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PHYS 512 Project: Transit Timing Variation

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1. INTRODUCTION

The transit method has revolutionized exoplanetary science: of the 5,788 confirmed exoplanets discovered to date, 4,310 have been discovered using this observational technique (NASA 2024). When an exoplanet passes between an observer (us on Earth) and its host star, the star's light is temporarily dimmed. The transit method measures the dip in the star's light curve caused by that dimming. Remarkably, by analyzing these transit light curves, we can ascertain many of the planet's characteristics, such as size (i.e., radius), orbital period, and even atmospheric composition (with multi-wavelength transit observations).

However, roughly $\frac{1}{5}$ of the exoplanets discovered to date are in multi-planet systems, where planets co-orbiting the star interact gravitationally and potentially perturb their respective rotational and orbital rates (NASA 2024). This interaction may nudge planets out of a periodic orbit, causing a **Transit Timing Variation** (TTV). The TTV observational method is advantageous and robust because it exploits this multi-planetary interaction to constrain the parameters listed for the single-planet case and obtain planetary mass. This is because the amplitude of the timing variations depends on the mass ratio between the two planets. Larger perturbing planets have a more significant gravitational influence and, therefore, cause a more pronounced timing variation. TTVs are especially relevant in the context of planets in mean-motion resonance—i.e., planets whose orbital periods have an integer ratio (e.g., 2:1, 3:2)—because such planets exert perturbations on one another that are periodic and, hence, predictable (Ketchum et al. 2012).

In both the single-planet and multi-planet scenarios, an important consideration when simulating and observing transit light curves is the effect called **limb darkening**, which we account for in our analysis (see section 2). Stellar limb darkening is a phenomenon in which the edge of a star appears darker than the center of the disk. This effect is essential to consider during exoplanet transits because when the planet passes in front of the darker "limb" at the beginning of the transit, the star's brightness decreases more gradually than if the star were uniformly bright. This leads to a smoother initial dip in the light curve, sharpening as the planet passes in front of the star's center. Accounting for limb darkening is crucial because it ensures that we correctly estimate how much light the planet is blocking at different parts of the star, which impacts the shape of the transit light curve and, hence, the properties inferred about the transiting planet.

Our project is split between three objectives. In Part I, we create an N-body simulation of 2-planet system to demonstrate the effect of gravitational interactions that causes the transit time variation. In Part II, we consider the transit of a single planet around its host star and use the Markov Chain Monte Carlo (MCMC) to constrain the best-fit orbital period and planetary radius parameters. Part II extends this analysis to consider the TTV resulting from a two-planet scenario by using MCMC again along with the N-body simulation developed in Part I.

Parameter	Meaning	Units
*eccentricity (e)	Describes shape of the orbit	$0 \leq e < 1$
semi-major axis (a)	Describes size of the orbit	R_{star}
[1]inclination (i)	The tilt of the orbital plane (90 = edge on orbit)	Degrees
*Argument of Periapsis (ω)	The orientation of the ellipse within the orbital plane.	°
*True Anomaly (ν)	The position of the object along the orbit at a given time.	°
Orbital Period (T)	Time of one orbit	Days
t_0	Time that the planet reaches the mid-transit point	Days
R_s	Planet radius	R_{star}
[2]limb darkening law	equation that characterizes limb darkening	n/a
limb darkening coefficients	u_1 and u_2	n/a

Table 1. Parameters of orbital dynamics **obtainable by transit observations and TTV analysis**.
*Parameters that are hard to constrain for both planets without other methods involved (i. e. Radial Velocity observations) or some bold assumptions.
[1]Often assumed to be $\approx 90^o$ for transiting planets.
[2]We use quadratic law: $I(r) = 1 - u_1(1-\mu) - u_2(1-\mu)^2$.

2. METHODS

2.1. *Part I: N-body simulation of a transit timing variation*

i. `Leapfrog`

In this part, we simulate a two-planet star system and explore how the planets interact with each other to produce the effect of TTVs. This simulation uses simple gravitational interactions between three bodies: the star, planet 1, and planet 2. Specifically, we observe the transit of planet 1, with its timing variations caused by the gravitational influence of planet 2. This code models the motion of the planets and demonstrates how planet-planet interactions lead to variations in transit times.

To implement this N-body simulation, we use the `leapfrog` method – a simple technique with good second-order accuracy and energy conservation. This method is similar to a basic (lazy) approach for calculating two-body gravitational interactions: at each time step, we calculate the force exerted based on the distance between two bodies, update the positions using the previous velocity, and then update the velocity based on the acceleration derived from the force. However, this straightforward approach is unsuitable for modeling planetary systems because it does not conserve energy and leads to cumulative errors and numerical instabilities over time. One improvement involves averaging positions and accelerations across two-time steps to update the system, but an even better method is to reorder the computation steps.

In the `leapfrog` method, positions are updated at full time steps, while velocities are updated at half time steps. This reordering means we update positions using velocities evaluated at the midpoint of the time step, while forces are computed based on the next position. This achieves second-order accuracy and better conserves energy.

Our simulation involves a large loop with extensive calculations. To optimize performance, we use the `Numba` just-in-time compiler, which significantly speeds up the code by precompiling key functions.

First, we demonstrate the core functions of the simulation and verify that energy is conserved. Then, we compare computation times between the standard implementation and the `Numba`-optimized version.

Given the coplanar nature of our leapfrog simulation, we make the following simplifications about parameters in Table 1: i and Ω are irrelevant since there is only a single rotational axis for the orbital plane; ω defines the orientation of the orbital plane; we assume the objects start at periapsis, setting ν to zero.

2.2. *Part II: MCMC modelling of the single planet transit*

i. `batman`

Given the increasing rate at which exoplanets are discovered yearly (Mathieu 2024), developing reliable light curve models is essential for interpreting observational data and extracting properties about the planets and their host stars. With this in mind, Professor Laura Kriedberg at the University of Chicago introduced a Python package `batman`: **BAsic Transit Model cAlculatioN**, which is designed to expedite light curve computations while maintaining their accuracy (Kreidberg 2015). Importantly, this package considers the role of limb darkening and can be applied to any radially symmetric limb darkening laws, including uniform, linear, quadratic, logarithmic, exponential, and nonlinear.

Underscoring the importance of code performance as discussed in class, `batman` uses C extension modules to speed up computation time by "a factor of 30 over pure Python implementation for quadratic limb darkening." Moreover, by using OpenMP, the `batman` code is efficiently improved with parallelization.

Our work with CAMB in Homework 4 and its role when fitting Cosmic Microwave Background parameters with MCMC influenced our choice to use `batman` for our MCMC. `batman` required nine input parameters to generate the model light curve: time of the mid-transit point (t_0), orbital period (T), planet radius in units of stellar radii (R_p/R_s), semi-major axis in units of stellar radii (a/R_s), orbital inclination (i), eccentricity (e), longitude of the periapsis (ϖ), limb darkening coefficients, and the limb darkening law. In exoplanetary science, inclination refers to the angle between the orbital plane of the planet and the plane of the sky as seen from Earth. Exoplanet transits can only be observed when the planet's inclination is close to 90°. Many planets in evolved systems, such as those in the TRAPPIST-1 system, have stabilized inclinations around this value. Consequently, in our simulations, we simplified the problem by considering motion in a single plane and fix i as 90°. We also fixed ϖ as 90° and e as 0°. Lasty, we chose "quadratic" as our limb-darkening model. We aimed to find the best-fit parameters for T, R_p/R_s, a/R_s, and the limb darkening coefficients, so that we could best characterize the nature of the transiting planet.

ii. `starry`

To run MCMC, we also needed "true" data, *i.e.*, a transit light curve from an observation. We simulated a true observation by generating a light curve using the python package `starry` (Luger 2024) and injecting Gaussian noise into the flux. We note that `starry` and `batman` are actually equivalent for transit modeling, but we were interested in being acquainted with multiple different exoplanet transit models and so used both in our project. In the case of `starry`, however, noise is added to the simulated light curve.

2.3. *Part III: MCMC modeling of a transit timing variation*

To estimate more parameters about our observed transiting exoplanet and to infer some of the new parameters of the second body (perturber), similarly to the previous part, we will utilize MCMC. In this case, however, we will rely completely on the N-body model we simulated in Part I.

From Part II, we find out that the mid-transit point (t_0) of each subsequent transit is not increasing periodically. In other words, we experience TTV and the period of the exoplanet is not constant, indicating a perturbation. In TTV research a common indicator of these perturbations is the Observed-Calculated (O-C) curve: the difference between the observed mid-transit point and the estimated mid-transit point from the period. In our case, the calculated period is the average period taken from all timing observations. The shape, magnitude, period, and slope of the curve can tell us a lot about the number of perturbers and their orbital and physical parameters. Thus, we perform MCMC sampling on the O-C curve with a given set of prior guesses.

As shown in Part I, the N-body simulation function takes a total of 10 parameters, and that's only if we consider 2D orbits. In terms of the O-C curve, this can be a degenerate problem and lead to correlations between some parameters. We restructure our prior parameters to reduce degeneracy while maintaining a physical sense. We assume that the mass of the star (M_{star}) is equal to 1 solar mass, let's just say that previous spectroscopic observations in combination with stellar evolution models and the fact that the star is in the stellar association provided us with age constraints. Instead of sampling the masses of both planets, we sample the mass of the second as a ratio of M_1/M_2. We assign the initial period of the first planet to 10 days, which we got from quasi-periodic transit observations. The eccentricity and the argument of periapsis are left to be equal to 0 from previous assumptions and from the fact that it's impossible to determine without the radial velocity method.

3. RESULTS

All the results and discussions are demonstrated in `Jupyter Notebooks` as follows:

1. Part I: `N-body part.ipynb`
2. Part II: `Transit part.ipynb`
3. Part III: `TTV part.ipynb`

REFERENCES

Ketchum, J., Adams, F. C., & Bloch, A. M. 2012, The Astrophysical Journal, doi: 10.1088/0004-637X/762/2/71

Kreidberg, L. 2015, Publications of the Astronomical Society of the Pacific, 127, 1161–1165, doi: 10.1086/683602

Luger, R. 2024, Examples Tutorials: A simple exoplanet system. https://starry.readthedocs.io/en/latest/notebooks/Exoplanet/

Mathieu, E. 2024, The number of identified exoplanets has dramatically increased thanks to the "transit method". https://ourworldindata.org

NASA. 2024, NASA Exoplanet Archive. https://exoplanetarchive.ipac.caltech.edu/docs/counts_detail.html