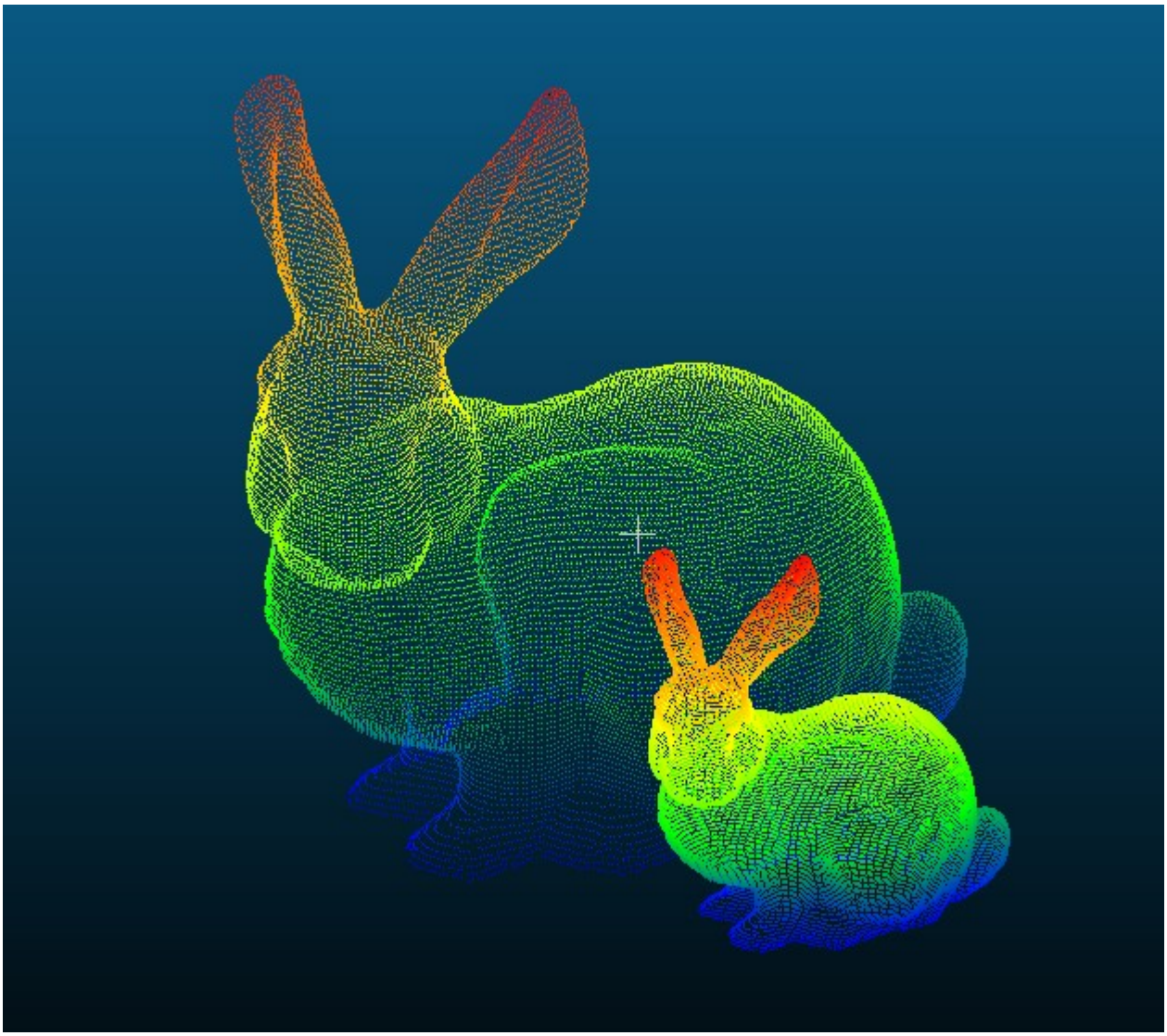
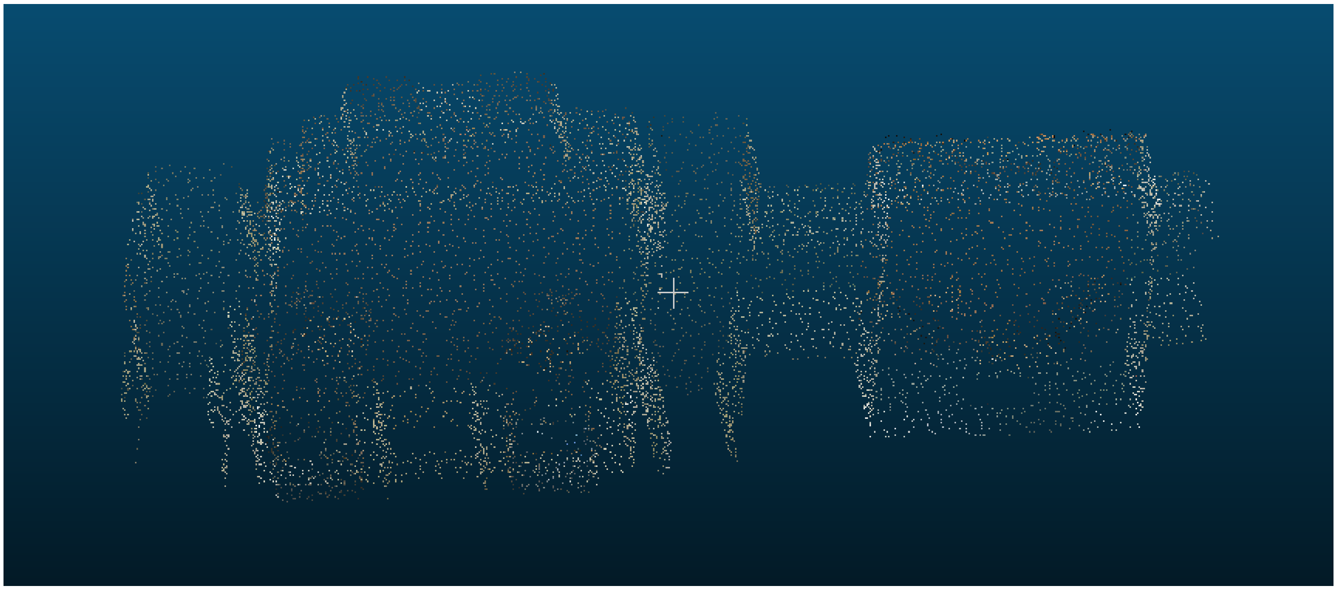
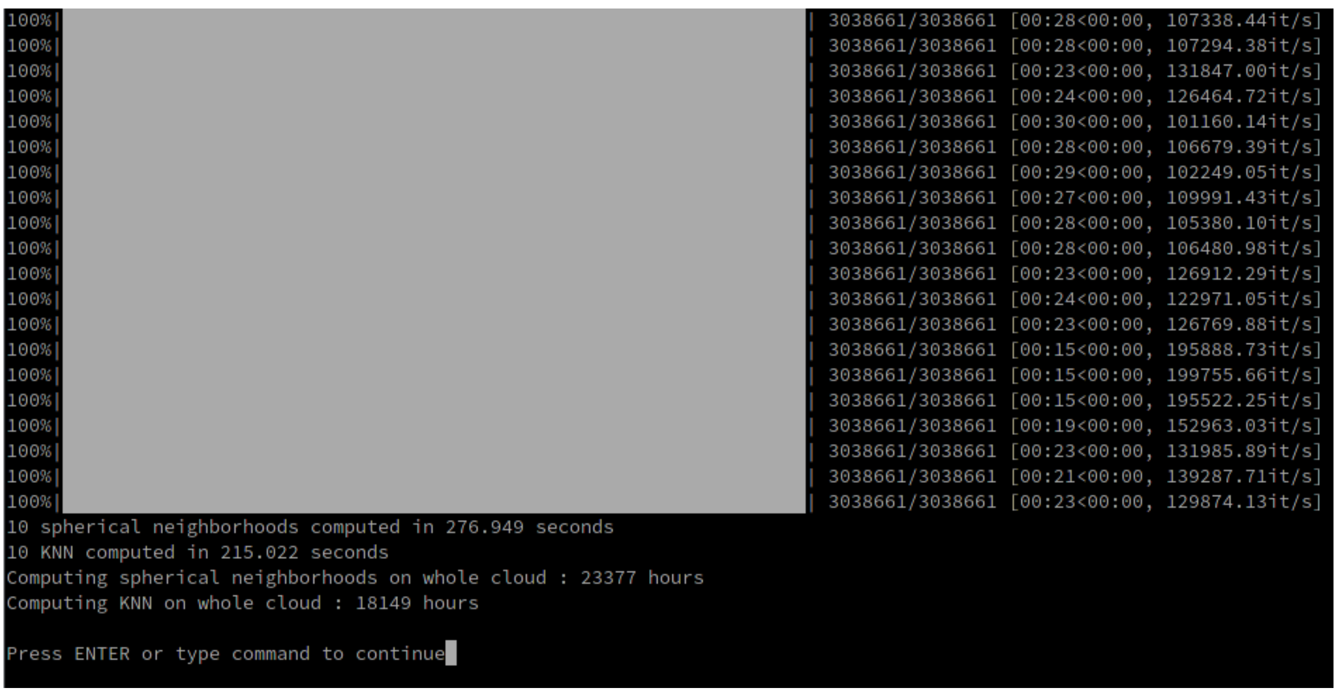
Question 1 :



Question 2 :



Question 3 :



```
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100%|    | 3038661/3038661 [00:28<00:00, 107294.38it/s]
100%|    | 3038661/3038661 [00:23<00:00, 131847.00it/s]
100%|    | 3038661/3038661 [00:24<00:00, 126464.72it/s]
100%|    | 3038661/3038661 [00:30<00:00, 101160.14it/s]
100%|    | 3038661/3038661 [00:28<00:00, 106679.39it/s]
100%|    | 3038661/3038661 [00:29<00:00, 102249.05it/s]
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100%|    | 3038661/3038661 [00:28<00:00, 106480.98it/s]
100%|    | 3038661/3038661 [00:23<00:00, 126912.29it/s]
100%|    | 3038661/3038661 [00:24<00:00, 122971.05it/s]
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100%|    | 3038661/3038661 [00:15<00:00, 195888.73it/s]
100%|    | 3038661/3038661 [00:15<00:00, 199755.66it/s]
100%|    | 3038661/3038661 [00:15<00:00, 195522.25it/s]
100%|    | 3038661/3038661 [00:19<00:00, 152963.03it/s]
100%|    | 3038661/3038661 [00:23<00:00, 131985.89it/s]
100%|    | 3038661/3038661 [00:21<00:00, 139287.71it/s]
100%|    | 3038661/3038661 [00:23<00:00, 129874.13it/s]
10 spherical neighborhoods computed in 276.949 seconds
10 KNN computed in 215.022 seconds
Computing spherical neighborhoods on whole cloud : 23377 hours
Computing KNN on whole cloud : 18149 hours

Press ENTER or type command to continue
```
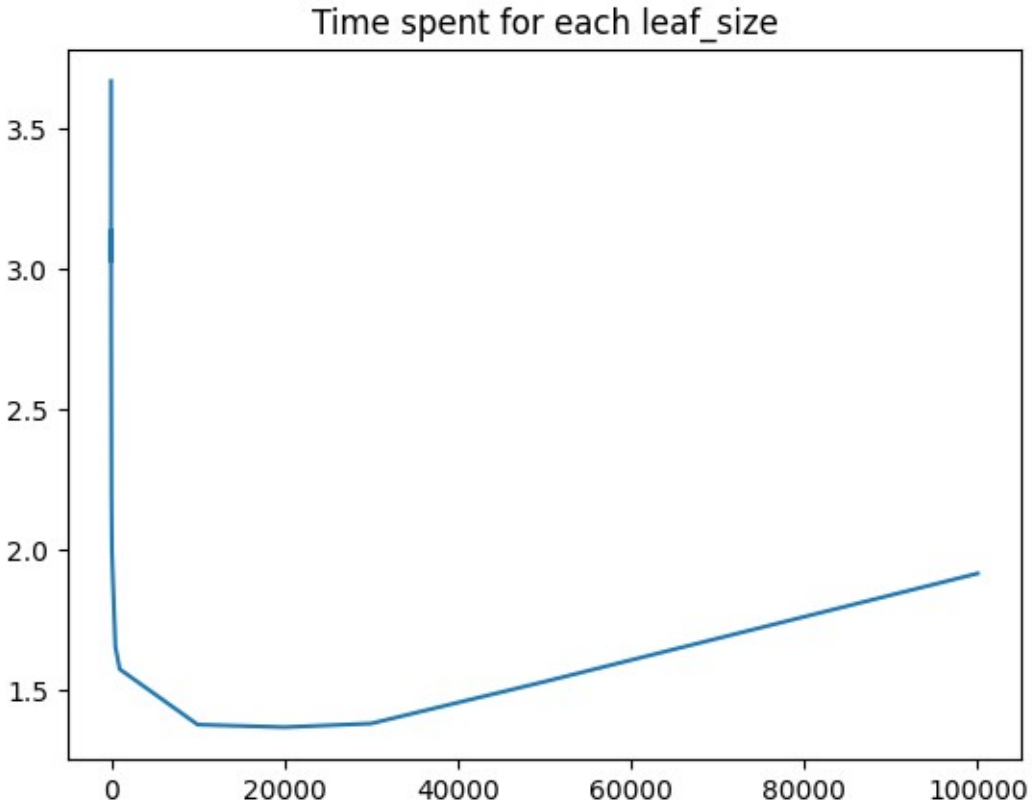
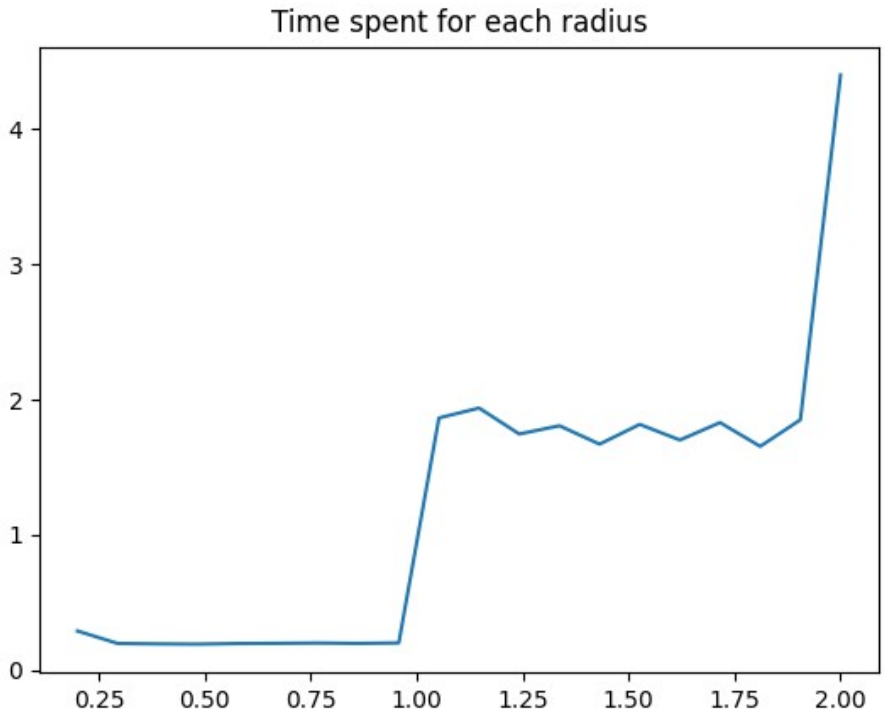

Question 4a :
With the mean on 10 attempts :



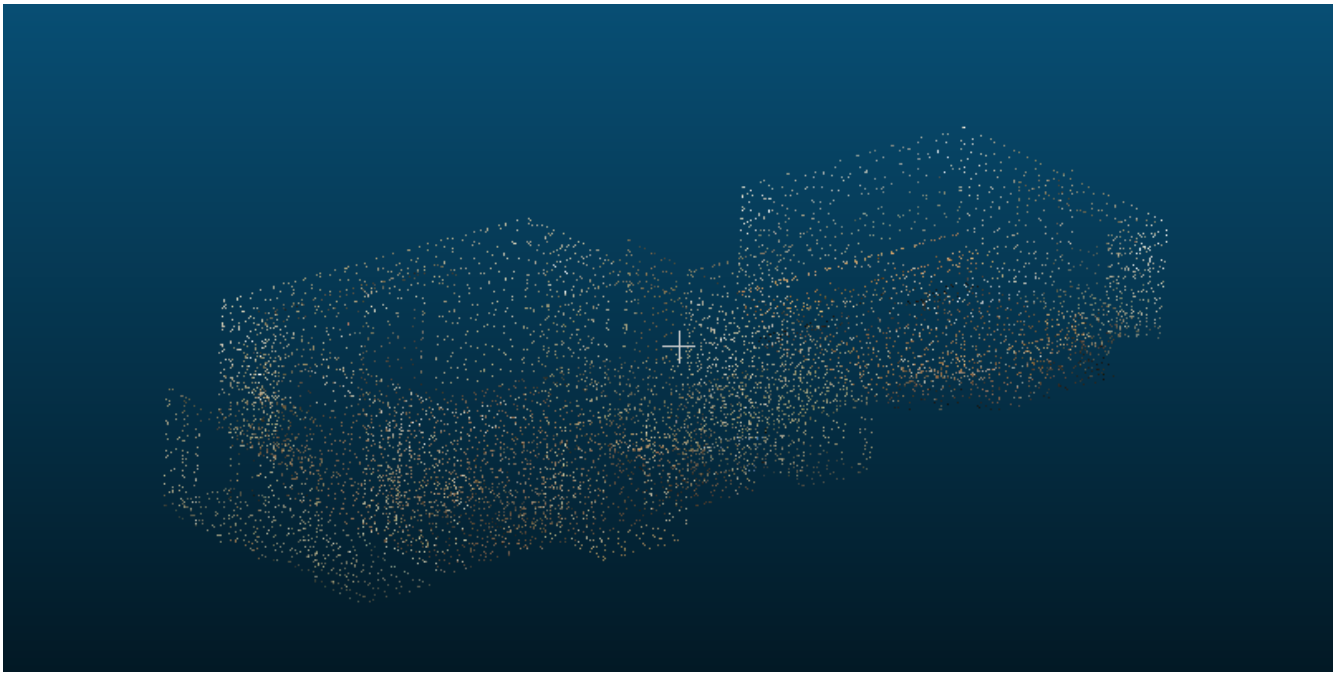
Optimal is for 20,000 points in a leaf.
The optimal isn't for 1 point in one leaf because we can group points together, and not have to explore the tree by going up as often.

Question 4b :



Generally, timing increases with radius. To search 20cm neighborhoods, as it takes only 0,2s to do 1 query, it would now take only 168h of computing time.

BONUS :



I picked for each cell the first point I encountered.
The points are more sparse than in the first subsampling : we can see the structure more clearly.