



# The ProbLemma's Channel Season 2 Guide

## Season 2 Episode 1: Seven Gallons Of Water On The Wall (Reinterpret And Conquer)

- Problem **S2M1** solved:
  - Mathematical Billiard
- Problem **S2M2** formulated:
  - Problem **S2M2**: an alternative expression for a finite sum of squares of consecutive whole positive numbers

## Season 2 Episode 2: A Weighty Question (Reinterpret And Conquer)

- Problem **S2M2** solved:
  - Center Of Mass
- Problem **S2M3**, Swan Lakes, formulated:
  - Problem **S2M3**: swans landing on lakes via the half of all swans plus half-a-swan rule



The above 2 episodes with S1E8 and S1E9 form the [“Reinterpret And Conquer” play list](#).

## Season 2 Episode 3: Swan Lakes (Reverse Order)

- The mechanics of the “Reverse Order” problem-solving approach explained
- Problem **S2M3** solved
- Problem **S2M4**, The Devil And A Loiterer, formulated:
  - **S2M4**: a loiterer crossing a bridge

## Season 2 Episode 4: The Devil And A Loiter (Reverse Order)

- The mechanics of the “Reverse Order” problem-solving approach explained again
- Problem **S2M4** solved
- Problem **S2M5** formulated:
  - **S2M5**: magic apples gathered by a peasant
- Problem **S2M6** formulated:
  - **S2M6**: apple injections
- Problem **S2M7** formulated:
  - **S2M7**: an equilateral triangle in a square

## Season 2 Episode 5: Apples Of Discord (Reverse Order)

- Problems **S2M5**, **S2M6**, **S2M7** solved
- Problems **S2M8** and **S2M9** formulated
  - **S2M8**: an isosceles triangle in a trapezoid
  - **S2M9**: external and internal tangents to two circles

Season 2 Episode 6: On The Tangent (Reverse Order)

- Problems **S2M8** and **S2M9** solved
- Problem **S2CS1** formulated:
  - **S2CS1**: 2 eggs versus 100-story building

Season 2 Episode 7: Two Eggs Versus One Building (Reverse Order)

- Problem **S2CS1** solved
- Problem **S2M10** formulated:
  - **S2M10**: horses and carrots, gamels and bananas

Season 2 Episode 8: Horses Eating Carrots, Discrete Rocket Propulsion (Reverse Order)

- Problem **S2M10** solved
- Problem **S2M11** formulated:
  - **S2M11**: An odd colony of infinitely excitable cells

Season 2 Episode 9: An Add Colony Of Infinitely Excitable Cells (Reverse Order)

- Problem **S2M11** solved
- Problem **S2M12** formulated:
  - **S2M12**: Zero in a recurrence relation

Season 2 Episode 10: Zero in a recurrence relation (Reverse Order)

- Problem **S2M12** solved
- Problem **S2M13** formulated:
  - **S2M13**: peasant, goat, cabbage, wolf crossing a river



The above eight episodes form the [“Reverse Order” play list](#).

Season 2 Episode 11: Peasant. Goat. Cabbage. Wolf (Space-Time)

- The mechanics of “Space-Time” problem-solving approach explained
- Problem **S2M13** solved
- Problem **S2M14** formulated:
  - **S2M14**: find a fake coin in a set 12 using 3 weighings on pan scales, an adaptive approach

Season 2 Episode 12: Not Blind Mathematical Justice (Space-Time)

- Problem **S2M14** solved (via an adaptive approach)
- Problem **S2M15** formulated:
  - **S2M15**: find a fake and heavy coin in a set of 18 using 3 non-adaptive weighings on pan scales

Season 2 Episode 13: Blind Mathematical Justice (Space-Time)

- Problem **S2M15** solved (via a non-adaptive approach)

- Problem **S2M16** formulated:
  - **S2M16**: put together at least one fake coin detection problem that admits at least one geometric solution

#### Season 2 Episode 14: Geometry In Fake Coin Detection Problems (Space-Time)

- Problem **S2M16** solved (a geometry of a non-adaptive approach)
- Problem **S2M17** formulated:
  - **S2M17**: decompose the  $\log(\Gamma(x))$  function into its Fourier series over the interval  $(0, 1]$



The above four episodes form the [“Space Time” play list](#).

#### Season 2 Episode 15: Fourier Series of $\log(\Gamma(x))$ over $(0, 1]$

- Problem **S2M17** solved
- Problem **S2M18** formulated:
  - **S2M18**: find the number of times a minute hand will rendezvous with the hour hand on the face of the standard analogue 12-hour clock in one 12-hour period starting from 12 o'clock

#### Season 2 Episode 16: A Chase Around The Clock (Equation)

- The mechanics of the “Equation” problem-solving approach explained
- Problem **S2M18** solved
- Problem **S2M19** formulated:
  - **S2M19**: generate a proof of the Pythagorean Theorem based on the Equation problem-solving approach

#### Season 2 Episode 17: Pythagorean Theorem Via Equations (Equation)

- Problem **S2M19** solved
- Problem **S2M20** formulated:
  - **S2M20**: solve an equation of order 4

#### Season 2 Episode 18: Now You Know Me, Now You Don't (Equation)

- Problem **S2M20** solved
- Problem **S2M21** formulated:
  - **S2M21**: effectiveness of advertisement

#### Season 2 Episode 19: Effectiveness Of Advertisement (Equation)

- Problem **S2M21** solved
- Problem **S2M22** formulated:
  - **S2M22**: Fresnel Integrals Via Equations

#### Season 2 Episode 20: Fresnel Integrals Via Equations (Equation)

- Problem **S2M22** solved

- Problem **S2M23** formulated:
  - **S2M23**: number of such 5-digit perfect squares that if each digit of that perfect square is increased by 1 then a new perfect square results (Scope Reduction)



The above five episodes form the [“Equation” play list](#).

#### Season 2 Episode 21: Heavy perfect 5-digit squares (Scope Reduction)

- The mechanics of the “Scope Reduction” problem-solving approach explained
- Problem **S2M23** solved
- Problem **S2M24** formulated:
  - **S2M24**: find the locus of points on a sphere each of which is equidistant from 3 given fixed distinct points on that sphere, no two of which are antipodal (Scope Reduction)

#### Season 2 Episode 22: Equidistant points on a sphere (Scope Reduction)

- Problem **S2M24** solved
- Problem **S2M25** formulated:
  - **S2M25**: explain how the Euclidean Greatest Common Divisor Algorithm from the perspective of the Scope Reduction problem-solving approach (Scope Reduction)

#### Season 2 Episode 23: the Euclidean GCD Algorithm via Scope Reduction (Scope Reduction)

- Problem **S2M25** solved
- Problem **S2M26** formulated:
  - **S2M26**: evaluate the following integral using the Scope Reduction problem-solving approach (Scope Reduction)

$$\int_c^{2c} \frac{x}{\sqrt{x^2 + cx - 2c^2}} dx$$

#### Season 2 Episode 24: Integral Evaluation via Scope Reduction (Scope Reduction)

- Problem **S2M26** solved
- Problem **S2M27** formulated:
  - **S2M27**: show that it is impossible to find the location of a circle using the Euclidean straightedge alone (Scope Expansion)



The above four episodes form the [“Scope Reduction” play list](#).

#### Season 2 Episode 25: Circle. Center. Straightedge. Nope (Scope Expansion)

- The mechanics of the “Scope Expansion” problem-solving approach explained

- Problem S2M27 solved
- Problem S2M28 formulated:
  - S2M28: find an alternative expression for the finite sums of consecutive positive whole numbers raised to a fixed positive whole power (Scope Expansion)

#### Season 2 Episode 26: Finite Integer Sums (Scope Expansion)

- Problem S2M28 solved
- Problem S2M29 formulated:
  - S2M29: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

#### Season 2 Episode 27: Integer Power Sums Revisited (Scope Expansion)

- Problem S2M29 solved
- Problem S2M30 formulated:
  - S2M30: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

#### Season 2 Episode 28: Is It A Hexagon? Or Is It A Triangle? (Scope Expansion)

- Problem S2M30 solved
- Problem S2M31 formulated:
  - S2M31: find a mechanical way to construct arbitrary magic squares of odd orders (Scope Expansion)

#### Season 2 Episode 29: Odd Magic Squares (Scope Expansion)

- Problem S2M31 solved
- Problem S2M32 formulated:
  - S2M32: evaluate the following indefinite integral via the Scope Expansion problem-solving approach without using the integration by parts (Scope Expansion)

$$\int e^{ax} \sin (bx) \cos (cx) dx$$

#### Season 2 Episode 30: One Integral? Two Integrals! (Scope Expansion)

- Problem S2M32 solved
- Problem S2M33 formulated:
  - S2M33: find the radius of a largest circle that passes only through the black squares of a standard  $8 \times 8$  chessboard (Eliminate And Conquer)



The above six episodes form the [“Scope Expansion” play list](#).

Season 2 Episode 31: Largest Circle On A Chessboard (Eliminate And Conquer)

- The mechanics of the “Eliminate And Conquer” problem-solving approach explained
- Problem S2M33 solved
- Problem S2M34 formulated:
  - S2M34: find two numbers given their LCM and the difference between them (Eliminate And Conquer)

Season 2 Episode 32: The LCM And The Difference (Eliminate And Conquer)

- Problem S2M34 solved
- Problem S2M35 formulated:
  - S2M35: show that no number of the form  $111 \dots 1$  is a perfect square (Eliminate And Conquer)

Season 2 Episode 33: A Non-Square Fence Of Ones (Eliminate And Conquer)

- Problem S2M35 solved
- Problem S2M36 formulated:
  - S2M36: construct a magic  $3 \times 3$  square with a given constant (Eliminate And Conquer)

Season 2 Episode 34: A  $3 \times 3$  Magic Square Construction (Eliminate And Conquer)

- Problem S2M36 solved
- Problem S2M37 formulated:
  - S2M37: recover the shape of a regular polygon given a relationship between the lengths of its sides and diagonals (Eliminate And Conquer)

Season 2 Episode 35: Regular Polygon Recognition (Eliminate And Conquer)

- Problem S2M37 solved
- Problem S2M38 formulated:
  - S2M38: find the magnitudes of the interior angles of a planar triangle given a relationship between the distances from the vertices of that triangle to the points chosen on its sides (Eliminate And Conquer)

Season 2 Episode 36: A Show Of Equal Distances (Eliminate And Conquer)

- Problem S2M38 solved
- Problem S2M39 formulated:
  - S2M39: cut an  $8 \times 3$  piece of wood into two pieces that fit perfectly inside of a  $12 \times 2$  whole (Divide And Conquer)



The above six episodes form the [“Eliminate And Conquer” play list](#).

Season 2 Episode 37: East Or West Divide And Conquer (Divide And Conquer)

- The mechanics of the “Divide And Conquer” problem-solving approach explained
- Problem S2M39 solved
- Problem S2M40 formulated:
  - S2M40: find the area of a Reuleaux Triangle (Divide And Conquer)

Season 2 Episode 38: The Area of The Reuleaux Triangle (Divide And Conquer)

- Problem S2M40 solved
- Problem S2M41 formulated:
  - S2M41: invent at least two distinct proofs of the Heron’s Formula (Divide And Conquer)

Season 2 Episode 39: Heron’s Formula Divided And Conquered (Divide And Conquer)

- Problem S2M41 solved
- Problem S2M42 formulated:
  - S2M42: show that the areas of two triangles whose vertices are located on the different branches of the unit hyperbola  $xy = 1$  are equal one another (Divide And Conquer)

Season 2 Episode 40: When Triangles Kiss A Hyperbola (Divide And Conquer)

- Problem S2M42 solved
- Problem S2M43 formulated:
  - S2M43: evaluate a finite product of cosines whose arguments are the whole positive numbers coprime with 100 and scaled by  $\pi$  and divided by 100 (Divide And Conquer)

Season 2 Episode 41: 5-Coprime Odd Numbers That Live On A Globe (Divide And Conquer)

- Problem S2M43 solved
- Problem S2M44 formulated:
  - S2M44: evaluate the following limit (Divide And Conquer)

$$\lim_{n \rightarrow +\infty} \frac{1}{n^3} \sum_{k=1}^n k \sqrt{n^2 - k^2}$$

Season 2 Episode 42: How A Hoof Can Compute A Limit (Divide And Conquer)

- Problem S2M44 solved
- Problem S2M45 formulated:
  - S2M45: evaluate the Poisson Integral  $P_r$  by the book (Divide And Conquer)

$$\int_0^\pi \log(1 - 2r \cos(x) + r^2) dx, \quad |r| \neq 1$$

Season 2 Episode 43: Poisson Integral Evaluation By The Book (Divide And Conquer)

- Problem S2M45 solved
- Problem S2M46 formulated:

- **S2M46**: the watermelons transportation problem (Invariant)



The above seven episodes form the [“Divide And Conquer” play list](#).

#### Season 2 Episode 44: Evaporating Watermelons (Invariant)

- The mechanics of the “Invariant” problem-solving approach explained
- Problem **S2M46** solved
- Problem **S2M47** formulated:
  - **S2M47**: bananas and pineapples grow on a magic tree. If two bananas or two pineapples are picked then they are replaced with one pineapple. If one banana and one pineapple are picked then these fruits are replaced with one banana. When someone was picking these magic fruits as explained above and only 1 fruit was left then what type of fruit was is, a banana or a pineapple, and why (Invariant)

#### Season 2 Episode 45: Bineapples and Pananas (Invariant)

- Problem **S2M47** solved
- Problem **S2M48** formulated:
  - **S2M48**: a  $4 \times 4$  chessboard is filled with 15 plus signs and 1 minus sign, one sign per square. In any one row or in any one column it is allowed to change all the signs to their opposite, pluses into minuses and minuses into pluses. Is it possible to fill this board with nothing but, 16, pluses in a finite number of the legal moves described? (Invariant)

#### Season 2 Episode 46: Minuses To Move And Never Lose (Invariant)

- Problem **S2M48** solved
- Problem **S2M49** formulated:
  - **S2M49**: 3 grasshoppers sit in the vertices of a unit square. Then, every second a random grasshopper jumps over a random grasshopper and flies the distance that separated the two before the jump (Invariant)

#### Season 2 Episode 47: Non-square Square Grasshoppers (Invariant)

- Problem **S2M49** solved
- Problem **S2M50** formulated:
  - **S2M50**: 66 flying squares sit on 66 trees arranged in a circle. Then, every second one, random, flying squirrel flies to the nearest tree clockwise and another random flying squirrel flies to the nearest tree counterclockwise (Invariant)

#### Season 2 Episode 48: A Circle Dance Of Flying Squirrels (Invariant)

- Problem **S2M50** solved
- Problem **S2M51** formulated:



- **S2M51**: on an island there live chameleons of three colors, red, green and blue. When two chameleons of opposite colors meet then they both change their color to the remaining one (Invariant)

#### Season 2 Episode 49: Modular Equilateral Chameleons (Invariant)

- Problem **S2M51** solved
- Problem **S2CS2** formulated:
  - **S2CS2**: find the number of ordered pairs  $(a, b)$  of whole non-negative numbers that satisfy  $a^2 + b^2 < n$ , where  $n$  is a fixed and given ahead of time whole positive number



The above six episodes form the “Invariant” play list.

#### Season 2 Episode 50: Diophantine Staircase (A mix of problem-solving approaches in CS)

- Problem **S2CS2** solved
- Problem **S2P1** formulated:
  - **S2P1**: which one of the two uniformly dense solids, a cube or a right circular cylinder of comparable size, is easier to tip over and why?

#### Season 2 Episode 51: A Cube Versus A Cylinder (Solving problems in Physics, How To)

- Problem **S2P1** solved
- Problem **S2M52** formulated:
  - **S2M52**: solve the 9-bottle and the 9-coin problems and determine whether a solution of one of these problems can be converted into a solution of the other problem mechanically

#### Season 2 Episode 52: Of Mice And Wine And Coins (Mathematical Equivalence)

- Problem **S2M52** solved
- Problem **S2M53** formulated:
  - **S2M53**: find the survivors count and the total annihilation time of two armies in a fully escalated mode under certain assumptions

#### Season 2 Episode 53: Lanchester's Square Law (Mathematical Equivalence)

- Problem **S2M53** solved
- Problem **S2P2** formulated:
  - **S2P2**: find the separation velocity and the separation time of a bead sliding off of a massless rod rotating in the horizontal plane

#### Season 2 Episode 54: Slitating Or Roliding Bead (Mathematical Equivalence)

- Problem **S2P2** solved
- Problem **S2M54** formulated:
  - **S2M54**: show how the solution of **S2M53** can be converted into a solution of **S2P2**

Season 2 Episode 55: Mathematical Equivalence

- Problem **S2M54** solved
- Problem **S3M1** formulated:
  - **S3M1**: show that for all real  $x > 1$  and all whole positive  $n$  it is the case that

$$\int_0^{\pi} \left( x + \sqrt{x^2 - 1} \cos \varphi \right)^n d\varphi = \int_0^{\pi} \frac{d\varphi}{\left( x - \sqrt{x^2 - 1} \cos \varphi \right)^{n+1}}$$



The last 4 episodes above form the [“Mathematical Equivalence” play list](#).

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