1. Exercise 1:

   (a) Let $X \sim \mathcal{N}(-1, 0.01)$, 0.01 being the variance. Compute:
      i. $P(X \leq -0.98)$
      ii. $P(X \leq -1.02)$
      iii. $P(X \geq -0.82)$
      iv. $P(X \in [-1.22; -0.96])$

   (b) Let $X \sim \mathcal{N}(0, 1)$. Determine t such that:
      i. $P(X \leq t) = 0.9$
      ii. $P(X \leq t) = 0.2$
      iii. $P(X \in [-t, t]) = 0.95$

2. Exercise 2:

   (a) Give the definition of a density function f

   (b) Let θ_n an estimator of a parameter θ. Give the definition of θ_n an unbiased estimator of θ.

   (c) Let $X_1, \ldots, X_n$ a n-sample. We denote by μ the expectation of X_1 and σ^2 its variance. Let $\bar{X}_n$ the empirical mean associated. Compute the expectation and the variance of $\bar{X}_n$.

   (d) Let $X_1, \ldots, X_n$ a n-sample with a $\mathcal{N}(\mu, \sigma^2)$ distribution. Give an unbiased estimator of σ^2 when we assume that μ is unknown. Prove the fact that it is unbiased.

3. Exercise 3:
   Let consider the observations that are in the file dataexam.txt. Those observations are the times where an event succeed.

   (a) Make a test to show that those times are distributed according to a uniform distribution.

   (b) Now if we consider the time between two events, how can you modelize this distribution?

   (c) Guess the value of the parameter and compute a confidence interval for it.

4. Exercise 4:
   Let X be a random variable whose distribution is an exponential with parameter $\lambda > 0$

   (a) We define the conditional probability $P(A|B)$ by:

   $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   if $P(B) \neq 0$.
   Prove that the exponential random variable is with no memory which means:

   $$\forall s, t > 0, \quad P(X > t + s | X > t) = P(X > s)$$

   (b) Let consider $Y = E(X) + 1$ where $E(x)$ is the biggest integer smaller or equal to x. Determine the distribution of Y.