

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

Hey! So, I'm not really sure where to start. I think we need to divide the seats based on the population, right? But how do I do that?



That's a great starting point! Indeed, you need to distribute the seats proportionally based on the population of each state. Let's break it into manageable steps.

First, you'll need to figure out the total population of the country. Can you calculate the sum of the populations of all the states?

Okay, I guess I just add them all up. So here goes:



$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$

Let me calculate...



Exactly, you will add those figures together. Take your time with the calculation; accuracy is important to ensure that the decisions you make later on are correct. What sum do you get?

I added them up and got:



$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$

So the total population of the country is 12,500,000 people.



Excellent! Now that you've got the total population, the next step is figuring out how many people each seat in the legislative body should represent on average. This is called finding the "standard divisor".

You can calculate the standard divisor by dividing the total population by the total number of available seats. What do you get when you divide 12,500,000 by 250?

Okay, I'm going to divide 12,500,000 by 250 to find the standard divisor.



$$\frac{12,500,000}{250} = 50,000$$

So each seat should represent 50,000 people on average.



That's right. Now, with the standard divisor, you can calculate the initial number of seats for each state by dividing the population of each state by 50,000. After dividing, you'll normally get a mix of whole numbers and fractions. Initially, assign the whole number of seats to each state and keep track of the remainders—we'll use them later.

Could you go ahead and calculate the initial number of seats for each state? Remember to round down to the nearest whole number for now.

Okay, let's do this for each state.



For state A, with 1,646,000 people:

$$\frac{1,646,000}{50,000} = 32.92$$

So, state A initially gets 32 seats.

For state B, with 6,936,000 people:

$$\frac{6,936,000}{50,000} = 138.72$$

State B gets 138 seats.

For state C, with 154,000 people:

$$\frac{154,000}{50,000} = 3.08$$

State C gets 3 seats.

For state D, with 2,091,000 people:

$$\frac{2,091,000}{50,000} = 41.82$$

State D gets 41 seats.

For state E, with 685,000 people:

$$\frac{685,000}{50,000} = 13.7$$

State E gets 13 seats.

For state F, with 988,000 people:

$$\frac{988,000}{50,000} = 19.76$$

50,000

State F gets 19 seats.

Now I'll add them up to see how many seats we've allotted so far...



Exactly, you're on the right track. After you've added those initial seats together, you'll be able to determine how many seats have been allotted and how many are still remaining to distribute. Will the sum of these initial seats be equal to 250, or will you have some seats left over? Let me know your findings.

Adding those up:



$$32 + 138 + 3 + 41 + 13 + 19 = 246$$

We've allocated 246 seats, but we need to fill all 250 seats. So we have 4 seats left over to distribute.



Well done! The final step is to distribute those 4 surplus seats. They are given to the states with the largest remainders from their initial seat calculation.

You already have the remainders from your earlier division. Look at the decimal part of each state's initial seat calculation and identify the four largest remainders. Those states will each receive one of the remaining seats.

Which states have the largest remainders?

Let's check the decimal parts (the remainders) I got from the initial calculation:



State A: .92
State B: .72
State C: .08
State D: .82
State E: .70
State F: .76

The four largest remainders are from states A (.92), D (.82), F (.76), and B (.72). So these states should get the remaining 4 seats. This means we add one more seat to each of these states. Let's update the seat counts:

State A: $32 + 1 = 33$ seats
State B: $138 + 1 = 139$ seats
State C: 3 seats (no change)
State D: $41 + 1 = 42$ seats
State E: 13 seats (no change)
State F: $19 + 1 = 20$ seats

Now, all 250 seats are allocated, keeping the representation as proportional as possible based on the populations of the states.



Perfect! You've distributed the seats fairly according to the populations using the largest remainder method. Just as a final step, do check your distribution adds up to the total number of 250 seats. Is there anything else regarding this problem that you might be wondering about or that you'd like to go over?

I'll double-check the total number of seats to make sure we've distributed all 250 seats:



State A: 33 seats
State B: 139 seats
State C: 3 seats
State D: 42 seats
State E: 13 seats
State F: 20 seats

Adding these up:

$$33 + 139 + 3 + 42 + 13 + 20 = 250$$

So, yes, we've allocated all 250 seats. I think that solves the problem! I don't have any other questions right now, thanks for your help!



You're welcome! It sounds like you've got a solid understanding of how to approach a proportional distribution problem like this one. If you ever have more questions or a new topic you'd like to discuss, feel free to reach out. Great job today, and happy studying!

Got it, thanks a lot! I'll totally reach out if I need help again. Happy studying to you too! Goodbye!



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