1. How many strings of three decimal digits

   (a) do not contain the same digit three times?

   (b) begin with an odd digit?

   (c) have exactly two digits that are 4 s ?

2. How many strings of four decimal digits

   (a) do not contain the same digit twice?

   (b) end with an even digit?

   (c) have exactly three digits that are 9 s?

3. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

4. How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

5. How many functions are there from the set $\{1, 2, \ldots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

   (a) that are one-to-one?

   (b) that assign 0 to both 1 and n?

   (c) that assign 1 to exactly one of the positive integers less than n?

6. How many subsets of a set with 100 elements have more than one element?

7. A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

8. How many 4 -element DNA sequences

   (a) do not contain the base T ?

   (b) contain the sequence ACG?

   (c) contain all four bases A, T, C, and G?

   (d) contain exactly three of the four bases A, T,C, and G?

9. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

10. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

    (a) How many balls must she select to be sure of having at least three balls of the same color?

    (b) How many balls must she select to be sure of having at least three blue balls?

11. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4 .

12. *. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

13. * Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

14. * Let $S = \{1, 2, 3, 4, 5\}$.

    (a) List all the 3-permutations of S.
    (b) List all the 3 -combinations of S.

15. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

    (a) are there in total?
    (b) contain exactly two heads?
    (c) contain at most three tails?
    (d) contain the same number of heads and tails?

16. How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45,4 are selected from a block of 57, and the others are selected from the remaining 69 countries?

17. How many ways are there for a horse race with three horses to finish if ties are possible? [Note: Two or three horses may tie.]

18. * . How many ways are there for a horse race with four horses to finish if ties are possible? [Note: Any number of the four horses may tie.)

19. * . There are six runners in the 100 -yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the runner or runners who finish with exactly one runner ahead receive silvermedals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)

20. Find the expansion of $(x + y)^4$

21. What is the coefficient of x^7 in $(1 + x)^{11}$?

22. * Show that if p is a prime and k is an integer such that $1 \le k \le p - 1$, then p divides $\begin{pmatrix} p \\ k \end{pmatrix}$