Introduction to Program Synthesis (WS 2024/25)

Chapter 3.1 - Traditional Methodologies (Optimization Fundamentals)

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Traditional Methodologies

Fundamentals of optimization

- Problem statement in PS $\rightarrow$ can be **formulated** as **optimization problem**
- **Optimization** $\rightarrow$ maximizing or minimizing a function w.t.r. to its parameter space θ
- Methods used for PS are derived from the field of optimization
- Consideration the task of optimization $\rightarrow$ **minimizing or maximizing** an **objective function**

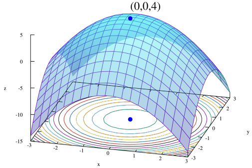

Figure: Global maximum of $z = f(x, y) = -(x^2 + y^2) + 4$ at $(x, y, z) = (0, 0, 4)$, Source: Wikimedia

Traditional Methodologies

Fundamentals of optimization: Terminology

- **Objective Function** $\rightarrow$ function to be minimized or maximized
- **Global Extrema** $\rightarrow$ its largest and smallest values
- **Local Extrema** $\rightarrow$ extrema of a specific neighborhood
- **Plateau** $\rightarrow$ flat area of the search space
- **Shoulder** $\rightarrow$ plateau that has an uphill edge
- **Feasibility** $\rightarrow$ Point that satisfies all constraints
- **Neighbourhood** $\rightarrow$ Local region of a point in the search space

Traditional Methodologies

Fundamentals of optimization: Definitions

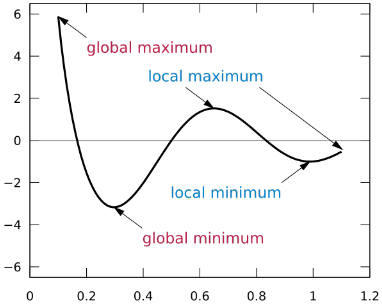

Figure: Extrema of an objective function (Source: Wikimedia)

Traditional Methodologies
Fundamentals of optimization: Definitions

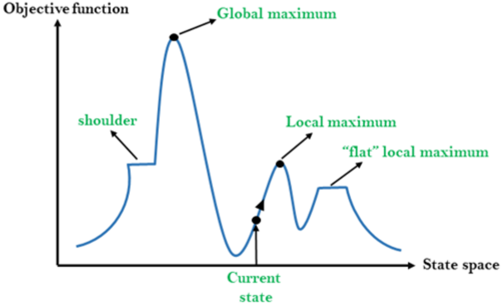

Figure: Landscape in random local search (Source: javapoint.com)

Definition (Continuous Optimization Problem)

$$
\begin{aligned}
\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & f_i(x) \quad i = 1, \ldots, M \\
\text{subject to} \quad & g_j(x) \leq 0, \quad j = 1, \ldots, J \\
& h_j(x) = 0, \quad k = 1, \ldots, K
\end{aligned}
$$

- $f_i : \mathbb{R}^n \mapsto \mathbb{R}$
- $g_j(x) \rightarrow$ inequality constraints
- $h_k(x) \rightarrow$ equality constraints
- $j \geq 0$ and $k \geq 0$
- $j = k = 0 \rightarrow$ unconstrained optimization problem
- $x = (x_1, x_2, \ldots, x_i)^T \rightarrow$ design vector

- Given a X and a function, $f : X \mapsto Y$ where X is arbitrary and Y has a total order:
  - Finding the arguments of the maxima $\rightarrow$ $\underset{x}{\operatorname{argmax}} f(x) := \{x \in X \ : \ f(y) \leq f(x) \ \forall y \in X\}$
  - Finding the arguments of the minima $\rightarrow$ $\underset{x}{\operatorname{argmin}} f(x) := \{x \in X \ : \ f(y) \geq f(x) \ \forall y \in X\}$

Definition (Combinatorial Optimization Problem)

A combinatorial optimization problem $\mathcal{P}$ is a 4-tuple ($\mathcal{I}$, $\mathcal{F}$, $\mathcal{M}$, $\mathcal{G}$) where the goal is to find an instance $x \in I$. [PS98]

- $\mathcal{I} \rightarrow$ set of instances
- $\mathcal{F} \rightarrow$ feasibility function
- $\mathcal{M} \rightarrow$ measure ($\mathcal{M} \in \mathbb{R}_{\geq 0}$)
- $\mathcal{G} \rightarrow$ goal

References I

[PS98] C.H. Papadimitriou and K. Steiglitz. *Combinatorial Optimization: Algorithms and Complexity*. Dover Books on Computer Science. Dover Publications, 1998. ISBN: 9780486402581. URL: https://books.google.de/books?id=cDY-joeCGoIC.