

$$J_{\text{MSE}}(\theta) = \frac{1}{2m} \sum_{i=1}^m \left(y^{(i)} - \underbrace{\left(\sum_{k=1}^n \omega_{2,k} \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) + b_2 \right)}_{h(x^{(i)})} \right)^2$$

$$\begin{aligned} \frac{\partial J_{\text{MSE}}}{\partial \omega_{2,k}} &= \frac{1}{2m} \sum_{i=1}^m \frac{\partial}{\partial \omega_{2,k}} \left(y^{(i)} - h(x^{(i)}) \right)^2 \\ &= \frac{1}{2m} \sum_{i=1}^m 2(y^{(i)} - h(x^{(i)})) \cdot \frac{\partial}{\partial \omega_{2,k}} (y^{(i)} - h(x^{(i)})) \\ &= -\frac{\omega_{2,k}}{m} \sum_{i=1}^m (y^{(i)} - h(x^{(i)})) \cdot \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \cdot (1 - \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k})) \cdot x^{(i)} \end{aligned}$$

$$\left\{ \begin{aligned} \frac{\partial h(x^{(i)})}{\partial \omega_{1,k}} &= \omega_{2,k} \cdot \frac{\partial}{\partial \omega_{1,k}} \left(\sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \right) \\ &= \omega_{2,k} \cdot \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \cdot (1 - \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k})) \cdot x^{(i)} \end{aligned} \right.$$

$$\begin{aligned} \frac{\partial J_{\text{MSE}}}{\partial \omega_{1,k}} &= \frac{1}{2m} \sum_{i=1}^m \frac{\partial}{\partial \omega_{1,k}} \left(y^{(i)} - h(x^{(i)}) \right)^2 \\ &= \frac{1}{2m} \sum_{i=1}^m 2(y^{(i)} - h(x^{(i)})) \cdot \frac{\partial}{\partial \omega_{1,k}} (y^{(i)} - h(x^{(i)})) \\ &= -\frac{1}{m} \sum_{i=1}^m (y^{(i)} - h(x^{(i)})) \cdot \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \end{aligned}$$

$$\left\{ \frac{\partial h(x^{(i)})}{\partial \omega_{2,k}} = \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \right.$$

$$\begin{aligned} \frac{\partial J_{\text{MSE}}}{\partial b_{1,k}} &= \frac{1}{2m} \sum_{i=1}^m \frac{\partial}{\partial b_{1,k}} \left(y^{(i)} - h(x^{(i)}) \right)^2 \\ &= \frac{1}{2m} \sum_{i=1}^m 2(y^{(i)} - h(x^{(i)})) \cdot \frac{\partial}{\partial b_{1,k}} (y^{(i)} - h(x^{(i)})) \\ &= -\frac{\omega_{2,k}}{m} \sum_{i=1}^m (y^{(i)} - h(x^{(i)})) \cdot \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \cdot (1 - \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k})) \end{aligned}$$

$$\left\{ \begin{aligned} \frac{\partial h(x^{(i)})}{\partial b_{2,k}} &= \omega_{2,k} \cdot \frac{\partial}{\partial b_{1,k}} \left(\sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \right) \\ &= \omega_{2,k} \cdot \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k}) \cdot (1 - \sigma(\omega_{1,k} \cdot x^{(i)} + b_{1,k})) \end{aligned} \right.$$

$$\begin{aligned} \frac{\partial J_{\text{MSE}}}{\partial b_2} &= \frac{1}{2m} \sum_{i=1}^m \frac{\partial}{\partial b_2} \left(y^{(i)} - h(x^{(i)}) \right)^2 \\ &= \frac{1}{2m} \sum_{i=1}^m 2(y^{(i)} - h(x^{(i)})) \cdot \frac{\partial}{\partial b_2} (y^{(i)} - h(x^{(i)})) \\ &= -\frac{1}{m} \sum_{i=1}^m (y^{(i)} - h(x^{(i)})) \end{aligned}$$