A practical introduction to Answer Set Programming

ENSEIRB — 2024-2025

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Organisation

- Objectives
- Roadmap
- Resources & Literature
- Systems

What are we going to talk about?

```
takeUmbrella :- rain, not stayAtHome.
rain.
```

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rain.  takeUmbrella.
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{x(M,N) : s(M,N)}.
:- s(R,_), not x(R,_).
#minimize {B,A:x(A,B) }.
#show x/2.
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Objectives of the course

- Discover a logic programming paradigm
- Understand the concept of ASP
- Learn ASP syntax and understand it through examples
- Use Clingo to solve ASP programs
- Model problems
- Design optimisations
- Applications to AI

In practice

Main ASP solver is Clingo

Online solver
https://potassco.org/clingo/run/

Conda installation (compiled with Python)

```
conda install -c conda-forge clingo
```

Python `pip` installation + Jupyter [doc]

```
pip install clyngor_with_clingo
pip install notebook
# and to use clingo in Jupyter notebooks
IPYLOC=$(ipython locate profile)
echo 'alias_magic clingo script -p "clingo
--no-raise-error"' >> $IPYLOC/startup/config.ipy
```

Roadmap

Credits & Teaching resources

This course is a foretaste of the great collection of teaching resources designed by the KRR group (led by Torsten Schaub) at the University of Potsdam. Some contents originate from the course material, some are adapted from the original teaching resources, some are my own.

Potassco is the Potsdam Answer Set Solving Collection.

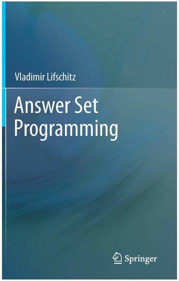
Visit potassco.org/teaching for additional content: videos, tutorials, slides...

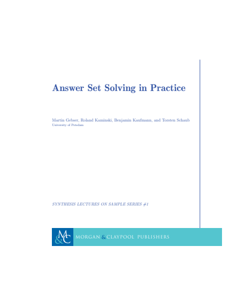
Potassco

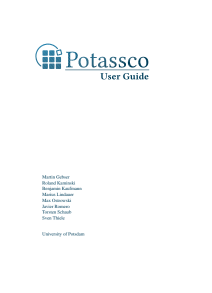
Resources

- A (short) paper on ASP: Lifschitz et al 2008.
- Course material
  - `https://github.com/potassco-asp-course`
  - `https://potassco.org/teaching`
- Videos
  - `https://youtube.com/c/potassco-live`
- Mailing lists
  - `https://sourceforge.net/projects/potassco/lists/potassco-users`
  - `https://sourceforge.net/projects/potassco/lists/potassco-announce`

The ASP and Potassco Book, and Guide







Resources

- http://potassco.org/book
- http://potassco.org/teaching

Literature

- Books [5], [25], [32], [38], [43]
- Surveys [40], [31], [19], [10], [36], [47]
- Magazines [9], [48]
- Articles [35], [34], [7], [45], [44], [39], [33], [23], etc.
- Guide [29]

- + bibliography on Github

Systems

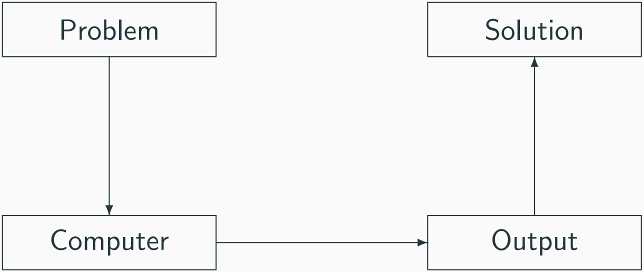
- Systems
  - *clingo* [27] https://potassco.org
  - *dlv* [37, 2] http://www.dlvsystem.com
- Grounders
  - *lparse* [50]
  - *gringo* [22, 28]+[24, 12] https://potassco.org
  - *idlv* [14]+[12] http://www.dlvsystem.com
- Solvers
  - *smodels* [46, 49]
  - *clasp* [26, 21] https://potassco.org
  - *wasp* [4] https://www.mat.unical.it/ricca/wasp
- Encodings
  - *asparagus* [8] https://asparagus.cs.uni-potsdam.de
  - competitions [30, 18, 16, 3, 15]

Motivation

- Traditional and declarative programming
- Nutshell
- Foundation
- Workflow and usage

Traditional programming

“How to solve the problem?”



Traditional programming

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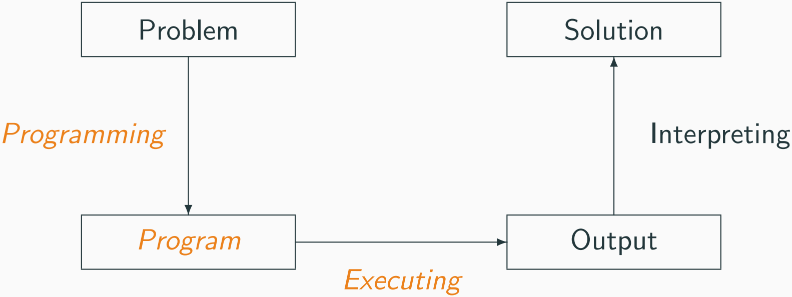
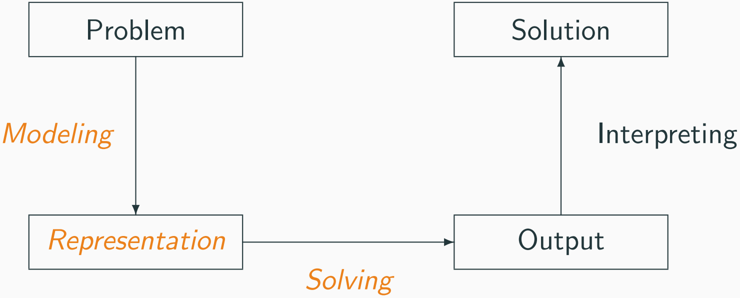
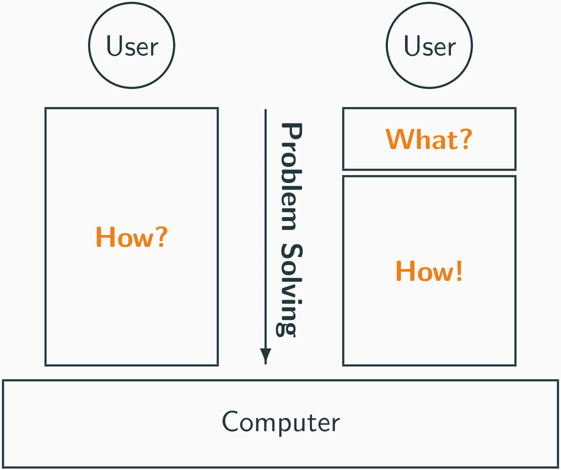


Declarative programming & problem solving

"What is the problem?"



From traditional to knowledge-driven software



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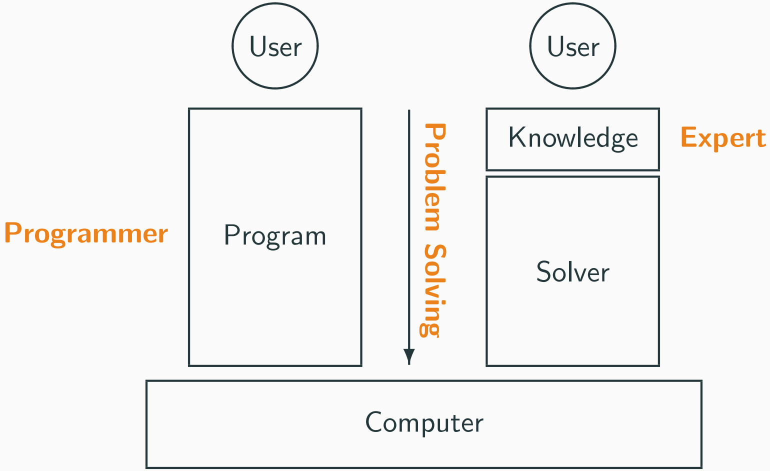
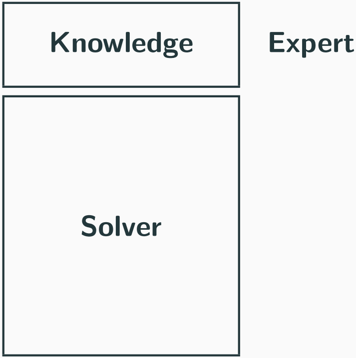


What is the benefit?

- \+ Transparency
- \+ Flexibility
- \+ Maintainability
- \+ Reliability

- \+ Generality
- \+ Efficiency
- \+ Optimality
- \+ Availability



Answer Set Programming (ASP)

- What is ASP?

  ASP is an approach for declarative problem solving

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- What is ASP? **ASP = DB+LP+KR+SAT !**

  ASP is an approach for declarative problem solving
- Where is ASP from?
  - Databases
  - Logic programming
  - Knowledge representation and reasoning
  - Satisfiability solving

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  Problems consisting of (many) decisions and constraints

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  Examples Sudoku, Configuration, Diagnosis, Music composition, Planning, System design, Time tabling, etc.

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  Problems consisting of (many) decisions and constraints
- What are ASP's distinguishing features?
  - High level, versatile modeling language
  - High performance solvers
  - Qualitative and quantitative optimization

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  Problems consisting of (many) decisions and constraints
  - Debian, Ubuntu: Linux package configuration
  - Exeura: Call routing
  - Fcc: Radio frequency auction
  - Gioia Tauro: Workforce management
  - Nasa: Decision support for Space Shuttle
  - Siemens: Partner units configuration
  - Variantum: Product configuration
  - US Navy: risk assessment

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  Problems consisting of (many) decisions and constraints
- Any industrial impact?
  - ASP Tech companies: DLV Systems and Potassco Solutions
  - Increasing interest in (large) companies

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- What problems are these? — **And industrial ones**
  Problems consisting of (many) decisions and constraints
- Any industrial impact?
  - ASP Tech companies: DLV Systems and Potassco Solutions
  - Increasing interest in (large) companies
- Anything not so good for ASP?
  - Number crunching

Logic programs

- A logic program is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_n}_{\text{body}}$$

where

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  1. (classical) models of the logic program
  2. requiring that each true atom is provable

Closed world assumption

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Closed world assumption

- Stable model semantics can be seen as a logic for reasoning under the Closed world assumption (CWA)
- In interpretations following the spirit of "classical" logic, an unknown proposition can be true or false
- Here: unknown information is treated as false
- This is a pretty "human" way, also referred to as *commonsense reasoning*
- e.g. bus stop, where a non-listed, and thus unknown timing, is interpreted as "false", in the sense that no bus is supposed to arrive
- Conclusions can change their truth value upon the arrival of new information

Default negation and strong negation

Default negation - Negation as failure
`p` does not not belong to the set and its negation either.

```
cross :- not train.
```

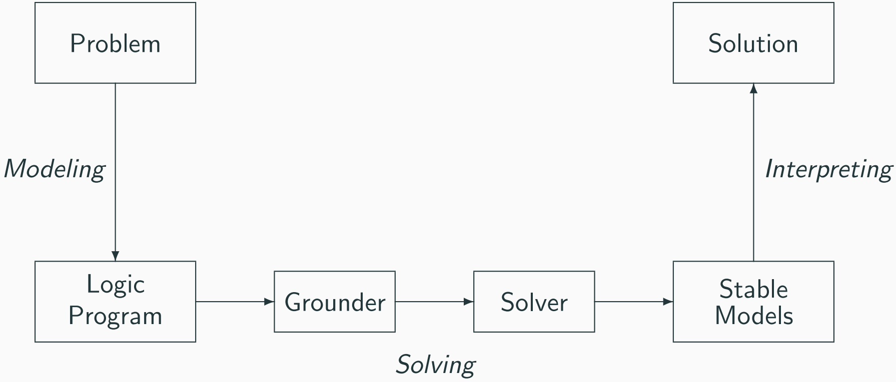
Answer set: `cross`

Strong negation
Also called "classical" negation. Denoted in the language by `-`.

```
cross :- - train.
```

Answer set: $\varnothing$

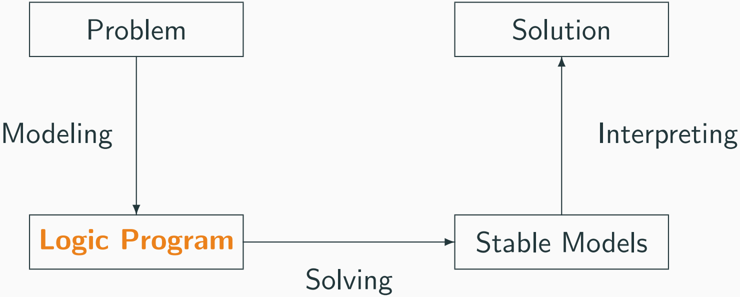
Modeling, grounding, and solving



Introduction to ASP

- Syntax
- Semantics: models and stable models
- Language
- Grounding and variables
- Summary

Syntax


Problem
Solution
Modeling
Interpreting
Logic Program
Stable Models
Solving

Atoms and terms

- Atoms
  - An atom is the elementary construct for representing knowledge
  - An atom represents a relation between objects
  - Examples
    - `answer(42)`
    - `bachelor(friend(joe))`
    - `hot`
  - An atom can be either *true* or *false*
- Terms
  - Terms are the subatomic components of atoms
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  - Examples
    - 42
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Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

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- Notation

$$\begin{aligned} h(r) &= a_0 \\ B(r) &= \{a_1, \dots, a_m, \neg a_{m+1}, \dots, \neg a_n\} \end{aligned}$$

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- Note We often assume that $\mathcal{A} = A(P)$

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Examples

- Example rules
  - $a \leftarrow b, \neg c$
  - $a \leftarrow \neg c, b$
  - $a \leftarrow$
  - $a \leftarrow b$
  - $a \leftarrow \neg c$
  - $bachelor(joe) \leftarrow male(joe), \neg married(joe)$
- Example literals
  - $a, b, c, bachelor(joe), male(joe), married(joe)$
  - $\neg c, \neg married(joe)$

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  - $a \leftarrow$
  - $a \leftarrow b$
  - $a \leftarrow \neg c$
  - $bachelor(joe) \leftarrow male(joe), \neg married(joe)$
- Example literals
  - $a, b, c, bachelor(joe), male(joe), married(joe)$
  - $\neg c, \neg married(joe)$

Examples

- Example rules
  - $a \leftarrow b, \neg c$
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Literal

- A literal is an atom or a negated atom

  also referred to as positive or negative literal

- Example
  - `answer(42)`, `not answer(42)`
  - `bachelor(friend(joe))`, `not bachelor(friend(joe))`, and
  - `hot`, `not hot`

Rules

- Rules are of the form

$$l_1 \,;\ldots\,; l_m \leftarrow l_{m+1}, \ldots, l_n \tag{1}$$

  where

  - l_i is a conditional literal for $1 \leq i \leq m$ and
  - l_i is a literal for $m+1 \leq i \leq n$

- Note Semicolons ';' must be used in (1) instead of commas ',' whenever some l_i is a (genuine) conditional literal for $1 \leq i \leq n$
- Example `a(X) :- b(X) : c(X), d(X); e(x).`
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Notational conventions

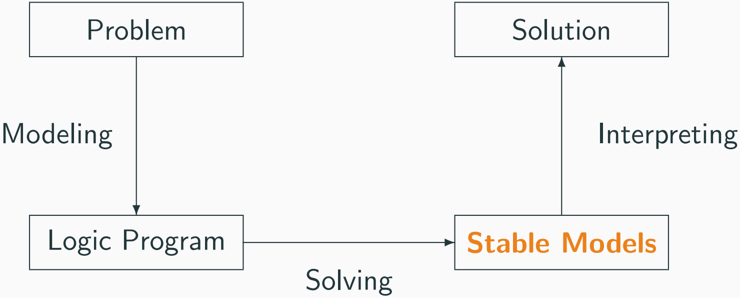
	false, true	if	and	or	iff	(default) negation	strong negation
source code		`:-`	`,`	`;`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\neg$	$\sim$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\neg$	$\sim$

Let's get practical

Identify stable models from given programs

- Programs without variables
  - positive programs
  - positive recursion
  - normal programs
  - negative recursion

Semantics



Problem
Solution
Modeling
Interpreting
Logic Program
Stable Models
Solving

What is a model?

- Assignment A function mapping variables to values
- Solution An assignment satisfying a set of constraints

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  - Example A is a solution of $\{2x < z, x + y < 2z\}$

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  - Example B is a model of $\{a \wedge b, a \vee c\}$

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  - Example We use $\{a, b\}$ to represent $\{a \mapsto \boldsymbol{T}, b \mapsto \boldsymbol{T}, c \mapsto \boldsymbol{F}\}$

Some "logical" remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

$$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model

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  Only negative body literals are evaluated!

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Example one

- $P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✘
$\{p\}$	$p \leftarrow p$	$\emptyset$	✘
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$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
$\{p\ \}$	$p \leftarrow$	$\{p\}$	✔
$\{\ q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✘

✔ stable model ✘ no stable model

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Stable model or Answer Set

- Theory - *cf* Potassco resources (esp. videos): consequences of positive programs, reducts of normal logic programs.
- In practice - some properties + examples.

Stable models vs open world reasoning

- $P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

X	*Classical logic*	*ASP*
$\{\quad\}$	✘	✘
$\{p\quad\}$	✔	✘
$\{\quad q\}$	✔	✔
$\{p, q\}$	✔	✘

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Some properties

- A logic program may have zero, one, or multiple stable models

- If X is a stable model of a logic program P, then $X \subseteq H(P)$

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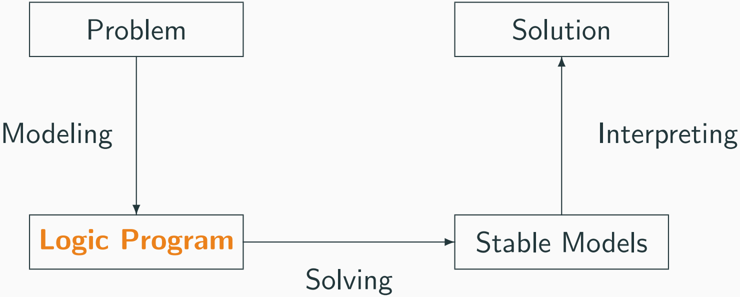
Exemplars

Logic program	Stable models
`a.`	{a}
`a :- b.`	{}
`a :- b.      b.`	{a,b}
`a :- b.      b :- a.`	{}
`a :- not c.`	{a}
`a :- not c.  c.`	{c}
`a :- not c.  c :- not a.`	{a}, {c}
`a :- not a.`	

Syntax


Problem
Solution
Modeling
Interpreting
Logic Program
Stable Models
Solving

Language constructs

- Facts `q(42).`
- Rules `p(X) :- q(X), not r(X).`
- Conditional literals `p :- q(X) : r(X).`
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- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y).`
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Grounding

- Ground instances of a rule r are obtained by replacing all variables in r by terms.
- Instanciation
- Grounding aims at reducing the ground instanciations by applying semantic principles
- More details in Potassco resources

Stable models of programs with Variables

- Let P be a normal logic program with variables
- A set X of (ground) atoms is a stable model of P,
  if X is a stable model of $\mathit{ground}(P)$

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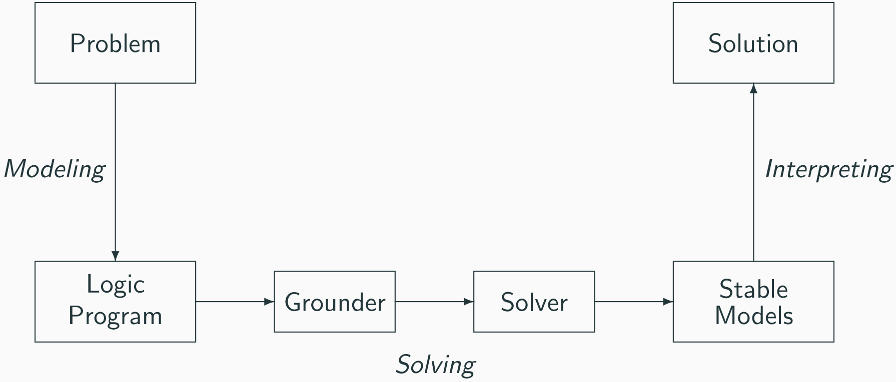
Modeling, grounding, and solving



Safety

- A normal rule is safe, if all its variables occur in its positive body
- Examples
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  - $p(X) \leftarrow$
  - $p(X) \leftarrow q(X)$
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ASP's syntax and semantics in a nutshell

- Syntax
  - A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
  - A (normal) rule, r, is of the form

    $$a_0 \leftarrow a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n$$

    where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

- Semantics
  - The reduct, P^X, of a program P relative to a set X of atoms is

    $$P^X = \{h(r) \leftarrow B(r)^+ \mid r \in P,\ B(r)^- \cap X = \emptyset\}$$

  - A set X of atoms is a stable model of a program P if $Cn(P^X) = X$

Exercises - Basic ASP

Let's get practical

Identify stable models from given programs

- Programs without variables
  - positive programs
  - positive recursion
  - normal programs
  - negative recursion
  - choice rules
  - integrity constraints
  - cardinality rules
- Programs with variables
  - normal logic programs
  - choices and constraints

Language

- Integrity constraint
- Choices
- Cardinality and weight
- Conditional literal
- Optimization
- Reasoning modes
- Exercises

Integrity constraint

- Purpose Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, \neg a_{m+1}, \dots, \neg a_n$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

- Example

```
:- edge(3,7), color(3,red), color(7,red).
```

Choice rule

- Purpose Provide choices over subsets of atoms
- Syntax A choice rule is of the form

$$\{a_1, \ldots, a_m\} \leftarrow a_{m+1}, \ldots, a_n, \neg a_{n+1}, \ldots, \neg a_o$$

  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model, any subset of $\{a_1, \ldots, a_m\}$ can be included in the stable model
- Example

```
{ buy(pizza);buy(wine);buy(corn) } :- at(grocery).
```

Cardinality rule

- Purpose Control (lower) cardinality of subsets of literals
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n \; \}$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

  and l is a non-negative integer called lower bound

- Informal meaning The head belongs to the stable model, if at least

  l positive/negative body literals are in/excluded in the stable model

- Example

```
pass(c42) :- 2 { pass(a1); pass(a2); pass(a3) }.
```

Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n \; \} \; u$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

  and l and u are non-negative integers

- Note The expression in the body of the cardinality rule is referred to as a cardinality constraint with lower and upper bound l and u

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Cardinality constraints as heads

- A rule of the form

  $$l \, \{a_1, \dots, a_m, \neg a_{m+1}, \dots, \neg a_n\} \, u \leftarrow a_{n+1}, \dots, a_o, \neg a_{o+1}, \dots, \neg a_p$$

  where $0 \le m \le n \le o \le p$ and each a_i is an atom for $1 \le i \le p$

  and l and u are non-negative integers

- Example

```
1 {color(2,red); color(2,green); color(2,blue)}
```

Conditional literals

- Syntax A conditional literal is of the form

$$l : l_1, \dots, l_n$$

  where l and l_i are literals for $0 \leq i \leq n$

- Informal meaning A (non-ground) conditional literal can be regarded as the collection of elements in the set $\{l \mid l_1, \dots, l_n\}$
- Example Assume '`p(1..3). q(2).`'
  - `r(X) : p(X), not q(X)` yields `r(1)` and `r(3)`

Optimization statement

- Purpose Express (multiple) cost functions subject to minimization
(and/or maximization)
- Syntax A minimize statement is of the form

$$minimize\ \{\ w_1@p_1 : l_{1_1}, \dots, l_{m_1}; \dots; w_n@p_n : l_{1_n}, \dots, l_{m_n}\ \}.$$

where each l_{j_i} is a literal and w_i and p_i are integers for $1 \leq i \leq n$

priority levels, p_i, allow for representing lexicographically ordered minimization objectives
- Meaning A minimize statement is a directive that instructs the
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  where each l_{j_i} is a literal and w_i and p_i are integers for $1 \leq i \leq n$

  priority levels, p_i, allow for representing lexicographically ordered minimization objectives
- Meaning A minimize statement is a directive that instructs the ASP solver to compute optimal stable models by minimizing a sum of weights (by descending levels)

Optimization statement

- A maximize statement of the form

$$maximize \; \{ \; w_1@p_1 : l_1, \ldots, w_n@p_n : l_n \; \}$$

stands for $minimize \; \{ \; -w_1@p_1 : l_1, \ldots, -w_n@p_n : l_n \; \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

```
#maximize { 250@1:hd(1); 500@1:hd(2); 750@1:hd(3) }.
#minimize { 30@2:hd(1);  40@2:hd(2);  60@2:hd(3) }.
```

The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

Optimization statement

- A maximize statement of the form

$$maximize \; \{ \; w_1@p_1 : l_1, \ldots, w_n@p_n : l_n \; \}$$

stands for $minimize \; \{ \; -w_1@p_1 : l_1, \ldots, -w_n@p_n : l_n \; \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

```
#maximize { 250@1:hd(1); 500@1:hd(2); 750@1:hd(3) }.
#minimize { 30@2:hd(1);  40@2:hd(2);  60@2:hd(3) }.
```

The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

Optimization statement

- A maximize statement of the form

$$maximize \{ \ w_1@p_1 : l_1, \ldots, w_n@p_n : l_n \ \}$$

stands for $minimize \{ \ -w_1@p_1 : l_1, \ldots, -w_n@p_n : l_n \ \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

```
#maximize { C@1:hd(I,P,C) }.
#minimize { P@2:hd(I,P,C) }.
```

The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

Reasoning modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

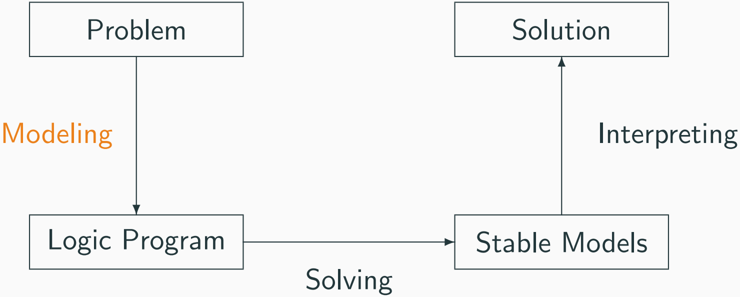
Let's get practical

- Programs with variables
  - count aggregates
  - sum aggregates
  - conditional litterals
- Going further with ASP
  - numbers
  - Python
  - Booleans
  - constants
  - intervals
  - show statements
  - projections
  - output

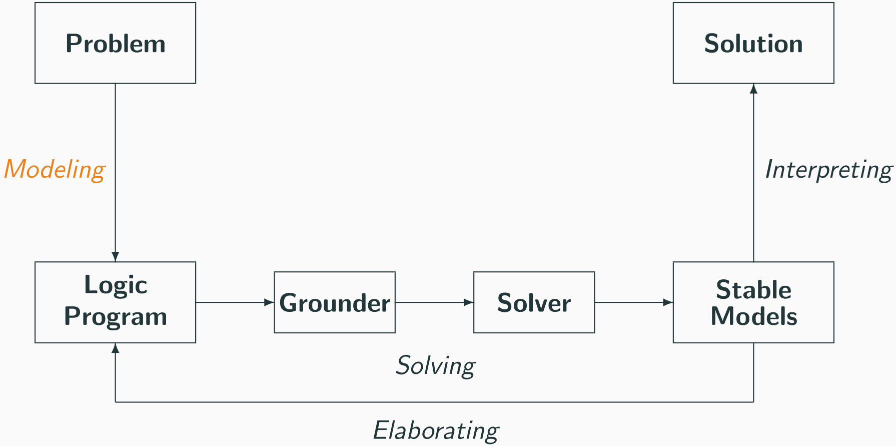
Modeling

- Example of graph coloring

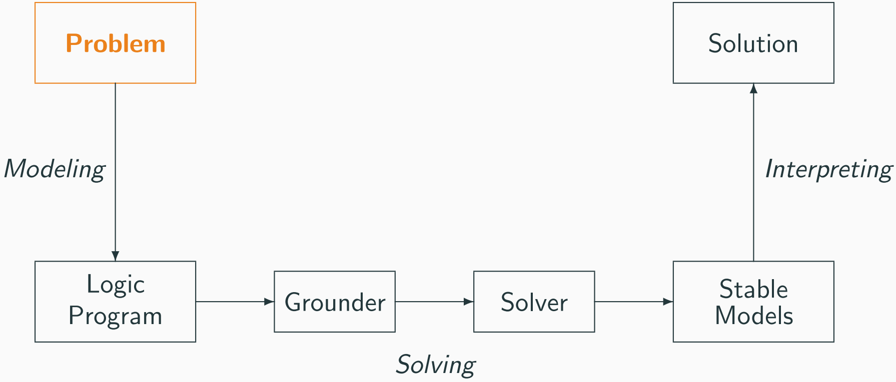
Modeling


Problem
Solution
Modeling
Interpreting
Logic Program
Stable Models
Solving

ASP workflow



ASP workflow: Problem



A case-study: Graph coloring

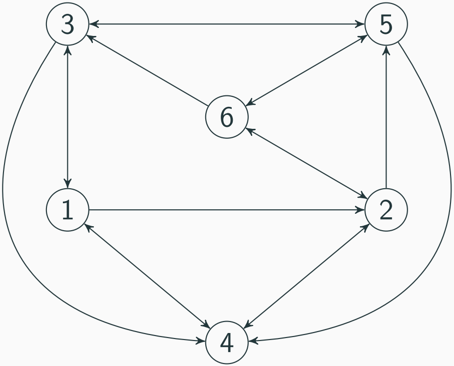
- Problem instance A graph consisting of nodes and edges

A case-study: Graph coloring

- Problem instance A graph consisting of nodes and edges

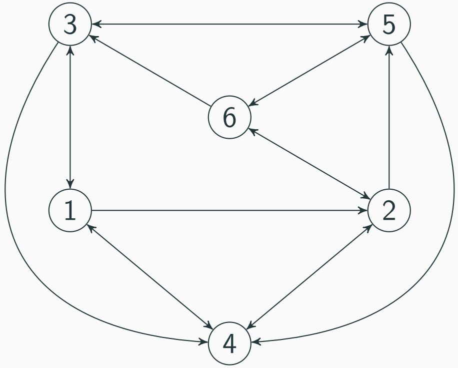
A case-study: Graph coloring

- Problem instance A graph consisting of nodes and edges



A case-study: Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`



A case-study: Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`
  - facts formed by predicate `color/1`

A case-study: Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`
  - facts formed by predicate `color/1`

- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

A case-study: Graph coloring

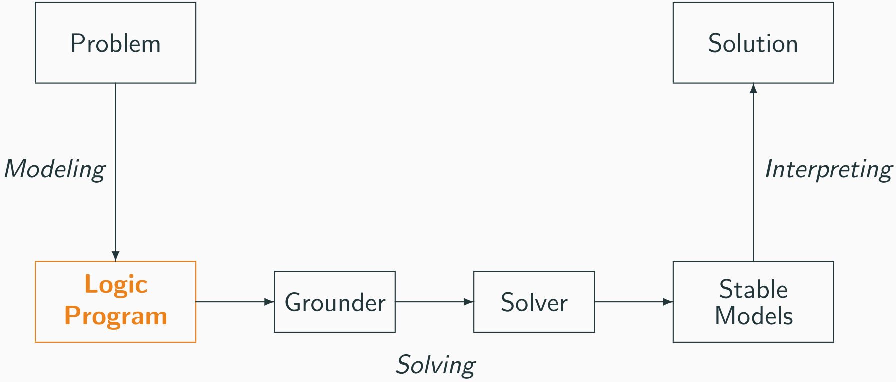
- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`
  - facts formed by predicate `color/1`

- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has one color
  2. Two connected nodes must not have the same color

ASP workflow: Problem representation



Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).

{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem instanc

Problem encoding

Graph coloring

```
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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

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{ assign(N,C) : color(C) } = 1 :- node(N).

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Problem encoding

Graph coloring

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Problem encoding

Graph coloring

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edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
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Problem instance

Problem encoding

Graph coloring

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Problem instance

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Problem encoding

Graph coloring

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edge(3,1).  edge(3,4).  edge(3,5).
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edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

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{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

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```

Problem encoding

Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```
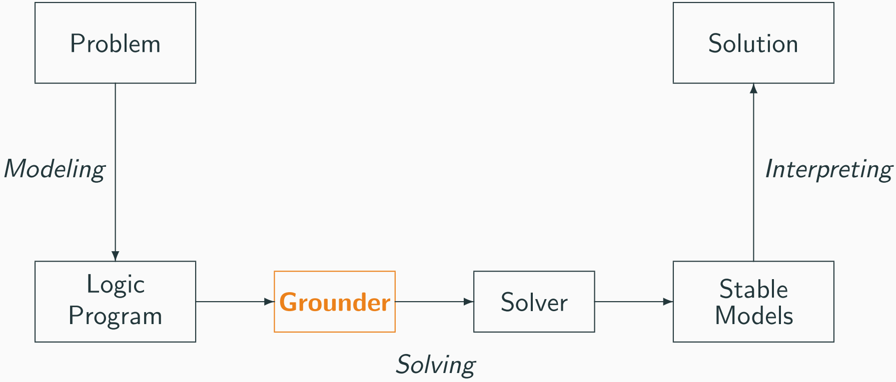

graph.lp

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

color.lp

ASP workflow: Grounding



Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{ assign(1,r); assign(1,b); assign(1,g) } = 1. { assign(4,r); assign(4,b); assign(4,g) } = 1.
{ assign(2,r); assign(2,b); assign(2,g) } = 1. { assign(5,r); assign(5,b); assign(5,g) } = 1.
{ assign(3,r); assign(3,b); assign(3,g) } = 1. { assign(6,r); assign(6,b); assign(6,g) } = 1.

:- assign(1,r), assign(2,r).  :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b).  :- assign(2,b), assign(4,b).        :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g).  :- assign(2,g), assign(4,g).        :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r).  :- assign(2,r), assign(5,r).        :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b).  :- assign(2,b), assign(5,b).        :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g).  :- assign(2,g), assign(5,g).        :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r).  :- assign(2,r), assign(6,r).        :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b).  :- assign(2,b), assign(6,b).        :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g).  :- assign(2,g), assign(6,g).        :- assign(6,g), assign(5,g).
```

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{ assign(1,r); assign(1,b); assign(1,g) } = 1. { assign(4,r); assign(4,b); assign(4,g) } = 1.
{ assign(2,r); assign(2,b); assign(2,g) } = 1. { assign(5,r); assign(5,b); assign(5,g) } = 1.
{ assign(3,r); assign(3,b); assign(3,g) } = 1. { assign(6,r); assign(6,b); assign(6,g) } = 1.

:- assign(1,r), assign(2,r).  :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b).  :- assign(2,b), assign(4,b).       :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g).  :- assign(2,g), assign(4,g).       :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r).  :- assign(2,r), assign(5,r).       :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b).  :- assign(2,b), assign(5,b).       :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g).  :- assign(2,g), assign(5,g).       :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r).  :- assign(2,r), assign(6,r).       :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b).  :- assign(2,b), assign(6,b).       :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g).  :- assign(2,g), assign(6,g).       :- assign(6,g), assign(5,g).
```

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
{assign(3,r);assign(3,b);assign(3,g)} = 1. {assign(6,r);assign(6,b);assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

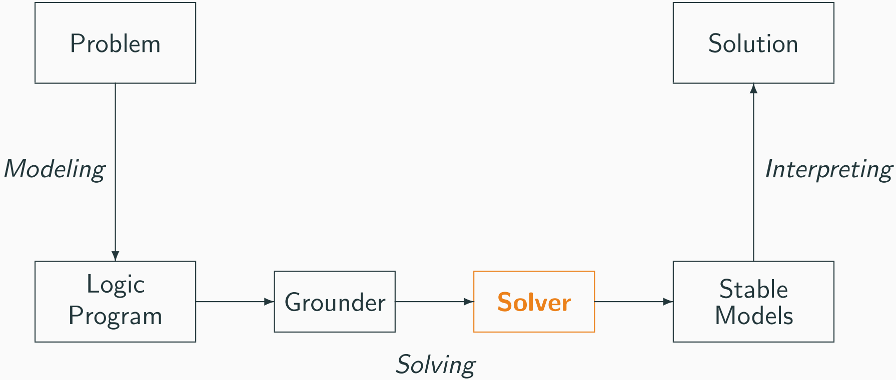
Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{ assign(1,r); assign(1,b); assign(1,g) } = 1. { assign(4,r); assign(4,b); assign(4,g) } = 1.
{ assign(2,r); assign(2,b); assign(2,g) } = 1. { assign(5,r); assign(5,b); assign(5,g) } = 1.
{ assign(3,r); assign(3,b); assign(3,g) } = 1. { assign(6,r); assign(6,b); assign(6,g) } = 1.

:- assign(1,r), assign(2,r).  :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b).  :- assign(2,b), assign(4,b).       :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g).  :- assign(2,g), assign(4,g).       :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r).  :- assign(2,r), assign(5,r).       :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b).  :- assign(2,b), assign(5,b).       :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g).  :- assign(2,g), assign(5,g).       :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r).  :- assign(2,r), assign(6,r).       :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b).  :- assign(2,b), assign(6,b).       :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g).  :- assign(2,g), assign(6,g).       :- assign(6,g), assign(5,g).
```

Graph coloring: Grounding

```
$ clingo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{ assign(1,r); assign(1,b); assign(1,g) } = 1. { assign(4,r); assign(4,b); assign(4,g) } = 1.
{ assign(2,r); assign(2,b); assign(2,g) } = 1. { assign(5,r); assign(5,b); assign(5,g) } = 1.
{ assign(3,r); assign(3,b); assign(3,g) } = 1. { assign(6,r); assign(6,b); assign(6,g) } = 1.

:- assign(1,r), assign(2,r).  :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b).  :- assign(2,b), assign(4,b).       :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g).  :- assign(2,g), assign(4,g).       :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r).  :- assign(2,r), assign(5,r).       :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b).  :- assign(2,b), assign(5,b).       :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g).  :- assign(2,g), assign(5,g).       :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r).  :- assign(2,r), assign(6,r).       :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b).  :- assign(2,b), assign(6,b).       :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g).  :- assign(2,g), assign(6,g).       :- assign(6,g), assign(5,g).
```

ASP workflow: Solving



Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Graph coloring: Solving

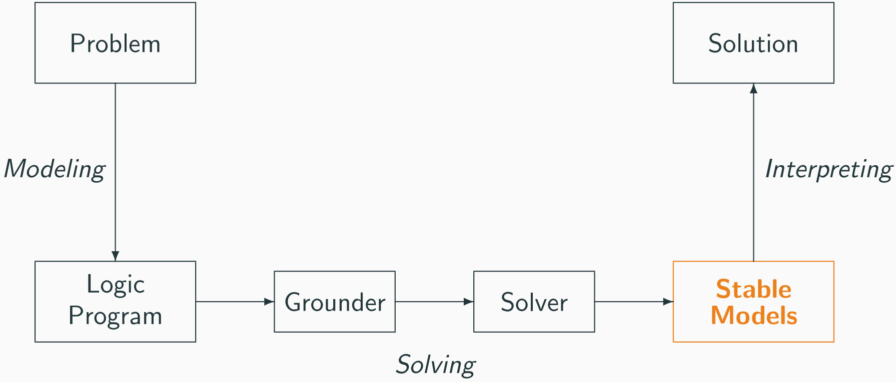
```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Graph coloring: Solving

```
$ clingo graph.lp color.lp 0

clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

ASP workflow: Stable models

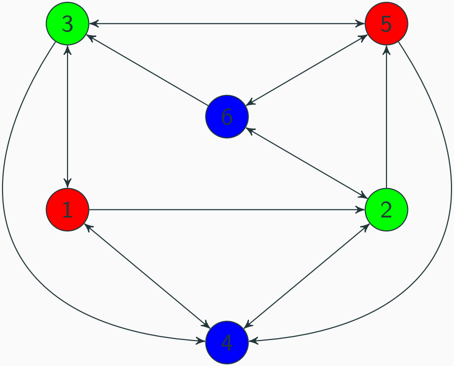


A coloring

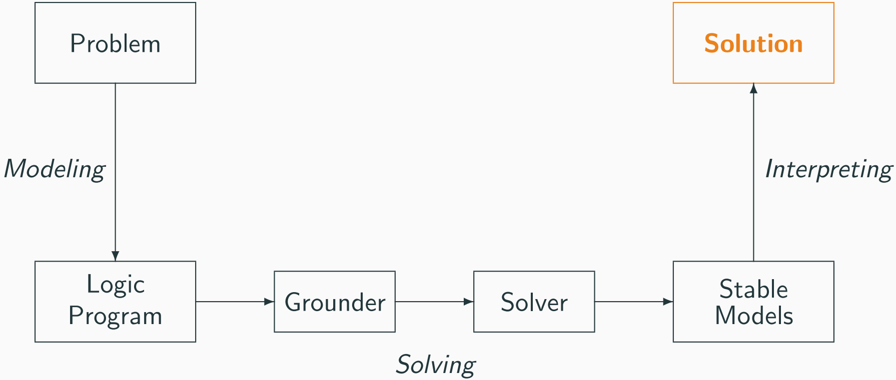
```
Answer: 6
node(1)   [...]   \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```

A coloring

```
Answer: 6
node(1)   [...]    \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



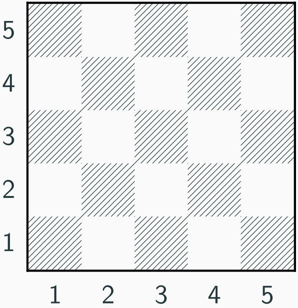
ASP workflow: Solutions



Hands-on

- n-queens problem
- Reviewer assignment
- Sudoku
- Traveling sales person

The n-queens problem

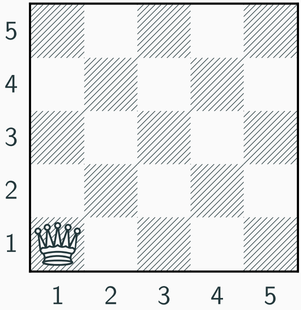


- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

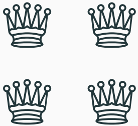
♕ ♕ ♕

♕ ♕

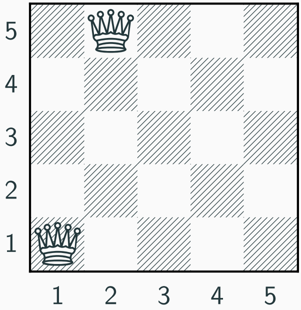
The n-queens problem



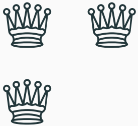
- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$



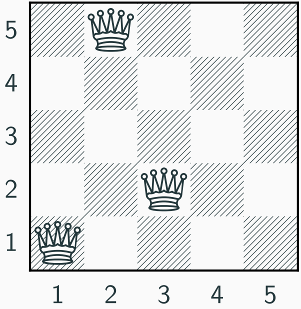
The n-queens problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

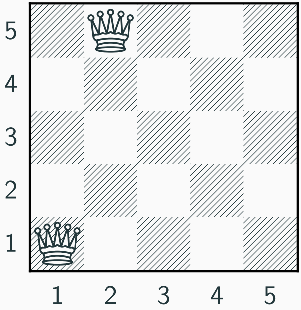


The n-queens problem

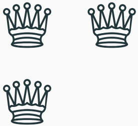


- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

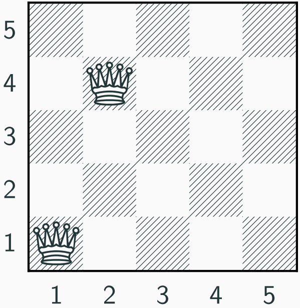
The n-queens problem



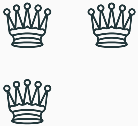
- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$



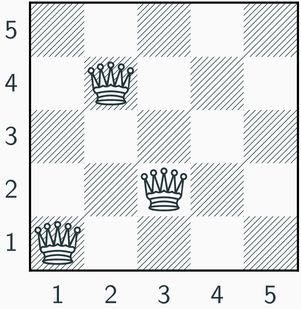
The n-queens problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

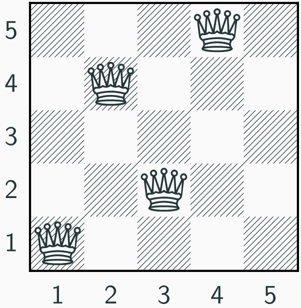


The n-queens problem



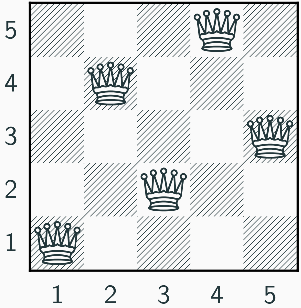
- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

The n-queens problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

The n-queens problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another
- Example $n = 5$

Defining the field

queens.lp

```
row(1..n).
col(1..n).
```

- Define the field
  - n rows
  - n columns

Defining the field

queens.lp

```
row(1..n).
col(1..n).
```

➥ Define the field

- n rows
- n columns

Defining the field

Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models       : 1
Time         : 0.000
```

Placing some queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
```

➥ Guess a solution candidate

by placing some queens on the board

Placing some queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
```

➥ Guess a solution candidate

by placing some queens on the board

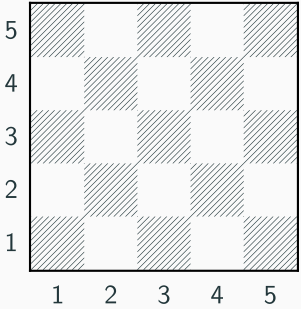
Placing some queens

Running ...

```
$ clingo queens.lp --const n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models       : 3+
```

Placing some queens

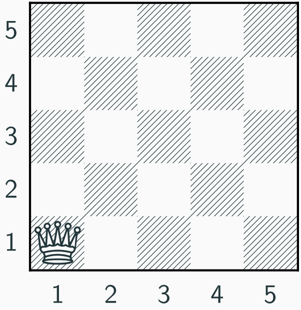
Answer: 1




```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
```

Placing some queens

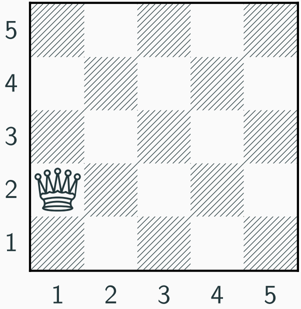
```
Answer: 2
```




```
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,1)
```

Placing some queens

```
Answer: 3
```



```
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(2,1)
```

Placing *n* queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
```

➥ Place exactly *n* queens on the board

Placing *n* queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
```

➥ Place exactly *n* queens on the board

Placing n queens directly

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) } = n.
```

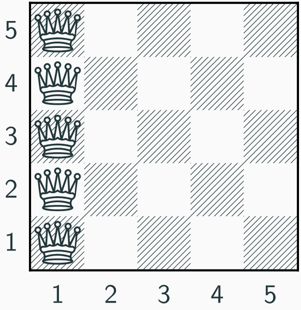
➥ Place exactly n queens on the board

Running . . .

```
$ clingo queens.lp --const n=5 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) queen(2,1)
queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) queen(2,1)
queen(1,1)
```

Placing *n* queens

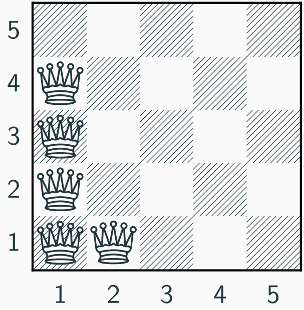
```
Answer: 1
```




```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
```

Placing *n* queens

Answer: 2




```
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
```

Horizontal and vertical attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

➥ Forbid horizontal and vertical attacks

Horizontal and vertical attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

➥ Forbid horizontal and vertical attacks

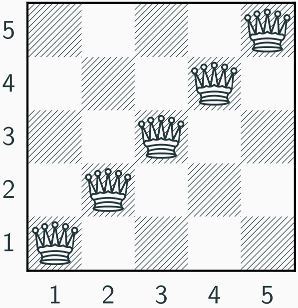
Horizontal and vertical attack

Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) queen(2,2)
queen(1,1)
```

Horizontal and vertical attack

```
Answer: 1
```




```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
```

Diagonal attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+
```

➥ Forbid diagonal attacks

Diagonal attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+
```

➥ Forbid diagonal attacks

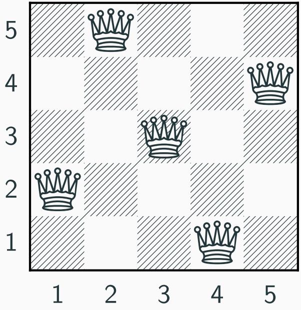
Diagonal attack

Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2)
queen(2,1)
SATISFIABLE

Models       : 1+
Time         : 0.000
```

Diagonal attack

```
Answer: 1
```



```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) \
queen(5,2) queen(2,1)
```

Optimizing

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+
```

- Encoding can be optimized
- Much faster to solve

Optimizing

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+
```

- Encoding can be optimized
- Much faster to solve

Optimizing

queens-opt.lp

```
{ queen(I,1..n) } = 1 :- I = 1..n.
{ queen(1..n,J) } = 1 :- J = 1..n.
 :- { queen(D-J,J) } > 1, D =   2..2*n.
 :- { queen(D+J,J) } > 1, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2

clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:    0 Sum:      0 Ratio:   0.00%)
```

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
```

```
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:   0 Sum:      0 Ratio:   0.00%)
```

Reviewer Assignment

- Problem Instance A set of papers and a set of reviewers along with their first and second choices of papers and conflict of interests
- Problem Class An assignment of three reviewers to each paper

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

#count { P,R : assigned(P,R) , reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment - by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

#count { P,R : assigned(P,R) , reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Sudoku

Solve a Sudoku puzzle using ASP.

- Fill a 9x9 grid with digits
- Each column, each row and each of the nine 3x3 sub-grids that compose the grid contains all numbers from 1 to 9.
- Partially filled grid as an input

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Representation in ASP

The initial state of the grid is represented by facts of predicate `initial/3`:

```
initial(X,Y,N). % initially cell [X,Y] contains number
N
```

The solution is represented by atoms of predicate `sudoku/3`:

```
sudoku(X,Y,N). % the cell [X,Y] contains number N
```

First tests

Start with a 4x4 grid

9x9 grid

```
initial(1,1,5). initial(1,2,3). initial(1,5,7). initial(2,1,6).
initial(2,4,1). initial(2,5,9). initial(2,6,5). initial(3,2,9).
initial(3,3,8). initial(3,8,6). initial(4,1,8). initial(4,5,6).
initial(4,9,3). initial(5,1,4). initial(5,4,8). initial(5,6,3).
initial(5,9,1). initial(6,1,7). initial(6,5,2). initial(6,9,6).
initial(7,2,6). initial(7,7,2). initial(7,8,8). initial(8,4,4).
initial(8,5,1). initial(8,6,9). initial(8,9,5). initial(9,5,8).
initial(9,8,7). initial(9,9,9).
```

A 4x4 grid example

```
1,_, _,_,
_,4, _,_,

_,_, _,3,
2,_, _,1,
```

```
initial(1,1,1).
initial(2,2,4).
initial(3,4,3).
initial(4,1,2). initial(4,4,1).
```

The traveling salesperson problem (TSP)

- Problem Instance A set of cities and distances among them, or simply a weighted graph
- Problem Class What is the shortest possible route visiting each city once and returning to the city of origin?
- Note
  - TSP extends the Hamiltonian cycle problem:
    Is there a cycle in a graph visiting each node exactly once
  - TSP is relevant to applications in logistics, planning, chip design,
    and the core of the vehicle routing problem

Traveling salesperson

```
start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40
road(b,d,30). road(d,c,25). road(c,a,35).
```

Traveling salesperson

```
{ travel(X,Y) } :- road(X,Y,_).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

:- city(X), 2 { travel(X,Y) }.
:- city(X), 2 { travel(Y,X) }.
```

Traveling salesperson

```
{ travel(X,Y) } :- road(X,Y,_).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

:- city(X), 2 { travel(X,Y) }.
:- city(X), 2 { travel(Y,X) }.

#minimize { D,X,Y : travel(X,Y), road(X,Y,D) }.
```

Running salesperson

```
$ clingo tsp.lp cities.lp
clingo version 5.3.1
Reading...
Solving...
Answer: 1
start(a) [...] road(c,a,35)
travel(a,b) travel(b,d) travel(d,c) travel(c,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 100

Answer: 2
start(a) [...] road(c,a,35)
travel(a,b) travel(b,c) travel(c,d) travel(d,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 95
OPTIMUM FOUND

Models       : 2
  Optimum    : yes
Optimization : 95
Calls        : 1
Time         : 0.005s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.002s
```

Traveling salesperson - Alternative problem encoding

```
{ travel(X,Y) : road(X,Y,_) } = 1 :- city(X).
{ travel(X,Y) : road(X,Y,_) } = 1 :- city(Y).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

#minimize { D,X,Y : travel(X,Y), road(X,Y,D) }.
```

Going further

- Python and ASP
- Preferences

Python & ASP

2 possibilities

- Use Python code directly in ASP e.g. to make calculations
- Hide ASP in Python (cf Clyngor)
- Completely hide the use of ASP from the user by installing clingo binaries in a Python environment (cf Clyngor-with-Clingo))

Preferences with Asprin

- Going further than classical optimisations
- Asprin on Github
- Various types of preferences already implemented (weight, cardinality, pareto...)
- Possibility to implement new ones

Applications

- Bioinformatics
- Planning, scheduling, timetabling
- Classification (e.g. of customers based on their preferences)
- Systems configuration

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- The following list of references is compiled from the open source bibliography available at

  `https://github.com/krr-up/bibliography`

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