



# The ProbLemma's Channel Season 2 Guide

## Season 2 Episode 1: Seven Gallons Of Water On The Wall (Reinterpret And Conquer)

- Problem **S2M1** solved:
  - Mathematical Billiard
- Problem **S2M2** formulated:
  - Problem **S2M2**: an alternative expression for a finite sum of squares of consecutive whole positive numbers

## Season 2 Episode 2: A Weighty Question (Reinterpret And Conquer)

- Problem **S2M2** solved:
  - Center Of Mass
- Problem **S2M3**, Swan Lakes, formulated:
  - Problem **S2M3**: swans landing on lakes via the half of all swans plus half-a-swan rule



The above 2 episodes with S1E8 and S1E9 form the [“Reinterpret And Conquer” play list](#).

## Season 2 Episode 3: Swan Lakes (Reverse Order)

- The mechanics of the “Reverse Order” problem-solving approach explained
- Problem **S2M3** solved
- Problem **S2M4**, The Devil And A Loiterer, formulated:
  - **S2M4**: a loiterer crossing a bridge

## Season 2 Episode 4: The Devil And A Loiter (Reverse Order)

- The mechanics of the “Reverse Order” problem-solving approach explained again
- Problem **S2M4** solved
- Problem **S2M5** formulated:
  - **S2M5**: magic apples gathered by a peasant
- Problem **S2M6** formulated:
  - **S2M6**: apple injections
- Problem **S2M7** formulated:
  - **S2M7**: an equilateral triangle in a square

## Season 2 Episode 5: Apples Of Discord (Reverse Order)

- Problems **S2M5**, **S2M6**, **S2M7** solved
- Problems **S2M8** and **S2M9** formulated
  - **S2M8**: an isosceles triangle in a trapezoid
  - **S2M9**: external and internal tangents to two circles

Season 2 Episode 6: On The Tangent (Reverse Order)

- Problems **S2M8** and **S2M9** solved
- Problem **S2CS1** formulated:
  - **S2CS1**: 2 eggs versus 100-story building

Season 2 Episode 7: Two Eggs Versus One Building (Reverse Order)

- Problem **S2CS1** solved
- Problem **S2M10** formulated:
  - **S2M10**: horses and carrots, gamels and bananas

Season 2 Episode 8: Horses Eating Carrots, Discrete Rocket Propulsion (Reverse Order)

- Problem **S2M10** solved
- Problem **S2M11** formulated:
  - **S2M11**: An odd colony of infinitely excitable cells

Season 2 Episode 9: An Add Colony Of Infinitely Excitable Cells (Reverse Order)

- Problem **S2M11** solved
- Problem **S2M12** formulated:
  - **S2M12**: Zero in a recurrence relation

Season 2 Episode 10: Zero in a recurrence relation (Reverse Order)

- Problem **S2M12** solved
- Problem **S2M13** formulated:
  - **S2M13**: peasant, goat, cabbage, wolf crossing a river



The above eight episodes form the [“Reverse Order”](#) play list.

Season 2 Episode 11: Peasant. Goat. Cabbage. Wolf (Space-Time)

- The mechanics of “Space-Time” explained
- Problem **S2M13** solved
- Problem **S2M14** formulated:
  - **S2M14**: find a fake coin in a set 12 using 3 weighings on pan scales, an adaptive approach

Season 2 Episode 12: Not Blind Mathematical Justice (Space-Time)

- Problem **S2M14** solved (via an adaptive approach)
- Problem **S2M15** formulated:
  - **S2M15**: find a fake and heavy coin in a set of 18 using 3 non-adaptive weighings on pan scales

Season 2 Episode 13: Blind Mathematical Justice (Space-Time)

- Problem **S2M15** solved (via a non-adaptive approach)

- Problem **S2M16** formulated:
  - **S2M16**: put together at least one fake coin detection problem that admits at least one geometric solution

#### Season 2 Episode 14: Geometry In Fake Coin Detection Problems (Space-Time)

- Problem **S2M16** solved (a geometry of a non-adaptive approach)
- Problem **S2M17** formulated:
  - **S2M17**: decompose the  $\log(\Gamma(x))$  function into its Fourier series over the interval  $(0, 1]$



The above four episodes form the [“Space Time” play list](#).

#### Season 2 Episode 15: Fourier Series of $\log(\Gamma(x))$ over $(0, 1]$

- Problem **S2M17** solved
- Problem **S2M18** formulated:
  - **S2M18**: find the number of times a minute hand will rendezvous with the hour hand on the face of the standard analogue 12-hour clock in one 12-hour period starting from 12 o'clock

#### Season 2 Episode 16: A Chase Around The Clock (Equation)

- Problem **S2M18** solved
- Problem **S2M19** formulated:
  - **S2M19**: generate a proof of the Pythagorean Theorem based on the Equation problem-solving approach

#### Season 2 Episode 17: Pythagorean Theorem Via Equations (Equation)

- Problem **S2M19** solved
- Problem **S2M20** formulated:
  - **S2M20**: solve an equation of order 4

#### Season 2 Episode 18: Now You Know Me, Now You Don't (Equation)

- Problem **S2M20** solved
- Problem **S2M21** formulated:
  - **S2M21**: effectiveness of advertisement

#### Season 2 Episode 19: Effectiveness Of Advertisement (Equation)

- Problem **S2M21** solved
- Problem **S2M22** formulated:
  - **S2M22**: Fresnel Integrals Via Equations

#### Season 2 Episode 20: Fresnel Integrals Via Equations (Equation)

- Problem **S2M22** solved
- Problem **S2M23** formulated:

- **S2M23**: number of such 5-digit perfect squares that if each digit of that perfect square is increased by 1 then a new perfect square results (Scope Reduction)



The above six episodes form the [“Equation” play list](#).

#### Season 2 Episode 21: Heavy perfect 5-digit squares (Scope Reduction)

- Problem **S2M23** solved
- Problem **S2M24** formulated:
  - **S2M24**: find the locus of points on a sphere each of which is equidistant from 3 given fixed distinct points on that sphere, no two of which are antipodal (Scope Reduction)

#### Season 2 Episode 22: Equidistant points on a sphere (Scope Reduction)

- Problem **S2M24** solved
- Problem **S2M25** formulated:
  - **S2M25**: explain how the Euclidean Greatest Common Divisor Algorithm from the perspective of the Scope Reduction problem-solving approach (Scope Reduction)

#### Season 2 Episode 23: the Euclidean GCD Algorithm via Scope Reduction (Scope Reduction)

- Problem **S2M25** solved
- Problem **S2M26** formulated:
  - **S2M26**: evaluate the following integral using the Scope Reduction problem-solving approach (Scope Reduction)

$$\int_c^{2c} \frac{x}{\sqrt{x^2 + cx - 2c^2}} dx$$

#### Season 2 Episode 24: Integral Evaluation via Scope Reduction (Scope Reduction)

- Problem **S2M26** solved
- Problem **S2M27** formulated:
  - **S2M27**: show that it is impossible to find the location of a circle using the Euclidean straightedge alone (Scope Expansion)



The above four episodes form the [“Scope Reduction” play list](#).

#### Season 2 Episode 25: Circle. Center. Straightedge. Nope (Scope Expansion)

- Problem **S2M27** solved
- Problem **S2M28** formulated:

- **S2M28**: find an alternative expression for the finite sums of consecutive positive whole numbers raised to a fixed positive whole power (Scope Expansion)

Season 2 Episode 26: Finite Integer Sums (Scope Expansion)

- Problem **S2M28** solved
- Problem **S2M29** formulated:
  - **S2M29**: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

Season 2 Episode 27: Integer Power Sums Revisited (Scope Expansion)

- Problem **S2M29** solved
- Problem **S2M30** formulated:
  - **S2M30**: determine if the sum of areas of yellow triangles is equal to the sum of areas of blue triangles that live in a regular hexagon (Scope Expansion)

Season 2 Episode 28: Is It A Hexagon? Or Is It A Triangle? (Scope Expansion)

- Problem **S2M30** solved
- Problem **S2M31** formulated:
  - **S2M31**: find a mechanical way to construct arbitrary magic squares of odd orders (Scope Expansion)

Season 2 Episode 29: Odd Magic Squares (Scope Expansion)

- Problem **S2M31** solved
- Problem **S2M32** formulated:
  - **S2M32**: evaluate the following indefinite integral via the Scope Expansion problem-solving approach without using the integration by parts (Scope Expansion)

$$\int e^{ax} \sin(bx) \cos(cx) dx$$

Season 2 Episode 30: One Integral? Two Integrals! (Scope Expansion)

- Problem **S2M32** solved
- Problem **S2M33** formulated:
  - **S2M33**: find the radius of a largest circle that passes only through the black squares of a standard  $8 \times 8$  chessboard (Eliminate And Conquer)



The above six episodes form the “Scope Expansion” play list.

Season 2 Episode 31: Largest Circle On A Chessboard (Eliminate And Conquer)

- Problem **S2M33** solved
- Problem **S2M34** formulated:

- **S2M34**: find two numbers given their LCM and the difference between them (Eliminate And Conquer)

Season 2 Episode 32: The LCM And The Difference (Eliminate And Conquer)

- Problem **S2M34** solved
- Problem **S2M35** formulated:
  - **S2M35**: show that no number of the form  $111 \dots 1$  is a perfect square (Eliminate And Conquer)

Season 2 Episode 33: A Non-Square Fence Of Ones (Eliminate And Conquer)

- Problem **S2M35** solved
- Problem **S2M36** formulated:
  - **S2M36**: construct a magic  $3 \times 3$  square with a given constant (Eliminate And Conquer)

Season 2 Episode 34: A  $3 \times 3$  Magic Square Construction (Eliminate And Conquer)

- Problem **S2M36** solved
- Problem **S2M37** formulated:
  - **S2M37**: recover the shape of a regular polygon given a relationship between the lengths of its sides and diagonals (Eliminate And Conquer)

Season 2 Episode 35: Regular Polygon Recognition (Eliminate And Conquer)

- Problem **S2M37** solved
- Problem **S2M38** formulated:
  - **S2M38**: find the magnitudes of the interior angles of a planar triangle given a relationship between the distances from the vertices of that triangle to the points chosen on its sides (Eliminate And Conquer)

Season 2 Episode 36: A Show Of Equal Distances (Eliminate And Conquer)

- Problem **S2M38** solved
- Problem **S2M39** formulated:
  - **S2M39**: cut an  $8 \times 3$  piece of wood into two pieces that fit perfectly inside of a  $12 \times 2$  whole (Divide And Conquer)



The above six episodes form the “Eliminate And Conquer” play list.

Season 2 Episode 37: East Or West Divide And Conquer (Divide And Conquer)

- Problem **S2M39** solved
- Problem **S2M40** formulated:
  - **S2M40**: find the area of a Reuleaux Triangle (Divide And Conquer)

Season 2 Episode 38: The Area of The Reuleaux Triangle (Divide And Conquer)

- Problem S2M40 solved
- Problem S2M41 formulated:
  - S2M41: invent at least two distinct proofs of the Heron's Formula (Divide And Conquer)

Season 2 Episode 39: Heron's Formula Divided And Conquered (Divide And Conquer)

- Problem S2M41 solved
- Problem S2M42 formulated:
  - S2M42: show that the areas of two triangles whose vertices are located on the different branches of the unit hyperbola  $xy = 1$  are equal one another (Divide And Conquer)

Season 2 Episode 40: When Triangles Kiss A Hyperbola (Divide And Conquer)

- Problem S2M42 solved
- Problem S2M43 formulated:
  - S2M43: evaluate a finite product of cosines whose arguments are the whole positive numbers coprime with 100 and scaled by  $\pi$  and divided by 100 (Divide And Conquer)

Season 2 Episode 41: 5-Coprime Odd Numbers That Live On A Globe (Divide And Conquer)

- Problem S2M43 solved
- Problem S2M44 formulated:
  - S2M44: evaluate the following limit (Divide And Conquer)

$$\lim_{n \rightarrow +\infty} \frac{1}{n^3} \sum_{k=1}^n k \sqrt{n^2 - k^2}$$

Season 2 Episode 42: How A Hoof Can Evaluate A Limit (Divide And Conquer)

- Problem S2M44 solved
- Problem S2M45 formulated:
  - S2M45: evaluate the Poisson Integral  $P_r$  by the book (Divide And Conquer)

$$\int_0^\pi \log(1 - 2r \cos(x) + r^2) dx, \quad |r| \neq 1$$

Season 2 Episode 43: Poisson Integral Evaluation By The Book (Divide And Conquer)

- Problem S2M45 solved
- Problem S2M46 formulated:
  - S2M46: the watermelons transportation problem (Invariant)



The above seven episodes form the “Divide And Conquer” play list.

Season 2 Episode 44: Evaporating Watermelons (Invariant)

- Problem S2M46 solved
- Problem S2M47 formulated:
  - S2M47: (Invariant)



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