Introduction to Program Synthesis (SS 2025)

Chapter 1 - Introduction

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Why program synthesis? Some major software bug issues...

1962	Mariner 1 Spacecraft → incorrect guidance signals → $320 million
1994	Pentium FDIV Bug → → lookup table bug → $475 million
1996	ESA Ariane 5 Flight V88 → conversion error → $370 million
2019	Boeing 737 Max → MCAS software bug → fatalities
2024	Intel Raptor Lake → stability issues
2024	CrowdStrike Falcon software update → world-wide outage

Why program synthesis?

- Software is **complex** and **fragile**
- Prone to human error
- Automated search and verification → Holy grail of software development?
- Well, its complicated[1]
  - ⇝ Despite the hype, LLMs lack of genuine formal reasoning capabilities[2][3]
  - ⇝ More issues and shortcomings will be addressed later in the course

[1] https://hackernoon.com/testing-llms-on-solving-leetcode-problems-in-2025

[2] https://garymarcus.substack.com/p/llms-dont-do-formal-reasoning-and

[3] https://arxiv.org/abs/2410.05229

General Idea

- Software development process → can be automated (to some extend)
- Synthesis of **correct** and **efficient** computer programs with respect to predefined specifications
- Universal approach → not limited to a specific programming language, paradigm or level

Definition and Problem Statement

Definition (Program Synthesis)

Automated discovery of executable programs that match predefined forms of constraints such as input-output relations.

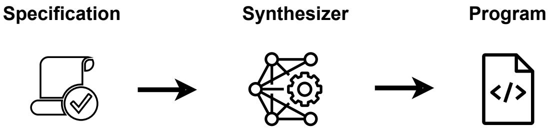
Fundamental Notations

Table: Notation

Symbol	Definition
$\mathcal{P}$	Program
$\mathcal{X}$	Set of inputs
$\mathcal{Y}$	Set of outputs
Φ	Constraints
ψ	Specification

Program Synthesis

- Generation of computer programs from a collection of **artifacts**
  - ⇝ **Automated search** in a space of possible programs
  - ⇝ Matching **semantic** and **syntactic requirements**
- Interaction between **synthesis** and **machine learning**

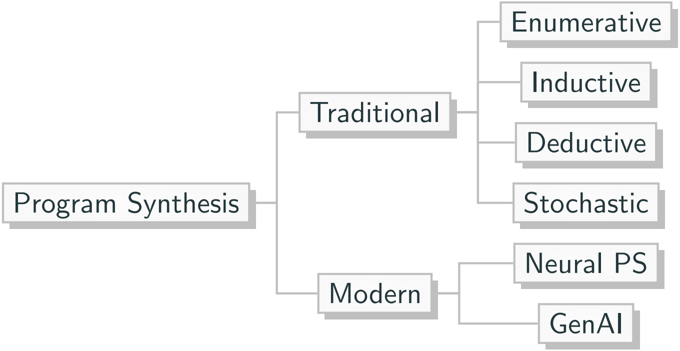


General Definition and Problem Statement

Definition (Program Statement)

Seek a program P that satisfies a specification ψ on a input set $\mathcal{X}$ and is subject to constraints Φ.

Taxonomy (excerpt)



Further Classification

- PS search methods are often performed directly on **high-level symbolic representations** of problems
  - $\rightsquigarrow$ Discovery of human-readable solutions
- Modern PS methodology → based on ML paradigms
- PS can be therefore nowadays considered as **symbolic AI/ML** methodology.

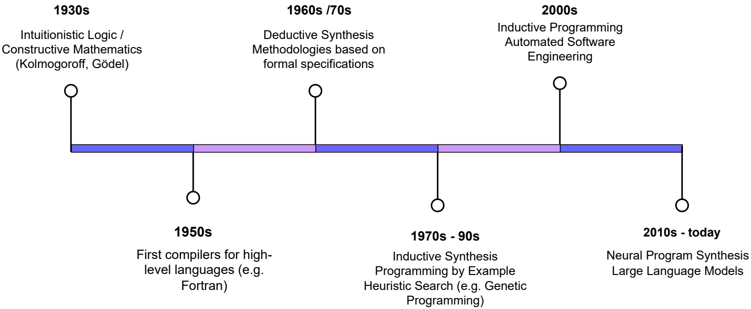
Scope and Distinction

- Synthesis of programs from a collection of given artifacts
  - $\leadsto$ **Artifacts** → represent semantic and syntactic requirements (for the programs)
- Program synthesis by its definition is not (merely):
  - $\leadsto$ **Compilation** → Thus, compilation and synthesis are very closely related as they share the similar goals
  - $\leadsto$ **Machine learning** → Incremental and transformative synthesis process that is nowadays *backed and guided* by ML concepts and techniques
  - $\leadsto$ **Optimization** → Definition of optimization objectives for the synthesis process

Historical Background

1932	Discovery of algorithms with proofs (Kolmogorov, 1932) [Kol32]
1954	FORTRAN Automatic Coding System (Backus et al., 1954) [Bac+57]
1957	Applications of recursive arithmetic to the problem of circuit synthesis (Alonzo Church) [Fri63]
1969	Solving Sequential Conditions by Finite State Strategies (Buchi and Landweber) [BL90]
1971	Toward automatic program synthesis (Mannar et al., 1971) [MW71]
1975	Transformational synthesis (Manna & Waldinger, 1975) [Kol32]
1977	*Synthesis-by-transformations* paradigm (Burstall & Darlington, 1977) [BD77]
1979-80	Automated Deduction (Manna & Waldinger, 1979; Manna & Waldinger, 1980; Bibel, 1980) [MW79; MW80; Bib80]
1980s	Efficient strategies for synthesis (generate & test, divide & conquer, problem reduction) (Smith, 1985b; Smith, 1987b) [Smi83; Smi86]
1985	Adaptive Generation of Simple Sequential Programs (Nichael L. Cramer) [Cra85]
1989	Hierarchical Genetic Algorithms Operating on Populations of Computer Programs (John R. Koza) [Koz89]
1992	Genetic Programming (John R. Koza) [Koz93]

Timeline



1930s

Intuitionistic Logic /
Constructive Mathematics
(Kolmogoroff, Gödel)

1950s

First compilers for high-level languages (e.g. Fortran)

1960s /70s

Deductive Synthesis
Methodologies based on formal specifications

1970s - 90s

Inductive Synthesis
Programming by Example
Heuristic Search (e.g. Genetic Programming)

2000s

Inductive Programming
Automated Software Engineering

2010s - today

Neural Program Synthesis
Large Language Models

Paradigms

- **Proofs-as-programs:**
  - Synthesis of an algorithm → constructive proof of the statement
- **Synthesis by transformations:**
  - Derivation of programs from given specifications by forward propagation
  - Originally based on rewrite rules that encode logical laws

Paradigms

- **Deductive**
  - Deductive reasoning → specific conclusions are drawn from general premises
  - General versions being transformed to a specified version that matches the specification
  - **Top-down approach** → from general information to the specific conclusions
- **Inductive**
  - Inductive reasoning → drawing general conclusions based on specific observations
  - Incremental synthesis based on given examples (i.e. input-output mappings)
  - **Bottom-up approach** → from specific to general
- **Stochastic** → Applying principles of randomized search heuristics
- **Neural** → Use of deep learning based methodologies

Search spaces

- **Symbolic:** Compositions from a finite set of functions $\mathcal{F}$ and set of finite terminals (e.g. variables or constants) $\mathcal{T}$
  - $\mathcal{P} \in \mathcal{F} \times \mathcal{T}$
  - Often represented as non-linear data structures such as trees and graphs
- **Syntactical:** Compositions from nonterminal symbols $\mathcal{N}$ and terminal symbols $\mathcal{T}$ in accordance to production rules $\mathcal{P}$
  - Syntax space is language specific
  - Terminal symbols $\rightarrow$ Functions or operators
  - Nonterminal symbols $\rightarrow$ Variables or constants
- **Semantic:** Discrete or continious output space of candidate programs
  - $\mathcal{P}(\mathcal{X}) \in \mathbb{R}^n$ or $\mathbb{N}^n$

Objectives

- **Feasibility:** Discovery whether a feasible program can be obtained that matches the predefined specification
    - Usually no constraints specified
    - No prior knowledge given → *Synthesis from scratch*
- **Efficiency:** Consideration of optimization objectives to obtain a more efficient solution
    - Minimization of runtime and/or complexity
- **Reliabitlity:** Construction of robust programs that guarantee the predefined specification

Challenges

- Search in symbolic and syntax space is often **ill-conditioned**:
  - Large search spaces
  - Roughness of the cost function landscape
  - No gradient descent → gradient-free methods needed
  - Poor local search features
- **Fragility** of syntactical compositions
- Programming requires **formal reasoning**

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