The ProbLemma Channel Sums

This document links all the sums evaluated on the ProbLemma channel to the episodes in which these evaluations were carried out.

In addition, all the sums related episodes are collected in the "Sums Are Us" play list.

1. Season 1 Episode 9 (S1M10): One Cotangent, Two Cotangents (Transform And Conquer)

$$\sum_{k=1}^{m} \cot^2\left(\frac{k\pi}{2m+1}\right) = \frac{m(2m-1)}{3}$$

2. Season 2 Episode 2 (S2M2): A Weighty Question (Reinterpret And Conquer)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Season 2 Episode 26 (S2M28): Finite Integer Sums (Scope Expansion)

$$S_1 = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$S_2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

4. Season 2 Episode 27 (S2M29): Integer power sums revisited (Scope Expansion)

The very same finite integer power sums shown above, S_1, S_2, S_3, etc., are evaluated in this episode as well but using a slightly different approach.

5. Season 2 Episode 43 (S2M45): A Poisson Integral By The Book (Divide And Conquer)

$$\sum_{k=1}^{n} \log \left(1 - 2r \cos \left(\frac{k\pi}{n} \right) + r^2 \right) = \log \left(\frac{r+1}{r-1} \cdot \left(r^{2n} - 1 \right) \right)$$

For the value of the corresponding finite product please see the "Products" index.

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6. Season 3 Episode 3 (S3M2): One Telescope, Two Telescopes

In this episode the following two finite trigonometric sums are evaluated using the basic product-to-sum and sum-to-product middle school trigonometric identities and the notion of "telescopic" sums:

$$\sum_{k=1}^{n} \cos (a + k\theta) = \cos \left(a + \frac{n}{2} \theta \right) \frac{\sin \left(\frac{n+1}{2} \theta \right)}{\sin \left(\frac{\theta}{2} \right)}$$

$$\sum_{k=0}^{n} \sin (a + k\theta) = \sin \left(a + \frac{n}{2} \theta \right) \frac{\sin \left(\frac{n+1}{2} \theta \right)}{\sin \left(\frac{\theta}{2} \right)}$$

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7. Season 3 Episode 4 (S3M3): A Complex Approach

The last two trigonometric sums shown above are evaluated in this episode using the machinery of complex numbers.

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8. Season 3 Episode 5 (S3M4): Euclid Says

The last two trigonometric sums shown above are evaluated in this episode using synthetic Euclidean plane geometry.

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9. Season 3 Episode 9 (S3M6): Finite Sums Evaluation Via Double-Counting

$$\sum_{n=0}^{+\infty} \frac{n}{e^n} = \frac{e}{(e-1)^2}$$

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10. Season 3 Episode 13 (S3M8): Binomial Differentials Slay Infinite Sums Before Breakfast

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+1)(n+2)\ldots(n+p)} = \frac{1}{p \cdot p!}$$

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11. Season 3 Episode xx (S3Mx): Lorem Ipsum