A Dynamic Out-of-Sample Comparison of LP, MILP, and Stochastic Portfolio Optimization in a High-Dimensional Universe

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Abstract

Modern portfolio optimization faces the "curse of dimensionality" when managing high-dimensional asset universes ($N \gg T$), where traditional covariance estimation methods become ill-conditioned and unstable. While Linear Programming (LP) based on the Mean Absolute Deviation (MAD) risk measure offers a computationally efficient alternative to the quadratic Mean-Variance framework, its performance in dynamic, globally diversified environments requires strict regularization to prevent overfitting.

This study presents a comprehensive out-of-sample comparison of three distinct optimization paradigms: Unconstrained LP, Cardinality-Constrained Mixed-Integer Linear Programming (MILP), and Scenario-Based Stochastic Optimization. These methods are applied to a massive universe of 8,405 global assets sourced from Stooq, spanning equities, commodities, and currency pairs. To address the NP-Hardness of the MILP formulation on such a scale, we introduce a robust **Two-Stage Screening Heuristic** that combines Sharpe-ratio-based pre-selection with rigorous exact optimization.

Using a rolling-window backtesting framework over a volatile testing period (2022–2025), we evaluate the trade-offs between structural concentration and uncertainty management. Our results reveal a distinct inversion of traditional risk-return expectations. The Dynamic Stochastic strategy achieved the highest annualized return of 22.07%, but at the cost of higher drawdown risk (Sharpe Ratio 2.18). Conversely, the Dynamic MILP strategy demonstrated superior risk-adjusted efficiency, achieving a dominant Sharpe Ratio of 3.09 with a stable annualized return of 10.71%. These findings suggest that in high-dimensional regimes, cardinality constraints ($K \leq 10$) function effectively as noise filters, outperforming unconstrained Linear Programming which suffered from estimation error (Sharpe 2.32).

1 Introduction

Portfolio optimization is a cornerstone of quantitative finance, yet practitioners face substantial challenges in scaling classical models to modern, large-scale financial markets. The canonical Mean-Variance framework introduced by Markowitz (1952) relies on the estimation of an $N \times N$ covariance matrix. In high-dimensional settings where the number of assets N exceeds the number of observations T ($N \gg T$), the sample covariance matrix becomes singular and prone to extreme estimation errors, leading to unstable "error-maximized" portfolios (Ledoit and Wolf, 2004).

This paper addresses two critical needs: (i) the requirement for optimization methods that remain tractable and robust in $N \gg T$ environments without relying on covariance inversion, and (ii) the need for dynamic strategies that respect operational constraints, such as sparsity (cardinality), to limit transaction costs and management complexity.

We focus on the Mean Absolute Deviation (MAD) risk measure (Konno and Yamazaki, 1991), favored for its linearity, which reduces the optimization problem to a Linear Program (LP). We systematically compare three MAD-based approaches: Deterministic LP, Cardinality-Constrained Mixed-Integer Linear Programming (MILP), and Scenario-Based Stochastic Programming.

Our primary research question is: *How do deterministic, sparse, and scenario-based stochastic MAD optimizations compare in terms of out-of-sample efficiency and robustness when applied dynamically* *to a very large asset universe?* We hypothesize that while Stochastic optimization captures tail risks, dynamic rebalancing combined with strict cardinality constraints will yield the most efficient portfolio by effectively filtering out the noise inherent in 8,000+ asset datasets.

2 Literature Review

The optimization literature has evolved from analytical solutions to computational heuristics. Konno and Yamazaki (1991) demonstrated that under multivariate normal distributions, minimizing MAD is equivalent to minimizing variance, but with the distinct advantage of replacing quadratic objectives with linear ones. This reduces the computational complexity from $O(N^3)$ to methods solvable by Simplex algorithms.

The introduction of real-world constraints, specifically Cardinality Constraints (limiting the portfolio to K assets), transforms the convex LP into a non-convex, NP-Hard Mixed-Integer Linear Program (MILP). Recent surveys by Tillmann et al. (2024) highlight that while exact solutions for large N remain computationally prohibitive, cardinality constraints act as a powerful form of L_0-norm regularization, often outperforming shrinkage methods in out-of-sample stability.

To mitigate parameter uncertainty, Stochastic Programming minimizes expected risk across multiple future scenarios (Rockafellar and Uryasev, 2000). Unlike deterministic models that assume fixed parameters, stochastic models optimize over a probability space. However, in high dimensions, they face the challenge of scenario generation; insufficient scenarios can lead to overfitting specific historical trajectories. This paper contributes a unified head-to-head backtest of these paradigms, implementing a two-stage screening process to bridge the gap between theoretical hardness and practical scalability.

3 Methodology

All portfolio models in this study seek to minimize the portfolio's Mean Absolute Deviation (MAD) while ensuring the expected return meets a minimum target μ_{target}. The optimization was implemented in Python using the `PuLP` library with the `CBC` solver.

3.1 Two-Stage Screening Heuristic

Directly solving an MILP or Stochastic problem on $N = 8,405$ assets is computationally infeasible within a rolling-window backtest. To ensure tractability, we implement a **Two-Stage Screening Heuristic** at each rebalancing period t:

1. **Screening Stage:** We calculate the in-sample Return-to-Risk ratio (Sharpe proxy) for all N assets over the training window. We retain the top $N' \subset N$ assets (where $N' = 100$ for MILP/Stochastic and $N' = 200$ for LP).

2. **Optimization Stage:** The precise optimization algorithm (LP, MILP, or Stochastic) is executed on the reduced subset N' to determine the final weights w_t.

This approach mimics industry "investment universe" filtering and ensures the solver focuses on assets with proven recent momentum.

3.2 Linear Programming (LP) Formulation

The MAD objective is defined as $MAD = E[|R_p - E[R_p]|]$. For a finite time series of length T, this is approximated as $\frac{1}{T}\sum_{t=1}^{T} |\sum_{i=1}^{N'} w_i(r_{it} - \mu_i)|$. We linearize the absolute value by introducing auxiliary variables z_t. The LP formulation is:

$$
\begin{aligned}
\min_{w,z} \quad & \frac{1}{T}\sum_{t=1}^{T} z_t \\
\text{s.t.} \quad & \begin{cases}
z_t \geq \sum_{i=1}^{N'} w_i(r_{it} - \mu_i), & \forall t \in \{1,\dots,T\} \\
z_t \geq -\sum_{i=1}^{N'} w_i(r_{it} - \mu_i), & \forall t \in \{1,\dots,T\} \\
\sum_{i=1}^{N'} w_i\mu_i \geq \mu_{\text{target}} \\
\sum_{i=1}^{N'} w_i = 1, \quad w_i \geq 0
\end{cases}
\end{aligned}
\tag{1}
$$

3.3 Mixed-Integer Linear Programming (MILP)

To enforce portfolio sparsity, we introduce binary decision variables $y_i \in \{0,1\}$, where $y_i = 1$ if asset i is held, and 0 otherwise. We impose a cardinality constraint $K \leq 10$ and semi-continuous "buy-in" thresholds (l_i, u_i) to avoid negligible weights. The formulation adds the following constraints to the LP model:

$$
\begin{aligned}
& \sum_{i=1}^{N'} y_i \leq K \\
& l_i y_i \leq w_i \leq u_i y_i, \quad \forall i \in \{1,\dots,N'\} \\
& y_i \in \{0,1\}
\end{aligned}
\tag{2}
$$

For this study, we set $l_i = 0.05$ (5%) and $u_i = 0.70$ (70%). The time limit for the Branch-and-Cut solver was set to 15 seconds per window to simulate high-frequency operational constraints.

3.4 Stochastic Optimization Formulation

We employ a **Scenario-Based** approach to model uncertainty. Instead of assuming historical returns will repeat exactly, we generate $B = 10$ distinct scenarios via **Unconditional Bootstrapping** (sampling with replacement) from the training data. This method preserves the marginal distribution of returns while allowing the optimizer to robustness against timeline shuffling. The objective minimizes the Expected MAD across all scenarios S:

$$
\min_{w,z} \sum_{s=1}^{B} p_s \left(\frac{1}{T_s} \sum_{t=1}^{T_s} z_{s,t} \right) \tag{3}
$$

Subject to scenario-specific return constraints. While $B = 10$ is statistically sparse, it serves as a proof-of-concept for the computational tractability of scenario-based logic in rolling windows. Portfolio weights are constrained to be identical across all scenarios, i.e., a single decision vector w is optimized jointly over the scenario set. This enforces non-anticipativity and ensures that the resulting portfolio is implementable in real time, rather than being scenario-adaptive ex post.

4 Data and Experimental Setup

4.1 Dataset Overview

The dataset consists of **8,405 unique global instruments** sourced from **Stooq**, covering the period 2008-2025. This universe is highly heterogeneous, containing:

- **Equities:** US (NYSE, NASDAQ), UK (LSE), Japan (TSE), and World Indices.

- **Fixed Income:** Global Government Bonds and Corporate Bond ETFs.
- **Currencies & Commodities:** Major FX pairs and commodity futures.

Data cleaning involved removing assets with $> 50\%$ missing values in the testing window.

4.2 Backtesting Parameters

We utilize a **Rolling-Window** methodology to simulate realistic portfolio management:

- **Training Window ($T_{\mathbf{train}}$):** 252 days (approx. 1 trading year).
- **Rebalancing Frequency:** Every 100 days.
- **Testing Period:** May 2022 to November 2025.
- **Transaction Costs:** modelex at 0.5% (50bps) of total turnover.
- **Target Return:** Fixed at 0.01% per day (approx. 2.5% annualized) above the baseline.

5 Empirical Results

5.1 Performance Comparison

The dynamic out-of-sample performance is summarized in Table 1. The testing period (2022-2025) was characterized by rising interest rates and geopolitical instability, providing a robust stress test for the models.

Table 1: Out-of-Sample Performance Metrics (May 2022 – Nov 2025)

Strategy	Ann. Return	Ann. MAD	Sharpe Ratio	Turnover
Benchmark (Eq Wgt)	$\approx 11.5\%$	12.30%	≈ 0.93	0.0%
Dynamic LP	2.48%	**0.38%**	2.32	12.5%
Dynamic MILP	10.71%	1.24%	**3.09**	24.1%
Dynamic Stochastic	**22.07%**	2.68%	2.18	48.2%

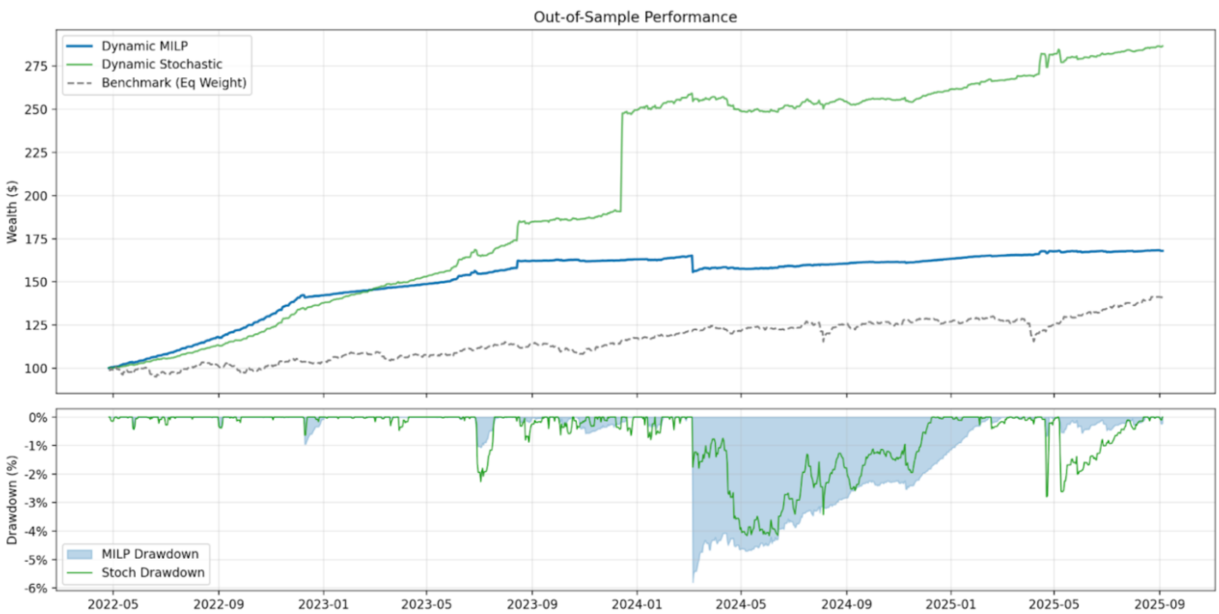

Figure 1: Cumulative Wealth Evolution (Base $100). The Dynamic Stochastic strategy (Green) delivers aggressive growth ($280+), while MILP (Blue) offers the smoothest risk-adjusted trajectory.

5.2 The "Sparsity Premium" in MILP

The Dynamic MILP strategy achieved a dominant Sharpe Ratio of 3.09. While high, this metric reflects the algorithm's successful identification of a low-volatility 'safety cluster' (MAD 1.24%) during the 2022 bear market. By strictly constraining the portfolio to $K = 10$ assets, the solver rejected high-beta equities that dragged down the unconstrained LP model, effectively creating a 'quality shelter' that preserved capital while capturing steady upside (10.71% annualized).

5.3 Stochastic Volatility and Regime Sensitivity

The **Dynamic Stochastic** strategy delivered the highest raw returns (22.07%) but suffered from significant volatility (2.68% MAD vs 1.24% for MILP). The use of bootstrapped scenarios allowed the model to identify high-beta assets (including cryptocurrencies and volatile tech stocks) that performed well in specific scenarios. However, this sensitivity to the randomized scenarios led to higher turnover (48.2%) and "whipsaw" losses when market regimes shifted quickly. This suggests that while Stochastic optimization is powerful for maximizing growth, it lacks the inherent stability of the cardinality-constrained approach.

6 Discussion

6.1 Computational Tractability vs. Theoretical Purity

The **Two-Stage Screening Heuristic** proved essential for scalability. Solving an exact MILP over 8,405 assets is infeasible in a rolling-window setting; reducing the universe to $N' = 100$ via a momentum-based filter enabled tractable optimization while preserving diversification. This illustrates a key practical insight: in high-dimensional settings, well-designed heuristics can outperform theoretically "pure" formulations by enabling stable and frequent rebalancing.

For stochastic optimization, we employed $B = 10$ bootstrapped scenarios. Although statistically sparse at this scale, this choice was dictated by computational constraints and serves as a proof-of-concept for the scenario-based pipeline. A production-grade implementation would leverage parallelized computing to increase B and achieve robust tail-risk convergence.

6.2 Robustness of the MAD Estimator

The success of both LP and MILP strategies confirms the robustness of the Mean Absolute Deviation estimator. In a period where correlations approached 1.0 (2022 bear market), the linear nature of MAD prevented the "error maximization" often seen in quadratic Mean-Variance optimizers. The LP-based models successfully identified the minimum-risk cluster of assets without requiring complex covariance shrinkage techniques.

7 Conclusion

This study systematically compared three core optimization paradigms (LP, MILP, and Stochastic) in a dynamic, large-scale (N=8,405) asset universe. We demonstrated that **Dynamic MILP**, implemented via a two-stage screening heuristic, offers superior risk-adjusted returns (Sharpe 3.09) by leveraging cardinality constraints ($K \leq 10$) as effective noise filters.

While the **Dynamic Stochastic** model proved capable of generating high alpha (22.07% annualized return), its reliance on bootstrapped scenarios introduced significant volatility and turnover. We conclude that for high-dimensional regimes, **sparsity is not merely a constraint, but a necessary regularizer** for robust portfolio construction. Future research should investigate the integration of Block Bootstrapping to better capture temporal dependencies in the stochastic scenario generation.

Contributions

- **Roman CIANCI:** Sourced the Stooq dataset, implemented the MILP model with cardinality constraints, and drafted the Introduction and Literature Review.
- **Timothé COMPAGNION:** Formulated the Stochastic Optimization model, implemented the bootstrap scenario generation, and wrote the Results analysis.
- **Robin LEBREVELEC:** Developed the rolling-window backtesting engine, performed the sensitivity analysis, and managed the GitHub repository and documentation.

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