MAD 2104 - HW04 - Romain Roux - PID: 6322237

Ex1. How many strings of three decimal digits

a. do not contain the same digit three times?

There are 1000 decimal strings with 3 digits, and 10 of them are containing three equal digits: 000, 111, . . . , 999.
Therefore, there are 1000 – 10 = 990 strings that do not contain the same digit three times.

b) begin with an odd digit?

There are 5 odd digits from 0 to 9, therefore we have 5 * 10 * 10 = 500 strings of three decimal digits beginning with an odd digit.

c) have exactly two digits that are 4 s ?

44x - 9 strings
4x4 - 9 strings
x44 - 9 strings

Total: 27 strings

Ex2. How many strings of four decimal digits

a) do not contain the same digit twice?

There are 10 ways to pick the first digit, then 9 for the second, 8 for the third, and 7 for the fourth.
Therefore there are 10 * 9 * 8 * 7 = 5040 strings of four decimal that do not contain the same digit twice.

b) end with an even digit?

There are 5 even digits from 0 to 9, therefore we have 5 * 10 * 10 * 10= 5000 strings of four decimal digits ending with an even digit.

c) have exactly three digits that are 9 s ?

999x - 9 strings
99x9 - 9 strings
9x99 - 9 strings
x999 - 9 strings

Total: 36 strings

Ex12. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

- We cannot have more than 2 boys seating together so we have 13 boys groups : 12 in pair and 1 alone.
- There are at least 2 girls between each group of boys, otherwise you will have boys as left and right neighbors.

But from the first statement, you can see that there are at least 13 places between boys that need to be filled with 2 girls, so a minnimum of 26 girls. But there is only 25 girls.

Ex13. Prove that at a party where there are at least two people, there are two people who know the same number of other people there

Let n be the number of party people.
The maximum number of people a person can know is n−1 and the minimum number he can know is 1, giving us n−1 possibilities for the number of people someone can know. Every single person must be assigned one of these n−1 possible numbers but since there are n party people one of these numbers must be used twice therefore two people know the same number of people.

Ex14. Let S = {1, 2, 3, 4, 5}.

(a) List all the 3-permutations of S.

5!/(3!*2!) = 10 permutations of S :
{1,2,3}
{1,2,4}
{1,2,5}
{1,3,4}
{1,3,5}
{1,4,5}
{2,3,4}
{2,3,5}
{2,4,5}
{3,4,5}

(b) List all the 3 -combinations of S.

5!/2! = 60 combinations of S
{1,2,3}
{1,2,4}

{1,2,5}
{1,3,2}
{1,3,4}
...
...
...

Ex18. How many ways are there for a horse race with four horses to finish if ties are possible? (Note: Any number of the four horses may tie.)

Solution by cases:
1- No horses tie :
Then there are P(4,4) = 24 possible ways for the horsesto finish.

2- Two horses that tie, but the others are differents.
There are C(4,2) = 6 ways to choose the horses to be tied, and P(3,3) = 6 ways to determine the order of finish for the three groups.
So there are 6*6 = 36 ways.

3- Two groups of two horses that tie.
There are C(4,2) = 6 ways to choose the winners.

4- Three horses that tie.
By the same reasoning, there are C(4,3)*P(2,2) = 4*2 = 8 ways.

5- There is only one way for all the horses to tie.

Adding all the possible ways, there is 24 + 36 + 8 + 6 + 1 = 75 differents ways.

Ex19. There are six runners in the 100 -yard dash. How many ways are there for three medals to be awarded if ties are possible?