

The student is working on the following problem:

A new country has six states, called A, B, C, D, E and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people.

There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation?

The solution of this problem is:

We assign seats proportionally to the population of each state. Since the results of the divisions are not integers, we round down the number and then distribute the remaining seats to states having the largest remainders.

$$\begin{aligned}\text{Total population} &= 1,646,000 \text{ (A)} + 6,936,000 \text{ (B)} + 154,000 \text{ (C)} + 2,091,000 \text{ (D)} + 685,000 \text{ (E)} + 988,000 \text{ (F)} \\ &= 12,500,000\end{aligned}$$

$$\text{Standard divisor} = \text{Total population} / \text{Number of seats} = 12,500,000 / 250 = 50,000$$

Initial quotas:

- A: $1,646,000 / 50,000 = 32.92 \rightarrow 32$ seats
- B: $6,936,000 / 50,000 = 138.72 \rightarrow 138$ seats
- C: $154,000 / 50,000 = 3.08 \rightarrow 3$ seats
- D: $2,091,000 / 50,000 = 41.82 \rightarrow 41$ seats
- E: $685,000 / 50,000 = 13.70 \rightarrow 13$ seats
- F: $988,000 / 50,000 = 19.76 \rightarrow 19$ seats

$$\text{Total initial seats assigned} = 32 + 138 + 3 + 41 + 13 + 19 = 246$$

$$\text{Seats left to distribute} = 250 - 246 = 4$$

Distribute the surplus seats based on largest remainders:

Remainders (from the divisions above):

- A: 0.92
- B: 0.72
- C: 0.08
- D: 0.82

- E: 0.70
- F: 0.76

The four highest remainders are from states A, B, D, and F. Give one extra seat to each.



Hello! I am your tutor. Can you walk me through your solution?

oh hey! so, um, i'm kinda stuck. i think we're supposed to divide the seats based on the population? but not sure how to start... 🤔 any hints?



Absolutely, dividing the seats based on the population is the right track. Think about what you need in order to distribute something fairly. If you were dividing a cake for a group of people and wanted everyone to get a piece proportional to their hunger, what would you need to know? How might that relate to dividing seats among states based on their population?

oh, i get it. so kinda like...each person's hunger is like the population of the state, and the size of each piece of cake is like the number of seats for that state. so, we need to find out how much each state gets based on its population compared to the total population, right? so first, do we add up all the populations of the states to get the total?



Precisely! You're on the right path. What would be your next step after finding the total population?

okay, cool. so after we find the total population, we divide it by the total number of seats to find out how many people each seat represents. then we divide each state's population by that number to get the number of seats for each state. does that sound right? 🤔



lemme start by adding up the populations... gimme a sec... 🤖

$1,646,000 + 6,936,000 + 154,000 + 2,091,000 + 685,000 + 988,000$... um... 🧮

...i think it's... 12,500,000? is that correct? 🤔



Great work calculating the total population. What do you think would be the next logical step in this process to ensure that each state gets a fair number of seats?

thanks! so now we divide the total population by the number of seats, right? let me do the math:



$$12,500,000 \div 250 = 50,000$$

looks like each seat should represent 50,000 people. now we divide each state's population by 50,000 to see how many seats they get... umm, let me try this:

For state A: $1,646,000 \div 50,000$... that's 32.92... i guess we round that? do we round up or down? 🤔



You've made a good observation about the fraction. How do you think you should handle these fractions initially before making decisions about rounding? Think about what would be the most equitable method for starting to assign the seats.

oh right, probably shouldn't round yet. so what we could do is take the whole number part for the initial seats and then look at the fractions afterwards to distribute the remaining seats. this way everyone gets a fair share to start? so, like, my internet says it's called the "largest remainder method" or something?



i'll jot down the initial whole numbers:

- state A: $1,646,000 \div 50,000 = 32$ whole and remainder 46,000
- state B: $6,936,000 \div 50,000 = 138$ whole and remainder 36,000
- state C: $154,000 \div 50,000 = 3$ whole and remainder 4,000
- state D: $2,091,000 \div 50,000 = 41$ whole and remainder 41,000
- state E: $685,000 \div 50,000 = 13$ whole and remainder 35,000
- state F: $988,000 \div 50,000 = 19$ whole and remainder 38,000

got that... now we gotta deal with those remainders 🤖



That's a good place to be. Considering the remainders, what do you think should be done next to ensure that all the seats are fairly distributed?

ok, so we gotta distribute the remaining seats based on the size of the remainders. since there are 250 seats, we need to make sure we distribute exactly 250 seats in total. we already have:



$$32 + 138 + 3 + 41 + 13 + 19 = 246$$

so we got 4 seats left to assign. we should give these to states with the largest remainders. let me check which ones those are...

looking at the remainders:

state A: 46,000

state B: 36,000

state C: 4,000

state D: 41,000

state E: 35,000

state F: 38,000

the largest remainders are from states A, B, D, and F which should each get one more seat. is that correct? 🤖



That sounds like a well-reasoned approach. Keep in mind the principles you've applied here as you tackle similar problems in the future. Best of luck, and if you have more questions in the future, don't hesitate to ask. You're doing great! Goodbye for now!

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