

SAT/SMT solvers

2. Conflict-Driven Clause Learning algorithm

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Moscow, 2023

Obvious algorithm

- Brute force
- $O(2^n)$

State of a clause under an assignment

A clause is satisfied if one or more of its literals are satisfied, conflicting if all of its literals are assigned but not satisfied, unit if it is not satisfied and all but one of its literals are assigned, and unresolved otherwise.

$$\{x_1 \mapsto 1, x_2 \mapsto 0, x_4 \mapsto 1\},$$

$(x_1 \vee x_3 \vee \neg x_4)$	is satisfied,
$(\neg x_1 \vee x_2)$	is conflicting,
$(\neg x_1 \vee \neg x_4 \vee x_3)$	is unit,
$(\neg x_1 \vee x_3 \vee x_5)$	is unresolved.

The clause $C := (\neg x_1 \vee \neg x_4 \vee x_3)$ and the partial assignment $\{x_1 \rightarrow 1, x_4 \rightarrow 1\}$ imply the assignment x_3 and $\text{Antecedent}(x_3) = C$

Implication graph

An implication graph is a labeled directed acyclic graph $G(V, E)$, where:

- V represents the literals of the current partial assignment (we refer to a node and the literal that it represents interchangeably). Each node is labeled with the literal that it represents and the decision level at which it entered the partial assignment
- E with $E = \{(v_i, v_j) | v_i, v_j \in V, \neg v_i \neg \text{Antecedent}(v_j)\}$ denotes the set of directed edges where each edge (v_i, v_j) is labeled with $\text{Antecedent}(v_j)$
- G can also contain a single conflict node labeled with k and incoming edges $\{(v, k) | \neg v \in c\}$ labeled with c for some conflicting clause c

Example

$$c_1 = (\neg x_1 \vee x_2)$$

$$c_2 = (\neg x_1 \vee x_3 \vee x_5)$$

$$c_3 = (\neg x_2 \vee x_4)$$

$$c_4 = (\neg x_3 \vee \neg x_4)$$

$$c_5 = (x_1 \vee x_5 \vee \neg x_2)$$

$$c_6 = (x_2 \vee x_3)$$

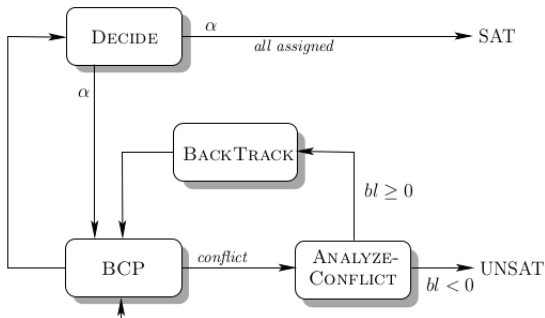
$$c_7 = (x_2 \vee \neg x_3)$$

$$c_8 = (x_6 \vee \neg x_5)$$

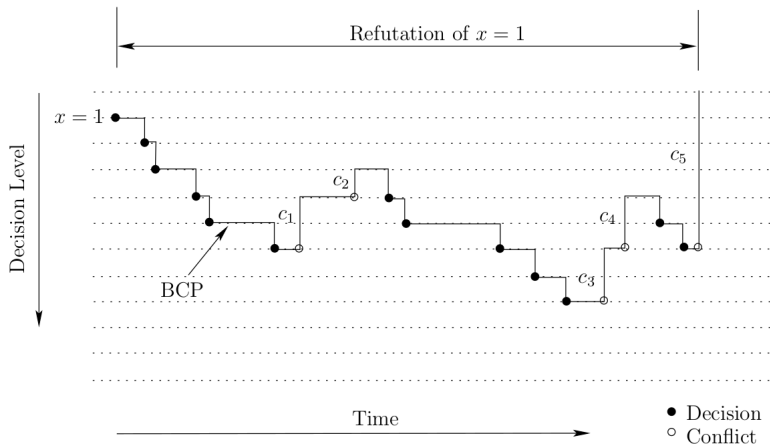
Let $x_1 @ 6$ and $\neg x_5 @ 3$

Conflict-driven clause learning

```
1. function CDCL
2.   while (TRUE) do
3.     while (BCP() = "conflict") do
4.       backtrack-level := ANALYZE-CONFLICT();
5.       if backtrack-level < 0 then return "Unsatisfiable";
6.       BackTrack(backtrack-level);
7.     if ¬DECIDE() then return "Satisfiable";
```



It is never the case that the solver enters decision level dl again with the same partial assignment

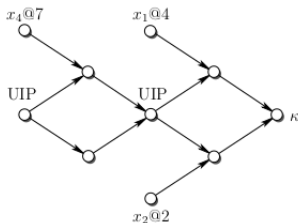


Unique Implication Point (UIP)

Given a partial conflict graph corresponding to the decision level of the conflict, a unique implication point (UIP) is any node other than the conflict node that is on all paths from the decision node to the conflict node

First UIP

A first UIP is a UIP that is closest to the conflict node



Binary resolution and related terms

$$\frac{(a_1 \vee \dots \vee a_n \vee \beta) \quad (b_1 \vee \dots \vee b_m \vee \neg\beta)}{(a_1 \vee \dots \vee a_n \vee b_1 \vee \dots \vee b_m)}$$

where

$a_1, \dots, a_n, b_1, \dots, b_m$ are literals and β is a variable. The variable β is called the resolution variable. The clauses $(a_1 \vee \dots \vee a_n \vee \beta)$ and $(b_1 \vee \dots \vee b_m \vee (\neg\beta))$ are the resolving clauses, and $(a_1 \vee \dots \vee a_n \vee b_1 \vee \dots \vee b_m)$ is the resolvent clause

Analyze conflict

1. **if** *current-decision-level* = 0 **then return** -1;
2. *cl* := *current-conflicting-clause*;
3. **while** (\neg STOP-CRITERION-MET(*cl*)) **do**
4. *lit* := LAST-ASSIGNED-LITERAL(*cl*);
5. *var* := VARIABLE-OF-LITERAL(*lit*);
6. *ante* := ANTECEDENT(*lit*);
7. *cl* := RESOLVE(*cl*, *ante*, *var*);
8. add-clause-to-database(*cl*);
9. **return** clause-asserting-level(*cl*); ▷ 2nd highest decision level in *cl*

Stop-criterion-met(*cl*) is to return true if and only if *cl* contains the negation of the first UIP as its single literal at the current decision level.

$$c_1 = (\neg x_4 \vee x_2 \vee x_5)$$

$$c_2 = (\neg x_4 \vee x_{10} \vee x_6)$$

$$c_3 = (\neg x_5 \vee \neg x_6 \vee \neg x_7)$$

$$c_4 = (\neg x_6 \vee x_7)$$

- Jeroslow–Wang: compute for each literal l
 $J(l) = \sum_{w \in B, l \in w} 2^{-|w|}$
- Dynamic Largest Individual Sum (DLIS): at each decision level, choose the unassigned literal that satisfies the largest number of currently unsatisfied clauses
- Variable State Independent Decaying Sum (VSIDS): A new conflict clause, adds 1 to the score of each literal that appears in it. Periodically divide all scores by 2.

