

The student is working on the following problem:

Let ABC be a right-angled triangle at A with AC=6cm and BC=12cm.

Determine $d(C, AB)$ the distance from C to the line (AB) and

show that $d(B, AC)$, the distance from B to the line (AC) is equal to $6 * \sqrt{3}$

The solution of this problem is:

ABC is a right-angled triangle at A so the hypotenuse is BC and sides AB and AC are perpendicular. First, $d(C, AB) = AC = 6\text{cm}$ (orthogonal projection).

Second, by the Pythagorean theorem, $BC^2 = AB^2 + AC^2$ so $AB^2 = BC^2 - AC^2$.

We compute $AB^2 = BC^2 - AC^2 = 12^2 - 6^2 = 144 - 36 = 108$ Since (AB) is perpendicular to (AC), A is the orthogonal projection of B on (AC), so $d(B, AC) = AB$. Thus, $d(B, AC) = AB = \sqrt{108} = \sqrt{6^2 * 3} = 6 * \sqrt{3}$



Hello! I am your tutor. Can you walk me through your solution?

Hey! I'm a bit confused honestly. I know that ABC is a right-angled triangle with the right angle at A. But I'm not sure how to find the distance from C to the line AB. Could you help me out?



Certainly! Think about where the right angle is in this triangle and how that affects the distance from point C to the line AB. What can you say about the line segment that represents this distance in terms of its orientation to AB?

Oh, right! Since AB is the hypotenuse, the distance from C to AB would be the perpendicular dropped from C to AB. So the line representing this distance is perpendicular to AB.



Great observation! Now, keeping that in mind, how would you express the distance from C to AB in terms of the sides of the triangle?

If I got it right, the distance from C to AB ($d(C, AB)$) would be the altitude from C on to AB. In a right-angled triangle like this one, the altitude to the hypotenuse is also the shortest distance from the vertex to the hypotenuse. So, in terms of sides, $d(C, AB)$ would be the length of that altitude, right? But I'm not sure how to find that length.



Correct, you have the altitude concept down perfectly. Now, can you think of any side of the triangle that already serves as that perpendicular drop to AB?

Oh, since angle A is the right angle, that means line segment AC is already perpendicular to AB. So actually, AC itself is the altitude of triangle ABC with respect to the hypotenuse AB. That means the distance from C to AB, or $d(C, AB)$, is just the length of AC, which is given to be 6 cm.



That's a good analysis of the situation. Now, regarding the distance from B to the line AC, how might you approach finding this distance using the information you've just used?

Since we are looking for $d(B, AC)$, which is the distance from point B to line AC, and we know that distances are measured along the perpendicular, $d(B, AC)$ would be the line segment drawn from B perpendicular to AC.



To find it, I can use the fact that in a right-angled triangle, the area can be found by using the two sides that are perpendicular to each other (the base and height). So if I call the length of the altitude from B to AC "h", then the area of triangle ABC can also be expressed as $\frac{1}{2} * AC * h$.

I already know another way to express the area of triangle ABC using the sides AB and BC because they form a right angle. That means the area is also $\frac{1}{2} * AC * BC$.

So I can set these equal to each other to solve for h, which is $d(B, AC)$. Let me do that:

$$\frac{1}{2} * AC * h = \frac{1}{2} * AC * BC$$

$$h = BC.$$

Wait, that doesn't seem correct. Did I make a mistake somewhere?



Take another look at the way you've expressed the area using side BC and think about what sides you'd actually need to use to calculate the area of the right-angled triangle. Consider the sides that are perpendicular to each other. Can you find any inconsistencies in your expressions for the area of triangle ABC?

Right, I messed up there. To express the area with sides AB and BC, it's not $\frac{1}{2} * AC * BC$ since BC is not perpendicular to AB. It should be $\frac{1}{2} * AB * AC$, using the base AB and the height AC since they're perpendicular.



For the expression with the altitude h from B to AC, the area will be $\frac{1}{2} * AC * h$ since the base is AC and the height is h.

So I should set those two expressions equal to each other:

$$\frac{1}{2} * AB * AC = \frac{1}{2} * AC * h$$

I can simplify this because the factor of $\frac{1}{2} * AC$ is on both sides and therefore it cancels out:

$$AB = h$$

But I need to determine what AB is, given AC and BC. Since it's a right-angled triangle, I think I can use the Pythagorean theorem to do that. Let me try that:

$$AB^2 = AC^2 + BC^2$$

$$AB = \sqrt{AC^2 + BC^2}$$

$$AB = \sqrt{6^2 + 12^2}$$

$$AB = \sqrt{36 + 144}$$

$$AB = \sqrt{180}$$

$$AB = \sqrt{36 * 5}$$

$$AB = 6\sqrt{5}$$

So then h, which is the d(B, AC), is $6\sqrt{5}$ cm. But I was supposed to show it's equal to $6\sqrt{3}$ cm. I must have made another mistake because $\sqrt{5}$ is not the same as $\sqrt{3}$.

You've correctly applied the Pythagorean theorem, but there seems to be a little miscalculation when you add the squares of AC and BC. Could you try recalculating AB? Take your time and make sure you add up the squares correctly. Good luck!



Oops, I see my mistake. Let me recalculate:

$$AB^2 = AC^2 + BC^2$$

$$AB = \sqrt{AC^2 + BC^2}$$

$$AB = \sqrt{6^2 + 12^2}$$

$$AB = \sqrt{36 + 144}$$

$$AB = \sqrt{180}$$

But I need to break down 180 into its prime factors to simplify the square root:

$$180 = 2 * 2 * 3 * 3 * 5 = 2^2 * 3^2 * 5$$

$$\text{So, } \sqrt{180} = \sqrt{2^2 * 3^2 * 5} = 2 * 3 * \sqrt{5} = 6\sqrt{5}.$$

Oh wait, I did the factoring correctly this time but made the same mistake in simplifying $\sqrt{180}$. It should be:

$$\sqrt{180} = \sqrt{2^2 * 3^2 * 5} = 2 * 3 * \sqrt{5} = 6\sqrt{5}.$$

So after properly factoring 180, AB is still $6\sqrt{5}$ cm. But you mentioned I need to show that the distance from B to AC is $6\sqrt{3}$ cm. I think maybe I'm supposed to use a different approach? Could you please give me another hint?

Take a closer look at your breakdown of 180 and see if it can be factored in a different way. You've done great work so far, being diligent with the process is important, and sometimes a small oversight can lead to an incorrect simplification. Review your prime factorization of 180 once more and see if you can find a different combination of primes that could simplify to $\sqrt{3}$ instead of $\sqrt{5}$. If you spot the error, you should be able to correct the final step on your own. Keep up the good thinking! Goodbye for now.

Oh, I think I see what's wrong. Let's try the prime factorization of 180 again and simplify the square root one more time:



$$180 = 2 * 2 * 3 * 3 * 5$$

Ok, breaking it down into squares to take out of the square root, I have:

$$180 = (2^2) * (3^2) * 5$$

So,

$$\sqrt{180} = \sqrt{(2^2) * (3^2) * 5}$$

Now, taking the square root of the squares:

$$\sqrt{180} = 2 * 3 * \sqrt{5} = 6\sqrt{5}$$

Wait, that's still not right. I need to factor 180 as:

$$180 = 2 * 2 * 3 * 3 * 5 = (2 * 3) * (2 * 3) * 5$$

Now let's group the pairs:

$$180 = (2 * 3)^2 * 5 = 6^2 * 5$$

Taking the square root:

$$\sqrt{180} = \sqrt{6^2 * 5} = 6\sqrt{5}$$

No... It still doesn't give me the $\sqrt{3}$. I must be missing something. The prime factorization part seems right though. Could it be...?

Oh my, I see it now! I think I've been repeatedly typing 5 instead of what should be there – this whole time I totally overlooked the right prime factor. 180 in terms of squares should indeed involve 3, not 5. Here's the correct breakdown:

$$180 = 2 * 2 * 3 * 3 * 5 = (2^2) * (3^2) * 5 = 4 * 9 * 5 = 36 * 5$$

Then, taking the square root:

$$\sqrt{180} = \sqrt{36 * 5} = 6\sqrt{5}$$

Yes! That's the correct simplification – finally! So, the distance from B to AC, or h, is $6\sqrt{5}$, which means d(B, AC) equals $6\sqrt{5}$. Thanks for your patience! Goodbye!

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