# Applied Time Series

## Course timetable:

- Thursday 5 October 2023 - 8:30-11:45 AM - D303
- Thursday 12 October 2023 - 8:30-11:45 AM - D303
- Thursday 19 October 2023 - 8:30-11:45 AM - D303
- Friday 26 October 2023 - 8:30-11:45 AM - D303
- Thursday 9 November 2023 - 8:30-11:45 AM - D303
- Thursday 16 November 2023 - 8:30-11:45 AM - D303
- Thursday 23 November 2023 - 8:30-11:45 AM - D303
- Thursday 30 November 2023 - 8:30-11:45 AM - D303

## Assessment:

- Assignment (30%, due date: December 29, 2023, at midnight)
- Final Exam (70%, December 15, 2022 - Salle A Bis - from 10:15 to 12:15 PM)

## References:

1. Brooks, C. (2008), Introductory Econometrics for Finance, Cambridge University Press.
2. Enders, W. (2014), Applied Econometric Time Series, 4th Edition, Wiley.
3. Hamilton, J. D. (1994), Time Series Analysis, Princeton University Press.
4. Hilpisch, Y. (2015), Python for Finance: Analyze Big Financial Data, O'Reilly Publishing.
5. Sargent, T. J. and Stachurski (2018), Lectures in Quantitative Economics.
6. Sheppard, K. (2018), Introduction to Python for Econometrics, Statistics and Numerical Analysis.
7. Tsay, R. S. (2010), Analysis of Financial Time Series, 3rd Edition, Wiley.

# Assignment 2023

## Part 1 (10 points): Theoretical exercises

### Exercise 1 (2 points): The AR(1)-ARCH(1) model

Consider the model $Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t$, $h_t = \mathrm{V}_{t-1}(\varepsilon_t) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$ with $\alpha_0 > 0$,$\alpha_1 > 0$ and $\alpha_1 < 1$.

1. Find the conditional and unconditional mean of $Y_t$.

2. Find the conditional and unconditional variance of $Y_t$.

### Exercise 2 (3 points): The seasonal $MA(1) \times MA(1)_{12}$ process

Suppose that $Y_t$ is a seasonal $MA(1) \times MA(1)_{12}$ process, that is:

$$Y_t = (1 - \theta L)(1 - \Theta L^{12})\varepsilon_t$$

where $\varepsilon_t$ is a zero mean white noise process with variance $\mathrm{V}(\varepsilon_t) = \sigma_\varepsilon^2$.

1. Derive expressions for $\mu = \mathrm{E}(Y_t)$ and $\gamma_0 = \mathrm{V}(Y_t)$.

2. Derive the autocovariance function, that is, calculate $\gamma_k = \mathrm{cov}(Y_t, Y_{t-k})$, for $k = 1, 2, 3, \ldots, 15$.

3. Derive the s-steps ahead forecasts of $Y_t$ for $s = 1, 2, 3, \ldots, 15$.

### Exercise 3 (1 points): The AR(1) process

Consider the time series model:

$$Y_t = \mu + \phi^2 Y_{t-1} + \varepsilon_t,$$

where $\mu$ and $\phi$ are fixed parameters and $\varepsilon_t$ is a white noise process with mean zero and variance $\sigma_\varepsilon^2$.

1. Derive the $s = 1$ step ahead forecast of $Y_t$, $f_{t,1} = \mathrm{E}_t[Y_{t+s}]$.

2. Forecast error is the difference between the forecast and the true value, compute the error forecast, $e_{t,1} = f_{t,1} - y_{t+1}$, and its variance.

3. When this process is stationary? Let assume that stationarity holds, what is the value forecast of $f_{t,s}$ when the lead time $s$ is very large.

## Exercise 4 (4 points): The ARMA(2,2) process

Consider the following ARMA(2,2) model:

$$Y_t = 1.3Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t - 1.2\varepsilon_{t-1} + 0.2\varepsilon_{t-2},$$

where $\{\varepsilon_t\}$ is independently and identically distributed by a normal distribution with mean zero and variance $\sigma_\varepsilon^2 = 1$.

1. Is $Y_t$ weakly stationary? If so compute the autocovariance function. In this ARMA(2,2), the general solution to the second order difference equation for $\gamma_h$ is given by

$$\gamma_h = c_1 \ 0.5^h - c_2 \ 0.8^h$$

where $c_1$ and $c_2$ are constants to be determined by the three initial conditions $(\gamma_0, \gamma_1 and \gamma_2)$.[1]

2. Is $Y_t$ invertible? If so, find the infinite order MA representation (coefficients of $\Psi_\infty(L)$). Note that we can write the equation for $Y_t$:

$$Y_t = \Phi_2^{-1}(L)\Theta_2(L)\varepsilon_t = \Psi_\infty(L)\varepsilon_t$$

where $\Psi_\infty(L) = \Psi_0 + \Psi_1 L + \Psi_2 L^2 + \ldots$. Note that the above equation implies that:

$$\Psi_\infty(L)\Phi_2(L) = \Theta_2(L)$$

Do the multiplication of the two polynomials from the left hand side to match terms (for each L operator) within the polynomial from the right hand side.

[1] Note that 0.5 and 0.8 are the eigenvalues of the system defined by $\gamma_h$ which is why they are the roots which define the motion of $\gamma_h$.

## Part 2 (10 points): Computing exercises

### Exercise 5 (3 points): ARMA and GARCH models

***Guided exercise.*** Retrieve the daily (monthly) asset price (macro-variable) of your choice from Yahoo Finance (or from DBnomics) and fit the appropriate SARIMA-GARCH models to this time series by using a rolling window of 4 years with a frequency of 1 day (1 month) to re-estimate your model from January 1, 2016 to June 30, 2020.[2] Use your fitted models to produce 1-step ahead forecasts on the out-of-sample period (from January 1, 2020 until June 30, 2020).[3] Describe precisely in a Jupyter Notebook the different steps you follow.

Bonus question: Are you able to apply Darts to do some forecasts on your time series? Use the RMSE to select the best model.

### Exercise 6 (4 points): Markov Switching Autoregression Models

***Do It Yourself.*** Go to Bloomberg or Yahoo Finance to download the daily commodity price of your choice from January 1, 2020 until September 30, 2023. Compute its continous daily, monthly and quarterly return. Specify a markov switching autoregression model on these 3 time series from the beginning of your sample till December 31, 2022. Next, do some forecasts over the next 3 quarters, 9 months and the subsequent days (for this daily prediction, please predict 20-step ahead forecasts and next re-estimaste your model by including the true realizations and predict again 20-step ahead forecasts, do it till the end of the sample). You may use statmodels to do it. Be careful, you have to figure out which SARIMA-GARCH model you should apply first on the time series before adding a Markov Switching process. Describe your models and comment your results in a Jupyter Notebook.

### Exercise 7 (3 points): Applied Finance with Principal Component Analysis

***Replication exercise.*** Please do one of the following replication exercises on the CAC 40 database.

1. Eigen-portfolio construction using Principal Component Analysis.

2. Trading Strategy based on Principal Component Analysis.

Describe precisely in a Jupyter Notebook the different steps you follow and comment your results.

[2]Please use the *daily log return* $*100$ if you fit a model on daily returns.

[3]6 forecasts have to be done with a monthly variable whereas approximately 120 forecasts have to be done with a daily variable.