Machine Learning for the geodynamo inverse problem

Physics-informed Neural Network

Romain Claveau

May 28, 2025

Table of Contents

1 Context

The geodynamo is a physical process behind the Earth's sustained magnetic field, stemming from complex fluid motions of the electrically conducting liquid metal within the outer core. Because these processes occur at extreme depth and conditions, which are entirely out-of-reach of direct observations or experimental replicas, our understanding of the geodynamo relies heavily on numerical simulations.

However, the Earth's physical parameters are also out-of-reach of numerical simulations as its outer core operates at low viscosity (as measured by $\mathrm{Ek} \simeq 10^{-15}$ and $\mathrm{Pm} \simeq 10^{-6}$). Also, the Earth's dynamo features a small ratio of the kinetic to magnetic energy (as measured by $\mathrm{Al}^2 \simeq 10^{-4}$). So, reproducing Earth-like conditions requires to reach an asymptotic regime, which afterwards could be extrapolated to the Earth.

We recall the definitions of the Ekman number (Ek), the magnetic Prandlt number (Pm) and the Alfvén number (Al):

$$\mathrm{Ek} = \frac{\tau_\Omega}{\tau_\nu} \;\;;\;\; \mathrm{Pm} = \frac{\tau_\eta}{\tau_\nu} \;\;;\;\; \mathrm{Al} = \frac{\tau_A}{\tau_U} \tag{1.1}$$

where τ_Ω the inverse rotation rate, τ_ν the viscous diffusion time, τ_η the magnetic diffusion time, τ_A the Alfvén time and τ_U the convective overturn time.

As a result, in addition to numerical simulations of the geodynamo, we rely on the induction equation relating the outer core motions and the magnetic field, to infers the flow through magnetic observations by solving the geodynamo inverse problem.

2 The geodynamo inverse problem

The induction equation, relating the core flow to the magnetic field, is obtained from Ohm's law (accounting for the Lorentz force)

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \tag{2.1}$$

where $\boldsymbol{J}$ is the electric current field, $\boldsymbol{E}$ the electrical field, $\boldsymbol{u}$ the velocity field and $\boldsymbol{B}$ the magnetic field. Because the Earth's mantle is electrically insulating (at least has a very low electrical conductivity), $\boldsymbol{J}$ vanishes. Then, taking the curl, and using the Faraday's law, one gets the ideal magneto-hydrodynamics equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) \tag{2.2}$$

where $\boldsymbol{\nabla}\times$ is the curl operator. Also, because we are considerating the mantle as insulating, the magnetic field is divergence-free and curl-free. As a result, it derives from a potential $\boldsymbol{B} = -\boldsymbol{\nabla} V$, with V the magnetic potential, and its evolution above the outer core surface is entirely prescribed by its radial component at the core-mantle boundary (CMB).

At the CMB, the outer core motions are constrained by the mantle as it cannot flow radially. As a result, only its tangential components are non-zero. Finally, because the radial magnetic field is the only one driven by these tangential motions, one gets the radial induction equation at the CMB, reading as

$$\frac{\partial B_r}{\partial t} = -\boldsymbol{\nabla}_H \cdot (\boldsymbol{u}_H B_r) \tag{2.3}$$

where $\boldsymbol{\nabla}_H \cdot$ is the horizontal (tangential) divergence operator, B_r the radial magnetic field and $\boldsymbol{u}_H$ the tangential flow.

Thus, the *geodynamo inverse problem* is precisely inferring the tangential outer core flow $\boldsymbol{u}_H$ from the magnetic observations B_r.

3 Tangential flow field

As the outer core flow is incompressible, it admits a unique toroidal–poloidal decomposition:

$$\boldsymbol{u} = \boldsymbol{\nabla} \times (\boldsymbol{r}\,\mathcal{T}) + \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times (\boldsymbol{r}\,\mathcal{S})) \tag{3.1}$$

with $\mathcal{T}$ and $\mathcal{S}$ the toroidal and poloidal scalar fields, respectively. The expression is further simplied as we are considering the tangential flow, and reads

$$\boldsymbol{u}_H = -\boldsymbol{r} \times \boldsymbol{\nabla}_H \mathcal{T} + \boldsymbol{\nabla}_H (r\,\mathcal{S}) \tag{3.2}$$

Using this expression for the tangential flow field $\boldsymbol{u}_H$ ensures the incompressibility condition (ie. $\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$).

4 Decomposition on the spherical harmonics basis