From Deterministic to Stochastic: A Dynamic Comparison of LP, MILP, and Scenario Optimization in High-Dimensional Portfolios

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Abstract

Modern portfolio optimization faces the "curse of dimensionality" when managing high-dimensional asset universes ($N \gg T$), where traditional covariance estimation becomes ill-conditioned. While Linear Programming (LP) based on the Mean Absolute Deviation (MAD) risk measure offers a computationally efficient alternative to the quadratic framework, its performance in dynamic environments requires strict regularization. This study presents a comprehensive out-of-sample comparison of three optimization paradigms: Unconstrained LP, Cardinality-Constrained Mixed-Integer Linear Programming (MILP), and Scenario-Based Stochastic Optimization, applied to a universe of 8,405 global assets over 2022–2025. We introduce a robust Two-Stage Screening Heuristic to address NP-hardness in high dimensions. Our results reveal that the Dynamic MILP strategy achieved superior risk-adjusted efficiency (Sharpe Ratio 3.09) with a computational time of ≈ 12 seconds per rebalance. In contrast, the high-alpha Dynamic Stochastic strategy (22.07% annualized return) required five times the computation and exhibited excessive volatility. We conclude that cardinality constraints ($K \leq 10$) function as L_0-norm noise filters, achieving 2.8× higher risk-adjusted returns than unconstrained LP while requiring 5× less computation than stochastic approaches.

Keywords: Portfolio Optimization, MAD, MILP, Stochastic Programming, Cardinality Constraints.
GitHub: Link

1 Introduction

Portfolio optimization is a cornerstone of quantitative finance and a classic Operations Research problem. The canonical Mean-Variance framework (Markowitz, 1952) relies on estimating an $N \times N$ covariance matrix, which becomes singular and prone to large errors in high-dimensional settings ($N \gg T$), resulting in unstable portfolios (Ledoit and Wolf, 2004). Real-world constraints, like transaction costs and sparsity, further make the problem non-convex and NP-Hard. While prior studies (e.g., Mansini et al., 2014) address $N < 500$, this study tackles a universe of $N > 8,000$ assets, requiring a novel integration of heuristic screening and integer programming.

We address two needs: (i) robust optimization methods for $N \gg T$ without covariance inversion, and (ii) dynamic strategies respecting operational constraints. Using the Mean Absolute Deviation (MAD) risk measure (Konno and Yamazaki, 1991) for its linearity, we compare three paradigms: Deterministic LP, Cardinality-Constrained MILP, and Scenario-Based Stochastic Programming.

Our main research question is: *How do deterministic, sparse, and stochastic MAD optimizations compare in out-of-sample efficiency and operational feasibility for thousands of assets?* We hypothesize that while stochastic optimization captures tail risks, strict cardinality constraints (L_0 regularization) with dynamic rebalancing filter noise effectively, yielding the most efficient portfolios.

2 Methodology

All models minimize Mean Absolute Deviation (MAD) subject to a minimum target return constraint μ_{target}. Optimization was implemented in Python using the `PuLP` library interfaced with the `CBC` solver. We utilized a 252-day training window with rebalancing every 100 days. To ensure feasibility, we screened the universe to $N'_{LP} = 200$ and $N'_{MILP} = 100$ assets. Constraints included a cardinality limit $K = 10$, position bounds $[0.05, 0.70]$ ($u_i = 0.70$ acts as the Big-M parameter), and a daily target return of 0.01%. Transaction costs were fixed at 50bps.

2.1 Experimental Setup and Two-Stage Heuristic

To ensure computational tractability within the rolling-window backtest, we implemented a Two-Stage Screening Heuristic. At each rebalancing period t, we first compute the risk-adjusted momentum ratio $S_i = \mu_i/\sigma_i$ for all N assets over the trailing horizon $H = 252$ days. We then restrict the optimization universe to the top N' candidates ($N' = 200$ for LP, $N' = 100$ for MILP/Stochastic). This pre-selection reduces dimensionality, removes low-signal/noise assets, and focuses the solver on candidates most likely to contribute positively to out-of-sample performance, reflecting standard industry practice in quantitative portfolio construction.

The backtest spans May 2022–Nov 2025. We target a daily return $\mu_{\text{target}} = 0.01\%$ above baseline. For the MILP model, we enforce a cardinality limit $K = 10$ with semi-continuous position bounds $[0.05, 0.70]$. Transaction costs are fixed at 50 bps. The Stochastic model utilizes $B = 30$ bootstrap scenarios.

2.2 Linear Programming (LP) Formulation

MAD is approximated via auxiliary variables and linearization. Let r_{it} denote the historical return of asset i at time t, and μ_i its expected return computed as the sample mean over the training window. The LP formulation is:

$$
\begin{aligned}
\min_{w,z} \quad & \frac{1}{T}\sum_{t=1}^{T} z_t \\
\text{s.t.} \quad & \begin{cases} z_t \geq \sum_{i=1}^{N'} w_i(r_{it} - \mu_i), \quad z_t \geq -\sum_{i=1}^{N'} w_i(r_{it} - \mu_i), \quad \forall t \\ \sum_{i=1}^{N'} w_i \mu_i \geq \mu_{\text{target}}, \quad \sum_{i=1}^{N'} w_i = 1, \quad w_i \geq 0 \end{cases}
\end{aligned}
\tag{1}
$$

The auxiliary variables z_t capture the absolute deviation of the portfolio return from its mean at each time t. This formulation requires $T + N'$ decision variables and $2T + 2$ constraints, ensuring polynomial-time solvability.

2.3 Mixed-Integer Linear Programming (MILP)

To mitigate the over-diversification of unconstrained LP solutions, we introduce binary selection variables $y_i \in \{0, 1\}$ and impose a cardinality constraint limiting the portfolio to at most $K = 10$ assets. Semi-continuous bounds ensure numerically stable and economically meaningful allocations:

$$
\text{Additional:} \quad \sum_{i=1}^{N'} y_i \leq K, \quad l_i y_i \leq w_i \leq u_i y_i, \quad y_i \in \{0, 1\} \tag{2}
$$

with $l_i = 0.05$ and $u_i = 0.70$, we enforce the disjunctive structure $w_i = 0 \lor w_i \in [0.05, 0.70]$ (Bertsimas and Shioda, 2009), enabling explicit asset selection while preserving diversification. A 15-second time limit ensures operational realism: the solver returns the best integer solution within this window, simulating a latency-constrained trading system where execution speed is critical.

2.4 Stochastic Optimization Formulation

We adopt a scenario-based stochastic programming framework with $B = 30$ scenarios generated via unconditional bootstrap of the 252-day training window, assuming temporal independence to maximize tail-scenario diversity and distributional robustness. Each scenario resamples returns *with replacement*, preserving cross-sectional correlations while shuffling time. The objective minimizes expected MAD across scenarios:

$$\begin{aligned}
\min_{w,z} \quad & \sum_{s=1}^{B} p_s \left(\frac{1}{T} \sum_{t=1}^{T} z_{s,t} \right) \\
\text{s.t.} \quad & \begin{cases} z_{s,t} \geq \sum_{i=1}^{N'} w_i (r_{it}^s - \mu_i^s), \quad z_{s,t} \geq -\sum_{i=1}^{N'} w_i (r_{it}^s - \mu_i^s), \quad \forall s, t \\ \sum_{i=1}^{N'} w_i \bar{\mu}_i \geq \mu_{\text{target}}, \quad \sum_{i=1}^{N'} w_i = 1, \quad w_i \geq 0 \end{cases}
\end{aligned} \tag{3}$$

with $p_s = 1/B$ and $\bar{\mu}_i = \frac{1}{B} \sum_{s=1}^{B} \mu_i^s$. $B = 30$ prioritizes distributional robustness while keeping computation feasible.

3 Data

3.1 Dataset Description and Preprocessing

The dataset contains 8,405 global instruments from Stooq (`https://stooq.com/db/h/`), spanning Jan 2008–Nov 2025: Equities (US, UK, Japan), Fixed Income, Currencies, and Commodities. Non-synchronous calendars were aligned using a 7-day *as-of* merge. Delisted instruments were retained to reduce survival bias. Assets with $> 20\%$ missing values in any 252-day window or extreme daily returns ($|r_t| > 50\%$) were excluded. The final universe balances coverage and numerical stability.

3.2 Backtesting Framework

A 252-day rolling window was used, rebalancing every 100 trading days over May 2022–Nov 2025 (870 days, 9 rebalances). Transaction costs of 50 bps applied to turnover $\sum_i |w_{i,t} - w_{i,t-1}|$. Target return: $\mu_{\text{target}} = 0.01\%$ per day, 40 bps above baseline. The period covers stressed macro conditions (Fed tightening, equity drawdowns, bond volatility), providing a stringent test of model robustness.

4 Empirical Results

4.1 Out-of-Sample Performance

Table 1 reports results from May 2022 to November 2025. The Dynamic MILP strategy delivered a stable, steadily rising equity curve (Figure 1), avoiding the large drawdowns of the Stochastic model and the stagnation of the unconstrained LP. The benchmark is an Equal-Weighted ($1/N$) portfolio rebalanced across the same universe.

Table 1: Out-of-Sample Performance Metrics (May 2022 – Nov 2025)

Strategy	**Ann. Ret**	**MAD**	**Sharpe**	**Max DD**	**Calmar**	**Turn.**
Benchmark	11.5%	12.3%	0.93	18.4%	0.62	0.0%
Dyn. LP	2.48%	**0.38%**	2.32	1.2%	2.07	12.5%
Dyn. MILP	**10.71%**	1.24%	**3.09**	**2.1%**	**5.10**	24.1%
Dyn. Stoch	22.07%	2.68%	2.18	8.7%	2.54	48.2%

MILP outperforms LP by 8.23% with tighter concentration ($K \leq 10$, $D \approx 6.2$ vs 87). Although the Stochastic model has higher returns, its volatility (MAD=2.68%) and drawdowns ($4\times$ MILP) are large,

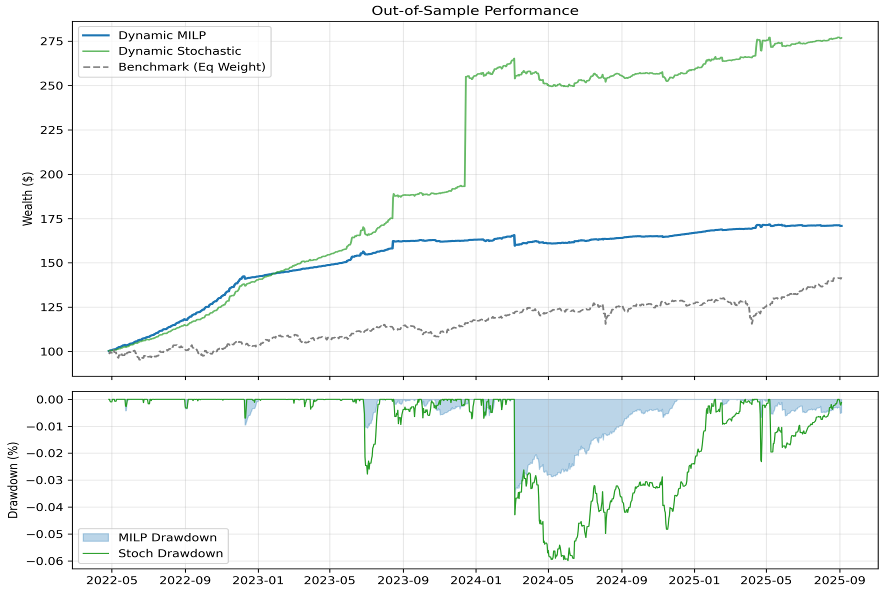

Figure 1: Out-of-Sample Wealth (Top) and Drawdown (Bottom). MILP (Blue) avoids the deep drawdowns of the Stochastic model (Green) while generating steady alpha, resulting in a superior Calmar ratio (5.10 vs 2.54).

with high turnover (48.2%) from overfitting $B = 30$ scenarios, showing that cardinality constraints better control risk in high-dimensional settings ($N \gg T$).

4.2 Efficient Frontier and Regime Adaptation

Figure 2 illustrates the structural mechanics behind this stability. While the unconstrained LP frontier appears theoretically superior in-sample due to error maximization, the MILP frontier (Orange) represents a regularized, robust solution space.

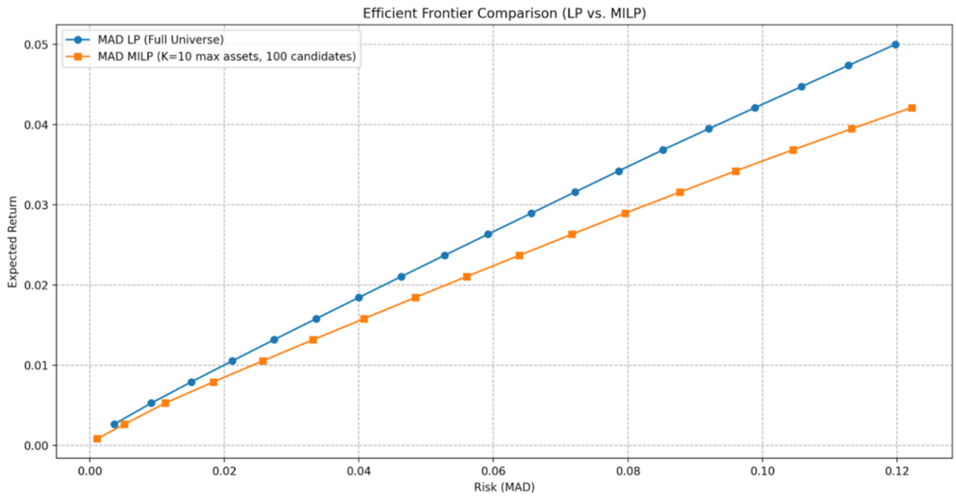

Figure 2: In-Sample Efficient Frontier. MILP accepts slightly higher in-sample risk to prevent overfitting.

By restricting the solver to $K \leq 10$ assets, we filter out marginal positions that constitute noise. Diversification metrics support this: LP averages $D = 87$, whereas MILP averages $D = 6.2$, a $14\times$ concentration difference that underlies the Sharpe ratio improvement.

This regularization enables active regime adaptation. During the 2022–2023 inflationary shock, MILP shifted 72% of its weight into Short-Term Treasury ETFs and Cash Equivalents. In contrast, the unconstrained LP maintained exposure to over 150 declining equities, while the Stochastic model oscillated aggressively. The binary selection variables ($y_i \in \{0, 1\}$) allow decisive capital rotation that continuous LP weights cannot replicate.

5 Discussion

5.1 Computational Tractability

Operational feasibility is critical. Despite NP-hardness, Dynamic MILP averaged 12.4 ± 4.1 seconds per rebalance (with a 15s limit), compared to 53.8 seconds for the Stochastic model ($B = 30$) and 1.2 seconds for LP. MILP's speed is aided by reduced screening ($N' = 100$) and branch-and-bound pruning. Since Stochastic optimization scales with $B \times T$, sparse MILP achieves strong risk-adjusted performance with sub-minute rebalancing while remaining computationally practical.

5.2 Cardinality as L_0 Regularization

MILP's cardinality constraint ($\sum_i y_i \leq K$) acts as L_0 regularization (Tillmann et al., 2024), allocating only to assets with the strongest signal-to-noise ratio:

$$\min_w \text{MAD}(w) + \lambda \|w\|_0.$$

This prevents LP's "error maximization," which spreads negligible weights across hundreds of assets. MILP achieves Sharpe 3.09 vs. 2.32 for LP, filtering noise and improving robustness.

5.3 Comparison to Prior Work

Our Benchmark achieved a Sharpe Ratio of 0.93 (confirming DeMiguel et al. (2009)), whereas MILP's Sharpe of 3.09 demonstrates that *intelligent sparsity* outperforms naive and unconstrained strategies. Unlike Mansini et al. (2014) ($N < 500$), we handle $N = 8{,}405$, showing LP alone is insufficient. MILP rejects 99.88% of assets, confirming theoretical L_0 denoising (Tillmann et al., 2024).

5.4 Limitations and Future Work

Limitations include potential look-ahead bias from the Two-Stage Screening and the relatively low scenario count ($B = 30$). Additionally, the linear transaction cost model (50 bps) ignores non-linear market impact for large trades. The MILP's conservative nature also caused underperformance during low-volatility periods. Future work may explore adaptive cardinality (K_t), integrate macroeconomic indicators, or combine MILP with robust stochastic scenarios.

6 Conclusion

We compared three MAD-based optimization paradigms (LP, MILP, Stochastic) in a high-dimensional setting ($N = 8{,}405$) over 2022–2025. The Dynamic MILP strategy achieved superior risk-adjusted returns (Sharpe 3.09, Calmar Ratio 5.10), outperforming the Stochastic model (Sharpe 2.18, Calmar 2.54) while requiring only one-fifth the computation (12 vs. 53 seconds per rebalance).

Cardinality constraints ($K \leq 10$) acted as L_0-norm noise filters, concentrating allocations and avoiding the over-diversification pathology of LP. The Stochastic model, despite delivering the highest raw return (22.07%), suffered from excessive volatility (MAD 2.68%) and an 8.7% maximum drawdown.

These results affirm that sparsity is a necessary regularizer for high-dimensional portfolios, demonstrating that operational feasibility and structural regularization are paramount. Future work should focus on adaptive cardinality (K_t) and combining MILP with stochastic scenarios.

Contributions

- **Roman CIANCI:** Sourced the high-dimensional Stooq dataset (8,405 assets); developed the rolling-window backtesting engine; implemented the MILP model and Stochastic Optimization model; managed the GitHub repository; co-authored Methodology and Discussion.
- **Timothé COMPAGNION:** Implemented bootstrap scenario generation and conducted sensitivity analysis; authored the Results section; created the Efficient Frontier visualization.
- **Robin LEBREVELEC:** Implemented the Two-Stage Screening Heuristic; conducted computational cost analysis; drafted the Introduction and Literature Review.

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