1. Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = (n+1)\ (2n+1)(2n+3)/3$ whenever n is a nonnegative integer.
2. Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = \left(1 - \ (-7)^{n+1}\right)/4$ whenever n is a nonnegative integer.
3. Prove that $3^n < n!$ if n is an integer greater than 6 .
4. Prove that $2^n > n^2$ if n is an integer greater than 4 .
5. For which nonnegative integers n is $n^2 \leq n!$? Prove your answer.
6. For which nonnegative integers n is $2n + 3 \leq 2^n$? Prove your answer.
7. Prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer.
8. $*$ Prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.
9. * What is wrong with this "proof"?

   "Theorem" For every positive integer n, $\sum_{i=1}^{n} i = \left(n + \frac{1}{2}\right)^2/2$

   Basis Step: The formula is true for $n = 1$

   Inductive Step: Suppose that $\sum_{i=1}^{n} i = \left(n + \frac{1}{2}\right)^2/2$. Then $\sum_{i=1}^{n+1} i = \left(\sum_{i=1}^{n} i\right) + (n+1)$. By the inductive hypothesis, $\sum_{i=1}^{n+1} i = \left(n + \frac{1}{2}\right)^2/2 + n + 1 = \left(n^2 + n + \frac{1}{4}\right)/2 + n + 1 = \left(n^2 + 3n + \frac{9}{4}\right)/2 = \left(n + \frac{3}{2}\right)^2/2 = \left[(n+1) + \frac{1}{2}\right]^2/2$, completing the inductive step.
10. What is wrong with this "proof"?

    "Theorem" For every positive integer n, if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

    Basis Step: Suppose that $n = 1$. If $\max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.

    Inductive Step: Let k be a positive integer. Assume that whenever $\max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $\max(x, y) = k + 1$, where x and y are positive integers. Then $\max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step
11. Suppose that m is a positive integer. Use mathematical induction to prove that if a and b are integers with $a \equiv b \ (\mathrm{mod} m)$, then $a^k \equiv b^k (\mathrm{mod} m)$ whenever k is a nonnegative integer.
12. * Let $a_1, a_2, \ldots, a_n$ be positive real numbers. The arithmetic mean of these numbers is defined by

    $$A = (a_1 + a_2 + \cdots + a_n)/n$$

    and the geometric mean of these numbers is defined by

    $$G = (a_1 a_2 \cdots a_n)^{1/n}$$

    Use mathematical induction to prove that $A \geq G$.