Introduction to Program Synthesis (SS 2025) - Exercise Sheet 1
(Proofs as Programs / Verification)

Deadline: 16.05.2025 - 23:59:59 (extended)
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Please compress your files and use the mail title template also for the file name. One submission per group is sufficient. Please do all implementation in Python.

Exercise 1: Constructive Verification

Theorem 1. *If n is an even integer, then n^2 is an even integer.*

a) Verify Theorem 1 in a constructive way by writing a function that only uses the four basic arithmetic operations ($+$, $-$, $*$, $\div$). For division, please only use the floor division operator (`//`).

b) Implement the verification in a Python `lambda` function.

Exercise 2: Inductive Verification

Theorem 2. *For* $n \geq 1 : 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

a) Verify Theorem 2 with a recursive function in an inductive way. Please use comments to highlight the definition of the base case the inductive hypothesis and the inductive step. Verify all cases from the base case to n.

b) Is there a possibility to avoid recalculation of the sum of powers for each case verification? If yes, implement an optimised version of your function.

Exercise 3: Exhaustive Verification

Theorem 3. *For* $n \in \mathbb{Z} : n^2 - 1$ *is a multiple of* 3 *if* n *is not a multiple of* 3.

Verify Theorem 3 by exhaustion for $-n \leq x \leq n$.