Time Series Analysis

- Hunter

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Quiz

- What is Time Series Data
- What is Stationarity
- What is Seasonality
- What is Moving Average
- What is Auto Regression
- What is Exponential Smoothing

Topics

- Time Series Data
- Smoothing Techniques
- Box-Jenkins Approach

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

1.

Time series data

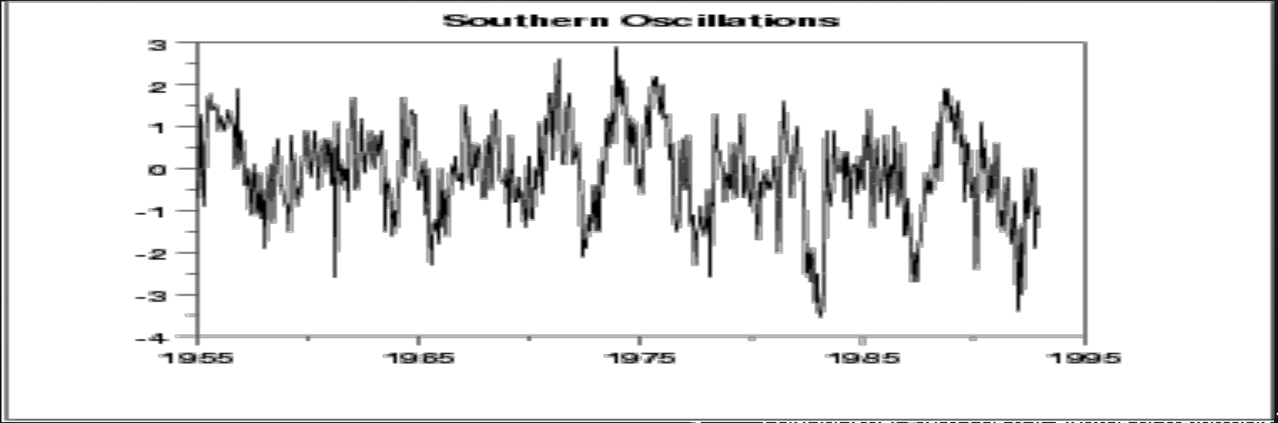
CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

What is Time series

"An ordered sequence of values of a variable at equally spaced time intervals."

- ✗ Obtain an understanding of the underlying forces and structure that produced the observed data
- ✗ Fit a model and proceed to forecasting, monitoring or even feedback and feedforward control.

The term "univariate time series" refers to a time series that consists of single (scalar) observations recorded sequentially over equal time increments.



CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

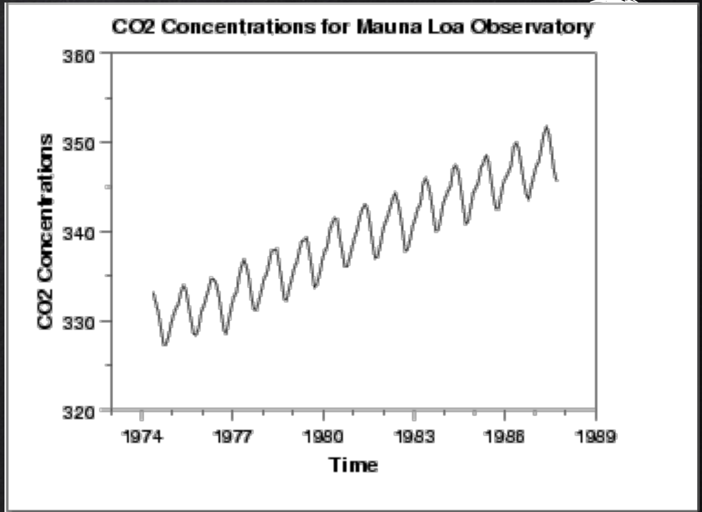
STATIONARITY

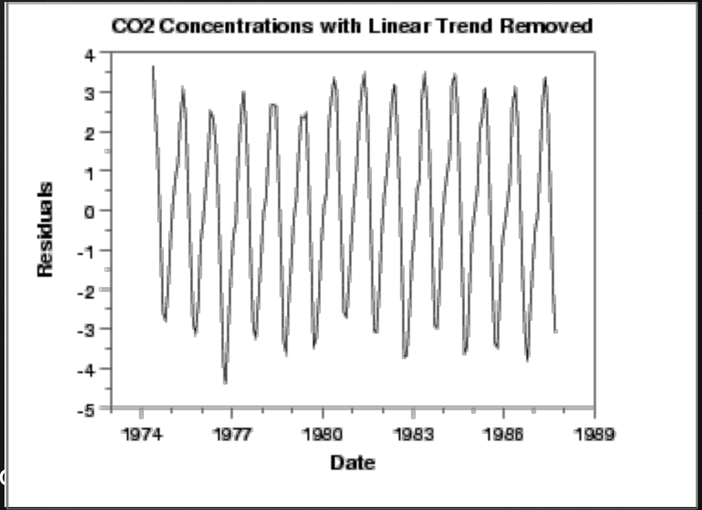
Assumption in many time series techniques is that the data are stationary.

A stationary process has the property that the mean, variance and autocorrelation structure do not change over time and no periodic fluctuations (seasonality).

Transform data to stationarity:

- ✗ difference the data. $Y_i = Z_i - Z_{i-1}$
- ✗ If the data contain a trend, we can fit some type of curve to the data and then model the residuals from that fit.
- ✗ For non-constant variance, taking the logarithm or square root of the series may stabilize the variance. For negative data, you can add a suitable constant to make all the data positive before applying the transformation..



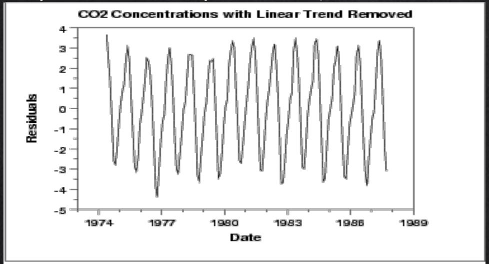


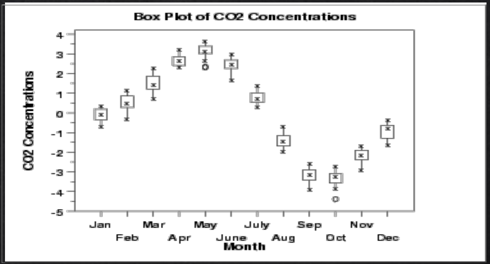
SEASONALITY

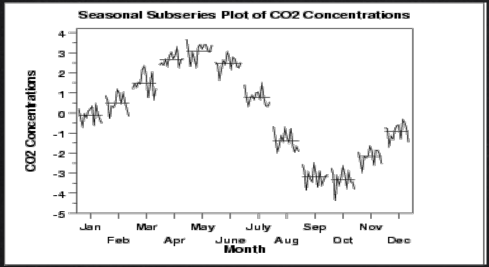
Many time series display seasonality. By seasonality, we mean periodic fluctuations.

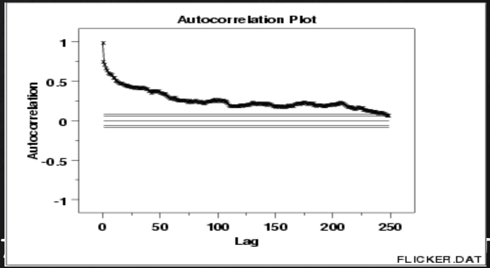
To detect seasonality:

- ✗ A run sequence plot will often show seasonality.
- ✗ A seasonal subseries plot is a specialized technique for showing seasonality.
- ✗ Multiple box plots can be used as an alternative to the seasonal subseries plot to detect seasonality.
- ✗ The autocorrelation plot can help identify seasonality.









2.

Smoothing techniques

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Single moving average

- Taking averages is the simplest way to smooth data. The computed mean or average of the data = 10
- Compute the mean of successive smaller sets of numbers of past data as follows.
- The MSE = 2.42 as compared to 3 by taking average.

Supplier	$	MA	Error	SE
1	9			
2	8			
3	9	8.667	0.33	0.111
4	12	9.667	2.33	5.444
5	9	10	-1	1
6	12	11	1	1
7	11	10.667	0.33	0.111
8	7	10	-3	9
9	13	10.333	2.667	7.111
10	9	9.667	-0.667	0.444
11	11	11	0	0
12	10	10	0	0

$$M_t = \frac{X_t + X_{t-1} + \cdots + X_{t-N+1}}{N}$$

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Centered moving average

- We could have placed the average in the middle of the time interval of three periods, that is, next to period 2. The MSE = 3.88
- This works well with odd time periods, but not so good for even time periods. We can smooth the smoothed values for even time periods.

Supplier	$	MA	Error	SE
1	9			
2	8	8.667	-0.667	0.444
3	9	9.667	-0.667	0.444
4	12	10	2	4
5	9	11	-2	4
6	12	10.667	1.333	1.777
7	11	10	1	1
8	7	10.333	-3.333	11.109
9	13	9.667	3.333	11.109
10	9	11	-2	4
11	11	10	1	1
12	10			

Supplier	$	MA	Centered
1	9		
1.5			
2	8		
2.5		9.5	
3	9		9.5
3.5		9.5	
4	12		10
4.5		10.5	
5	9		10.75
5.5		11	
6	12		
6.5			
7	11		

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Exponential Smoothing

- Exponential Smoothing assigns exponentially decreasing weights as the observation get older.
- In other words, recent observations are given relatively more weight in forecasting than the older observations.

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Single Exponential Smoothing

1. For any time period t, the smoothed value S_t is found by computing

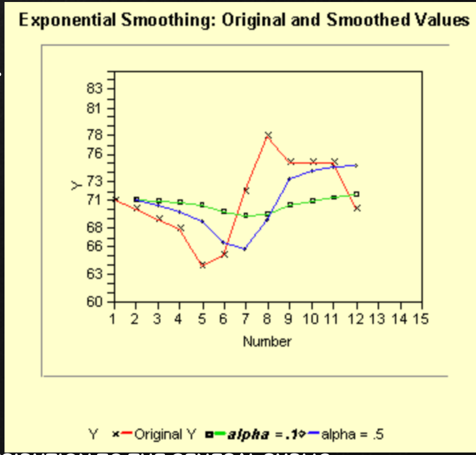
$$S_t = \alpha y_{t-1} + (1-\alpha)S_{t-1} \quad 0 < \alpha \le 1 \quad t \ge 3$$

2. Why is it called "Exponential"?

$$\begin{aligned} S_t &= \alpha y_{t-1} + (1-\alpha)\left[\alpha y_{t-2} + (1-\alpha)S_{t-2}\right] \\ &= \alpha y_{t-1} + \alpha(1-\alpha)y_{t-2} + (1-\alpha)^2 S_{t-2} \,. \end{aligned}$$

3. What is the “best” value for α? When α is close to 1, dampening is quick and when α is close to 0, dampening is slow. choose the best value for α so the value which results in the smallest MSE.

α	$(1-\alpha)$	$(1-\alpha)^2$	$(1-\alpha)^3$	$(1-\alpha)^4$
0.9	0.1	0.01	0.001	0.0001
0.5	0.5	0.25	0.125	0.0625
0.1	0.9	0.81	0.729	0.6561



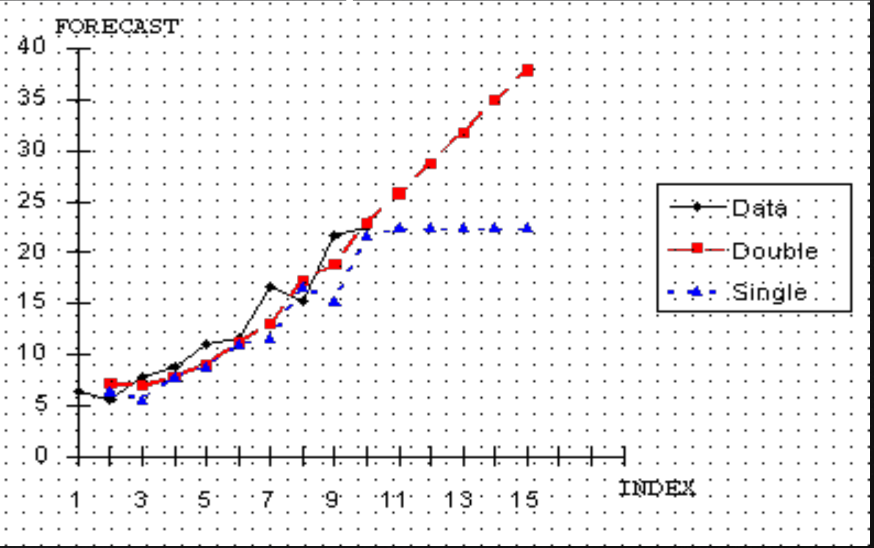
CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Double Exponential Smoothing

- Single Smoothing does not excel in following the data when there is a trend. This situation can be improved by the introduction of a second equation with a second constant, γ, which must be chosen in conjunction with α.

$$S_t = \alpha y_t + (1-\alpha)(S_{t-1} + b_{t-1}) \quad 0 \leq \alpha \leq 1$$

$$b_t = \gamma(S_t - S_{t-1}) + (1-\gamma)b_{t-1} \quad 0 \leq \gamma \leq 1$$



- Here are three suggestions for b_1.

$$b_1 = y_2 - y_1$$

$$b_1 = \frac{1}{3}[(y_2 - y_1) + (y_3 - y_2) + (y_4 - y_3)]$$

$$b_1 = \frac{y_n - y_1}{n-1}$$

Triple Exponential Smoothing

- Triple Exponential Smoothing is introduced to take care of seasonality. The resulting set of equations is called the "Holt-Winters" (HW) method after the names of the inventors.

$S_t = \alpha \frac{y_t}{I_{t-L}} + (1-\alpha)(S_{t-1} + b_{t-1})$	OVERALL SMOOTHING
$b_t = \gamma(S_t - S_{t-1}) + (1-\gamma)b_{t-1}$	TREND SMOOTHING
$I_t = \beta \frac{y_t}{S_t} + (1-\beta)I_{t-L}$	SEASONAL SMOOTHING
$F_{t+m} = (S_t + mb_t)I_{t-L+m}$	FORECAST ,

- To initialize the HW method we need at least one complete season's data to determine initial estimates of the seasonal indices I_{t-L}.

$$I_1 = (y_1/A_1 + y_5/A_2 + y_9/A_3 + y_{13}/A_4 + y_{17}/A_5 + y_{21}/A_6)/6$$

$$I_2 = (y_2/A_1 + y_6/A_2 + y_{10}/A_3 + y_{14}/A_4 + y_{18}/A_5 + y_{22}/A_6)/6$$

$$I_3 = (y_3/A_1 + y_6/A_2 + y_{11}/A_3 + y_{15}/A_4 + y_{19}/A_5 + y_{23}/A_6)/6$$

$$I_4 = (y_4/A_1 + y_6/A_2 + y_{12}/A_3 + y_{16}/A_4 + y_{20}/A_5 + y_{24}/A_6)/6$$

- Initial values for the Seasonal Indices. we work with data that consist of 6 years with 4 periods (that is, 4 quarters) per year.
  1. Compute the averages of each of the 6 years.

$$A_p = \frac{\sum_{i=1}^{4} y_i}{4}, \quad p = 1, 2, \ldots, 6.$$

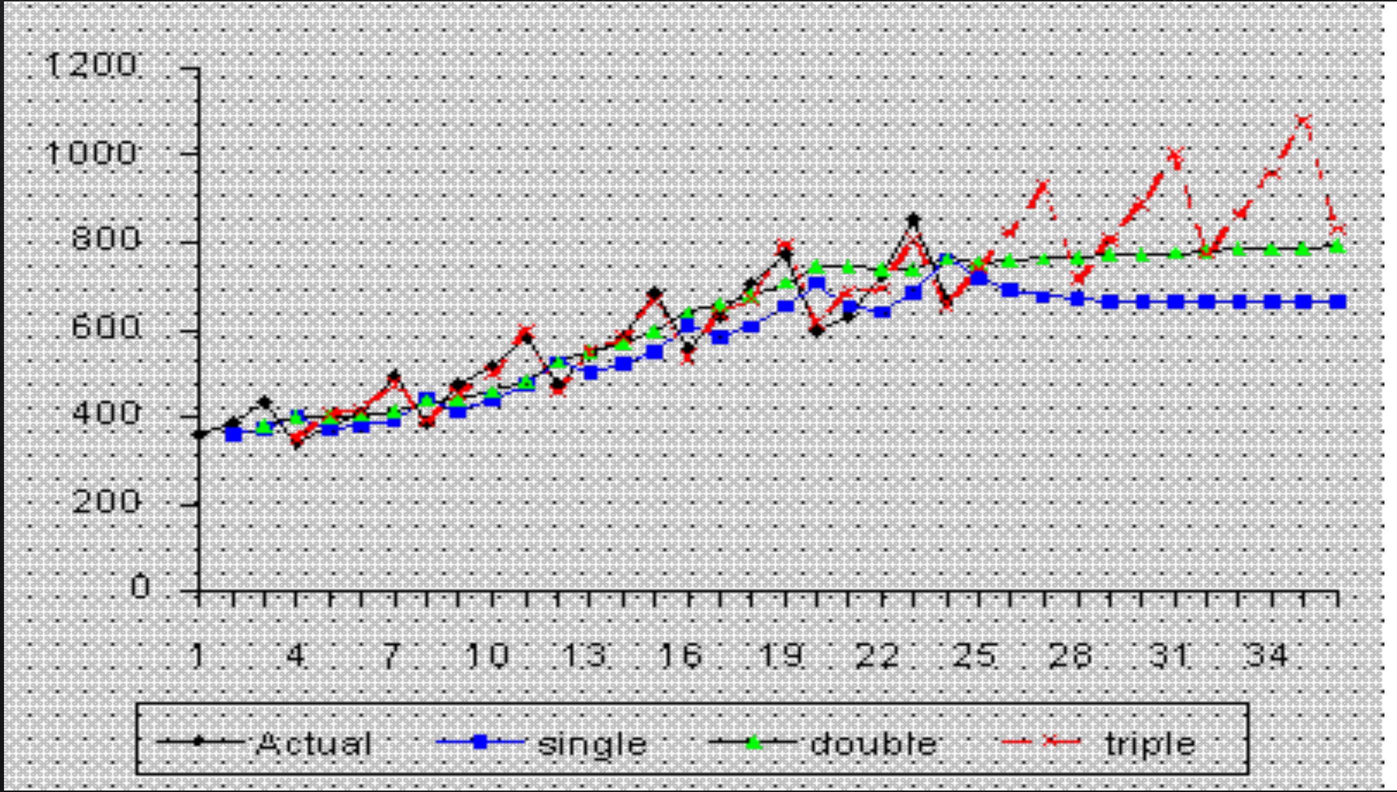
  2. Divide the observations by the appropriate yearly mean.

1	2	3	4	5	6
y_1/A_1	y_5/A_2	y_9/A_3	y_{13}/A_4	y_{17}/A_5	y_{21}/A_6
y_2/A_1	y_6/A_2	y_{10}/A_3	y_{14}/A_4	y_{18}/A_5	y_{22}/A_6
y_3/A_1	y_7/A_2	y_{11}/A_3	y_{15}/A_4	y_{19}/A_5	y_{23}/A_6
y_4/A_1	y_8/A_2	y_{12}/A_3	y_{16}/A_4	y_{20}/A_5	y_{24}/A_6

  3. Compute the average of each row:

AL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Compare Different Smoothing Methods



CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

3.

Box-Jenkins Approch

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Auto Regression

An autoregressive model is simply a linear regression of the current value of the series against one or more prior values of the series.

$$X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-1} + A_t$$

Where X_t is the time series, A_t is white noise, with μ denoting the process mean.

$$\delta = \left(1 - \sum_{i=1}^{p} \phi_i\right)\mu,$$

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Moving Average

Moving average model is conceptually a linear regression of the current value of the series against the white noise or random shocks of one or more prior values of the series.

$$X_t = \mu + A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - \cdots - \theta_q A_{t-q}$$

where X_t is the time series, μ is the mean of the series, A_{t-i} are white noise terms, and $\theta_1,\ldots,\theta_q$ are the parameters of the model. The value of q is called the order of the MA model.

$$\delta = \left(1 - \sum_{i=1}^{p} \phi_i\right)\mu\,,$$

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Box and Jenkins Method

Box and Jenkins popularized an approach that combines the moving average and the autoregressive approaches

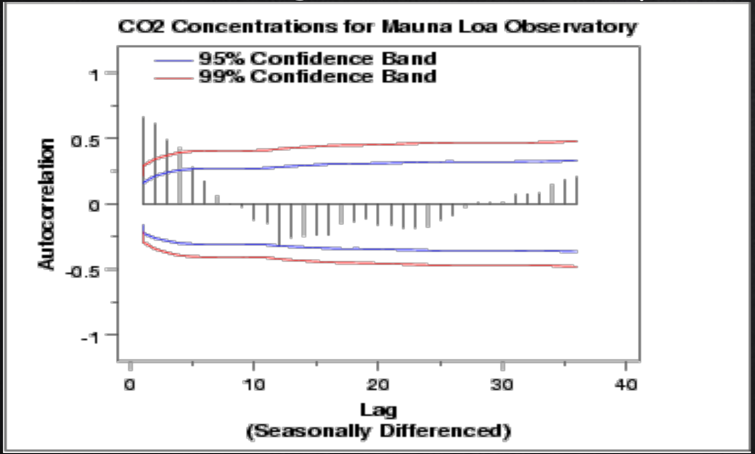
$$X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - \cdots - \theta_q A_{t-q},$$

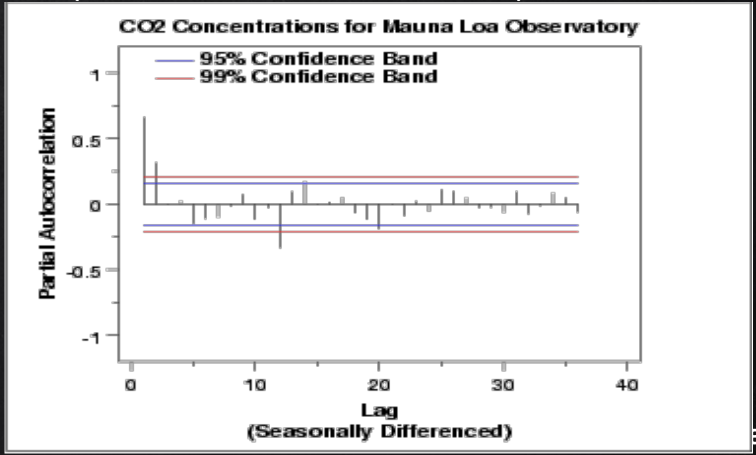
1. The Box-Jenkins model assumes that the time series is stationary. Box and Jenkins recommend differencing non-stationary series one or more times to achieve stationarity. Doing so produces an ARIMA model, with the "I" standing for "Integrated".
2. Box-Jenkins models can be extended to include seasonal autoregressive and seasonal moving average terms.
3. The most general Box-Jenkins model includes difference operators, autoregressive terms, moving average terms, seasonal difference operators, seasonal autoregressive terms, and seasonal moving average terms.

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC

Box and Jenkins Method

1. Determine if the series is stationary and if there is any significant seasonality that needs to be modeled.
2. Differencing to achieve stationarity
3. Seasonal differencing
4. Identify the order (i.e., the p and q) of the autoregressive and moving average terms using autocorrelation plot (ACF) and the partial autocorrelation plot (PACF).





BLIC

Recommended Readings

https://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4.htm

CONFIDENTIAL – NOT FOR USE AND/OR DISTRIBUTION TO THE GENERAL PUBLIC