Financial Econometrics II - 2024

Homework: Core European Stock Portfolio

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Contents

1. Building the reduced form factor model.

We first build a reduced form model of the 47 European stocks returns universe encompassed in the SX5E index, using the Principal Component Analysis method. This procedure allows us to easily build a multi-factor model with the data being summarized by a limited number of orthogonal features explaining most of the variance of the data. We start by computing the full-sample variance-covariance matrix of the stock returns $\hat{\Omega}$, and standardizing each return data series. Hence, we obtain the standardized stock returns:

$$r_{i,t}^{std} = \frac{r_{i,t} - \mu_i}{\sigma_i}$$

We then, build our PCA model with 47 features, one for each stock, and obtain the vectors V_i with $i = 1, ..., 47$, which are the eigenvectors of the variance-covariance matrix of the standardized returns $r_{i,t}^{std}$ (PC loadings), and the factors $F_{i,t}$ with $i = 1, ..., 47$ and $t = 1, ..., 216$ (real scale value of each PC i at time t). We find that the first eigenvector V_1 contains a large majority of negative values indicating a negative correlation between the first core factor and most of the stock returns. This correlation being negative, we will not be able to find the weights of the replicating portfolio by solving the optimization problem defined in question 3 (2), due to the absence of authorized leverage (the optimized portfolio obtained contains only 1 stock). Hence, we change the sign of the factor $F_{1,t}$, as advised in the project statement. Furthermore, we observe that the variances of the factors are important, especially during the "2007-2008" and "2020-2021" periods. This was predictable as stock returns were highly volatile during these crisis periods and our factors are built to capture this variance. Nevertheless, to stabilize our factors and ensure that our optimization problem (2) works, we re-scale the factors to the benchmark volatility, as suggested.

$$F_{i,t} = F_{i,t} \frac{\sigma_{bench}}{\sigma_{F_i}}$$

2. Estimation of the stock returns' exposure to the selected factors using OLS estimators.

We opt to keep a restricted number of factors k, effectively encapsulating the majority of variance within the dataset. Using the eigenvalues, obtained through the eigen-decomposition of the standardized returns variance-covariance matrix, we can determine how much variance of the data is explained by each principal component j through:

$$\frac{\lambda_j}{\sum_{i=1}^{47} \lambda_i}$$

To determine the number of factors to retain in the model, we use the information criterion provided by Bai and Ng (2002)[1]. Hence, we select a number m of factors, $1 \leq m \leq M = 47$, such that either $C_{p_1}(m)$ or $C_{p_2}(m)$ are minimized.

$$\hat{\sigma}^2(m) = \frac{1}{k}\sum_{i=1}^{k} \hat{\sigma_i}^2(m)$$
$$C_{p_1}(m) = \hat{\sigma}^2(m) + m\hat{\sigma}^2(M)\left(\frac{k+T}{kT}\right) ln\left(\frac{kT}{k+T}\right)$$
$$C_{p_2}(m) = \hat{\sigma}^2(m) + m\hat{\sigma}^2(M)\left(\frac{k+T}{kT}\right) ln\left(min\left(\sqrt{k}, \sqrt{T}\right)^2\right)$$

By taking into account the two information criteria for the models with up to 20 factors, we find that both $C_{p_1}(m)$ and $C_{p_2}(m)$ attain their minimum values when $m = 3$, as shown in (1). Thus, we choose to keep only the first 3 factors, which alone explain 54% of the data variance. As our factors are orthogonal, by construction, we can use OLS estimators without any multicollinearity issues and obtain the estimated sensitivities $\hat{b}_{i,k}$ of each stock i returns to the core factor k. Hence, for each of the 47 stock returns data series, we estimate the following model:

$$r_{i,t} = \alpha_i + \sum_{k=1}^{3} b_{i,k}F_{k,t} + \epsilon_t \tag{1}$$

Descriptive statistics for the R-squared and the estimated sensitivities of the stock returns to the first core factor $\hat{b}_{i,1}$, obtained through these regressions, are available in appendices (2).

3. First Core Equity Factor Replicating Portfolio.

Having previously obtained the estimated sensitivities of each stock returns $\hat{b}_{i,1}$ to the first core factor F_1. We solve the optimization problem defined below to find the weights of the minimum variance portfolio

[1] Jushan Bai and Serena Ng. Determining the number of factors in approximate factor models in Econometrica: 70.1 (2002), pages 191–221.

that replicates this factor:

$$
\begin{aligned}
\underset{w_1}{\arg\min} \quad & w_1' \hat{\Omega} w_1 \\
\text{s.t.} \quad & \sum_{i=1}^{47} w_{i,1} = 1 \\
& w_{i,1} \geq 0 \\
& \sum_{i=1}^{47} w_{i,1} \hat{b}_{i,1} = 1
\end{aligned} \tag{2}
$$

Hence, we obtain the optimized weights w_1^* of the portfolio that replicates F_1. We observe, that this portfolio is highly diversified across the stock universe and outperforms significantly the benchmark. The replicating portfolio returns on average 11% per annum versus 5.7% for the benchmark. In addition, we observe that our portfolio exhibits a strong correlation to the market ($\beta = 0.98$) and as it is built, such that, it replicates the first core factor, which has been re-scaled to a volatility equal to that of the benchmark (0.175% p.a.), our portfolio has an annualized volatility $\sigma_{ptf} = 0.1755\%$, securing a decent Sharpe ratio: $S_{ptf} = \frac{\mu_{ptf} - \mu_{rf}}{\sigma_{ptf}} = 0.5098$ (For the risk-free rate, we have taken the 10Y OAT yield).

4. Estimation of the Replicating Portfolio's Alpha against the Benchmark.

To obtain an estimation of the alpha of our first core equity factor replicating portfolio, we decide to run a simple OLS regression of the portfolio returns on the benchmark returns.

$$
\begin{aligned}
& w_1^* \cdot r_t = \alpha + \beta_1 \cdot r_{bench,t} + \epsilon_t \\
& \hat{\alpha} = w_1^* \cdot r_t - \hat{\beta}_1 \cdot r_{bench,t}
\end{aligned}
$$

Where w_1^* is the vector of size (1×47) containing the optimized first core factor replicating portfolio weights, obtained through (2), r_t is a vector of size (47×1) containing the stock returns at time t, and $r_{\text{bench},t}$ is the return at time t of the benchmark. As conjectured in the previous section, we observe that our replicating portfolio significantly outperforms the market, allowing us to extract an estimated annual alpha of 5.41%, indicating a strong performance. However, the reliability of such metrics may be questioned, given that the replicating portfolio is obtained by optimizing the weights based on the estimated sample variance-covariance matrix. Furthermore, the alpha is then derived from an OLS regression, introducing additional estimation error.

5. Impact of estimation errors on the Replicating Portfolio's Alpha.

We want to demonstrate the impact of the estimation error in the variance-covariance on the estimated alpha of our replicating portfolio. To do so, we choose to randomly permute the rows of our return data set and keep 95% of the resampled data set obtained. We then use the re-sampled dataset to compute a new perturbed variance-covariance matrix $\tilde{\Omega}_n$. The perturbed variance-covariance being estimated, we then solve again the optimization problem to find the new first core equity factor replicating portfolio weights:

$$\begin{aligned} \underset{\tilde{w}_{1,n}}{\arg\min} \quad & \tilde{w}'_{1,n}\tilde{\Omega_n}\tilde{w}_{1,n} \\ \text{s.t.} \quad & \sum_{i=1}^{47} \tilde{w}_{i,n,1} = 1 \\ & \tilde{w}_{i,n,1} \geq 0 \\ & \sum_{i=1}^{47} \tilde{w}_{i,n,1}\hat{b}_{i,1} = 1 \end{aligned} \tag{3}$$

Having obtained these new optimized weights $\tilde{w}^*_{1,n}$, we can compute the replicating portfolio returns and estimate its associated alpha against the benchmark through:

$$\begin{aligned} & \tilde{w}^*_{1,n} \cdot r_t = \alpha_n + \beta_{1,n} r_{bench,t} + \epsilon_{t,n} \\ & \hat{\alpha}_n = w^*_{1,n} \cdot r_t - \hat{\beta}_{1,n} \cdot r_{bench,t} \end{aligned}$$

We repeat this process $n = 1000$ times, obtaining a set of 1000 simulated first core equity factor replicating portfolio with the different variance-covariance matrix. We choose to employ this re-sampling method to avoid making any assumptions about the joint distribution of stock returns. Hence, using this non-parametric approach, we obtain a distribution of the estimated alphas see (6). We can observe that, on average, our first core equity factor replicating portfolio extracts an estimated annual alpha of 5.4% over the benchmark, which is consistent with what we found in question 4. Nevertheless, the mean-variance profiles of the replicating portfolios vary across the simulations see (5). These simulations exhibit well the impact of the estimation error in the variance-covariance matrix on the construction of the replicating portfolio and its associated final performance.

Appendices

$m-1$	$C_{p_1}(m-1)$	$C_{p_2}(m-1)$
0	9.2254	9.2254
1	8.7498	8.7549
2	8.7346	8.7448
3	8.7350	8.7503
4	8.7538	8.7742
5	8.7803	8.8058
6	8.8103	8.8409
7	8.8402	8.8760
8	8.8722	8.9130
9	8.9044	8.9503
10	8.9349	8.9859
11	8.9644	9.0205
12	8.9904	9.0516
13	9.0195	9.0858
14	9.0478	9.1192
15	9.0754	9.1519
16	9.1008	9.1824
17	9.1243	9.2110
18	9.1456	9.2374
19	9.1652	9.2621

Table 1: Values for C_{p_1} and C_{p_2}, Bai and Ng (2002) criteria.

Statistic	**Value**
Count	47
Mean	0.539
Standard Deviation	0.156
Minimum	0.19
25th Percentile	0.436
Median (50th Percentile)	0.515
75th Percentile	0.649
Maximum	0.822

(a) R-squared.

Statistic	**Value**
Count	47
Mean	0.994
Standard Deviation	0.348
Minimum	0.388
25th Percentile	0.785
Median (50th Percentile)	0.917
75th Percentile	1.21
Maximum	1.943

(b) First Core Sensitivity.

Table 2: Descriptive Statistics for the OLS regressions (1).

Statistic	Value (%)
Average return (annualized)	0.1097
Total cumulative return	4.4186
Volatility (annualized)	0.1755
Alpha (annualized)	0.0541
Beta	0.9819
Sharpe Ratio	0.5098

Table 3: 1st Core Factor Replicating Portfolio Statistics.

Company	**Weight (%)**
ANHEUSER-BUSCH INBEV SA/NV	0.0164
KONINKLIJKE AHOLD DELHAIZE N	0.0134
ADIDAS AG	0.0174
AIR LIQUIDE SA	0.0260
AIRBUS SE	0.0182
ALLIANZ SE-REG	0.0274
ASML HOLDING NV	0.0179
BASF SE	0.0287
BAYER AG-REG	0.0238
BANCO BILBAO VIZCAYA ARGENTA	0.0237
BAYERISCHE MOTOREN WERKE AG	0.0226
DANONE	0.0234
BNP PARIBAS	0.0200
CRH PLC	0.0175
AXA SA	0.0239
DEUTSCHE BOERSE AG	0.0208
VINCI SA	0.0276
DHL GROUP	0.0260
DEUTSCHE TELEKOM AG-REG	0.0208
ESSILORLUXOTTICA	0.0206
ENEL SPA	0.0255
ENGIE	0.0219
ENI SPA	0.0242
FRESENIUS SE & CO KGAA	0.0173
SOCIETE GENERALE SA	0.0179
IBERDROLA SA	0.0218
ING GROEP NV	0.0206
INTESA SANPAOLO	0.0211
INDUSTRIA DE DISENO TEXTIL	0.0223
KERING	0.0204
MERCEDES-BENZ GROUP AG	0.0222
LVMH MOET HENNESSY LOUIS VUITTON SE	0.0243
MUENCHENER RUECKVER AG-REG	0.0231
NOKIA OYJ	0.0086
L'OREAL	0.0265
ORANGE	0.0163
KONINKLIJKE PHILIPS NV	0.0193
SAFRAN SA	0.0219
SANOFI	0.0192
BANCO SANTANDER SA	0.0181
SAP SE	0.0231
SIEMENS AG-REG	0.0264
SCHNEIDER ELECTRIC SE	0.0264
TELEFONICA SA	0.0226
TOTAL SE	0.0224
VIVENDI SE	0.0195
VOLKSWAGEN AG-PREF	0.0110

Table 4: 1st Core Factor Replicating Portfolio Composition.

Statistic	Value
Count	1000
Mean	0.110
Standard Deviation	0.004
Minimum	0.097
25th Percentile	0.107
Median (50th Percentile)	0.110
75th Percentile	0.113
Maximum	0.125

(a) Annualized average return

Statistic	Value
Count	1000
Mean	0.174
Standard Deviation	0.004
Minimum	0.157
25th Percentile	0.172
Median (50th Percentile)	0.175
75th Percentile	0.177
Maximum	0.18

(b) Annualized volatility

Table 5: Statistics of the average return and volatility of the simulated replicating portfolios (3).

Statistic	Value (%)
Mean	0.0544
Standard Deviation	0.0149
95% Confidence Interval	(0.0535, 0.0553)

Table 6: Simulated Alpha Statistics.