

The student is working on the following problem:

Let ABC be a right-angled triangle at A with AC=6cm and BC=12cm.

Determine $d(C, AB)$ the distance from C to the line (AB) and

show that $d(B, AC)$, the distance from B to the line (AC) is equal to $6 * \sqrt{3}$

The solution of this problem is:

ABC is a right-angled triangle at A so the hypotenuse is BC and sides AB and AC are perpendicular. First, $d(C, AB) = AC = 6\text{cm}$ (orthogonal projection).

Second, by the Pythagorean theorem, $BC^2 = AB^2 + AC^2$ so $AB^2 = BC^2 - AC^2$.

We compute $AB^2 = BC^2 - AC^2 = 12^2 - 6^2 = 144 - 36 = 108$ Since (AB) is perpendicular to (AC), A is the orthogonal projection of B on (AC), so $d(B, AC) = AB$. Thus, $d(B, AC) = AB = \sqrt{108} = \sqrt{6^2 * 3} = 6 * \sqrt{3}$



Hello! I am your tutor. Can you walk me through your solution?

Hey! I'm not really sure where to start. I think the distance from C to AB is like a perpendicular distance maybe? But I'm not sure how to calculate it or prove it. And for the second part, I have no idea how to show that the distance from B to AC is $6 * \sqrt{3}$. Can you help me out?



Absolutely, you're on the right track thinking about the perpendicular distances. Since we're dealing with a right-angled triangle at A, let's consider the sides and the angles they form with one another. What can you tell me about the angle formed between the side AC and the line AB?

Oh, since triangle ABC is a right-angled triangle and A is the right angle, that means the angle between AC and AB is 90 degrees. So AC is perpendicular to AB.



That's correct. Now, how does that piece of information relate to finding the distance from point C to the line AB?

If AC is perpendicular to AB and we want the distance from point C to line AB, then that distance is just the length of AC itself, because the shortest distance from a point to a line is the perpendicular distance. So, that means the distance is just 6 cm.



Excellent observation! Now that you've determined the distance from C to line AB, what about the second part of the problem involving the distance from B to the line AC? Can you think of a property or theorem that might help with finding the length of side AB?

Yes, since we have a right triangle, we can use the Pythagorean theorem to find the length of AB. The Pythagorean theorem says that in a right triangle, the square of the length of the hypotenuse (which is AB here) is equal to the sum of the squares of the lengths of the other two sides. So if we let AB be x , we would have $x^2 = AC^2 + BC^2$. We can plug in the values for AC and BC, which are 6 cm and 12 cm, and solve for x . So that's $x^2 = 6^2 + 12^2$. Let me calculate that...



Take your time doing the calculation, and make sure to review the relationship between the sides of the triangle carefully when applying the Pythagorean theorem.

Alright, so we have $x^2 = 6^2 + 12^2$, which means $x^2 = 36 + 144$. That gives us $x^2 = 180$. Then, we take the square root of both sides to find x . So the length of AB, which is the hypotenuse, would be $\sqrt{180}$. Simplifying that, it's the same as $\sqrt{36 \cdot 5}$ which is $6\sqrt{5}$ cm.



Hold on, I think I made a mistake. Let me try that calculation again...

It should be $x^2 = 36 + 144 = 180$, and the square root of 180 is $6\sqrt{5}$. But we're talking about a distance to a line, so that doesn't seem right for $d(B, AC)$... I'm confused now.

Take your time to reflect on your calculations and consider if each step follows logically from the previous. Remember to think about the relationships between the lines and the angles they make with each other within the triangle.

Right, sorry about that. The relationship is between the sides of the triangle itself. Let's try the Pythagorean theorem again correctly.



We have $AC^2 + BC^2 = AB^2$. With AC being 6 cm and BC being 12 cm, we plug those values in to get $6^2 + 12^2 = AB^2$, which is $36 + 144 = AB^2$. So $AB^2 = 180$.

Taking the square root of both sides, we then find $AB = \sqrt{180}$, and breaking down 180, we get $AB = \sqrt{36 \cdot 5}$, which further simplifies to $AB = 6\sqrt{5}$ cm.

Now that we have the length of AB, we can look for the height of the triangle from B to AC, which is the line segment perpendicular to AC and passing through B. This segment is the same as the distance $d(B, AC)$. We know the area of a triangle can also be found by using the formula $\frac{1}{2} \times \text{base} \times \text{height}$.

The area can also be computed using the sides AC and BC because they are perpendicular. So the area is $\frac{1}{2} \times AC \times BC = \frac{1}{2} \times 6 \times 12 = 36 \text{ cm}^2$.

We can also express the area in terms of side AB and the distance $d(B, AC)$ we are looking for.

So we have $36 = \frac{1}{2} \times AB \times d(B, AC)$.

Now, substituting AB with $6\sqrt{5}$ cm, we solve for $d(B, AC)$:

$$36 = \frac{1}{2} \times 6\sqrt{5} \times d(B, AC).$$


Divide both sides by $3\sqrt{5}$ to solve for $d(B, AC)$:

$$d(B, AC) = \frac{36}{3\sqrt{5}} = \frac{12}{\sqrt{5}}.$$

To rationalize the denominator, we multiply the numerator and denominator by $\sqrt{5}$, so we get

$$d(B, AC) = \frac{12\sqrt{5}}{5}.$$

Let me verify that



You're re-evaluating your steps, which is good practice. Look closely at your calculations and the triangle's properties, and consider if there are any simplifications or alternative expressions you might use. Reflect on whether you've applied the Pythagorean theorem and the area formulas correctly and what the results imply.



Okay, let's start by calculating the area of the triangle again since it seems I got a bit lost in the calculations. The area (A) of a triangle can be calculated as $A = \frac{1}{2} \times \text{base} \times \text{height}$. Here, we can use AC and BC as the base and height since they are perpendicular.

So the area A is $A = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 6 \times 12 = 36 \text{ cm}^2$. That's correct.

Now the area can also be expressed in terms of side AB and the height from B, which is the distance $d(B, AC)$. Using AB as the base, we have $A = \frac{1}{2} \times AB \times d(B, AC)$. We've already found out that the length of AB is $6\sqrt{5}$ cm from the Pythagorean theorem.

Plugging in the values we have:

$$36 = \frac{1}{2} \times 6\sqrt{5} \times d(B, AC).$$

After simplifying we get:

$$36 = 3\sqrt{5} \times d(B, AC).$$

And then we solve for $d(B, AC)$:

$$d(B, AC) = \frac{36}{3\sqrt{5}}.$$

$$d(B, AC) = \frac{36}{3} \times \frac{1}{\sqrt{5}}.$$

$$d(B, AC) = 12 \times \frac{1}{\sqrt{5}}.$$

To rationalize the denominator, we multiply by $\sqrt{5}$ both numerator and denominator:

$$d(B, AC) = 12 \times \frac{\sqrt{5}}{5}.$$

Actually, I made a mistake here. We've got to simplify this correctly. Multiplying the numerator and the denominator by $\sqrt{5}$:

$$d(B, AC) = \frac{12\sqrt{5}}{5}.$$

But to get the denominator to be a perfect square root, we can multiply both by $\sqrt{5}$:

$$d(B, AC) = \frac{12\sqrt{5}}{5} \times \frac{\sqrt{5}}{\sqrt{5}}.$$

Now, simplifying the numerator

That seems like a good approach to follow through. What will the new expression for $d(B, AC)$ look like when you simplify the numerator?

Right, when we multiply the numerator by $\sqrt{5}$, we get $12\sqrt{5} \times \sqrt{5}$, which simplifies to 12×5 because $\sqrt{5} \times \sqrt{5} = 5$. So the expression for $d(B, AC)$ simplifies to:



$$d(B, AC) = \frac{12 \times 5}{5}.$$

Simplifying further, we cancel out the 5 in the numerator and the denominator:

$$d(B, AC) = \frac{60}{5} = 12.$$

But when I look at it, I realize I have ignored the crucial step of rationalizing the denominator properly. Let's do that part again:

$$d(B, AC) = 12 \times \frac{\sqrt{5}}{\sqrt{5}} \text{ gives us:}$$

$$d(B, AC) = 12 \times 1 = 12 \text{ cm.}$$

I made an error earlier, but this simplification shows that the distance from B to AC, $d(B, AC)$, is just 12 cm, not $12\sqrt{5}$ cm as I incorrectly wrote before. The calculation of $12 \times \sqrt{5}$ was erroneous because when we perform the rationalization correctly, we multiply both the numerator and denominator by $\sqrt{5}$ which gives us 12×5 in the numerator, not $12 \times \sqrt{5}$.

My bad, let's rationalize it correctly:


$$d(B, AC) = \frac{12\sqrt{5}}{5} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{12 \cdot 5}{5}.$$

Now, simplifying the expression:

$$d(B, AC) = \frac{60}{5} = 12.$$

Thus, $d(B, AC) = 12$ cm. But we want to show that it is equal to $6\sqrt{3}$.

After rechecking, we see that the correct factorial for the square root is missed. The square root of 180 should be simplified by factoring into perfect squares, which is $\sqrt{36} \times \sqrt{5}$, thus $\sqrt{180} = 6\sqrt{5}$. Our target distance here $d(B, AC)$ is related to the



Consider the relationship between the lengths of the sides in a right triangle when you use the Pythagorean theorem to express them. Try to evaluate how you determined the length of AB from the equation involving the Pythagorean theorem. What does the Pythagorean theorem dictate about the relationship between the sides in our right triangle?

Okay, let's look again at the Pythagorean theorem relation. We have a right triangle, and the theorem states that for a right triangle with sides of lengths a , b , and the hypotenuse c , the relation $a^2 + b^2 = c^2$ holds.



For our triangle ABC, with sides AC and BC, and hypotenuse AB, we have:

$$AC^2 + BC^2 = AB^2.$$

We have AC = 6 cm and BC = 12 cm, so plugging these values in gives us:

$$\begin{aligned} 6^2 + 12^2 &= AB^2, \\ \text{which is:} \\ 36 + 144 &= AB^2. \end{aligned}$$

$$\begin{aligned} \text{Therefore:} \\ 180 &= AB^2. \end{aligned}$$

Now let's find the value of AB by taking the square root:

$$AB = \sqrt{180}.$$

To simplify $\sqrt{180}$, we can break 180 into its prime factors:

$$\begin{aligned} 180 &= 2^2 \times 3^2 \times 5, \\ \text{so that means:} \\ \sqrt{180} &= \sqrt{2^2 \times 3^2 \times 5} = 2 \times 3\sqrt{5} = 6\sqrt{5}. \end{aligned}$$

$$\text{Now we have } AB = 6\sqrt{5} \text{ cm.}$$

Moving on to the second part, to find the distance $d(B, AC)$, we can use the formula for the area of triangle ABC again. The area that we calculated before, 36 cm^2 , can be expressed using the height from B to AC, which is the distance we're looking for.

$$\begin{aligned} \text{So:} \\ \text{Area} &= \frac{1}{2} \times AB \times d(B, AC), \\ \text{thus:} \\ 36 &= \frac{1}{2} \times 6\sqrt{5} \times d(B, AC). \end{aligned}$$

If we solve this for $d(B, AC)$, we get:


$$\begin{aligned} d(B, AC) &= \frac{36 \times 2}{6\sqrt{5}}, \\ \text{and simplifying gives us:} \end{aligned}$$

$$d(B, AC) = \frac{72}{6\sqrt{5}},$$

which further simplifies to:

$$d(B, AC) = \frac{12}{\sqrt{5}}.$$

To rationalize the denominator, multiply the numerator and the denominator by



Remember to carefully consider your last step. What would be your approach to rationalize the denominator?

My apologies for the oversight. To rationalize the denominator when we have $\frac{12}{\sqrt{5}}$, we should multiply the numerator and the denominator by $\sqrt{5}$ to get rid of the square root in the denominator. Here's how we do it:



$$d(B, AC) = \frac{12}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}.$$

When we multiply the numerators together and the denominators together, we get:

$$d(B, AC) = \frac{12 \times \sqrt{5}}{5}.$$

So, the distance from B to the line AC, $d(B, AC)$, is:

$$d(B, AC) = \frac{12 \times \sqrt{5}}{5} = \frac{12}{5} \sqrt{5}.$$

But we need to show that this is equal to $6\sqrt{3}$.

If we compare this result with $6\sqrt{3}$, there is an apparent discrepancy. Let's recheck our calculation of the length of AB using the Pythagorean theorem:

$6^2 + 12^2 = AB^2$ gives us $36 + 144 = AB^2$, so $AB^2 = 180$. Then,

$$AB = \sqrt{180}.$$

Factoring 180 we have:

$$180 = 2^2 \times 3^2 \times 5, \text{ which means}$$

$$\sqrt{180} = \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}.$$

Now, we've established that AB is indeed $6\sqrt{5}$ cm. Now I see the error in my previous calculation. We should actually have:

$d(B, AC) = \frac{36 \times 2}{6\sqrt{5}} = 12\sqrt{5}$ divided by $\sqrt{5} \times \sqrt{5}$, which indeed gives us:

$$d(B, AC) = 12\sqrt{5} \text{ divided by } 5 = \frac{12\sqrt{5}}{5} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{12 \times 5}{5} =$$

Now that we've multiplied the numerator and the denominator by $\sqrt{5}$, what does the expression become?

Multiplying the numerator and the denominator by $\sqrt{5}$, we have:



$$d(B, AC) = \frac{12\sqrt{5}}{5} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{12*5}{5}$$

Simplifying further by multiplying inside the numerator and cancelling the 5 in the denominator with the 5 in the numerator gives:

$$d(B, AC) = \frac{12*5}{5} = 12$$

So the distance from B to the line AC, $d(B, AC)$, is 12 cm. But we needed to show that it's equal to $6\sqrt{3}$. Let's compare the two expressions:

$$12 \quad \text{and} \quad 6\sqrt{3}.$$

A quick computation shows that $\sqrt{3} \approx 1.732$. If we multiply 6 by $\sqrt{3}$, the result is approximately:

$$6 * 1.732 \approx 10.392,$$

which is not equal to 12. It seems we're looking at an incorrect statement here, because based on our calculations, $d(B, AC)$ is indeed 12 cm, not $6\sqrt{3}$ cm.

Now, consider how the lengths relate and what properties of a right triangle we might use. Is there another approach we could take to verify the relationship between the sides of the triangle and the distance from B to the line AC?



Right, my mistake. Let's rethink the approach by using the properties of a right-angled triangle. We know that in a right-angled triangle, the altitude to the hypotenuse divides the triangle into two smaller similar right-angled triangles. Each of these triangles is similar to the original right-angled triangle as well as to each other.

From the given triangle ABC, where angle A is the right angle, the altitude from point B to side AC (which is the distance $d(B, AC)$) will divide the original right-angled triangle into two smaller triangles, ABC and BCD, where D is the foot of the altitude from B on AC.

Each of the smaller triangles (ABC and BCD) is similar to the original triangle (ABC) because they all share the same angles and therefore, the sides are proportional.

For the original right triangle ABC:

$$\frac{AB}{BC} = \frac{BC}{AC}$$

Let's insert the known lengths:

$$\frac{AB}{12} = \frac{12}{6}$$

Since we previously computed $AB = 6\sqrt{5}$, we can write:

$$\frac{6\sqrt{5}}{12} = \frac{12}{6}$$

Now solving for AB :

$$\frac{\sqrt{5}}{2} = 2$$

It is at this step that it becomes clear we can equate the length of the altitude with the geometric mean of the segments it divides on the hypotenuse. So we should have:

$$AC \cdot AD = BC^2$$

Given $AC = 6\text{cm}$, AD is the segment of AB that extends from point A to point D, and $BC = 12\text{cm}$, we can now create an equation for AD:

$$6 \cdot AD = 12 \cdot 12$$


$$AD = \frac{144}{6}$$

$$AD = 24\text{cm}$$

This value does not seem right as it implies that AD is longer than AB which is impossible. Let's correct that calculation by finding AD using

the geometric mean property correctly:

Since, in the same right-angled triangle, the altitude (let's call it h for clarity) is also the geometric mean of the two segments it cuts the hypotenuse into. Considering that AD is the



Reflect on the geometric mean relationship in the context of triangle ABC . How would you apply the relationship between the lengths of the segments on the hypotenuse and the altitude from B to AC ?

You're right. Let's use the geometric mean property properly here. In a right-angled triangle, the altitude to the hypotenuse has a special property: the length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse it creates.



Consider our right triangle ABC with the right angle at A, and AB as the hypotenuse. When we draw the altitude BD from vertex B to side AC, it meets side AC at point D, dividing AB into two segments, AD and DB.

The geometric mean relationships are as follows:

1. The length of the altitude BD (which is our $d(B, AC)$) is the geometric mean of the lengths of the segments of the hypotenuse it determines, so $BD^2 = AD \cdot DB$.
2. Also, the length of AD is the geometric mean of the lengths of AB and AD, so $AD^2 = AB \cdot AC$.
3. Similarly, the length of DB is the geometric mean of the lengths of AB and DB, so $DB^2 = AB \cdot BC$.

Since we want to find BD (aka $d(B, AC)$), then we should concentrate on the first relationship:

Here's how we should proceed:

Let $AB = 6\sqrt{5}$, $AC = 6$, and $BC = 12$ as previously calculated. $BD^2 = AD \cdot DB$. It's also true that the altitude BD is the same as $d(B, AC)$, which we are trying to find.

Since we also have the similar triangles ABD and CBD to the larger triangle ABC, we set up the two relationships:

$$AD/AB = AC/BC \text{ and } DB/AB = BC/AC.$$

We get AD and DB by solving these proportions: $AD = (AB \cdot AC) / BC$ and $DB = (AB \cdot BC) / AC$.

After calculating AD and DB, we can solve for BD by using the geometric means:

$$BD = \sqrt{AD \cdot DB}.$$

Let's start by finding AD and DB.

Let's start by finding AD and DB.

$$AD = (AB * AC)/BC = (6\sqrt{5} * 6)/12,$$

$$AD = 36\sqrt{5}/12,$$

$$AD = 3\sqrt{5}.$$

For DB, we have:



Now that the expression for AD has been established, can you find DB using a similar process?

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