figure out how many seats each state gets based on population? but I have no idea how to divide the seats fairly. any tips?



Absolutely, you're on the right track thinking about distributing seats based on population. Imagine you had just 1 seat to assign, how might you decide which state gets it based on the populations you've got? What about if you had to distribute just 10 seats? How do you think that could extend to 250 seats?

ok, so if I had 1 seat, I guess it would go to the state with the most people, which is state B. And with 10 seats...i think u divide the total population by 10 and give out seats to the states based on that?



but like, for 250 seats that feels kinda complicated. there's probably some sort of proportion to figure out for each? 🤔



That's a good intuition about the larger state getting attention first. Yes, think about proportions. If you had a certain number of seats to distribute based on the whole population, how would you determine the fair share for each state? What kind of calculation involves the total amount and individual parts to find a proportion?

ohhh, i think you're talking about ratios, right? so i'd add up all the populations to get the total population of the country. then i'd divide each state's population by the total population to get the proportion of the total that each state makes up. after that, i'd multiply each state's proportion by the 250 seats to find out how many seats they should get. is that it? 🤓👉





That's an insightful approach! You've outlined a strong method for proportional distribution. What do you think you'd do once you've calculated each state's proportion of the total population? How would these proportions help you in the next step to finding out the number of seats per state?

oh, after getting each state's proportion, we multiply it by 250 since that's the total seats. that'll tell us how many seats each state should get based on their proportion. rounding might be a thing, since we can't have, like, half a seat. do I have to round numbers up or down, or is there a special way to do it so that the total seats end up being exactly 250?



That's a very good point about the rounding! What ideas do you have about dealing with fractions of seats? How might you ensure that after rounding, the total number of seats still adds up to 250?

umm, i think you'd normally just round to the nearest whole number, but that might not add up to 250 exactly. i guess one way is to give out the seats by rounding down, and then see if there are any leftover seats? those could go to the states that were closest to getting another seat? 🤔 i feel like there's a term for this... is it, like, "largest remainder method" or something?



You've hit upon an important concept in the apportionment process. Now, thinking about that idea, how could it be applied to this situation so that every seat is filled while keeping the distribution as fair as possible? What would be your next step?

ah, so if we use the largest remainder method, I think that means we:



1. divide each state's population by the total population,
2. multiply that number by 250 to get each state's initial number of seats,
3. round that number down to get a whole number of seats for each state,
4. add up all the seats to see if we have less than 250,
5. if we do have leftover seats, we give them to the states with the largest remainders until we reach 250 seats.

guess i should calculate the proportions now. lemme grab a calculator real quick... 🧮 👁



That plan sounds like a good approach to allocate the seats. As you get your calculator, what's the first calculation you intend to perform?

first, gonna add up all the state populations to find the total population of the country. then i'll divide each state's population by the total to get the proportion. after that, multiply each proportion by 250. let me do that real quick...



ok so the total population is $1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 = 12,500,000$.

now for the proportions... here goes:

A: $1,646,000 / 12,500,000 = 0.13168$

B: $6,936,000 / 12,500,000 = 0.55488$

C: $154,000 / 12,500,000 = 0.01232$

D: $2,091,000 / 12,500,000 = 0.16728$

E: $685,000 / 12,500,000 = 0.0548$

F: $988,000 / 12,500,000 = 0.07904$

and then multiplying those by 250 gives the initial number of seats. lemme calculate those... 🧮



Great job doing the calculations! Once you have each state's initial number of seats, what do you think is the next thing you should consider?

ok, next i'd round down the initial number of seats for each state to get the whole number of seats. and then i'd see how many seats are left and give those to the states with the largest remainders. let me do the first part...



A: $0.13168 \times 250 = 32.92$ so, 32 seats

B: $0.55488 \times 250 = 138.72$ so, 138 seats

C: $0.01232 \times 250 = 3.08$ so, 3 seats

D: $0.16728 \times 250 = 41.82$ so, 41 seats

E: $0.0548 \times 250 = 13.7$ so, 13 seats

F: $0.07904 \times 250 = 19.76$ so, 19 seats

now I add them up: $32 + 138 + 3 + 41 + 13 + 19 = 246$

we got 4 seats left! so now i give those to the states with the biggest remainders after rounding down. let me figure those out... one sec...



You've calculated the initial seat allocation. It seems like you're considering what to do with those additional seats. What are you thinking?

thinking i need to check the decimal parts of the initial seat numbers to find the largest remainders. I'm gonna list them real quick:



A: 0.92
B: 0.72
C: 0.08
D: 0.82
E: 0.7
F: 0.76

So the seats should go to states D, A, F, and B since they have the largest remainders. Adding 1 seat to each of those states, we get:

A: 33 seats
B: 139 seats
D: 42 seats
F: 20 seats

The others stay the same:

C: 3 seats
E: 13 seats

Now all the seats add up to 250. So I think that's it! 🎉



That was quite a methodical process you went through! Could you take a moment to recapitulate what you've done? Moreover, do you see any implications or connections to more general situations or problems one might encounter outside of this specific example?

Sure! So here's the recap:



1. I found the total population of the country by adding up all the states' populations.
2. I calculated each state's proportion of the total population.
3. I found the initial seat numbers by multiplying each proportion by 250, the total number of seats.
4. I rounded down each state's seat number to get a whole number.
5. I distributed the remaining seats one by one to the states with the largest remainders after rounding down, until all 250 seats were assigned.

About implications, this method could apply to any situation where you need to divide something up fairly based on size or amount. So it could be like assigning budgets based on population, or even slicing a pizza and giving people bigger or smaller pieces based on how hungry they are, lol. It's all about making sure everyone gets a fair share based on some proportionate value. 🌍💡



You've captured the essence of the process and its broader implications quite well. Your analogy with dividing a pizza is particularly spot-on and relatable—it's all about proportionality and fairness, whether we're talking about legislative seats or sharing food fairly. These concepts are deeply rooted in many aspects of life where equity and fairness are important.

Great work today! I hope you feel more confident about approaching problems like this in the future. If you have further questions down the road or tackle new challenges, don't hesitate to reach out. Goodbye and take care!

Thanks a lot for all the help! I definitely understand it better now, and it was kinda cool to see how it's done. Will do, I'll reach out if I get stuck with something else in the future. Goodbye! 🙌😊



ok/BY2

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