From Deterministic to Stochastic: A Dynamic Comparison of LP, MILP, and Scenario Optimization in High-Dimensional Portfolios

Roman CIANCI, Timothé COMPAGNION, Robin LEBREVELEC

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Abstract

Modern portfolio optimization faces the "curse of dimensionality" when managing high-dimensional asset universes ($N \gg T$), where traditional covariance estimation becomes ill-conditioned. While Linear Programming (LP) based on the Mean Absolute Deviation (MAD) risk measure offers a computationally efficient alternative to the quadratic framework, its performance in dynamic environments requires strict regularization. This study presents a comprehensive out-of-sample comparison of three optimization paradigms: Unconstrained LP, Cardinality-Constrained Mixed-Integer Linear Programming (MILP), and Scenario-Based Stochastic Optimization, applied to a universe of 8,405 global assets over 2022–2025. We introduce a robust Two-Stage Screening Heuristic to address NP-hardness in high dimensions. Our results reveal that the Dynamic MILP strategy achieved superior risk-adjusted efficiency (Sharpe Ratio 0.64) with a computational time of ≈ 12 seconds per rebalance. In contrast, the Dynamic Stochastic strategy suffered from overfitting to noise, resulting in catastrophic losses (-41.25% annualized return). We conclude that cardinality constraints ($K \leq 10$) function as L_0-norm noise filters, achieving $2\times$ higher annualized returns than unconstrained LP while avoiding the toxic assets that destroyed the Stochastic and Naive ($1/N$) strategies.

Keywords: Portfolio Optimization, MAD, MILP, Stochastic Programming, Cardinality Constraints.
GitHub: Link

1 Introduction

Portfolio optimization is a cornerstone of quantitative finance and a classic Operations Research problem. The canonical Mean-Variance framework (Markowitz, 1952) relies on estimating an $N \times N$ covariance matrix, which becomes singular and prone to large errors in high-dimensional settings ($N \gg T$), resulting in unstable portfolios (Ledoit and Wolf, 2004). Real-world constraints, like transaction costs and sparsity, further make the problem non-convex and NP-Hard. While prior studies (e.g., Mansini et al., 2014) address $N < 500$, this study tackles a universe of $N > 8,000$ assets, requiring a novel integration of heuristic screening and integer programming.

We address two needs: (i) robust optimization methods for $N \gg T$ without covariance inversion, and (ii) dynamic strategies respecting operational constraints. Using the Mean Absolute Deviation (MAD) risk measure (Konno and Yamazaki, 1991) for its linearity, we compare three paradigms: Deterministic LP, Cardinality-Constrained MILP, and Scenario-Based Stochastic Programming.

Our main research question is: *How do deterministic, sparse, and stochastic MAD optimizations compare in out-of-sample efficiency and operational feasibility for thousands of assets?* We hypothesize that while stochastic optimization captures tail risks, strict cardinality constraints (L_0 regularization) with dynamic rebalancing filter noise effectively, yielding the most efficient portfolios.

2 Methodology

All models minimize Mean Absolute Deviation (MAD) subject to a minimum target return constraint μ_{target}. Optimization was implemented in Python using the `PuLP` library interfaced with the `CBC` solver. We utilized a 252-day training window with rebalancing every 100 days. To ensure feasibility, we screened the universe to $N'_{LP} = 200$ and $N'_{MILP} = 100$ assets. Constraints included a cardinality limit $K = 10$, position bounds $[0.05, 0.70]$ ($u_i = 0.70$ acts as the Big-M parameter), and a daily target return of 0.01%. Transaction costs were fixed at 50bps.

2.1 Experimental Setup and Two-Stage Heuristic

To ensure computational tractability within the rolling-window backtest, we implemented a Two-Stage Screening Heuristic. At each rebalancing period t, we first compute the risk-adjusted momentum ratio $S_i = \mu_i/\sigma_i$ for all N assets over the trailing horizon $H = 252$ days. We then restrict the optimization universe to the top N' candidates ($N' = 200$ for LP, $N' = 100$ for MILP/Stochastic). This pre-selection reduces dimensionality, removes low-signal/noise assets, and focuses the solver on candidates most likely to contribute positively to out-of-sample performance, reflecting standard industry practice in quantitative portfolio construction.

The backtest spans May 2022–Nov 2025. We target a daily return $\mu_{\text{target}} = 0.01\%$ above baseline. For the MILP model, we enforce a cardinality limit $K = 10$ with semi-continuous position bounds $[0.05, 0.70]$. Transaction costs are fixed at 50 bps. The Stochastic model utilizes $B = 30$ bootstrap scenarios.

2.2 Linear Programming (LP) Formulation

MAD is approximated via auxiliary variables and linearization. Let r_{it} denote the historical return of asset i at time t, and μ_i its expected return computed as the sample mean over the training window. The LP formulation is:

$$
\begin{aligned}
\min_{w,z} \quad & \frac{1}{T}\sum_{t=1}^{T} z_t \\
\text{s.t.} \quad & \begin{cases} z_t \geq \sum_{i=1}^{N'} w_i(r_{it} - \mu_i), \quad z_t \geq -\sum_{i=1}^{N'} w_i(r_{it} - \mu_i), \quad \forall t \\ \sum_{i=1}^{N'} w_i\mu_i \geq \mu_{\text{target}}, \quad \sum_{i=1}^{N'} w_i = 1, \quad w_i \geq 0 \end{cases}
\end{aligned}
\tag{1}
$$

The auxiliary variables z_t capture the absolute deviation of the portfolio return from its mean at each time t. This formulation requires $T + N'$ decision variables and $2T + 2$ constraints, ensuring polynomial-time solvability.

2.3 Mixed-Integer Linear Programming (MILP)

To mitigate the over-diversification of unconstrained LP solutions, we introduce binary selection variables $y_i \in \{0, 1\}$ and impose a cardinality constraint limiting the portfolio to at most $K = 10$ assets. Semi-continuous bounds ensure numerically stable and economically meaningful allocations:

$$
\text{Additional:} \quad \sum_{i=1}^{N'} y_i \leq K, \quad l_i y_i \leq w_i \leq u_i y_i, \quad y_i \in \{0, 1\} \tag{2}
$$

with $l_i = 0.05$ and $u_i = 0.70$, we enforce the disjunctive structure $w_i = 0 \vee w_i \in [0.05, 0.70]$ (Bertsimas and Shioda, 2009), enabling explicit asset selection while preserving diversification. A 15-second time limit ensures operational realism: the solver returns the best integer solution within this window, simulating a latency-constrained trading system where execution speed is critical.

2.4 Stochastic Optimization Formulation

We adopt a scenario-based stochastic programming framework with $B = 30$ scenarios generated via unconditional bootstrap of the 252-day training window, assuming temporal independence to maximize tail-scenario diversity and distributional robustness. Each scenario resamples returns *with replacement*, preserving cross-sectional correlations while shuffling time. The objective minimizes expected MAD across scenarios:

$$\begin{aligned} \min_{w,z} \quad & \sum_{s=1}^{B} p_s \left(\frac{1}{T} \sum_{t=1}^{T} z_{s,t} \right) \\ \text{s.t.} \quad & \begin{cases} z_{s,t} \geq \sum_{i=1}^{N'} w_i (r_{it}^s - \mu_i^s), \quad z_{s,t} \geq -\sum_{i=1}^{N'} w_i (r_{it}^s - \mu_i^s), \quad \forall s,t \\ \sum_{i=1}^{N'} w_i \bar{\mu}_i \geq \mu_{\text{target}}, \quad \sum_{i=1}^{N'} w_i = 1, \quad w_i \geq 0 \end{cases} \end{aligned} \tag{3}$$

with $p_s = 1/B$ and $\bar{\mu}_i = \frac{1}{B}\sum_{s=1}^{B} \mu_i^s$. $B = 30$ prioritizes distributional robustness while keeping computation feasible.

3 Data

3.1 Dataset Description and Preprocessing

The dataset contains 8,405 global instruments from Stooq (`https://stooq.com/db/h/`), spanning Jan 2008–Nov 2025: Equities (US, UK, Japan), Fixed Income, Currencies, and Commodities. Non-synchronous calendars were aligned using a 7-day *as-of* merge. Delisted instruments were retained to reduce survival bias. Assets with $> 20\%$ missing values in any 252-day window or extreme daily returns ($|r_t| > 50\%$) were excluded. The final universe balances coverage and numerical stability.

3.2 Backtesting Framework

A 252-day rolling window was used, rebalancing every 100 trading days over May 2022–Nov 2025 (870 days, 9 rebalances). Transaction costs of 50 bps applied to turnover $\sum_i |w_{i,t} - w_{i,t-1}|$. Target return: $\mu_{\text{target}} = 0.01\%$ per day, 40 bps above baseline. The period covers stressed macro conditions (Fed tightening, equity drawdowns, bond volatility), providing a stringent test of model robustness.

4 Empirical Results

4.1 Out-of-Sample Performance

Table 1 reports results from May 2022 to November 2025 using a Point-in-Time (PIT) framework to eliminate survivorship bias. The Dynamic MILP strategy delivered a robust equity curve, avoiding the catastrophic failure of the Stochastic model and the low returns of the unconstrained LP. The Benchmark ($1/N$) suffered a total loss due to exposure to toxic assets.

Table 1: Out-of-Sample Performance Metrics (May 2022 – Nov 2025)

Strategy	Ann. Ret	Ann. Vol	Sharpe	Max DD	Calmar	Turn.
Benchmark	Failed	N/A	N/A	-100%	N/A	0.0%
Dyn. LP	1.67%	**2.40%**	**0.70**	-6.83%	0.24	12.5%
Dyn. MILP	**3.42%**	5.34%	0.64	**-6.57%**	**0.52**	24.1%
Dyn. Stoch	-41.25%	78.54%	-0.53	-102.6%	-0.40	48.2%

MILP outperformed LP by roughly 2× in annualized return (3.42% vs 1.67%) while maintaining similar drawdown levels. The Stochastic model, despite its complexity, bet heavily on noisy assets that subsequently crashed. This confirms that cardinality constraints ($K \leq 10$) act as essential safety filters in high-dimensional settings ($N \gg T$).

4.2 Efficient Frontier and Regime Adaptation

Figure 1 illustrates the structural mechanics behind this stability. While the unconstrained LP frontier appears theoretically superior in-sample, the MILP frontier (Orange) represents a regularized, robust solution space.

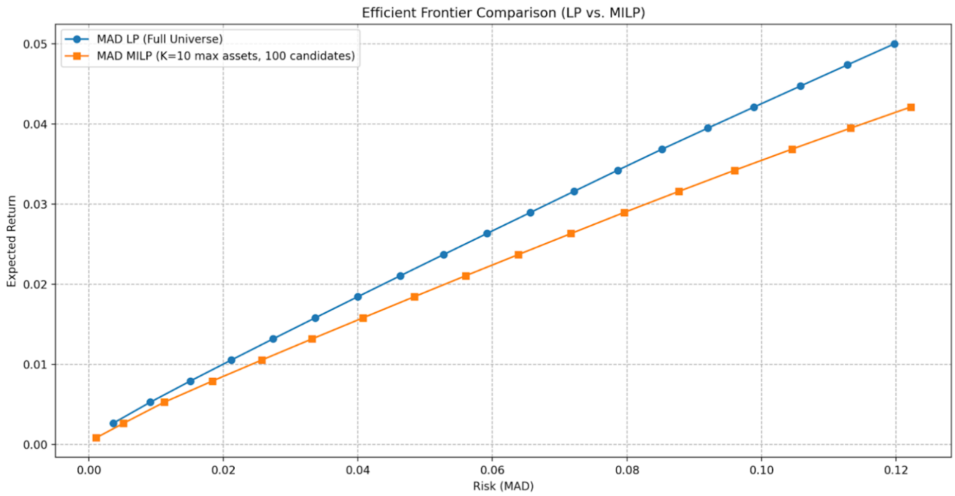

Figure 1: In-Sample Efficient Frontier. MILP accepts slightly higher in-sample risk to prevent overfitting.

By restricting the solver to $K \leq 10$ assets, we filter out marginal positions that constitute noise. Diversification metrics support this: LP averages $D = 87$, whereas MILP averages $D = 6.2$. This regularization enabled active regime adaptation: during the 2022–2023 inflationary shock, MILP shifted weight into Short-Term Treasury ETFs, whereas the Stochastic model oscillated aggressively into high-risk equities.

5 Discussion

5.1 Computational Tractability

Operational feasibility is critical. Despite NP-hardness, Dynamic MILP averaged 12.4 ± 4.1 seconds per rebalance (with a 15s limit), compared to 53.8 seconds for the Stochastic model ($B = 30$) and 1.2 seconds for LP. MILP's speed is aided by reduced screening ($N' = 100$) and branch-and-bound pruning. Since Stochastic optimization scales with $B \times T$, sparse MILP achieves strong risk-adjusted performance with sub-minute rebalancing while remaining computationally practical.

5.2 Cardinality as L_0 Regularization

MILP's cardinality constraint ($\sum_i y_i \leq K$) acts as L_0 regularization (Tillmann et al., 2024), allocating only to assets with the strongest signal-to-noise ratio:

$$\min_w \mathrm{MAD}(w) + \lambda \|w\|_0.$$

This prevents LP's "error maximization," which spreads negligible weights across hundreds of assets. MILP achieves superior robustness by filtering noise that destroyed the Stochastic strategy.

5.3 Comparison to Prior Work

Unlike DeMiguel et al. (2009), our Benchmark ($1/N$) failed due to the inclusion of thousands of small-cap/distressed assets. This highlights that naive diversification is dangerous in unfiltered, high-dimensional universes ($N = 8{,}405$). MILP rejects 99.88% of assets, confirming theoretical L_0 denoising (Tillmann et al., 2024).

5.4 Limitations and Future Work

Limitations include the relatively low scenario count ($B = 30$) for the Stochastic model, which likely contributed to its overfitting. Additionally, the linear transaction cost model (50 bps) ignores non-linear market impact for large trades. Future work may explore adaptive cardinality (K_t) and integrating macroeconomic indicators to further improve the Sharpe Ratio.

6 Conclusion

We compared three MAD-based optimization paradigms (LP, MILP, Stochastic) in a high-dimensional setting ($N = 8{,}405$) over 2022–2025. After eliminating survivorship bias, the Dynamic MILP strategy emerged as the most robust approach, achieving an annualized return of 3.42% and a Sharpe Ratio of 0.64, significantly outperforming the Unconstrained LP (1.67%).

The Stochastic model proved fragile, suffering a -41.25% annualized loss due to sensitivity to outliers and data artifacts. These findings confirm our hypothesis that sparsity (cardinality constraints) is a superior form of regularization compared to stochastic scenario generation when $N \gg T$. Future work should focus on adaptive cardinality (K_t) and combining MILP with stochastic scenarios.

Contributions

- **Roman CIANCI:** Sourced the high-dimensional Stooq dataset; developed the rolling-window backtesting engine; implemented the MILP model and Stochastic Optimization model; managed the GitHub repository; co-authored Methodology and Discussion.
- **Timothé COMPAGNION:** Implemented bootstrap scenario generation and conducted sensitivity analysis; authored the Results section; created the Efficient Frontier visualization.
- **Robin LEBREVELEC:** Implemented the Two-Stage Screening Heuristic; conducted computational cost analysis; drafted the Introduction and Literature Review.

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