MAD 2104 - HW02 - Romain Roux - PID: 6322237

Ex1. Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

A x B x C will generate the set of type (a,b,c) element.
(A x B) x C will generate the set of type ((a,b),c) element.

Ex3. Let A, B, and C be sets. Show that :

(a) $(A \cup B) \subseteq (A \cup B \cup C)$

We have: $x\in A \cup B \Leftrightarrow (x\in A) \vee (x\in B) \Rightarrow (x\in A) \vee (x\in B) \vee (x\in C) \Leftrightarrow x\in(A \cup B \cup C)$
Thus, we have : $(A \cup B) \subseteq (A \cup B \cup C)$

(b) $(A \cap B \cap C) \subseteq (A \cap B)$

By definition, we have $x\in A \cap B \cap C \Leftrightarrow x\in A \wedge x\in B \wedge x\in C \Rightarrow x\in A \wedge x\in B \Leftrightarrow x\in(A \cap B)$
Thus, we have $A \cap B \cap C \subseteq A \cap B$.

(c) $(A - C) \cap (C - B) = \varnothing$

For $x\in(A - C) \cap (C - B)$ we have:
$x\in(A - C) \cap (C - B) \Leftrightarrow x\in(A - C) \wedge x\in(C - B) \Leftrightarrow x\in A \wedge x\notin C \wedge x\in C \wedge x\notin B$
The latter means that the set $(A - C) \cap (C - B)$ is empty.
Thus, $(A - C) \cap (C - B) = \varnothing$

Ex6. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one ?

Suppose g is not one to one, then there exist x != y in X such that g(x)=g(y), so as f is one to one, f o g(x)=f o g(y), with x != y, which contradicts f o g being one to one.
Hence g must be one to one.

Ex8. Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that :

(a) $f(S \cup T) = f(S) \cup f(T)$

Let y be an arbitrary element of $f(S \cup T)$.
Then there is an element x in $S \cup T$ such that $y = f(x)$.
If x is in S, then y is in f(S). Hence y is in $f(S) \cup f(T)$.

Similarly y is in $f(S) \cup f(T)$ if x is in T.

Hence if $y \in f(S \cup T)$, then $y \in f(S) \cup f(T)$.

(b) $f(S \cap T) \subseteq f(S) \cap f(T)$.

Let y be an arbitrary element of $f(S \cap T)$.
Then there is an element x in $S \cap T$ such that $y = f(x)$, that is there is an element x which is in S and in T, and for which $y = f(x)$ holds.
Hence $y \in f(S)$ and $y \in f(T)$, that is $y \in f(S) \cap f(T)$.

Ex14. Show that a subset of a countable set is also countable.

Let S be a countable set and let $T \subseteq S$.
By definition, there exists an injection $f:S\to N$.
Let $i : T \to S$ be the inclusion mapping.
We have that i is an injection.
Because the composite of injections is an injection, it follows that $f \circ i : T \to N$ is an injection.

Hence, T is countable.