№ 18

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma'(z) = -\frac{1}{(1+e^{-z})^2} \cdot (-e^{-z}) =$$

$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} =$$

$$= \sigma \cdot (1-\sigma)$$

№ 19.

$$g_k(s_1 \dots s_K) = \frac{e^{s_k}}{\sum\limits_{l=1}^{K} e^{s_l}}$$

$$R^{(i)} = -\sum_{k=1}^{K} I(y^{(i)} = k) \ln g_k(s_1 \dots s_K)$$

1) доказать: $\frac{\partial g_k}{\partial s_l} = g_k \cdot (I(k=l) - g_l)$

$$\cancel{\frac{\partial g_k}{\partial s_l} = \frac{\partial}{\partial s_l}(e^{s_k}) \cdot \frac{\partial}{\partial s_l}\left(\frac{1}{\sum e^{s_l}}\right) =}$$

$$\cancel{= I(k=l) \cdot e^{s_k} \cdot -\frac{1}{(\sum e^{s_l})^2} \cdot e^{s_l}}$$

$$\frac{\partial g_k}{\partial s_l} = \frac{\partial (e^{s_k})}{\partial s_l} \cdot \frac{1}{\sum e^{s_l}} + \frac{\partial}{\partial s_l}\left(\frac{1}{\sum e^{s_l}}\right) \cdot e^{s_k} =$$

$$= g_k \cdot I(s=k) + e^{s_k} \cdot -\frac{1}{(\sum e^{s_l})^2} \cdot e^{s_l} =$$

$$= g_k \cdot I(s=k) - \left(\frac{e^{s_k}}{\sum e^{s_l}}\right) \cdot \frac{e^{s_l}}{\sum e^{s_l}} =$$

$$= g_k(I(k=l) - g_l)$$

2) $\dfrac{\partial R^{(i)}}{\partial g_k} = -\dfrac{I(y^{(i)})}{g_k}$ — Доказать.

$$R^{(i)} = \begin{cases} -\ln g_k(s_1 \ldots s_K), & y^{(i)} = k \\ 0, & y \end{cases}$$

$$R^{(i)} = -\ln g_{y^{(i)}}(s_1 \ldots s_K)$$

$$\frac{\partial R^{(i)}}{\partial g_k} = -\frac{I(y^{(i)} = k)}{g_k}$$

3) $\dfrac{\partial R^{(i)}}{\partial s_l} = g_l - I(l = y^{(i)})$ — Доказать

$$\frac{\partial R^{(i)}}{\partial s_l} = \sum_{k=1}^{K} \frac{\partial R^{(i)}}{\partial g_k} \cdot \frac{\partial g_k}{\partial s_l} =$$

$$= \sum_{k=1}^{K} -\frac{I(y^{(i)} = k)}{g_k} \cdot g_k \left(I(k = l) - g_l \right) =$$

$$= \sum_{k=1}^{K} g_\ell \cdot I(y^{(i)} = k) - \sum_{k=1}^{K} I(y^{(i)} = k) \cdot I(\ell = k) =$$

$$= \cancel{g_\ell}\, g_\ell - I(y^{(i)} = \ell)$$