# Introduction to Program Synthesis (WS 2024/25)

## Chapter 2.1 - Foundations (Program Representation: Tree)

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# Computer Programs: Representations

- Data structures can be used to represent programs on a syntactical level:
  - Non-linear:
    - Tree
    - Graph
  - Linear:
    - List
    - Stack
- Another way are grammars:
  - Backus-Naur form

# Computer Programs: Representations
## Expression Tree

- Trees are commonly used to represent programs and expression for different purposes:
  - Symbolic expressions $\rightarrow$ **expression tree**
  - Syntactic structures $\rightarrow$ **program tree**
  - Context-free grammars $\rightarrow$ **parse tree**

# Computer Programs: Representations
## Expression Tree

### Definition (Computer Program)

A computer program $\mathcal{P}$ can be defined as a finite **sequence** of **operations** (or instructions) executed with respective **arguments** and in specific order. Let

- $\mathcal{F}$ a finite non-empty set of operations
- $\mathcal{T}$ a finite non-empty set of arguments
- $\Omega$ an order of operations (of the programming language)

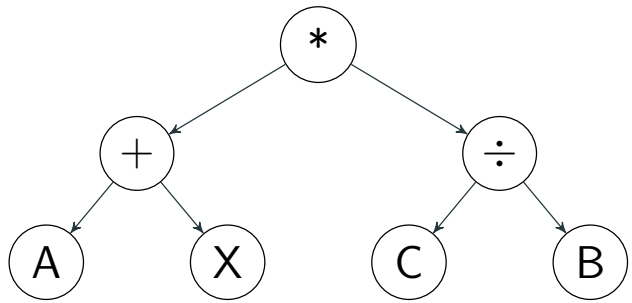
$\mathcal{P}$ is then a sequence of elements from $\mathcal{F} \times \mathcal{T}$ w.r.t. $\Omega$

# Computer Programs: Representations
## Expression Tree

- Trees are a suitable way to represent **symbolic expressions**
- Each node is annotated either with a function/operator or terminal symbol
  - **Inner nodes** represent **operators**
  - Each **leaf node** represents an **operand**
- Concept is also known in as **syntax tree** in literature

# Computer Programs: Representations
## Expression Tree

### Example (Expression Tree)




$$\mathcal{F} = \{ + , - , *, \div \}$$

$$\mathcal{T} = \{ A, X, C, B \}$$

$$\mathcal{E} = \text{Edges}$$

$$\Psi = (A + X) * (C \div B)$$

## Definition (Expression Tree)

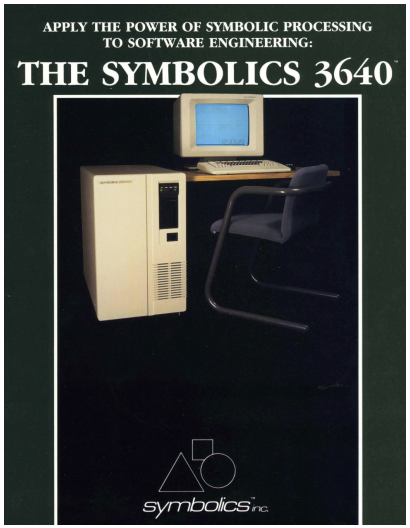
An expression tree $\mathcal{P}$ is an element of $\mathcal{T} \times \mathcal{F} \times \mathcal{E}$:

- $\mathcal{F}$ is a finite non-empty set of operators
- $\mathcal{T}$ is a finite non-empty set of terminals
- $\mathcal{E}$ is a finite non-empty set of edges

Each leaf node represents an **operand** and each inner node represents an **operator** or **function**. Let $\phi : \mathcal{P} \mapsto \Psi$ be a **decode function** which maps $\mathcal{P}$ to a expression $\Psi$

# Computer Programs: Representations
## Expression Tree

- LISP (*List Processor*) $\rightarrow$ family of programming languages
  - Developed in the late 1950s
  - Second oldest high-level programming language (besides Fortran)
  - Follows functional and procedural paradigm
- Influenced by $\lambda$-calculus
- Popular representative of the Lisp family $\rightarrow$ Common Lisp
  - Often applied for symbolic computation $\rightarrow$ *symbolic programming language*
  - Originally designed for **symbolic data processing**




# Computer Programs: Representations
## Expression Tree

```
1  ; Function which iteratively calculates the GCD by using Euclidean algorithm
2  (defun euclidean (a, b)
3    (if (< a b)
4        (let* (tmp a))
5        (setq ((a b)
6               (b tmp))))
7    (let (r 0) (d 0))
8    (loop
9          (setq (r (mod a b)))
10         (when (= r 0) (return a))
11         (setq (a (floor (/ a b))))
12         (setq (b r))))
```

Listing: Iterative implementation of Euclidean's algorithm in LISP

```
1  ; Function which recursively calculates the GCD by using Euclidean algorithm
2  (defun euclidean (a b)
3      (if (< a b)
4        (progn
5          (let* (tmp a)
6          (setq a b b tmp))))
7      (if (= b 0)
8          (return a))
9      euclidean(b, a mod b))
```

Listing: Recursive implementation of Euclidean's algorithm in LISP

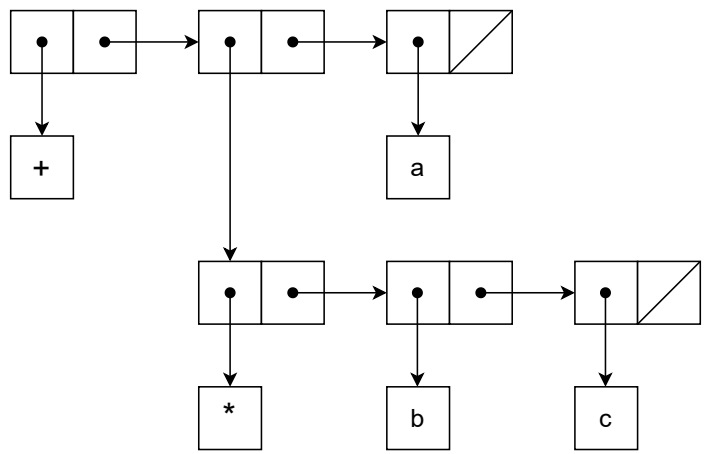
# Computer Programs: Representations
## Expression Tree

- LISP programs can be composed entirely of expressions with no use of statements
- Programs are made of three major building blocks:
  - **Atoms**
  - **Lists**
  - **Symbolic expressions**
- Lists $\rightarrow$ **sequence** of **atoms** or **nested sublists** encapsulated in the parentheses

# Computer Programs: Representations
## Expression Tree

### Example (Nested List)

```
1 (defun foobar (a b c) (+ a (* b c)))
```



# Computer Programs: Representations
## Expression Tree

- Representation of LISP code and data $\rightarrow$ **S-expression** (symbolic expression)
  - Also known as sexpr or s-exp
  - Storage unit for LISP programmes
- LISP statements are represented in **prefix notation** (Polish notation)
- S-expressions are **nested lists** of symbols $\rightarrow$ can be represented as **tree-like structures**

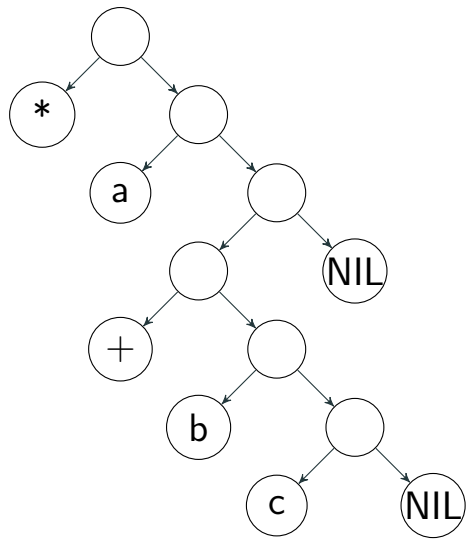
# Computer Programs: Representations
## Expression Tree

- A S-expression can be considered a composition of:
  - Atoms $\rightarrow$ Symbols, numbers or strings
  - Cons cell $\rightarrow$ Connection of subexpressions: $(S_1 \ . \ S_2)$
  - List termination symbol $\rightarrow$ NIL

# Computer Programs: Representations
## Expression Tree

**Example (S-Expression)**

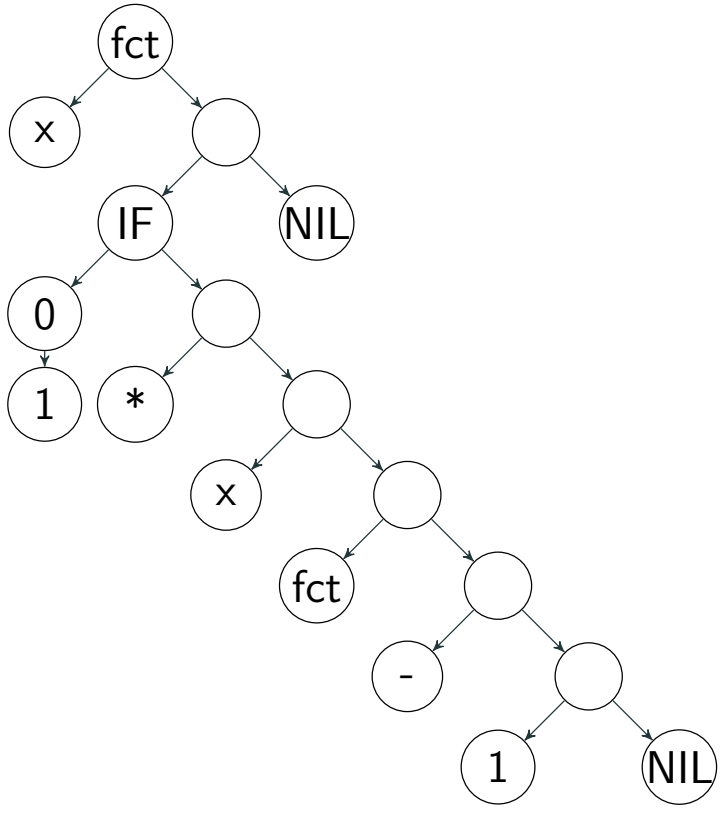
```
(* a (+ b c))
```



# Computer Programs: Representations
## Expression Tree

### Example (S-Expression)

```
(defun fct (x)
        (if (zerop x)
         1
        (* x (fct (- x 1)))))
```



# Computer Programs: Representations
## Parse Tree

- **Abstract Syntax Tree** (AST) $\rightarrow$ abstract representation of a program's syntactic structure
  - Expression and program trees can be considered AST
- **Parse Tree** $\rightarrow$ syntactic structure that relates to some context-free grammar
- Compilers transform the **high-level code** of the respective language into a **parse tree**
  - Sometimes called a concrete syntax tree (CST)

# Computer Programs: Representations

## Parse Tree

### Definition (Parse Tree)

A parse tree is a **tree-like hierarchical representation** of the derivation of a character string in accordance with a **formal grammar**.

# Computer Programs: Representations

## Parse Tree

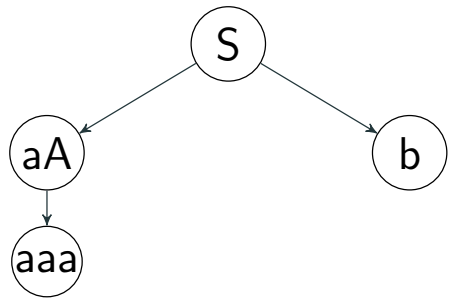
- **Parsing** $\rightarrow$ Breaking down a syntactic element into its component parts
  - Describing their **syntactic components**
  - Performing syntactic analysis
  - Establishing hierarchical structures
- **Tree** $\rightarrow$ Suitable format for representing hierarchical structures
- **Grammar** $\rightarrow$ Set of rules that defines how to write valid statements (for the respective programming language)

**Example (Parse Tree)**

Let $G$ be a grammar with vocabulary $V = \{ A, B, a, b \}$, set of terminal symbols $T = \{a, b \}$, start symbol $S$ and production rules $P = \{\mathcal{S} \to aA, \mathcal{S} \to b, \mathcal{A} \to aa \}$

# Computer Programs: Representations
## Parse Tree

**Example (Parse Tree)**




$$\mathcal{S} \rightarrow aA$$

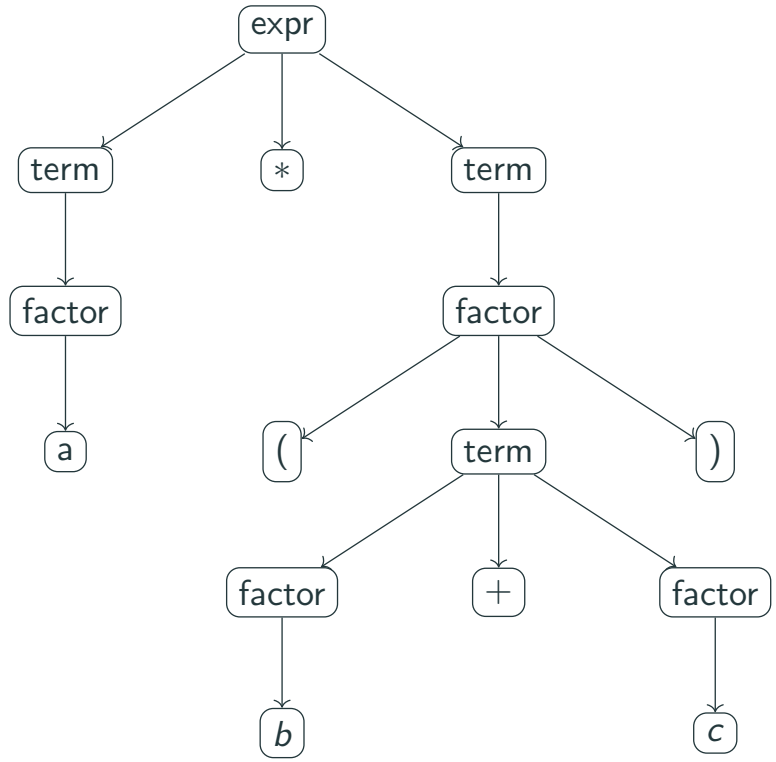
$$\mathcal{S} \rightarrow b$$

$$\mathcal{A} \rightarrow aa$$

# Computer Programs: Representations
## Parse Tree

### Example (Parse Tree)

a * ( b + c )



# Computer Programs: Representations

## Parse Tree

- **AST**
  - Do not represent every detail from the real syntax (abstract)
  - Represents an **abstract syntactic structure** of a symbolic or language construct
- **Parse Tree**
  - Respect the **syntactical** and **grammatical definitions** of the respective language
  - Represents an **concrete syntactic** construct