

The student is working on the following problem:

Table 1. shows the number of goals that each football player striker scored between 2007 and 2011. Who is the most consistent player? Immagine a measure of "consistency" and justify why you think your method is correct.

Year	Mike Arwen	Dave Backhand	Ivan Right
2007	13	12	14
2008	12	14	10
2009	15	16	18
2010	17	15	18
2011	13	13	15

The solution of this problem is:

The concept of standard deviation is the usual way to measure consistency but it is not expected that the student knows it. Any measure proposed by the student to measure consistency is acceptable as long as it can be justified.

Example of canonical solution: computing the variance (or standard deviation) for each player: First, compute the mean:

Mean number of goals for Mike: 14

Mean number of goals for Dave: 14

Mean number of goals for Ivan: 15

Then, compute the sum of square deviations from the mean for each player.

Then devide by the number of data points to get the variance:

Variance for Mike: $12/5 = 3.2$

Variance for Dave: $10/5 = 2$

Variance for Ivan: $44/5 = 8.8$

So according to the variance, Dave is the most consistent player.



Hello! I am your tutor. Can you walk me through your solution?

Hey! So I've never really done this before. I guess we need to figure out who is the most consistent, right? But I don't really know how to measure consistency. What do you think we should do first?



Exactly, we need to measure consistency. A good starting point might be to look at the data for each player and see if there's a pattern - how do their scores vary from year to year? When thinking about consistency, what comes to mind regarding the way someone performs over time?

Hmm, I guess when I think about someone being consistent, I think about them doing pretty much the same thing every time, without a lot of big changes. So maybe for the players, it would mean scoring close to the same number of goals each year? Like if someone scored 15 goals every year, they would be super consistent. But if they scored like 10 one year, then 20 the next, that would be less consistent. How do we turn that into like a math thing, though?



Precisely! If a player scores a similar number of goals each year, they're consistent. What you're talking about is essentially the variation in their performance. A mathematical way to capture variation is to calculate how much the number of goals scored each year diverges from the player's average number of goals over the period in question.

Do you know how to calculate an average, and have you heard about the concepts of variance or standard deviation before?

Yep, I know how to calculate an average. You just add up all the numbers and then divide by how many numbers there are. But no, I've never heard about variance or standard deviation. What are those? Are they hard to calculate?





Great, since you understand how to calculate an average, that's a good starting place. Variance and standard deviation are measures that tell us how spread out numbers are in a set of data. We won't dive deeply into the technical definitions right now, but variance measures the average degree to which each number is different from the mean.

To put it simply, variance is like finding an average of how much each score differs from the player's average score. Here's how you could start:

1. Calculate the average score for each player over the 5 years.
2. Then, look at each score and see how much it differs from the average you found.

Would you like to start by calculating the averages for Mike Arwen, Dave Backhand, and Ivan Right based on the data provided?

Sure thing! Let me calculate the average number of goals for each player:



For Mike Arwen:

$$\text{Average} = \frac{13+12+15+17+13}{5} = \frac{70}{5} = 14$$

For Dave Backhand:

$$\text{Average} = \frac{12+14+16+15+13}{5} = \frac{70}{5} = 14$$

For Ivan Right:

$$\text{Average} = \frac{14+10+18+18+15}{5} = \frac{75}{5} = 15$$

Alright, got the averages. What's next?



Well done on calculating the averages. The next step will be to see how much each player's yearly goals deviate from their average. This is a step towards measuring consistency – the less deviation there is, the more consistent the player.

For each player and for each year, you'll want to:

1. Subtract the average number of goals from the goals scored in that year.
2. Square that difference (this is to ensure all values are positive and to emphasize larger differences).

Would you like to try this out for one of the players to see how it goes?

Sure, I can try that for Mike Arwen first.



Mike Arwen's average is 14 goals. So the deviation for each year would be:

For 2007:

$$13 - 14 = -1$$

$$(-1)^2 = 1$$

For 2008:

$$12 - 14 = -2$$

$$(-2)^2 = 4$$

For 2009:

$$15 - 14 = 1$$

$$1^2 = 1$$

For 2010:

$$17 - 14 = 3$$

$$3^2 = 9$$

For 2011:

$$13 - 14 = -1$$

$$(-1)^2 = 1$$

So we have the squared differences: 1, 4, 1, 9, 1. What should I do with these numbers?



You're on the right track. Now we sum up those squared differences. Adding these together gives us the total amount Mike's scores deviate from his mean. After you've summed up these squared differences, the next step is to average this sum by dividing it by the number of data points (which is 5 in this case). This will give us Mike's variance.

Can you calculate the total of the squared differences and then find the variance for Mike Arwen?

Okay, we've got the squared differences of 1, 4, 1, 9, and 1. Let's add them up:



$$\text{Total squared differences} = 1 + 4 + 1 + 9 + 1 = 16$$

To find the variance, we divide by the number of years:

$$\text{Variance for Mike Arwen} = \frac{16}{5} = 3.2$$

So that means the variance for Mike Arwen is 3.2. What's next?



You've done that correctly. With a variance of 3.2 for Mike Arwen, you have a measure of his consistency. A lower variance would indicate a higher consistency in the number of goals scored each year.

You can now repeat this process for Dave Backhand and Ivan Right to find their variances. After that, you can compare the variances of all three players to determine who the most consistent player is based on this measure. The player with the lowest variance will be the most consistent. Would you like to proceed with this?

Got it! I'll do the same for Dave Backhand and Ivan Right.



For Dave Backhand, his average is also 14 goals. So here are the squared differences:

For 2007:

$$(12 - 14)^2 = (-2)^2 = 4$$

For 2008:

$$(14 - 14)^2 = 0^2 = 0$$

For 2009:

$$(16 - 14)^2 = 2^2 = 4$$

For 2010:

$$(15 - 14)^2 = 1^2 = 1$$

For 2011:

$$(13 - 14)^2 = (-1)^2 = 1$$

Total squared differences for Dave = $4 + 0 + 4 + 1 + 1 = 10$

$$\text{Variance for Dave Backhand} = \frac{10}{5} = 2.0$$

Now for Ivan Right, with an average of 15 goals:

For 2007:

$$(14 - 15)^2 = (-1)^2 = 1$$

For 2008:

$$(10 - 15)^2 = (-5)^2 = 25$$

For 2009:

$$(18 - 15)^2 = 3^2 = 9$$

For 2010:

$$(18 - 15)^2 = 3^2 = 9$$

For 2011:

$$(15 - 15)^2 = 0^2 = 0$$

Total squared differences for Ivan = $1 + 25 + 9 + 9 + 0 = 44$

$$\text{Variance for Ivan Right} = \frac{44}{5} = 8.8$$

So, looking at these variances, Dave Backhand has the lowest variance of 2.0, which means he is the most consistent player based on our measure! Did I do that right?



You've done an excellent job, and you've applied the method correctly! By comparing the variances, you've assessed the consistency of each player, and your conclusion is based on the calculated variances. Dave Backhand indeed has the lowest variance at 2.0, which suggests he's the most consistent player of the three according to this specific mathematical measure of consistency.

How do you feel about the process and the result? Is there anything about it that you find particularly interesting or would like to discuss further?

Thanks! The process actually makes sense once you break it down. It's interesting to see how you can use math to make decisions like this. I guess with this method, even if someone scored more goals overall, they might not win the award if their performance was up and down. Anyway, I'm good with the result. Math is cool when it works out like this. Is there anything else you think I should know or do with this problem?



I'm glad to hear you find the process and results meaningful. It is indeed an interesting observation that high performance does not necessarily equate to consistency, and mathematics can help us parse these nuances.

This utilization of variance is just one of many ways to measure consistency. In statistics, this method has a related concept called standard deviation, which is the square root of the variance. Standard deviation is often used because it is in the same unit as the original data, making it sometimes more intuitive to understand than variance.

It's also worth noting that there are other statistical methods and measures to consider consistency and performance, including the coefficient of variation, median absolute deviation, and others, which could provide additional insights.

If you wish to explore this topic further, you could investigate these other measures, or consider the possible influence of outliers on the variance, and how different measures of consistency handle such anomalies.

But for this problem, you've done what was asked, and you've done it well. If you have any more questions or if there's another topic you're curious about, feel free to ask!

Yeah, the standard deviation sounds important to know about, especially if it makes things easier to understand. It's cool that there are even more ways to look at this stuff. I don't have any more questions right now, but I might take a look at those other measures you talked about later. I want to say thanks for helping me out with this math problem. It was kind of fun figuring this all out. Goodbye!



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