MAD 2104 - HW01 - Romain Roux - PID: 6322237

Ex1. Show that each of these conditional statements is a tautology by using truth tables.

a) [~p /\ (p V q)] → q

p	~p	q	p V q	~p /\ (p V q)	[~p /\ (p V q)] → q
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	F	F	T

b) [(p → q) /\ (q → r)] → (p → r)

p	q	r	p → q	q → r	(p → q)/\(q → r)	p → r	[(p → q)/\(q → r)] → (p→r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
F	T	T	T	T	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c) [p /\ (p → q)] → q

p	q	p → q	p /\ (p → q)	[p /\ (p → q)] → q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d) [(p V q) /\ (p → r) /\ (q → r)] → r

p	q	r	p V q	p → r	(pVq) /\ (p→r)	q → r	(pVq) /\ (p→r) /\ (q→r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	F	T	F	F	F	F
F	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	T	F

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T
T
T
T
T
T
T
T

Ex2. Show that the following proposition are logical equivalent.

a) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

p	q	$p \leftrightarrow q$	~p	~q	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

b) Show that $\sim(p \leftrightarrow q)$ and $p \leftrightarrow \sim q$ are logically equivalent.

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	~q	$p \leftrightarrow \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

c) Show that $p \to q$ and $\sim q \to \sim p$ are logically equivalent.

p	q	$p \to q$	~q	~p	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

d) Show that $\sim p \leftrightarrow a$ and $p \leftrightarrow \sim a$ are logicallv equivalent.

p	~p	a	~a	$\sim p \leftrightarrow a$	$p \leftrightarrow \sim a$
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	F	F

Ex4.

a) Explain why the question ”Are you a liar?” does not work.

The question 'Are you a liar' does not work because if the canibal always lies, he will answer NO because he will be lying.
If the canibal always tells the truth, then his answer will also be NO because he will be telling the truth that he is not a liar.

(b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

"Are you a cannibal ?"

Ex5. Express these system specifications using the propositions p "The user enters a valid password" q "Access is granted" and r "The user has paid the subscription fee" and logical connectives

a) "The user has paid the subscription fee, but does not enter a valid password."

r /\ ~p

b) ”Access is granted whenever the user has paid the subscription fee and enters a valid password.”

(r /\ q) → q

c) "Access is denied if the user has not paid the subscription fee."

~r → ~q

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

(~p /\ r) → q

Ex8. Determine whether each of these compound propositions is satisfiable.

a) (p V ~q) /\ (~p V q) /\ (~p V ~q)

p	~p	q	~q	pV~q	~p V q	(p V ~q)/\(~p V q)	~p V ~q	(pV~q)/\(~pVq)/\(~pV~q)
T	F	T	F	T	T	T	F	F
T	F	F	T	T	F	F	T	F
F	T	T	F	F	T	F	T	F
F	T	F	T	T	T	T	T	T

(p V ~q) /\ (~p V q) /\ (~p V ~q)
F
F
F
T

(p V ~q) /\ (~p V q) /\ (~p V ~q) is true when p and q are False.
Hence, it is satisfiable.

b) (p → q) /\ (p → ~q) /\ (~p → q) /\ (~p → ~q)

p	~p	q	~q	p → q	p → ~q	(p → q) /\ (p → ~q)	~p → q
T	F	T	F	T	F	F	T
T	F	F	T	F	T	F	T
F	T	T	F	T	T	T	T
F	T	F	T	T	T	T	F

(p → q) /\ (p → ~q) /\ (~p → q)	~p→~q	(p → q) /\ (p → ~q)/\(~p → q) /\ (~p → ~q)
F	T	F
F	T	F
T	F	F
F	T	F

(p → q) /\ (p → ~q) /\ (~p → q) /\ (~p → ~q) is never true.
Hence, it is unsatisfiable.

c) (p ↔ q) /\ (~p ↔ q)

p	q	~p	p ↔ q	~p ↔ q	(p ↔ q) /\ (~p ↔ q)
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	T	F
F	F	T	T	F	F

(p ↔ q) /\ (~p ↔ q) is never true.
Hence, it is unsatisfiable.

Ex9. Translate these specifications into English where F(p) is "Printer p is out of ser vice," B(p) is "Printer p is busy" L(j) is "Print job j is lost" and Q(j) is "Print job j is queued."

a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

If there is a printer that s out of service and busy, then there is a print job that s lost

b) b) $\forall p B(p) \rightarrow \exists j Q(j)$

If all printers are busy, then there is a print job that s queued.

c) c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

If there exists a print job that is lost and queued, then there exists a printer that s out of service

d) d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

If all printers are busy and all print jobs are queued, then there exists a print job that is lost

Ex11. Let P(x), Q(x), and R(x) be the statements " x is a professor," "x is ignorant" and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), and R(x), where the domain consists of all people

a) No professors are ignorant.

$\forall x(P(x) \rightarrow \sim Q(x))$

b) All ignorant people are vain.

$\forall x(Q(x) \rightarrow R(x))$

c) No professors are vain.

$\forall x(P(x) \rightarrow \sim R(x))$

d) Does (c) follow from (a) and (b)?