

1 Extended fields

Let F be a totally ordered field. Let $F_\infty = F \cup \{\perp, \top\}$. We define operations $+$ and \cdot on F_∞ so that (for all $a \in F_\infty$ and for all $p \in F, p > 0$) from top down:

$$\begin{aligned} \perp &< a < \top \\ \perp + a &= \perp = a + \perp & (\text{includes } \perp + \top = \perp) \\ \top + a &= \top = a + \top \\ \perp \cdot p &= \perp = p \cdot \perp \\ \top \cdot p &= \top = p \cdot \top \\ \perp \cdot 0 &= \perp = 0 \cdot \perp & (\text{the “weird” rule}) \\ \top \cdot 0 &= 0 = 0 \cdot \top \end{aligned}$$

All other cases of all operations and relations preserve their behavior from F . We keep the product between negative numbers and $\{\perp, \top\}$ undefined.

2 Farkas-like conjecture

Let $A \in F_\infty^{m \times n}$ and $b \in F^m$. Exactly one of the following is true:

- $\exists x \in F^n$ such that $0 \leq x$ and $Ax \leq b$
- $\exists y \in F^m$ such that $0 \leq y$ and $(-A^T)y \leq 0$ and $b^T y < 0$

3 Counterexample

$$A = \begin{pmatrix} \perp & \top \\ \top & \perp \end{pmatrix} \quad b = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

Both are true.

4 Remark

For all other versions of Farkas lemma that I tried to generalize to F_∞ similar counterexample still applies. However, they might work if it was forbidden for a row of A to contain both \perp and \top and alike for a column.

The problem is when the conversion to the finite version requires us to delete both rows and columns, the rows must be deleted first both in the primal and in the dual, and so, if deleting a row stops deleting a column from triggering, it leads to a different result in the primal than in the dual because of the matrix transposition.