

# PN JUNCTION THEORY

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# PN Homojunction

- Non linear device
- rectifier devices (composants redresseur)
- 2 devices reach the same results:
  - PN Junction(this chapter)
  - Schottky barrier or Metal / SC contact (next chapter)

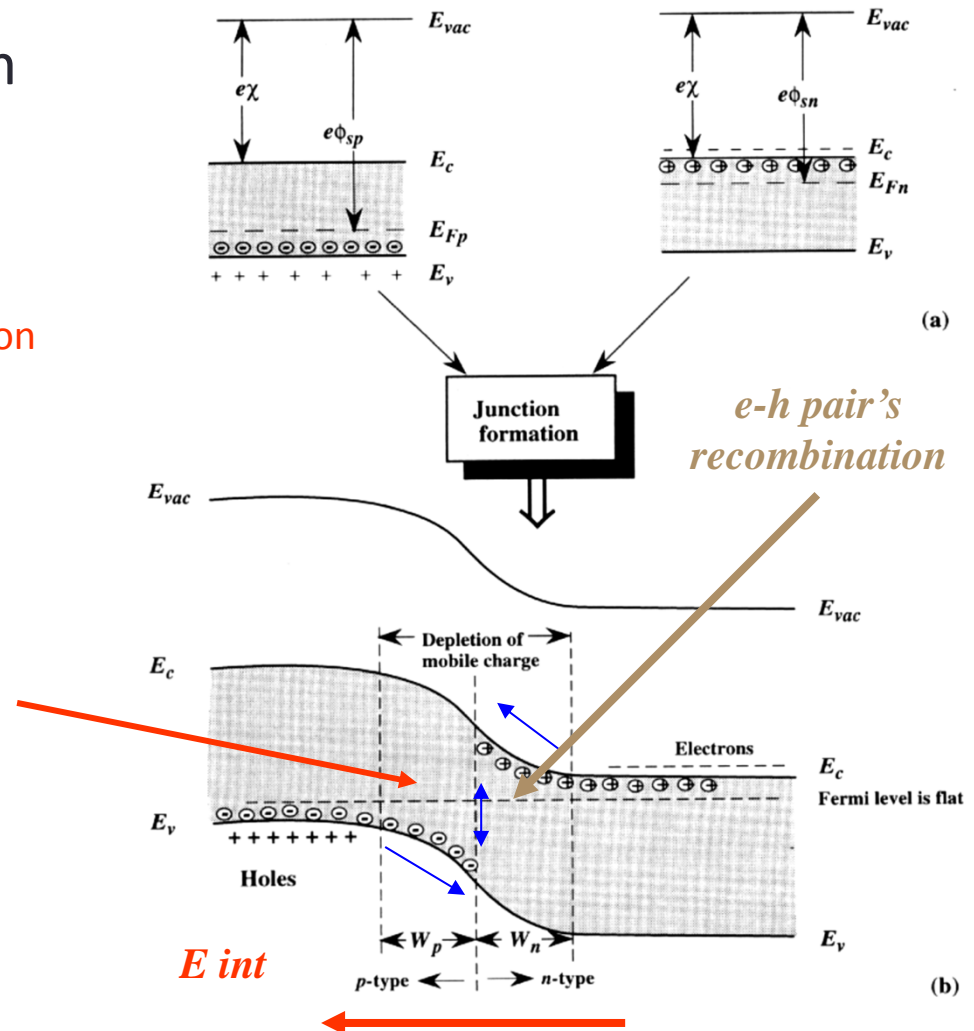
# The Junction's formation mechanism

## • PN Junction at equilibrium

1st Step: diffusion mechanism

2<sup>nd</sup> Step: built in Electric Field  
appears  $\Leftrightarrow$  compensates diffusion  
forces

Flat Fermi level:  
No current / thermal equilibrium



## « built in potential $V_{bi}$ »

- Definition : Potential drop between N and P regions

$$V_{bi} = V_N - V_P$$

Holes current equation:

$$J_p(x) = e \left[ \mu_p p(x) E(x) - D_p \frac{dp(x)}{dx} \right] = 0$$

or

$$\frac{\mu_p}{D_p} E(x) = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

or

$$\frac{-e}{kT} \frac{dV(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

Integrating from P to N region:

$$V_{bi} = \frac{kT}{e} \ln\left(\frac{p_p}{p_n}\right)$$

finally:

$$V_D = V_{bi} = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

# • *Field, potential and Space Charge width(1)*

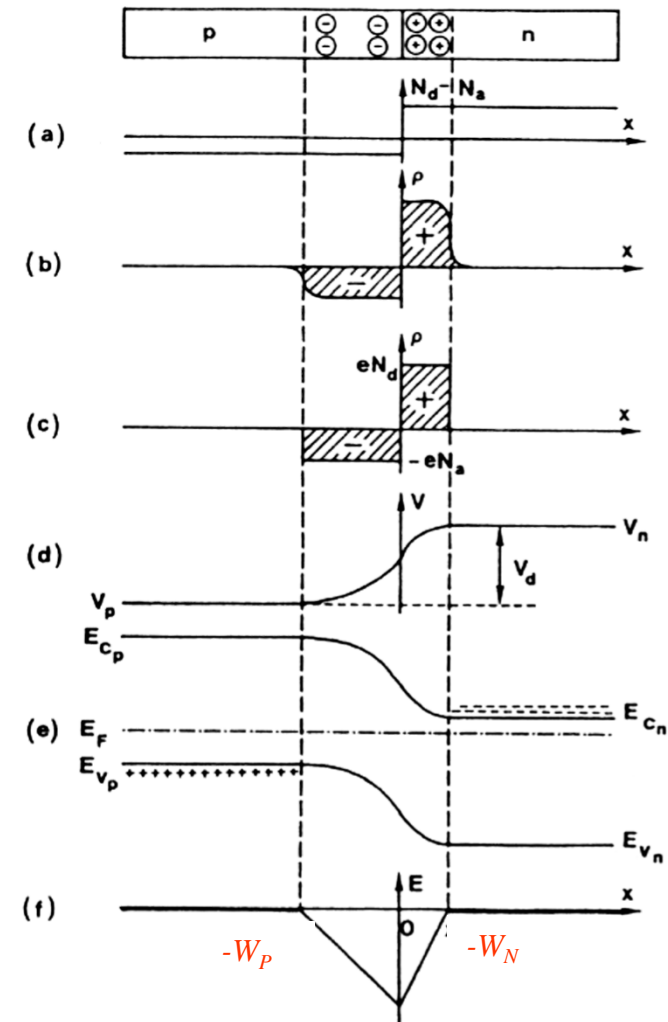
## • *Poisson's equation:*

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_{sc}}$$

## ■ *In N and P region:*

$$\frac{d^2V(x)}{dx^2} = -\frac{e}{\epsilon_{sc}} N_D \quad 0 < x < W_N$$

$$\frac{d^2V(x)}{dx^2} = +\frac{e}{\epsilon_{sc}} N_A \quad -W_P < x < 0$$



## • *Field, potential and Space Charge width(2)*

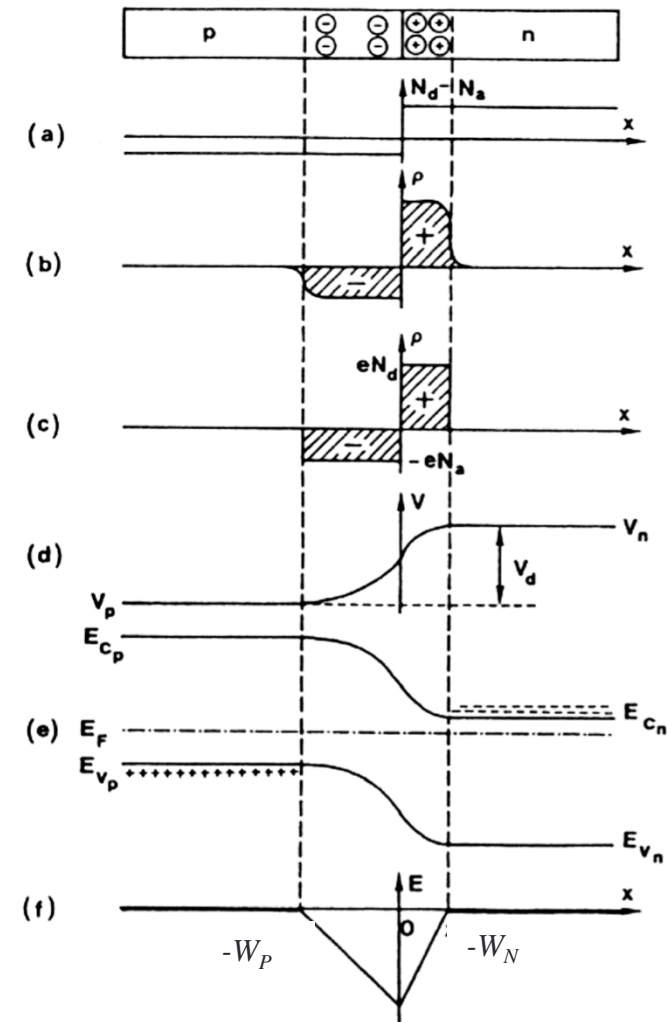
### ■ Electric Field E(x)

$$E_n(x) = +\frac{eN_D}{\epsilon_{sc}}(x - W_N) \quad E_p(x) = -\frac{eN_A}{\epsilon_{sc}}(x + W_P)$$

### ■ Continuity of Field on x=0:

$$N_D W_N = N_A W_P$$

$$E_M = -\frac{eN_D W_N}{\epsilon_{sc}} = -\frac{eN_A W_P}{\epsilon_{sc}}$$



## • *Field, potential and Space Charge width(3)*

### ■ Built in potential $V(x)$

$$V_n(x) = -\frac{eN_D}{2\epsilon_{sc}}(x - W_N)^2 + V_n$$

$$V_p(x) = \frac{eN_A}{2\epsilon_{sc}}(x + W_p)^2 + V_p$$

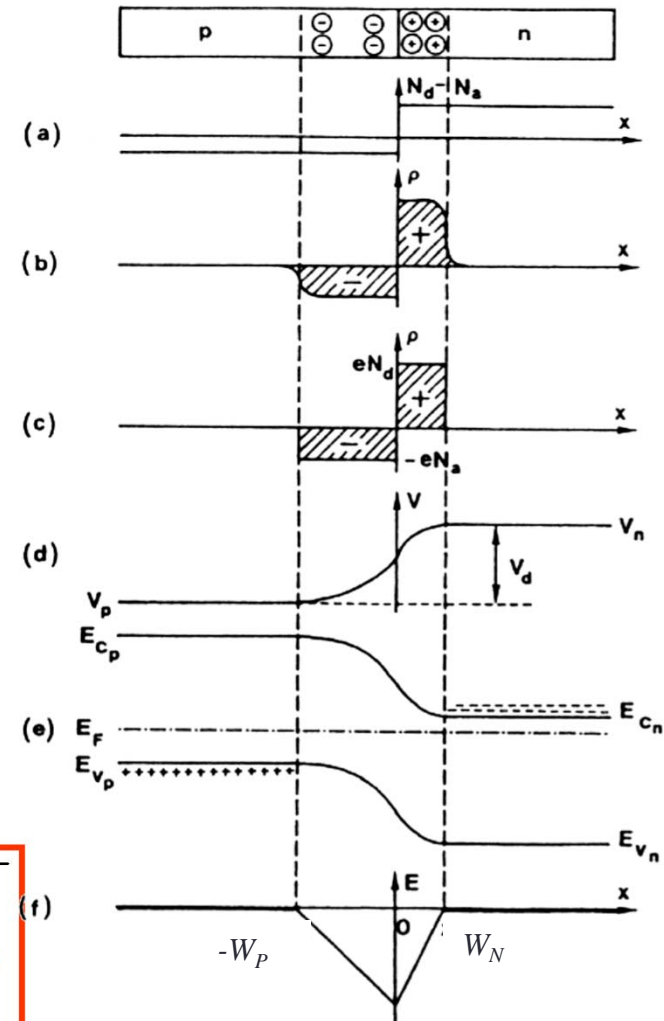
### ■ Depletion layer (ZCE)

$$V(W_n) - V(-W_p) = V_d = \frac{eN_D W_n^2}{2\epsilon_{sc}} + \frac{eN_A W_p^2}{2\epsilon_{sc}}$$

$$W_p(V_d) = \sqrt{\frac{2\epsilon_{sc}}{e} \frac{N_D}{N_A(N_A + N_D)} V_d}$$

$$W_n(V_d) = \sqrt{\frac{2\epsilon_{sc}}{e} \frac{N_A}{N_D(N_A + N_D)} V_d}$$

$$W(V_d) = \sqrt{\frac{2\epsilon_{sc}}{e} \frac{N_D + N_A}{N_A N_D} V_d}$$



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WARNING: WHEN A VOLTAGE  $V$  IS  
APPLIED ON P SIDE,  $V_{BI}$  HAVE TO  
BE REPLACED BY  $V_{BI} - V$

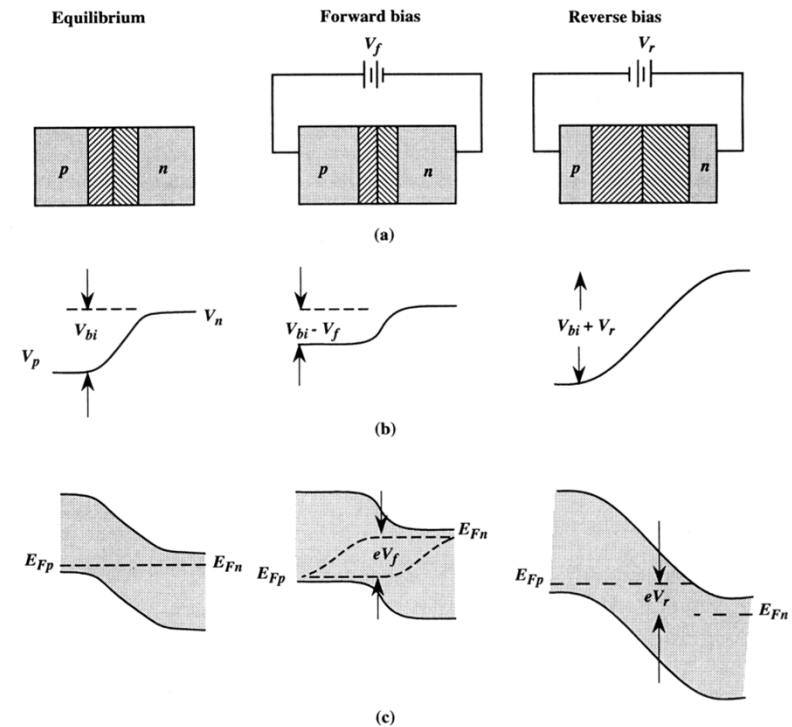


## BIASED PN JUNCTION

- When a positive voltage is applied on p side, the equilibrium is destroyed and a net current can flow
- simplifying assumptions :
  - Depletion layer with no free carriers ( $e^-$  and  $h^+$ )
  - Low injection
  - Boltzmann's approximation
  - Drop voltage only in depletion layer
  - No generation-recombination mechanisms present

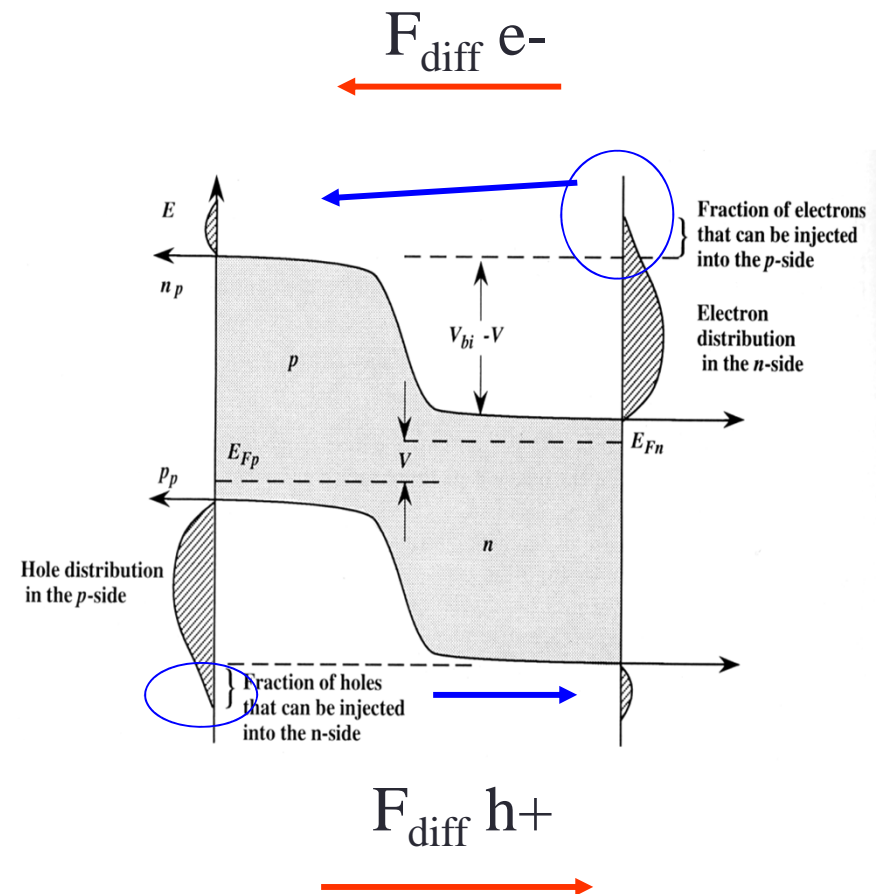
# BIASED PN JUNCTION

- Forward Biasing
  - Positive voltage on P
  - lowering of built in potential
  - Diffusion mechanism dominates
  - High current

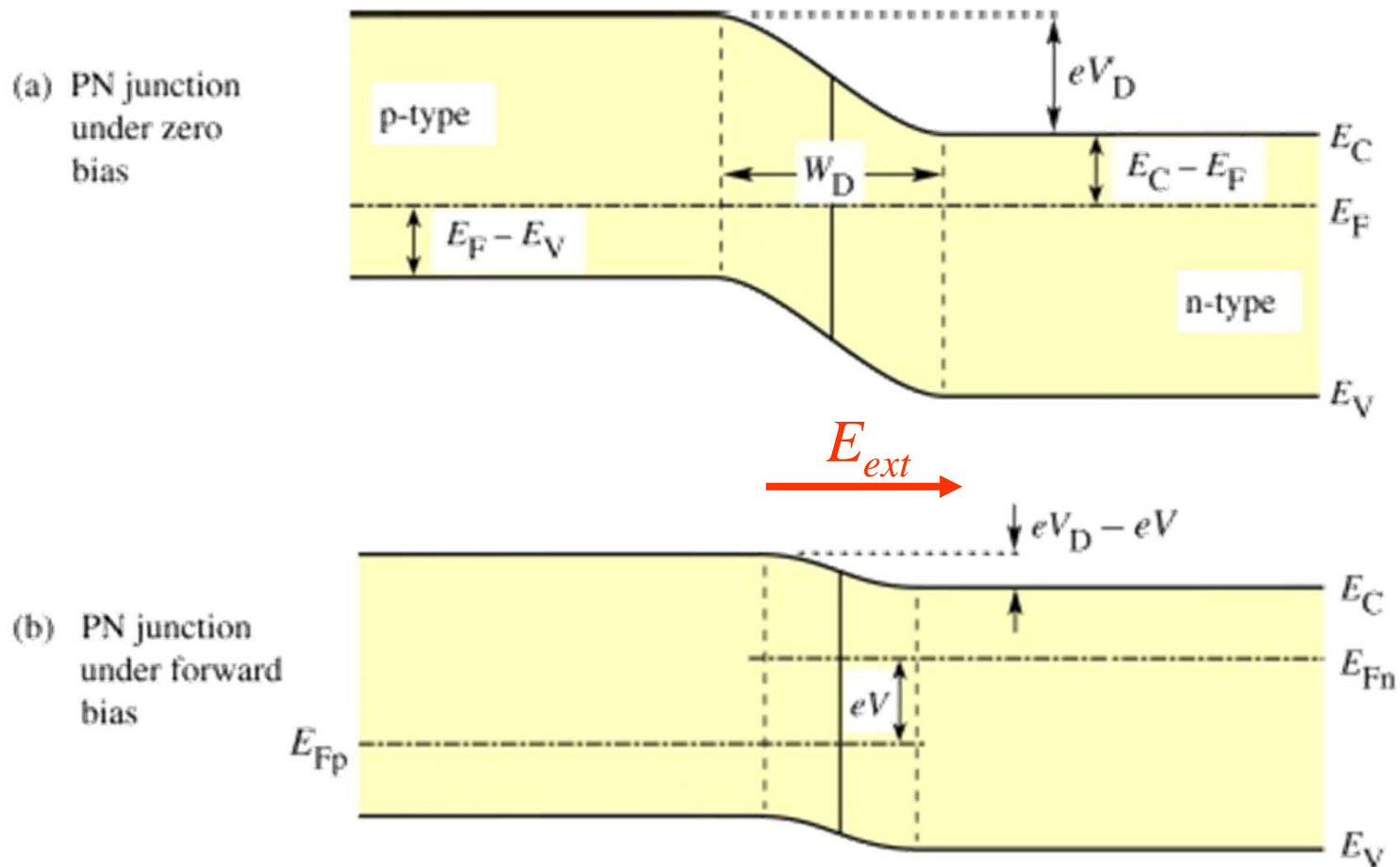


# BIASED PN JUNCTION

- Forward biasing
  - Lowering of built in Field due to opposite external field
  - Electrons injected from N to P regions: minority carriers injection
  - High current due to full « reservoir »



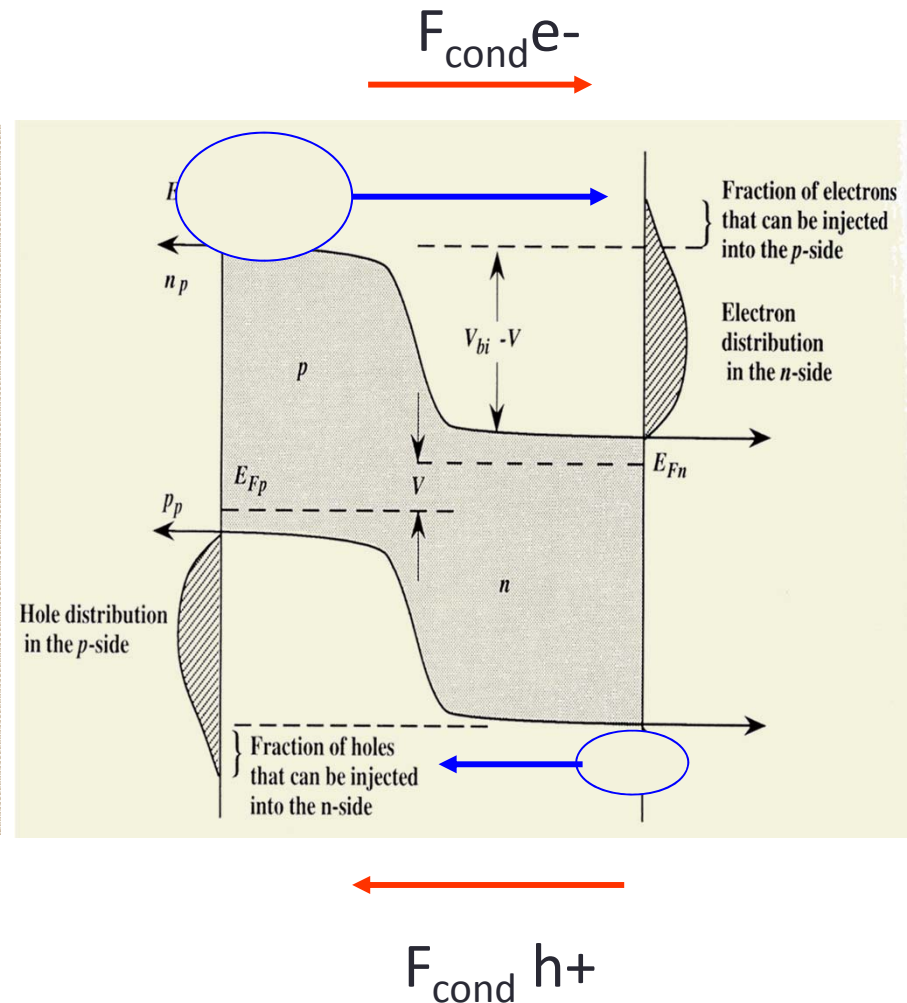
## Forward Biasing



# Jonction PN sous polarisation

- Reverse Biasing

- Global Electric Field increases (External Field added to built in Field)
- Injection of electrons from P to N and holes injection from N to P: **majority carriers injection**
- Low current ( leak current) due to empty « reservoir »



## PN Junction under biasing

At equilibrium, null current  $\Leftrightarrow$  two components compensate between it. Taken separately, the magnitude of these components  $10^4 \text{ A / cm}^2$  (ie 1A for typical diode) and at low injection  $I$  is of the order of few mA (max 10mA)

• **Boltzmann's Approximation:** *The Boltzmann approximation is to say that the resulting current being small compared with the components of this current, we consider that we are still in quasi equilibrium and therefore that the current's equation is still valid by replacing  $V_{bi}$  by  $V_{bi} - V_A$  :*

$$\frac{-e}{kT} \frac{dV(x)}{dx} \approx \frac{1}{p(x)} \frac{dp(x)}{dx}$$

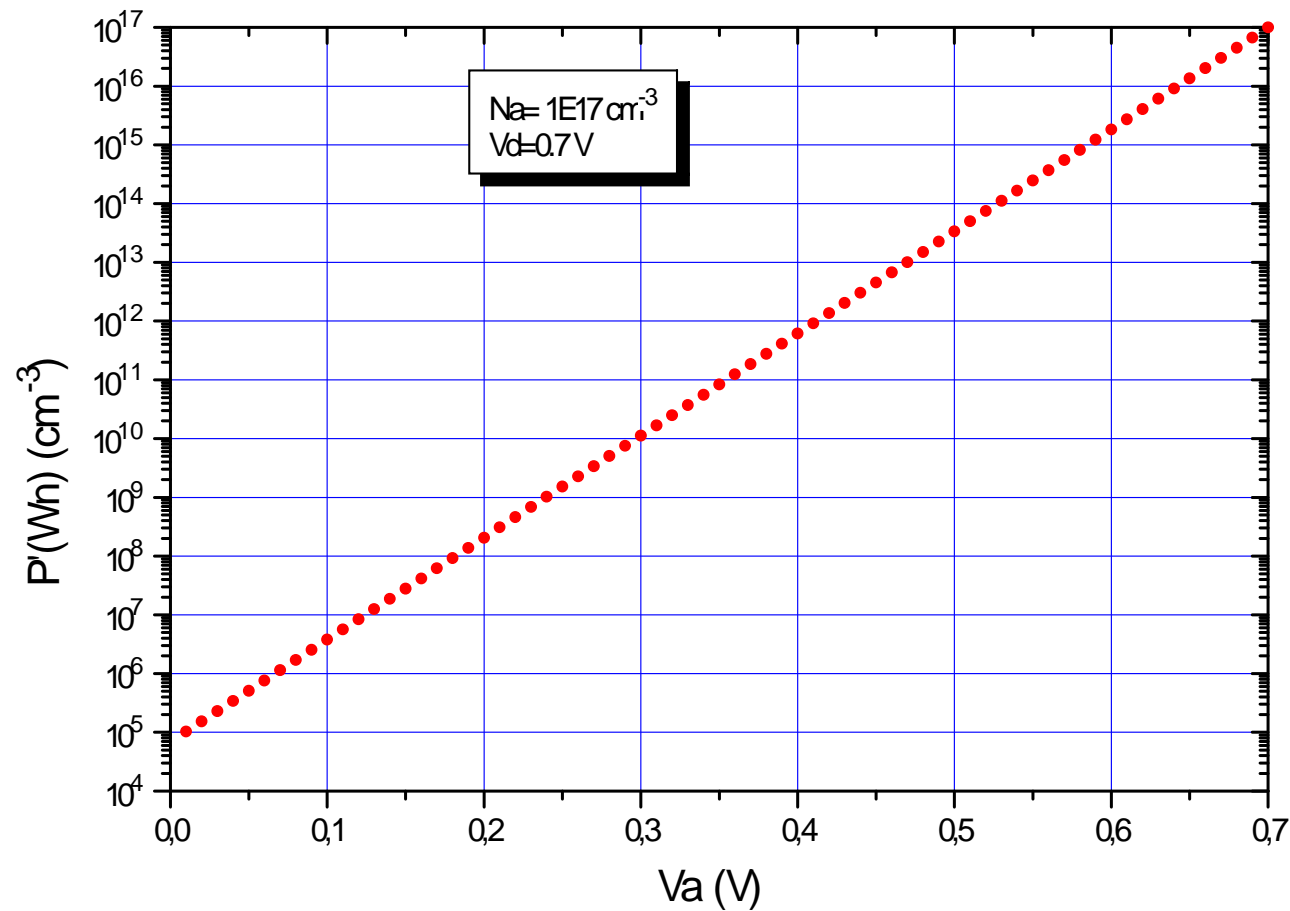
## Density of carriers injected to the limits of depletion layer

- If  $V_a=0$   $\frac{p(W_N)}{p_p} = \frac{p_n}{p_p} \exp(-\frac{eV_{bi}}{kT})$
- If  $V_a \neq 0$   $\frac{p'(W_N)}{p_p} = \frac{p'_n}{p_p} = \exp(-\frac{e(V_{bi} - V_A)}{kT}) = \frac{p_n}{p_p} \exp(\frac{eV_A}{kT})$

$$p'_n = p_n \exp(\frac{eV_A}{kT}) = \frac{n_i^2}{N_D} \exp(\frac{eV_A}{kT}) \qquad n'_p = n_p \exp(\frac{eV_A}{kT}) = \frac{n_i^2}{N_A} \exp(\frac{eV_A}{kT})$$

$$n'_p * p_p = p'_n * n_n = n_i^2 \exp(\frac{eV_a}{kT})$$

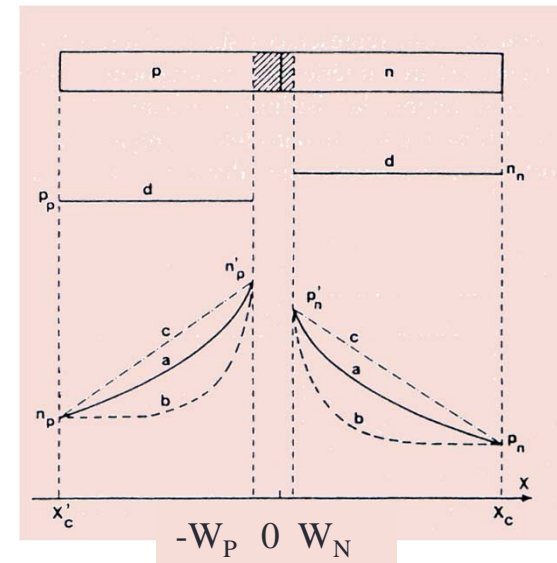
## Holes density injected versus bias voltage $V_a$





# Minority carriers distribution in neutral region

- Due to gradient concentration, carriers will diffuse and produce diffusion current (no electric field in neutral region !)
- Distribution is geometry dependant
- Discriminatory parameter :  
length diffusion  $L_{Dn,p}$  of electrons and holes and  
neutral region widths  $d_{n,p}$



## Minority carriers distribution in neutral region

- Long regions ( $d_{n,p} \gg L_{p,n}$ )

$$p'(x) - p_n = p_n \left( e^{\frac{eV_a}{kT}} - 1 \right) e^{(W_N - x)/L_p}$$

$$n'(x) - n_p = n_p \left( e^{\frac{eV_a}{kT}} - 1 \right) e^{(x + W_p)/L_n}$$

- Short (narrow) regions ( $d_{n,p} \ll L_{p,n}$ )

$$p'(x) - p_n = \frac{p_n}{d_n} \left( e^{\frac{eV_a}{kT}} - 1 \right) (x_c - x)$$

$$n'(x) - n_p = \frac{n_p}{d_p} \left( e^{\frac{eV_a}{kT}} - 1 \right) (x'_c + x)$$

- General case

$$p'(x) - p_n = \frac{p_n}{sh\left(\frac{d_n}{L_p}\right)} \left( e^{\frac{eV_a}{kT}} - 1 \right) sh\left[\frac{(x_c - x)}{L_p}\right]$$

$$n'(x) - n_p = \frac{n_p}{sh\left(\frac{d_p}{L_n}\right)} \left( e^{\frac{eV_a}{kT}} - 1 \right) sh\left[\frac{(x + x'_c)}{L_n}\right]$$

## Minority currents in neutral region

Knowing minority distribution we are in position to calculate the current which is a **diffusion current** (very low field in neutral region):

$$J_p(x) = -eD_p \frac{dp(x)}{dx} \quad J_n(x) = eD_n \frac{dn(x)}{dx}$$

- Hypothesis : **no G-R process in depletion layer (ZCE)**

$$J(V) = J_p(-W_p) + J_n(W_p) = J_p(W_n) + J_n(-W_p)$$

- We get the classical and well known diode equation:

$$J(V) = J_s (e^{eV/kT} - 1)$$

$J_s$  is the theoretical saturation current or reverse current

## Minority currents in neutral region

- Short (Narrow) region

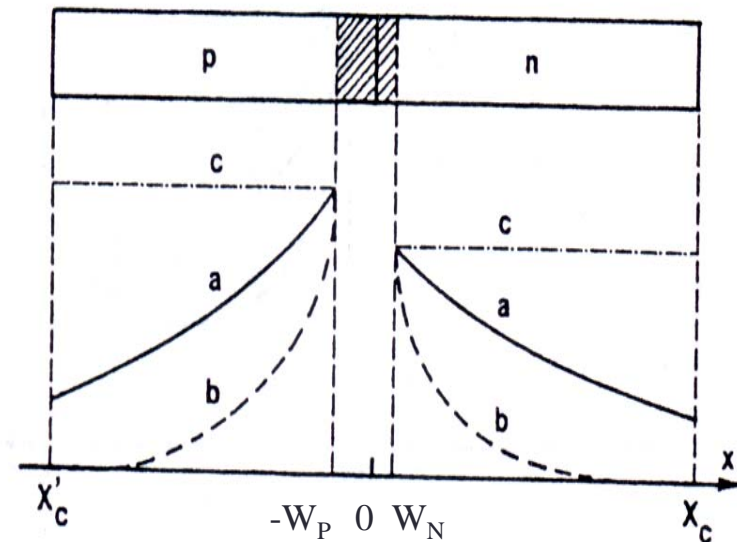
$$J_S = \frac{en_i^2 D_P}{N_D d_n} + \frac{en_i^2 D_n}{N_A d_p}$$

- Long region

$$J_S = \frac{en_i^2 D_P}{N_D L_P} + \frac{en_i^2 D_n}{N_A L_n}$$

- General case

$$J_S = \frac{en_i^2 D_P}{N_D L_P \operatorname{th}\left(\frac{d_n}{L_P}\right)} + \frac{en_i^2 D_n}{N_A L_n \operatorname{th}\left(\frac{d_p}{L_n}\right)}$$



## The real diode: génération-recombinaison mechanism in depletion layer

- The model is refined  $\Leftrightarrow$  we take into account G-R process in depletion layer
- Well understood mechanism (Shockley-Read)

$$r = \frac{1}{\tau} \frac{pn - n_i^2}{2n_i + p + n}$$

- We know that  $p(W_N)n(W_N) = p(W_P)n(W_P) = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$
- If we suppose  $np$  constant in depleted region and  $np \gg n_i^2$  (in forward bias), the rate  $r$  is max when  $n=p$ , and it can be rewritten

$$r_{\max} = \frac{n_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

## The real diode: génération-recombinaison mechanism in depletion layer

- Generation-Recombinaison current in depletion layer can be expressed as:

$$J_n(W_n) - J_n(-W_p) = J_{GR} = e \int_{W_p}^{W_n} r dx$$

- For **reverse biasing** (  $pn \ll n_i^2$  ), we have a negative rate (  $r = -\frac{n_i}{2\tau} < 0$  ). It means dans we have a net **generation** process
- For **forward bias**,  $r_{max} = cte > 0$  and the current is due to **recombinations**.

## The real diode: génération-recombinaison mechanism in depletion layer

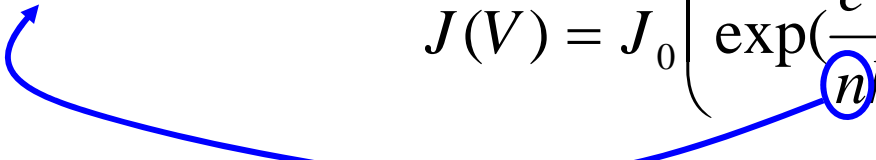
- Finally GR current present in depletion layer can be expressed as:

$$J_{GR} = J_{GR}^0 \left( \exp\left(\frac{eV_a}{2kT}\right) - 1 \right) \qquad J_{GR}^0 = \frac{en_i}{2\tau} W_T$$

- If we take into account the diffusion current, we get:

$$J(V_a) = J_s \left( \exp\left(\frac{eV_a}{kT}\right) - 1 \right) + J_{GR}^0 \left( \exp\left(\frac{eV_a}{2kT}\right) - 1 \right)$$

- The expression above can be generalized by introducing **Ideality factor**:

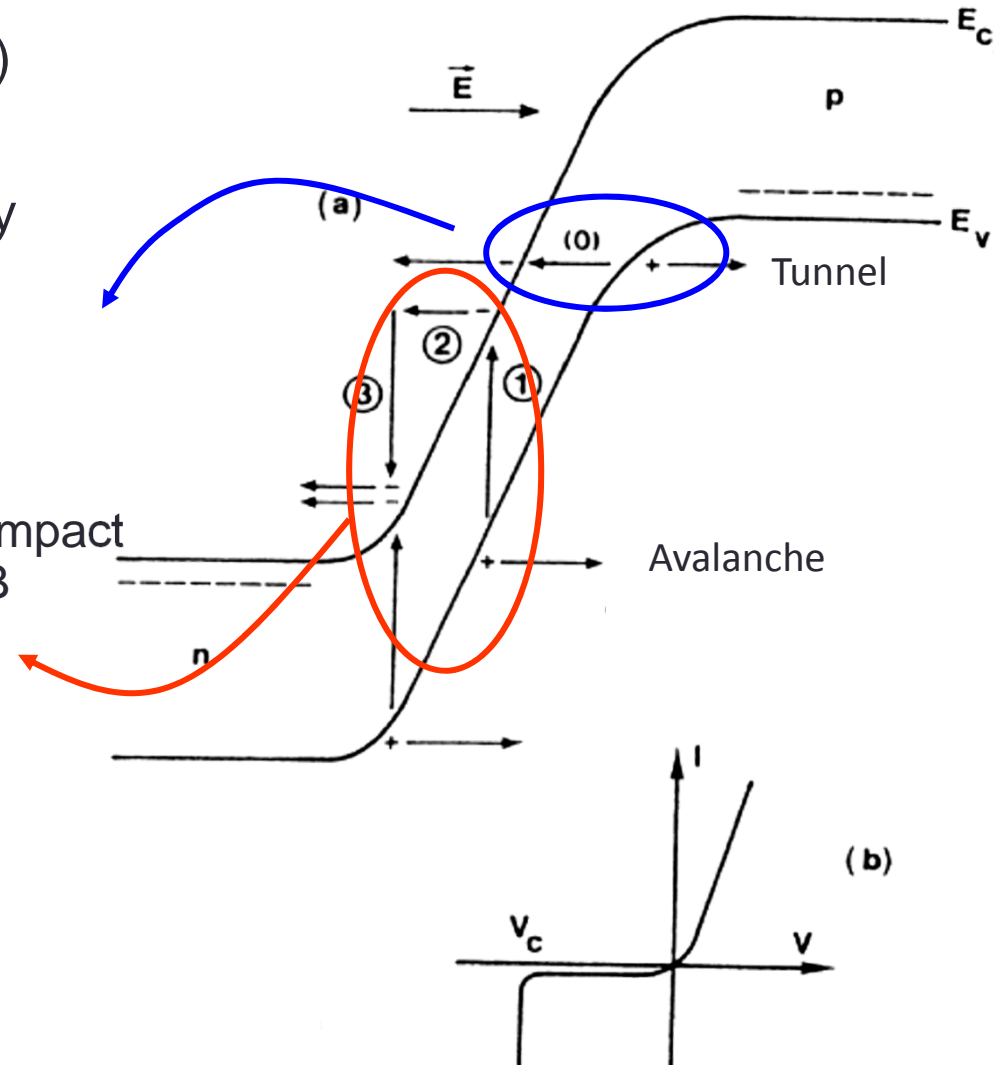
$$J(V) = J_0 \left( \exp\left(\frac{eV_a}{nkT}\right) - 1 \right)$$


## Reverse biasing: Junction breakdown

- Thermal Effect (Narrow bandgap)
- **Zener Effect:**
  - direct flowing from VB to CB by tunnel effect (0), if electric field above critical Field  $E_c$
- **Avalanche Effect:**
  - before « tunneling », hot electrons (accelerated electrons) excite by impact ionisation electrons from VB to CB (1,2,3) etc....

$$V_{BD} = \frac{\epsilon \cdot E_c^2}{2eN_B}$$

- « Punchthrough »





## Small signal model of the diode: capacitances

- Capacitance associated to charges
- 2 types of charges present in the junction
  - Fixed charges (ionised dopants) in depletion layer
  - Mobiles ( $e^-$  et  $h^+$ ) injected when forward biasing
- 2 types of capacitance
  - Junction (or Transition) Capacitance
  - Charge Storage (or diffusion) Capacitance

## Junction capacitance

Simply associated to charges present in depletion layer

$$C_T = C_j = \left| \frac{dQ}{dV} \right| \quad |Q| = eAN_A W_P = eAN_D W_N$$

or:

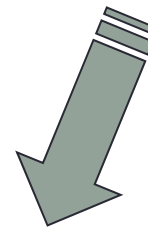
$$C_T = C_j = \frac{A}{2} \sqrt{\frac{2e\epsilon}{(V_D - V_A)} \frac{N_A N_D}{(N_A + N_D)}} = \frac{A\epsilon}{W_T}$$

## Charge Storage (diffusion) capacitance

- Reflects the delay between the voltage and current
- Associated with charges injected into the neutral regions Traduit le retard entre la tension et le courant

$$Q_{Sp} = A \int_{W_N}^{X_c} e \underbrace{(p'(x) - p_n)}_{\text{Holes density in excess present in N region}} dx$$

Holes density in excess present in N region

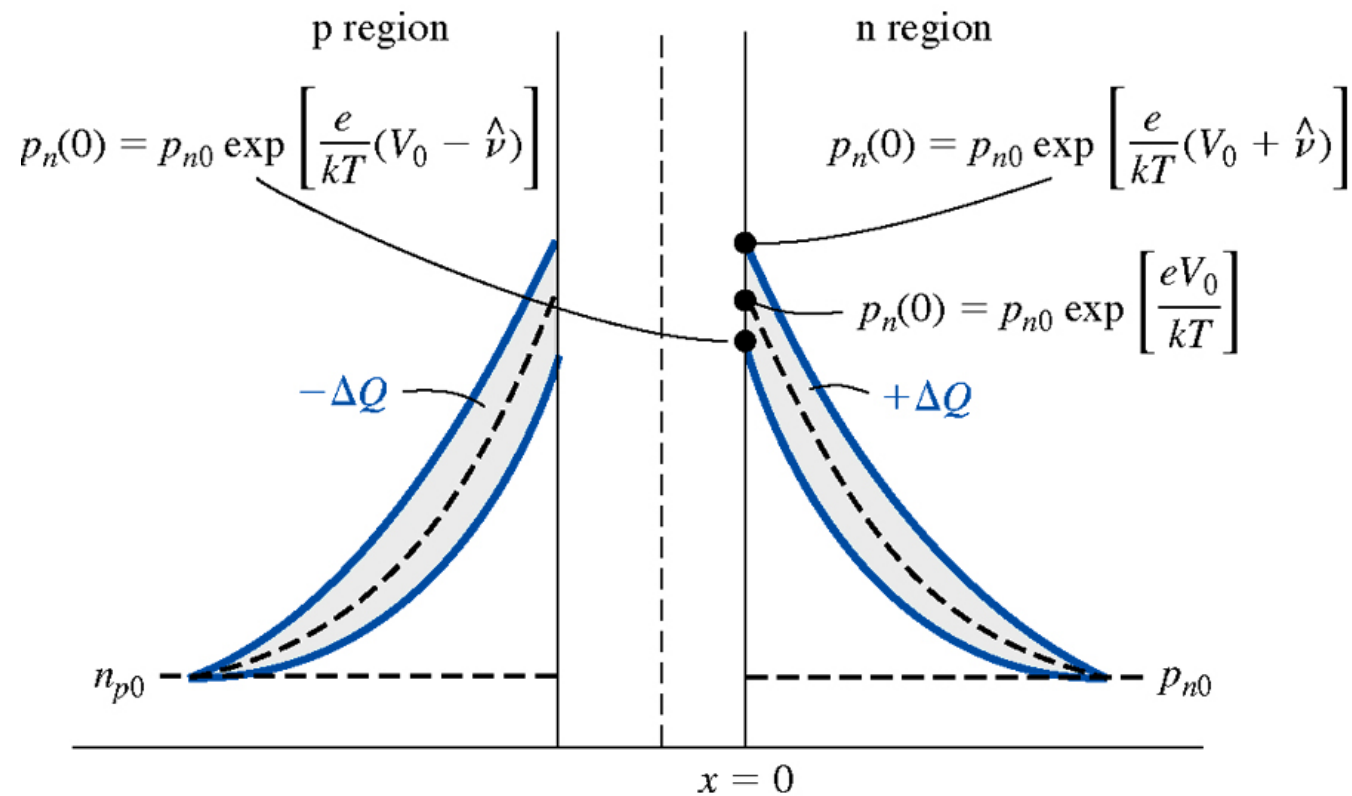


$$Q_{Sp} = \tau_p J_p$$

$$Q_{Sn} = \tau_n J_n$$

$$Q_{Sp} = e(p'(0) - p_n)L_p \left[ \coth\left(\frac{d_n}{L_p}\right) - \frac{1}{\operatorname{sh}\left(\frac{d_n}{L_p}\right)} \right]$$

## Charge Storage (diffusion) capacitance



## Charge Storage (diffusion) capacitance

- We can transform the previous expression by:

$$Q_{Sp} = \tau J_P(W_N) \quad \text{avec} \quad \tau = \tau_P \left( 1 - \frac{1}{ch\left(\frac{d_n}{L_P}\right)} \right)$$

- Time expression can be simplified, depending of the neutral geometry:

- *Narrow diode*:  $\tau_t = \frac{d_n^2}{2D_P} \Leftrightarrow$  transit time

- *Long diode*:  $\tau = \tau_P \Leftrightarrow$  lifetime

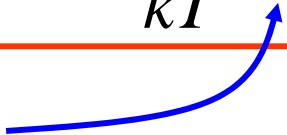
## Charge Storage (diffusion) capacitance

- The previous expression, valid in N region, can be generalized in P region and we obtain for the whole diode:

$$Q_S = Q_{Sn} + Q_{Sp} = \tau_{(n)} J_n (-W_P) + \tau_{(p)} J_p (W_N)$$

If we use:

$$C_S = C_d = \frac{dQ_S}{dV}$$

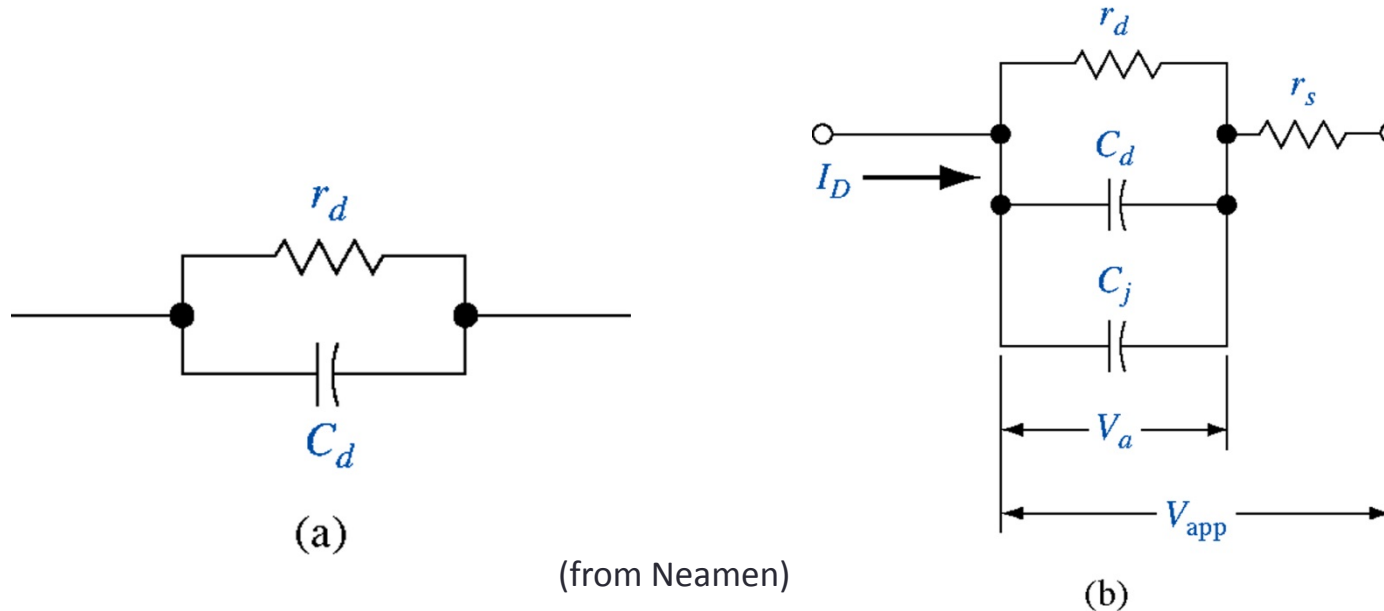
$$C_S = C_d = C_{Sn} + C_{Sp} = \frac{e}{kT} K (\tau_{(n)} J_n + \tau_{(p)} J_p)$$


K : Geometry dependant Factor

(2/3  $\Leftrightarrow$  narrow)

(1/2  $\Leftrightarrow$  long)

# Equivalent circuit of forward diode

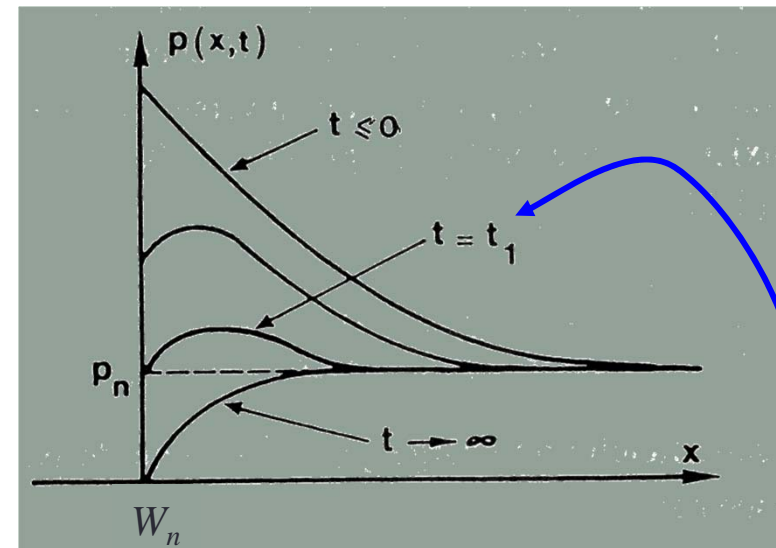
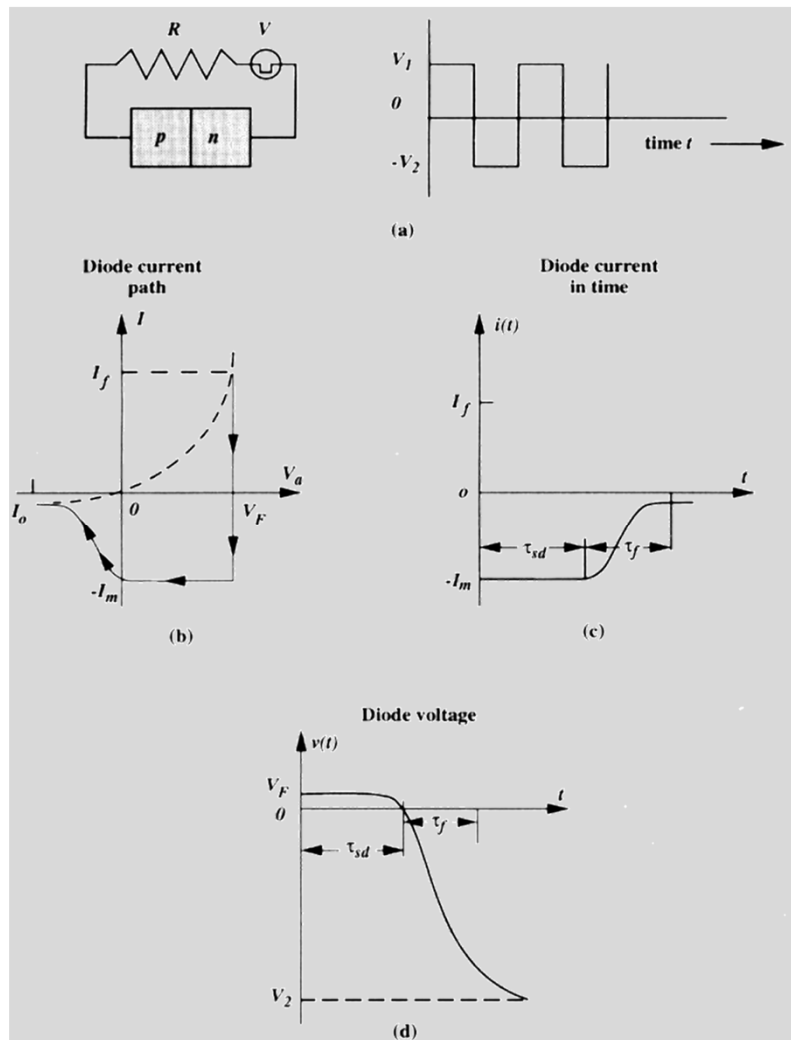


$r_d$  : diode resistance (dynamic resistance) given by the differential slope of the I-V characteristics

$$r_d = \frac{kT}{e} \frac{1}{I}$$

$r_s$  : serie resistance of neutral region n and p

# Large signal switching of diode



As long as the stored charge is positive  
 $\Leftrightarrow$  forward bias diode  $\Leftrightarrow$  voltage across diode is small (few 10 mV)

$$p'_n - p_n = p_n \left( e^{\frac{eV_a}{kT}} - 1 \right)$$

$\tau_{sd} \Leftrightarrow$  Storage time ie  $p'(W_N) = p_n$



## Large signal switching of diode

- Storage time : The main problem in minority carriers devices:

- Storage time:

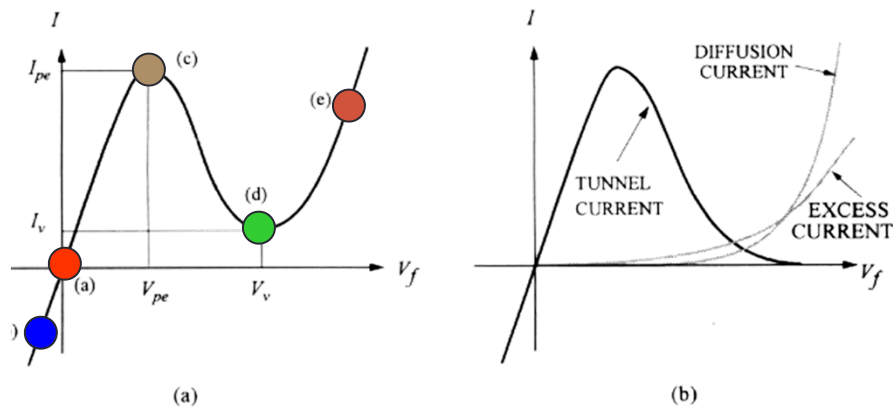
$$\tau_{sd} \cong \tau_p \left[ \ln\left(1 + \frac{I_f}{I_m}\right) - \ln\left(1 + \frac{I_f}{I_f + I_m}\right) \right]$$

- Rise (or fall) time :

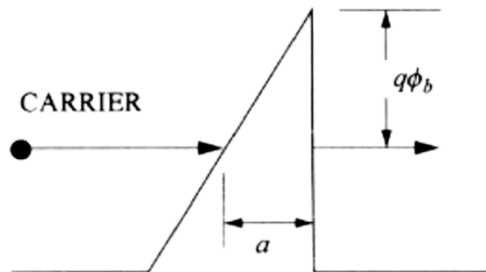
$$\tau_f \cong 2.3 \left[ \frac{\tau_F \delta + RC_j}{1 + \delta} \right] \text{ avec } \delta = \frac{I_f}{I_f + I_m}$$

- $C_j$ : mean value of capacitance between zero and  $-V_2$

# Diode Tunnel – diode Backward



$$I_t = I_{pe} \left( \frac{V_a}{V_{pe}} \right) \exp \left( 1 - \frac{V_a}{V_{pe}} \right)$$



$$T_t \approx \exp \left( - \frac{4a\sqrt{2m^*e\phi_b}}{3\hbar} \right)$$

