Computational geometry : Homework 5

Vandemaele Romain

November 23, 2015

1 Introduction

We consider a simple polygon P with n vertices. We can place a light in a point q inside P. A light emits ray in all direction. A ray can only be stopped by an edge. The question are how many lights are sometimes necessary and how many lights are always sufficient ?

2 Necessary case

We need more than $mathcalO(\log(n))$ lights. Indeed figure 1 is an examples, we have 6 "components" in which we need one light with 28 points which $\log_2(n) = 4.807355$. Moreover, we can add a new "components" by adding a constant number of vertices(here 4 cf. dotted edges) while the log grow by one only when n is multiplied by 2.

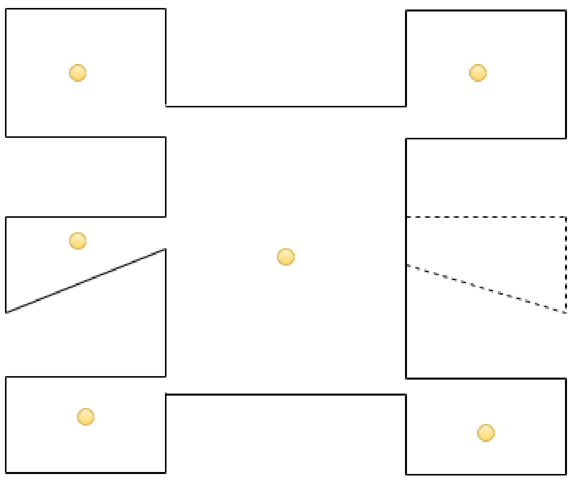

Figure 1: Example of polygon needing more than $\log(n)$ lights

3 Sufficient case

$\mathcal{O}(n)$ lights are always sufficient. If there is one light at each vertex, the whole polygon is illuminated.

Proof. (By contradiction). If the poygon is not illuminated, $\exists p \in P$ s.t. p is not illuminated. Thereby, $\forall\ v \in V(P)$[1], $\nexists\ s$ a segment between v and p that do not intersect an edge. By Meisters' two ears theorem, we know that there exists a triangle formed by 3 vertices v_1, v_2, v_3 that contains p. Thereby, there exists a segment s between v_1 and p that do not intersect any edges $\Rightarrow$ Contradiction. $\square$

4 Conclusion

The number of lights needed to illuminate a polygon with n vertices is between $\mathcal{O}(\log(n)$ and $\mathcal{O}(n)$.

[1] set of vertices of P