

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Hey! tbh I'm kinda lost on where to start 😊 Can you give me some pointers?



Sure, no problem at all—it's usual to feel lost with a new type of problem. Let's take this step by step. Given that you have the populations of the states and a fixed number of legislative seats, what do you think is the first piece of information you need to find out to begin solving this problem?

umm, i guess we'd need to figure out how many people each seat represents? So like a ratio of people to seats?



That sounds like you're on the right track. Can you express your idea using a formula or calculation?

Yeah, we should probably find the total population of the country first and then divide by the number of seats. So I guess it would be something like:



Total population = population of A + population of B + population of C + population of D + population of E + population of F

Then, do People per seat = Total population / 250

I think that's right, yeah?



That's a good foundation. Once you have calculated the total population and found out how many people each seat represents, what would be your next step?

After that, I think we'd divide the population of each state by the number of people each seat represents to find out how many seats each state should get.



Exactly. How would you go about dividing the populations? What will you do if you encounter a non-whole number?

For dividing the populations, I'd just use a calculator. But if I get non-whole numbers, I'm not really sure what to do. Maybe round the numbers? But that might not add up to 250 exactly... 🤔 Hmm, is there a specific way to handle that?



You've raised a good point. Rounding might not give the most fair distribution since rounding each state's seats could lead to a total different from 250 seats available.

It's very important that the total number of seats comes out exactly to 250.

Think about the issue with rounding—how else could you make sure every state gets a fair number of seats and you use all 250 available seats? There might be something you can do with those non-whole numbers. Consider the remainders after you assign the initial whole number of seats.

Oh, I think I get what you mean. Maybe we can give each state the whole number of seats they get from dividing, and then we take the decimal part, the remainder, to decide who gets the extra seats. Like, maybe the states with the larger remainders get the additional seats until we reach 250. Does that sound right?



Yes, can you write down this intuition mathematically and see how it would work with your data?

Ok, let me try.



First, we add up all the populations:

$$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$$

Then, calculate the people per seat:

$$\text{People per seat} = (\text{Total population}) / 250$$

I'll use these two things to find how many seats each state should initially get, which is:

$$\text{Seats for State X} = (\text{Population of State X}) / (\text{People per seat})$$

Then for the non-whole numbers that result, I'd assign the whole number of seats first and list out the remainders. The states with the biggest remainders will get the extra seats until we hit 250.

I'll start with calculating the total population. Lemme grab my calculator... Alright got it!

$$\begin{aligned} \text{Total population} &= 1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000 \\ \text{Total population} &= 12,500,000 \end{aligned}$$

$$\begin{aligned} \text{So, People per seat} &= 12,500,000 / 250 \\ \text{People per seat} &= 50,000 \end{aligned}$$

Now I'll divide the population of each state by the People per seat and see what I get. Just a sec...



Great progress! Once you have those numbers, what will you do next?

Alright, so after dividing the population of each state by 50,000, I'll write down the whole number of seats for each state and keep track of the decimal parts, the remainders. The whole numbers are guaranteed seats. Then we use the decimal parts to allocate the remaining seats. The states with the highest remainders get one additional seat each until all 250 seats are allocated.



I'm just gonna calculate the exact number of seats for each state now. Let's see...



What do you find? Take your time with the calculations and make sure to keep track of both the whole numbers and remainders for each state.

Ok, here we go:



State A: $1,646,000/50,000 = 32.92$ so, they get 32 seats, and the remainder is 0.92.

State B: $6,936,000/50,000 = 138.72$ so, they get 138 seats, and the remainder is 0.72.

State C: $154,000/50,000 = 3.08$ so, they get 3 seats, and the remainder is 0.08.

State D: $2,091,000/50,000 = 41.82$ so, they get 41 seats, and the remainder is 0.82.

State E: $685,000/50,000 = 13.7$ so, they get 13 seats, and the remainder is 0.7.

State F: $988,000/50,000 = 19.76$ so, they get 19 seats, and the remainder is 0.76.

Totalling up the guaranteed seats:

$$32 + 138 + 3 + 41 + 13 + 19 = 246$$

So we have 246 seats allocated, and we need to allocate 4 more seats based on the highest remainders.

The order of remainders from highest to lowest is:

State A: 0.92

State D: 0.82

State F: 0.76

State B: 0.72

State E: 0.7

State C: 0.08

So, the first four states on this list get one more seat each. That's state A, D, F, and B.

Final seat counts will be:

$$\text{State A: } 32 + 1 = 33$$

$$\text{State B: } 138 + 1 = 139$$

$$\text{State C: } 3$$

$$\text{State D: } 41 + 1 = 42$$

$$\text{State E: } 13$$

$$\text{State F: } 19 + 1 = 20$$

And now the total seats should add up to 250. Does this look good?



That's a well-organized approach! You've identified the method for fairly distributing the legislative seats based on the population. It looks like you've worked through the problem systematically and reached a solution.

Would you like to summarize what you've done in your words? This could help consolidate your understanding of the method you've just applied.

Yeah, sure! So like, first I added all the populations of the states together to get the total population. Then I divided that number by 250 to figure out how many people each seat represents.



Next up, I divided the population of each state by that "people per seat" number to get the initial amount of seats for each state. Since we'd probably get fractions, I kept the whole numbers cause those are the guaranteed seats, and then I made a list of the decimal parts to sort out the remaining seats.

Last step was to look at the remainders and give the extra seats to the states with the biggest remainders, one by one, until all the 250 seats were allocated. Made sure the states with the highest fractions got the first pick for those extra seats. So in the end, we get a pretty fair division of seats that matches the populations.

That about sums it up, I think! 😊



Fantastic recap! You've not only solved this particular problem but also learned a method for proportional representation that is used in real-world situations, such as assigning seats in legislative bodies based on population. This method ensures that each group is represented fairly, reflecting a cornerstone of democratic systems.

Good work today! Keep in mind that the method you applied can be used in various scenarios where proportionate distribution is necessary. If you ever encounter a similar problem again, you'll have a strategy ready to go.

Well done, and goodbye. If you have more questions in the future, don't hesitate to ask. Have a great day!

Thank you so much! 😊 I definitely understand it better now. Goodbye, and have an awesome day too!



ok/BY1

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