The ProbLemma Channel Integrals

This document links all the integrals evaluated on the ProbLemma channel to the episodes in which these evaluations were carried out.

In addition, all the integral related episodes are collected in the "Integrals Are Us" play list.

1. Season 2 Episode 15 (S2M17): Fourier Series of $\log(\Gamma(x))$ over $(0, 1]$

$$\int_0^1 \log(\Gamma(x))\,dx = \log(\sqrt{2\pi})$$

$$\int_0^1 \log(\Gamma(x))\cos(2n\pi x)\,dx = \frac{1}{4n},\ n = 1, 2, 3, \ldots$$

$$\int_0^1 \log(\Gamma(x))\sin(2n\pi x)\,dx = \frac{\gamma + \log(2n\pi)}{2n\pi},\ n = 1, 2, 3, \ldots$$

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2. Season 2 Episode 19 (S2M21): Effectiveness Of Advertisement (Equation)

$$\int \frac{1}{x(a-x)}\,dx = \frac{1}{a}\log\left(\left|\frac{x}{a-x}\right|\right) + C$$

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3. Season 2 Episode 20 (S2M22): Fresnel Integrals Via Equations (Equation)

$$\int_0^{+\infty} e^{-ax^2}\cos(bx^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}} \cdot \sqrt{\frac{a + \sqrt{a^2+b^2}}{a^2+b^2}},\ a > 0, b \geqslant 0$$

$$\int_0^{+\infty} e^{-ax^2}\sin(bx^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}} \cdot \sqrt{\frac{-a + \sqrt{a^2+b^2}}{a^2+b^2}},\ a > 0, b \geqslant 0$$

$$\int_{0}^{+\infty} \cos\left(x^2\right) dx = \int_{0}^{+\infty} \sin\left(x^2\right) dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

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4. Season 2 Episode 24 (**S2M26**): Integral Evaluation via Scope Reduction (Scope Reduction)

$$\int_{c}^{2c} \frac{x}{\sqrt{x^2 + cx - 2c^2}}\, dx = 2c - \frac{c}{2}\log(3)$$

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5. Season 2 Episode 30 (**S2M32**): One Integral? Two Integrals! (Scope Expansion)

$$\int e^{ax} \sin(bx) \cos(cx)\, dx =$$

$$= e^{px}\left(\frac{a\sin((b+c)x) - (b+c)\cos((b+c)x)}{a^2 + (b+c)^2} + \frac{a\sin((b-c)x) - (b-c)\cos((b-c)x)}{a^2 + (b-c)^2}\right) + C$$

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6. Season 2 Episode 43 (**S2M45**): A Poisson Integral By The Book (Divide And Conquer)

$$P_{|r|\leqslant 1} = \int_{0}^{\pi} \log\left(1 - 2r\cos(x) + r^2\right) dx = 0, \quad |r| \leqslant 1$$

$$P_{|r|>1} = \int_{0}^{\pi} \log\left(1 - 2r\cos(x) + r^2\right) dx = 2\pi\log(|r|), \quad |r| > 1$$

$$\int_{0}^{\pi} \log\left(2 - 2\cos(x)\right) dx = 0$$

$$\int_{0}^{\pi} \log\left(2 + 2\cos(x)\right) dx = 0$$

The Euler's "log-sin" integral is evaluated in this episode not "by the book", via a limit of an integral sum, but via Euler's clever and witty substitutions.

The evaluation of the Euler's integral "by the book" is done in a different episode, see below:

$$\int_0^{\frac{\pi}{2}} \log\left(\sin\left(t\right)\right) dt = -\frac{\pi}{2} \log\left(2\right)$$

$$\int_0^{\frac{\pi}{2}} \log\left(\cos\left(t\right)\right) dt = -\frac{\pi}{2} \log\left(2\right)$$

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7. Season 2 Episode 53 (**S2M53**): Lanchester's Square Law (Mathematical Equivalence)

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(\frac{x}{a} + \sqrt{\frac{x^2}{a^2}+1}\right) + C$$

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8. Season 2 Episode 54 (**S2P2**): Slitating Or Roliding Bead (Mathematical Equivalence)

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left(\frac{x}{a} \pm \sqrt{\frac{x^2}{a^2}-1}\right)$$

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9. Season 3 Episode 2 (**S3M1**): An Integral Transformation

For any whole positive number n and any real numbers $x > 1$ and θ it is the case that:

$$\int_0^{\pi} \left(x + \sqrt{x^2-1}\cos\left(\theta\right)\right)^n \mathrm{d}\theta = \int_0^{\pi} \frac{\mathrm{d}\theta}{\left(x - \sqrt{x^2-1}\cos\left(\theta\right)\right)^{n+1}}$$

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10. Season 3 Episode 12 (**S3M7**): Integration Of Binomial Differentials

If:

$$J(p,q) = \int (a+bz)^p z^q \, dz, \quad p,q \in \mathbb{Q}$$

then, for $p \neq -1$:

$$J(p,q) = -\frac{(a+bz)^{p+1} z^{q+1}}{a(p+1)} + \frac{p+q+2}{a(p+1)} J(p+1,q)$$

for $q \neq -1$:

$$J(p,q) = \frac{(a+bz)^{p+1} z^{q+1}}{a(q+1)} - b\,\frac{p+q+2}{a(q+1)} J(p,q+1)$$

for $p+q \neq -1$:

$$J(p,q) = \frac{(a+bz)^{p} z^{q+1}}{p+q+1} + \frac{ap}{p+q+1} J(p-1,q)$$

and:

$$J(p,q) = \frac{(a+bz)^{p+1} z^{q}}{b(p+q+1)} - \frac{aq}{b(p+q+1)} J(p,q-1)$$

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11. Season 3 Episode x (S3Mx): lorem ipsum