The ProbLemma's Channel Season 2 Guide

[Season 2 Episode 1: Seven Gallons Of Water On The Wall (Reinterpret And Conquer)]

- Problem S2M1 solved:
  - Mathematical Billiard
- Problem S2M2 formulated:
  - Problem S2M2: an alternative expression for a finite sum of squares of consecutive whole positive numbers

[Season 2 Episode 2: A Weighty Question (Reinterpret And Conquer)]

- Problem S2M2 solved:
  - Center Of Mass
- Problem S2M3 , Swan Lakes, formulated:
  - Problem S2M3: swans landing on lakes via the half of all swans plus half-a-swan rule

The above 2 episodes with S1E8 and S1E9 form the ["Reinterpret And Conquer" play list].

[Season 2 Episode 3: Swan Lakes (Reverse Order)]

- The mechanics of the "Reverse Order" problem-solving approach explained
- Problem S2M3 solved
- Problem S2M4, The Devil And A Loiterer, formulated:
  - S2M4: a loiterer crossing a bridge

[Season 2 Episode 4: The Devil And A Loiter (Reverse Order)]

- The mechanics of the "Reverse Order" problem-solving approach explained again
- Problem S2M4 solved
- Problem S2M5 formulated:
  - S2M5: magic apples gathered by a peasant
- Problem S2M6 formulated:
  - S2M6: apple injections
- Problem S2M7 formulated:
  - S2M7: an equilateral triangle in a square

[Season 2 Episode 5: Apples Of Discord (Reverse Order)]

- Problems S2M5, S2M6, S2M7 solved
- Problems S2M8 and S2M9 formulated
  - S2M8: an isosceles triangle in a trapezoid
  - S2M9: external and internal tangents to two circles

Season 2 Episode 6: On The Tangent (Reverse Order)

- Problems S2M8 and S2M9 solved
- Problem S2CS1 formulated:
  - S2CS1: 2 eggs versus 100-story building

Season 2 Episode 7: Two Eggs Versus One Building (Reverse Order)

- Problem S2CS1 solved
- Problem S2M10 formulated:
  - S2M10: horses and carrots, gamels and bananas

Season 2 Episode 8: Horses Eating Carrots, Discrete Rocket Propulsion (Reverse Order)

- Problem S2M10 solved
- Problem S2M11 formulated:
  - S2M11: An odd colony of infinitely excitable cells

Season 2 Episode 9: An Add Colony Of Infinitely Excitable Cells (Reverse Order)

- Problem S2M11 solved
- Problem S2M12 formulated:
  - S2M12: Zero in a recurrence relation

Season 2 Episode 10: Zero in a recurrence relation (Reverse Order)

- Problem S2M12 solved
- Problem S2M13 formulated:
  - S2M13: peasant, goat, cabbage, wolf crossing a river

The above eight episodes form the "Reverse Order" play list.

Season 2 Episode 11: Peasant. Goat. Cabbage. Wolf (Space-Time)

- The mechanics of "Space-Time" problem-solving approach explained
- Problem S2M13 solved
- Problem S2M14 formulated:
  - S2M14: find a fake coin in a set 12 using 3 weighings on pan scales, an adaptive approach

Season 2 Episode 12: Not Blind Mathematical Justice (Space-Time)

- Problem S2M14 solved (via an adaptive approach)
- Problem S2M15 formulated:
  - S2M15: find a fake and heavy coin in a set of 18 using 3 non-adaptive weighings on pan scales

Season 2 Episode 13: Blind Mathematical Justice (Space-Time)

- Problem S2M15 solved (via a non-adaptive approach)

- Problem S2M16 formulated:
  - S2M16: put together at least one fake coin detection problem that admits at least one geometric solution

Season 2 Episode 14: Geometry In Fake Coin Detection Problems (Space-Time)
- Problem S2M16 solved (a geometry of a non-adaptive approach)
- Problem S2M17 formulated:
  - S2M17: decompose the $\log(\Gamma(x))$ function into its Fourier series over the interval $(0, 1]$


The above four episodes form the "Space Time" play list.

Season 2 Episode 15: Fourier Series of $\log(\Gamma(x))$ over $(0, 1]$
- Problem S2M17 solved
- Problem S2M18 formulated:
  - S2M18: find the number of times a minute hand will rendezvous with the hour hand on the face of the standard analogue 12-hour clock in one 12-hour period starting from 12 o'clock

Season 2 Episode 16: A Chase Around The Clock (Equation)
- The mechanics of the "Equation" problem-solving approach explained
- Problem S2M18 solved
- Problem S2M19 formulated:
  - S2M19: generate a proof of the Pythagorean Theorem based on the Equation problem-solving approach

Season 2 Episode 17: Pythagorean Theorem Via Equations (Equation)
- Problem S2M19 solved
- Problem S2M20 formulated:
  - S2M20: solve an equation of order 4

Season 2 Episode 18: Now You Know Me, Now You Don't (Equation)
- Problem S2M20 solved
- Problem S2M21 formulated:
  - S2M21: effectiveness of advertisement

Season 2 Episode 19: Effectiveness Of Advertisement (Equation)
- Problem S2M21 solved
- Problem S2M22 formulated:
  - S2M22: Fresnel Integrals Via Equations

Season 2 Episode 20: Fresnel Integrals Via Equations (Equation)
- Problem S2M22 solved

- Problem S2M23 formulated:
  - S2M23: number of such 5-digit perfect squares that if each digit of that perfect square is increased by 1 then a new perfect square results (Scope Reduction)

The above five episodes form the "Equation" play list.

Season 2 Episode 21: Heavy perfect 5-digit squares (Scope Reduction)
- The mechanics of the "Scope Reduction" problem-solving approach explained
- Problem S2M23 solved
- Problem S2M24 formulated:
  - S2M24: find the locus of points on a sphere each of which is equidistant from 3 given fixed distinct points on that sphere, no two of which are antipodal (Scope Reduction)

Season 2 Episode 22: Equidistant points on a sphere (Scope Reduction)
- Problem S2M24 solved
- Problem S2M25 formulated:
  - S2M25: explain how the Euclidean Greatest Common Divisor Algorithm from the perspective of the Scope Reduction problem-solving approach (Scope Reduction)

Season 2 Episode 23: the Euclidean GCD Algorithm via Scope Reduction (Scope Reduction)
- Problem S2M25 solved
- Problem S2M26 formulated:
  - S2M26: evaluate the following integral using the Scope Reduction problem-solving approach (Scope Reduction)

$$\int_c^{2c} \frac{x}{\sqrt{x^2+cx-2c^2}}\,dx$$

Season 2 Episode 24: Integral Evaluation via Scope Reduction (Scope Reduction)
- Problem S2M26 solved
- Problem S2M27 formulated:
  - S2M27: show that it is impossible to find the location of a circle using the Euclidean straightedge alone (Scope Expansion)

The above four episodes form the "Scope Reduction" play list.

Season 2 Episode 25: Circle. Center. Straightedge. Nope (Scope Expansion)
- The mechanics of the "Scope Expansion" problem-solving approach explained

- Problem S2M27 solved
- Problem S2M28 formulated:
  - S2M28: find an alternative expression for the finite sums of consecutive positive whole numbers raised to a fixed positive whole power (Scope Expansion)

Season 2 Episode 26: Finite Integer Sums (Scope Expansion)
- Problem S2M28 solved
- Problem S2M29 formulated:
  - S2M29: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

Season 2 Episode 27: Integer Power Sums Revisited (Scope Expansion)
- Problem S2M29 solved
- Problem S2M30 formulated:
  - S2M30: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

Season 2 Episode 28: Is It A Hexagon? Or Is It A Triangle? (Scope Expansion)
- Problem S2M30 solved
- Problem S2M31 formulated:
  - S2M31: find a mechanical way to construct arbitrary magic squares of odd orders (Scope Expansion)

Season 2 Episode 29: Odd Magic Squares (Scope Expansion)
- Problem S2M31 solved
- Problem S2M32 formulated:
  - S2M32: evaluate the following indefinite integral via the Scope Expansion problem-solving approach without using the integration by parts (Scope Expansion)

$$\int e^{ax} \sin(bx) \cos(cx)\, dx$$

Season 2 Episode 30: One Integral? Two Integrals! (Scope Expansion)
- Problem S2M32 solved
- Problem S2M33 formulated:
  - S2M33: find the radius of a largest circle that passes only through the black squares of a standard 8×8 chessboard (Eliminate And Conquer)


The above six episodes form the "Scope Expansion" play list.

Season 2 Episode 31: Largest Circle On A Chessboard (Eliminate And Conquer)

- The mechanics of the "Eliminate And Conquer" problem-solving approach explained
- Problem S2M33 solved
- Problem S2M34 formulated:
  - S2M34: find two numbers given their LCM and the difference between them (Eliminate And Conquer)

Season 2 Episode 32: The LCM And The Difference (Eliminate And Conquer)

- Problem S2M34 solved
- Problem S2M35 formulated:
  - S2M35: show that no number of the form $111\ldots1$ is a perfect square (Eliminate And Conquer)

Season 2 Episode 33: A Non-Square Fence Of Ones (Eliminate And Conquer)

- Problem S2M35 solved
- Problem S2M36 formulated:
  - S2M36: construct a magic 3×3 square with a given constant (Eliminate And Conquer)

Season 2 Episode 34: A 3×3 Magic Square Construction (Eliminate And Conquer)

- Problem S2M36 solved
- Problem S2M37 formulated:
  - S2M37: recover the shape of a regular polygon given a relationship between the lengths of its sides and diagonals (Eliminate And Conquer)

Season 2 Episode 35: Regular Polygon Recognition (Eliminate And Conquer)

- Problem S2M37 solved
- Problem S2M38 formulated:
  - S2M38: find the magnitudes of the interior angles of a planar triangle given a relationship between the distances from the vertices of that triangle to the points chosen on its sides (Eliminate And Conquer)

Season 2 Episode 36: A Show Of Equal Distances (Eliminate And Conquer)

- Problem S2M38 solved
- Problem S2M39 formulated:
  - S2M39: cut an 8×3 piece of wood into two pieces that fit perfectly inside of a 12×2 whole (Divide And Conquer)

The above six episodes form the "Eliminate And Conquer" play list.

Season 2 Episode 37: East Or West Divide And Conquer (Divide And Conquer)

- The mechanics of the "Divide And Conquer" problem-solving approach explained
- Problem S2M39 solved
- Problem S2M40 formulated:
  - S2M40: find the area of a Reuleaux Triangle (Divide And Conquer)

Season 2 Episode 38: The Area of The Reuleaux Triangle (Divide And Conquer)

- Problem S2M40 solved
- Problem S2M41 formulated:
  - S2M41: invent at least two distinct proofs of the Heron's Formula (Divide And Conquer)

Season 2 Episode 39: Heron's Formula Divided And Conquered (Divide And Conquer)

- Problem S2M41 solved
- Problem S2M42 formulated:
  - S2M42: show that the areas of two triangles whose vertices are located on the different branches of the unit hyperbola $xy = 1$ are equal one another (Divide And Conquer)

Season 2 Episode 40: When Triangles Kiss A Hyperbola (Divide And Conquer)

- Problem S2M42 solved
- Problem S2M43 formulated:
  - S2M43: evaluate a finite product of cosines whose arguments are the whole positive numbers coprime with 100 and scaled by π and divided by 100 (Divide And Conquer)

Season 2 Episode 41: 5-Coprime Odd Numbers That Live On A Globe (Divide And Conquer)

- Problem S2M43 solved
- Problem S2M44 formulated:
  - S2M44: evaluate the following limit (Divide And Conquer)

$$\lim_{n\to+\infty} \frac{1}{n^3} \sum_{k=1}^{n} k\sqrt{n^2-k^2}$$

Season 2 Episode 42: How A Hoof Can Compute A Limit (Divide And Conquer)

- Problem S2M44 solved
- Problem S2M45 formulated:
  - S2M45: evaluate the Poisson Integral P_r by the book (Divide And Conquer)

$$\int_0^{\pi} \log\left(1 - 2r\cos(x) + r^2\right) dx, \ |r| \neq 1$$

Season 2 Episode 43: Poisson Integral Evaluation By The Book (Divide And Conquer)

- Problem S2M45 solved
- Problem S2M46 formulated:

- S2M46: the watermelons transportation problem (Invariant)


The above seven episodes form the "Divide And Conquer" play list.

Season 2 Episode 44: Evaporating Watermelons (Invariant)

- The mechanics of the "Invariant" problem-solving approach explained
- Problem S2M46 solved
- Problem S2M47 formulated:
  - S2M47: bananas and pineapples grow on a magic tree. If two bananas or two pineapples are picked then they are replaced with one pineapple. If one banana and one pineapple are picked then these fruits are replaced with one banana. When someone was picking these magic fruits as explained above and only 1 fruit was left then what type of fruit was is, a banana or a pineapple, and why (Invariant)

Season 2 Episode 45: Bineapples and Pananas (Invariant)

- Problem S2M47 solved
- Problem S2M48 formulated:
  - S2M48: a 4×4 chessboard is filled with 15 plus signs and 1 minus sign, one sign per square. In any one row or in any one column it is allowed to change all the signs to their opposite, pluses into minuses and minuses into pluses. Is it possible to fill this board with nothing but, 16, pluses in a finite number of the legal moves described? (Invariant)

Season 2 Episode 46: Minuses To Move And Never Lose (Invariant)

- Problem S2M48 solved
- Problem S2M49 formulated:
  - S2M49: 3 grasshoppers sit in the vertices of a unit square. Then, every second a random grasshopper jumps over a random grasshopper and flies the distance that separated the two before the jump (Invariant)

Season 2 Episode 47: Non-square Square Grasshoppers (Invariant)

- Problem S2M49 solved
- Problem S2M50 formulated:
  - S2M50: 66 flying squares sit on 66 trees arranged in a circle. Then, every second one, random, flying squirrel flies to the nearest tree clockwise and another random flying squirrel flies to the nearest tree counterclockwise (Invariant)

Season 2 Episode 48: A Circle Dance Of Flying Squirrels (Invariant)

- Problem S2M50 solved
- Problem S2M51 formulated:

- S2M51: on an island there live chameleons of three colors, red, green and blue. When two chameleons of opposite colors meet then they both change their color to the remaining one (Invariant)

Season 2 Episode 49: Modular Equilateral Chameleons (Invariant)

- Problem S2M51 solved
- Problem S2CS2 formulated:
  - S2CS2: find the number of ordered pairs (a, b) of whole non-negative numbers that satisfy $a^2 + b^2 < n$, where n is a fixed and given ahead of time whole positive number

The above six episodes form the "Invariant" play list.

Season 2 Episode 50: Diophantine Staircase (A mix of problem-solving approaches in CS)

- Problem S2CS2 solved
- Problem S2P1 formulated:
  - S2P1:which one of the two uniformly dense solids, a cube or a right circular cylinder of comparable size, is easier to tip over and why?

Season 2 Episode 51: A Cube Versus A Cylinder (Solving problems in Physics, How To)

- Problem S2P1 solved
- Problem S2M52 formulated:
  - S2M52: solve the 9-bottle and the 9-coin problems and determine whether a solution of one of these problems can be converted into a solution of the other problem mechanically

Season 2 Episode 52: Of Mice And Wine And Coins (Mathematical Equivalence)

- Problem S2M52 solved
- Problem S2M53 formulated:
  - S2M53: find the survivors count and the total annihilation time of two armies in a fully escalated mode under certain assumptions

Season 2 Episode 53: Lanchester's Square Law (Mathematical Equivalence)

- Problem S2M53 solved
- Problem S2P2 formulated:
  - S2P2: find the separation velocity and the separation time of a bead sliding off of a massless rod rotating in the horizontal plane

Season 2 Episode 54: Slitating Or Roliding Bead (Mathematical Equivalence)

- Problem S2P2 solved
- Problem S2M54 formulated:
  - S2M54: show how the solution of S2M53 can be converted into a solution of S2P2

Season 2 Episode 55: Mathematical Equivalence

- Problem S2M54 solved
- Problem S3M1 formulated:
  - S3M1: show that for all real $x > 1$ and all whole positive n it is the case that

$$\int_0^\pi \left(x + \sqrt{x^2-1}\cos\varphi\right)^n \, \mathrm{d}\varphi = \int_0^\pi \frac{\mathrm{d}\varphi}{\left(x - \sqrt{x^2-1}\cos\varphi\right)^{n+1}}$$

The last 4 episodes above form the "Mathematical Equivalence" play list.

The ProbLemma's Channel Season 2 Index

Problem Number	Formulated In	Solved In
S2M1	Season 1 Episode 9	Season 2 Episode 1
S2M2	Season 2 Episode 1	Season 2 Episode 2
S2M3	Season 2 Episode 2	Season 2 Episode 3
S2M4	Season 2 Episode 3	Season 2 Episode 4
S2M5	Season 2 Episode 4	Season 2 Episode 5
S2M6	Season 2 Episode 4	Season 2 Episode 5
S2M7	Season 2 Episode 4	Season 2 Episode 5
S2M8	Season 2 Episode 5	Season 2 Episode 6
S2M9	Season 2 Episode 5	Season 2 Episode 6
S2CS1	Season 2 Episode 6	Season 2 Episode 7
S2M10	Season 2 Episode 7	Season 2 Episode 8
S2M11	Season 2 Episode 8	Season 2 Episode 9
S2M12	Season 2 Episode 9	Season 2 Episode 10
S2M13	Season 2 Episode 10	Season 2 Episode 11
S2M14	Season 2 Episode 11	Season 2 Episode 12
S2M15	Season 2 Episode 12	Season 2 Episode 13
S2M16	Season 2 Episode 13	Season 2 Episode 14
S2M17	Season 2 Episode 14	Season 2 Episode 15
S2M18	Season 2 Episode 15	Season 2 Episode 16
S2M19	Season 2 Episode 16	Season 2 Episode 17
S2M20	Season 2 Episode 17	Season 2 Episode 18
S2M21	Season 2 Episode 18	Season 2 Episode 19
S2M22	Season 2 Episode 19	Season 2 Episode 20
S2M23	Season 2 Episode 20	Season 2 Episode 21
S2M24	Season 2 Episode 21	Season 2 Episode 22
S2M25	Season 2 Episode 22	Season 2 Episode 23
S2M26	Season 2 Episode 23	Season 2 Episode 24
S2M27	Season 2 Episode 24	Season 2 Episode 25

S2M28	Season 2 Episode 25	Season 2 Episode 26
S2M29	Season 2 Episode 26	Season 2 Episode 27
S2M30	Season 2 Episode 27	Season 2 Episode 28
S2M31	Season 2 Episode 28	Season 2 Episode 29
S2M32	Season 2 Episode 29	Season 2 Episode 30
S2M33	Season 2 Episode 30	Season 2 Episode 31
S2M34	Season 2 Episode 31	Season 2 Episode 32
S2M35	Season 2 Episode 32	Season 2 Episode 33
S2M36	Season 2 Episode 33	Season 2 Episode 34
S2M37	Season 2 Episode 34	Season 2 Episode 35
S2M38	Season 2 Episode 35	Season 2 Episode 36
S2M39	Season 2 Episode 36	Season 2 Episode 37
S2M40	Season 2 Episode 37	Season 2 Episode 38
S2M41	Season 2 Episode 38	Season 2 Episode 39
S2M42	Season 2 Episode 39	Season 2 Episode 40
S2M43	Season 2 Episode 40	Season 2 Episode 41
S2M44	Season 2 Episode 41	Season 2 Episode 42
S2M45	Season 2 Episode 42	Season 2 Episode 43
S2M46	Season 2 Episode 43	Season 2 Episode 44
S2M47	Season 2 Episode 44	Season 3 Episode 45
S2M48	Season 2 Episode 45	Season 2 Episode 46
S2M49	Season 2 Episode 46	Season 2 Episode 47
S2M50	Season 2 Episode 47	Season 2 Episode 48
S2M51	Season 2 Episode 48	Season 2 Episode 49
S2CS2	Season 2 Episode 49	Season 2 Episode 50
S2P1	Season 2 Episode 50	Season 2 Episode 51
S2M52	Season 2 Episode 51	Season 2 Episode 52
S2M53	Season 2 Episode 52	Season 2 Episode 53
S2P2	Season 2 Episode 53	Season 2 Episode 54
S2M54	Season 2 Episode 54	Season 2 Episode 55