① a) $y = 2 - \frac{1}{2} tg(2x - \frac{\pi}{4})$

D(f):

$2x - \frac{\pi}{4} \neq \frac{\pi}{2}$

$2x \neq \frac{3\pi}{4}$

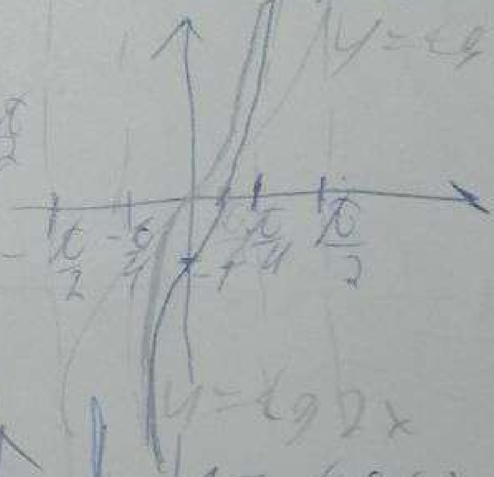
$2x \neq -\frac{\pi}{4}$

$x \neq \frac{3\pi}{8}$

$x \neq -\frac{\pi}{8}$

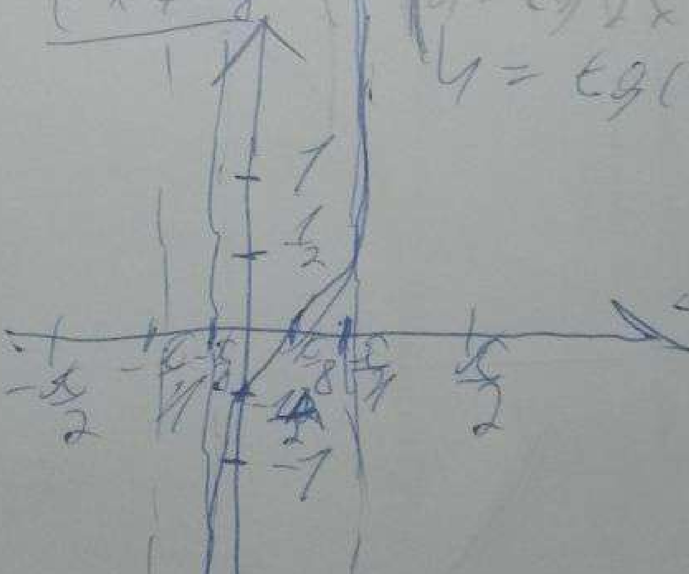
$y = tg\,x$

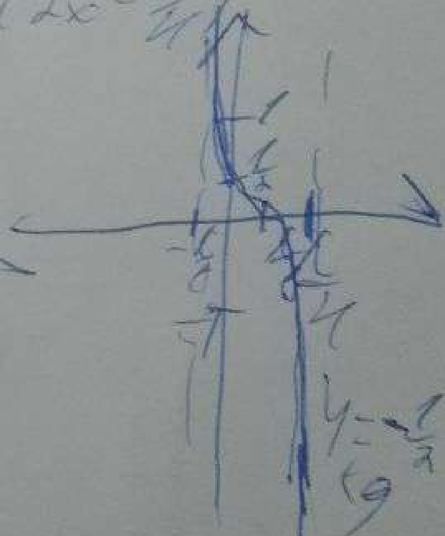
E(f)



$y = tg\,2x$

$y = tg(2x - \frac{\pi}{4})$





$y = -\frac{1}{2} tg$

$y = \frac{1}{2} tg(2x - \frac{\pi}{4})$

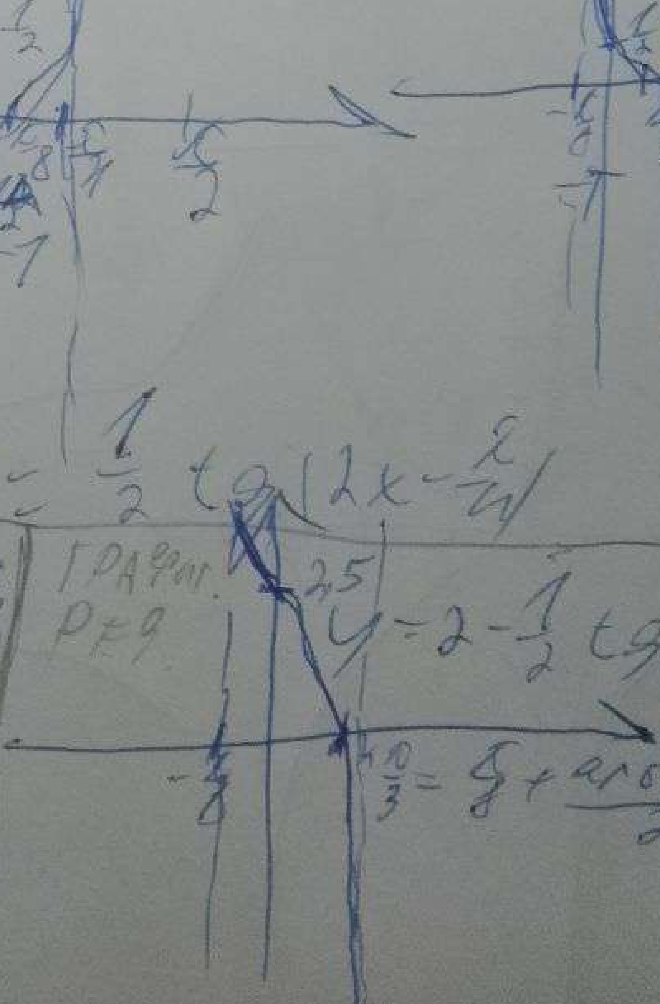
D(f): $x \neq -\frac{\pi}{8} + 2\pi k$

$x \neq \frac{3\pi}{8} + 2\pi n$

E(f): $(-\infty; +\infty)$

График. РГЗ

$y = 2 - \frac{1}{2} tg(2x - \frac{\pi}{4})$



$\frac{\pi}{3} = \frac{\pi}{8} + \frac{arctg\,4}{2}$

② б) $y = -\left|\dfrac{x}{2-|x-1|}\right|$

$D(f): 2-|x-1| \neq 0$

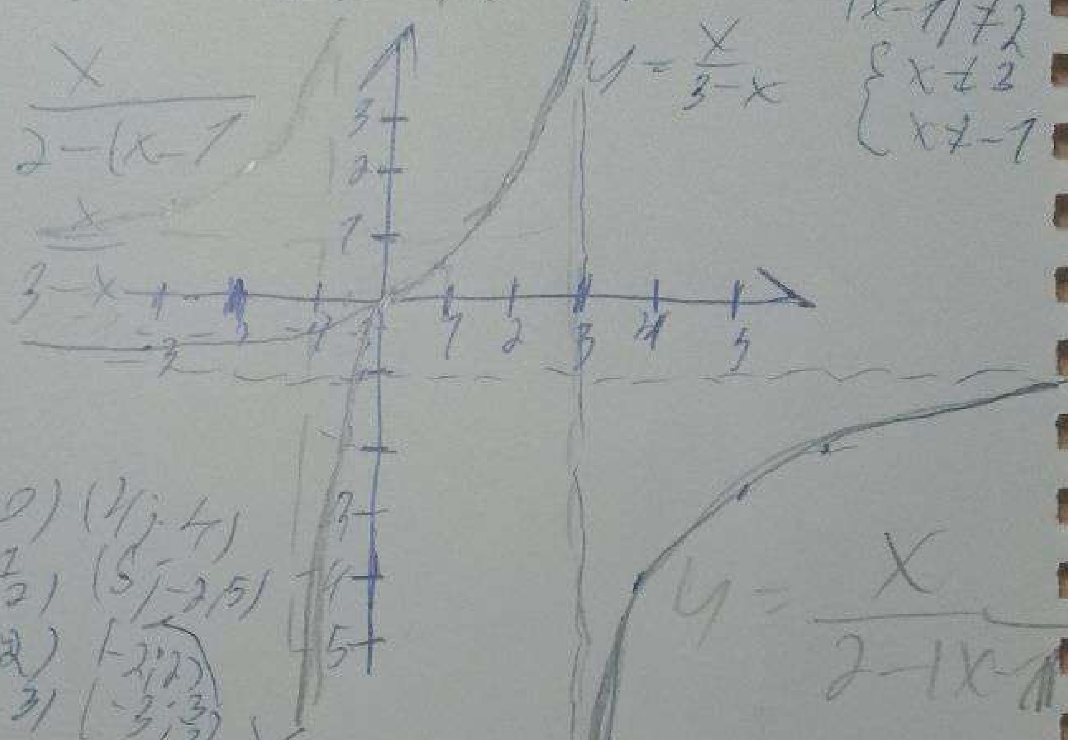
$|x-1| \neq 2$

$\begin{cases} x \neq 3 \\ x \neq -1 \end{cases}$

1) $y = \dfrac{x}{2-|x-1|}$

$y = \dfrac{x}{3-x}$

$y = \dfrac{x}{3-x}$



(0;0) (4;-4)

(1;1/2) (5;-2,5)

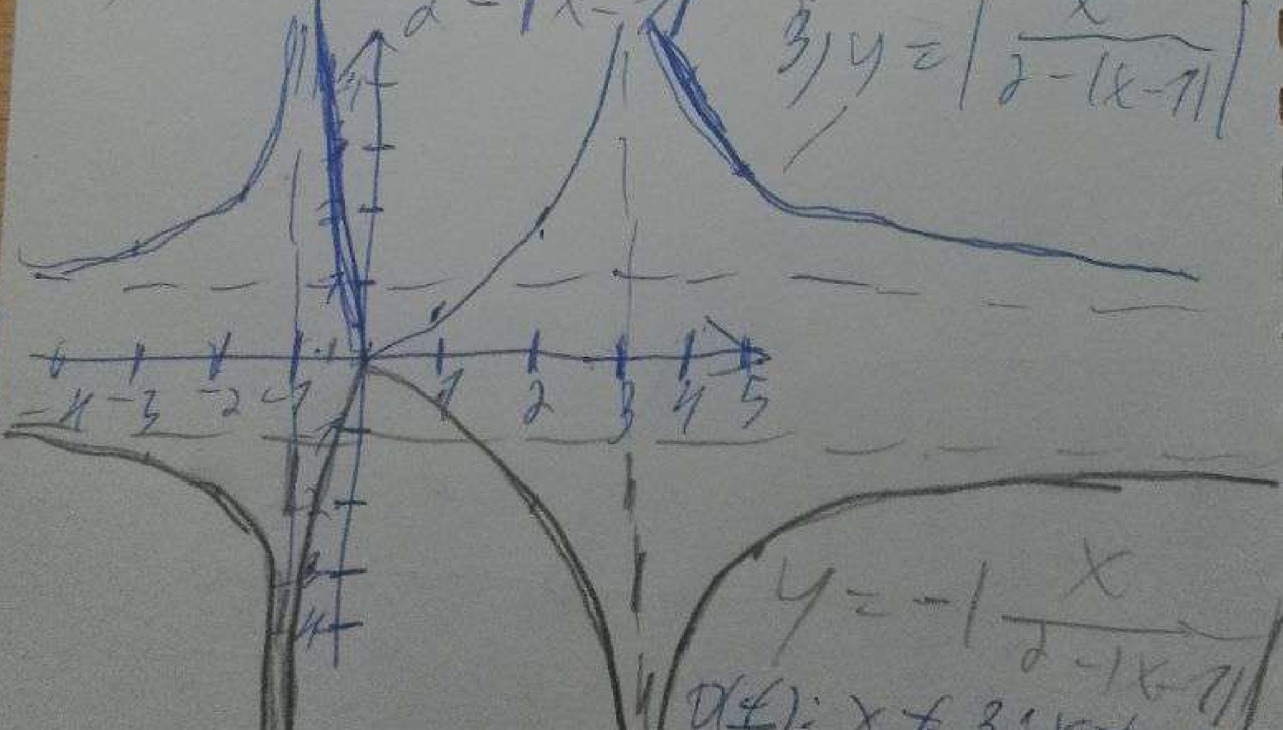
(2;2) (-2;2/3)

(-1/2;-3) (-3;3/2)

$y = \dfrac{x}{2-|x-1|}$

2) $y = \dfrac{x}{2-|x-1|}$

3) $y = \left|\dfrac{x}{2-|x-1|}\right|$



$y = -\left|\dfrac{x}{2-|x-1|}\right|$

$D(f): x \neq 3; x \neq -1$

$E(f): [0; -\infty)$

в) $y = \ln\frac{x+1}{x} = \ln(1+\frac{1}{x})$

$y = \ln x$

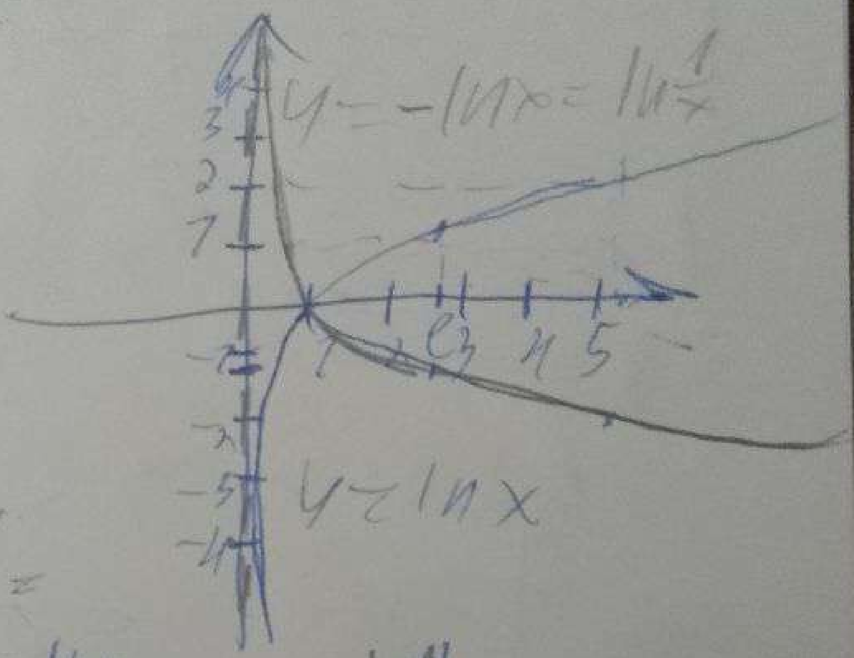
при $x \to 0$

$\ln x \to -\infty$

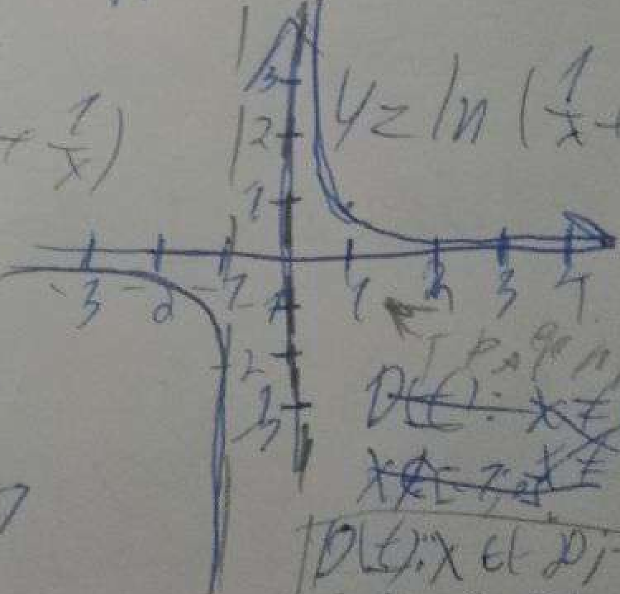
$\ln 1 = 0$

при $x < 1$

$\ln x < 0$



г) $y = \ln\frac{1}{x} = \ln x^{-1} = -\ln x$



з) $y = \ln(1+\frac{1}{x})$

$D(f)$: $1+\frac{1}{x} \neq 0$

$1+\frac{1}{x} > 0$ $\quad \frac{1}{x} \neq -1$

$\frac{1}{x} > -1$ $\quad x \neq -1$

$\begin{cases} x > 0 \\ x > -1 \end{cases}$ $\neq 0$

~~$D(f)$: $x \neq 0$~~

$\boxed{D(f): x \in (-\infty; -1) \cup (0; +\infty) \quad E(f): (-\infty; 0) \cup (0; +\infty)}$

при $x > 0$

$\ln(1+\frac{1}{x}) > 0$ | $\downarrow$ при $x \to 0$

при $x \to -1$ $\ln(1+\frac{1}{x}) \to -\infty$

при $x \to -\infty$, $1+\frac{1}{x} \to 1$ $\ln \to 0$

② 1. $\lim\limits_{x\to -3} \frac{2x^2+5x-3}{x+3} = -7$

$$\left|\frac{2x^2+5x-3}{x+3}+7\right|<\varepsilon$$

$$\left|\frac{(x+3)(2x-1)}{\cancel{x+3}}+7\right|<\varepsilon$$

$x\ne -3$

$$|2x-1+7|<\varepsilon$$

$$|2x+6|<\varepsilon$$

$$|x+3|<\frac{\varepsilon}{2}$$

Доказать: $\exists\,\delta>0: 0<|x-\overset{-3}{\cancel{2}}|<\delta$

$$0<|x+3|<\delta$$

$\exists\ x: |f(x)+7|<\varepsilon$

Если взять $\delta=\frac{\varepsilon}{2}$

тогда $0<x+3<\frac{\varepsilon}{2}$ будет

выполняться $|f(x)+7|<\varepsilon$ ч.т.д.

(3) $\lim\limits_{x\to 0} \cos\left(\frac{1}{x}\right)$

Если найдутся 2 последов. $\{x'_n\}$ и $\{x''_n\}$, сход. к числу 0 при $n \to \infty$ и такие, что $\lim\limits_{n\to\infty} f(x'_n) \neq \lim\limits_{n\to\infty} f(x''_n)$, то это будет означать, что $\lim\limits_{x\to 0} f(x)$ не сущ.

$\exists\ x'_n = \frac{1}{\pi n} \to 0$, $x''_n = \frac{1}{\pi n + \frac{\pi}{2}} \to 0$ очевидно, что они сходятся к 0.

1. $\lim\limits_{n\to\infty} f(x'_n) =$
$\lim\limits_{n\to\infty} \cos(\pi n) = \pm 1$ в зав. от чет.

2. $\lim\limits_{n\to\infty} \cos(\pi n + \frac{\pi}{2}) = 0$

т.к. эти пределы не совпадают, то очевидно предела $\lim\limits_{x\to 0} f(x)$ не будет.
ч.т.д.

(1) 1) $\lim\limits_{x\to -1} \dfrac{(x^2+3x+2)^2}{x^3+2x^2-x-2} =$

$\lim\limits_{x\to -1} \dfrac{(x^2+3x+2)^2}{x(x^2-1)+2(x-1)} =$

$= \lim\limits_{x\to -1} \dfrac{(x+2)^2(x+1)^2}{(x+2)(x+1)(x-1)} = \lim\limits_{x\to -1} \dfrac{(x+2)(x+1)}{(x-1)} = 0$

Ответ: 0

2) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+13}-2\sqrt{x+1}}{\sqrt[3]{x^2-9}} =$

$\lim\limits_{x\to 3} \dfrac{x+13-4x-4}{\sqrt[3]{x^2-9}\,(\sqrt{x+13}+2\sqrt{x+1})} =$

$\lim\limits_{x\to 3} \dfrac{-3\cancel{(x-3)}(x-3)^{\frac{2}{3}}}{\cancel{\sqrt[3]{x-3}}\,\sqrt[3]{x+3}\,(\sqrt{x+13}+2\sqrt{x+1})}$

$= 0$ Ответ: 0

3) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt[10]{x}-1} = \lim\limits_{t\to 0} \dfrac{\sqrt[3]{t+1}-1}{\sqrt[10]{t+1}-1}$

$= \lim\limits_{t\to 0} \dfrac{\frac{1}{3}t}{\frac{1}{10}t} = \dfrac{10}{3}$

т.к. при $x\to 0$ $(1+x)^\alpha - 1 \sim \alpha x$

4) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cancel{(x^2+3x+2)}\,2^{\cos^2 x}-1}{\ln\sin x} =$ $t = \frac{\pi}{2}+x$

$\lim\limits_{t\to 0} \dfrac{2^{\cos^2(t+\frac{\pi}{2})}-1}{\ln\sin(\frac{\pi}{2}+t)} = \lim\limits_{t\to 0} \dfrac{2^{\sin^2 t}-1}{\ln\cos t} =$

$= \lim\limits_{t\to 0} \dfrac{2^{t^2}-1}{\ln\cos t} = \lim\limits_{t\to 0} \dfrac{\ln 2\, 2^{t^2}}{\ln 1} = +\infty$

5) $\lim\limits_{x\to 0} \left(\dfrac{1+x^2\cdot 2^x}{1+x^2\cdot 5^x}\right)^{1/\sin^3 x} =$

$\cancel{e^{\lim}}\ e^{\lim\limits_{x\to 0} \frac{\ln\left(\frac{1+x^2\cdot 2^x}{1+x^2\cdot 5^x}\right)}{\sin^3 x}}$

$\lim\limits_{x\to 0} \dfrac{\ln(1+x^2\cdot 2^x)-\ln(1+x^2\cdot 5^x)}{\sin^3 x} =$

$\lim\limits_{x\to 0} \dfrac{x^2\cdot 2^x - x^2\cdot 5^x}{\sin^3 x} = \dfrac{x^2(2^x-5^x)}{x^3} =$

$\lim\limits_{x\to 0}\dfrac{2^x-5^x+1-1}{x} = \lim\limits_{x\to 0}\dfrac{\ln 2\, 2^x - \ln 5\, 5^x}{x} = \dfrac{\ln 2 - \ln 5}{0} =$

$= -\infty$

$e^{-\infty} = 0$

6) $\lim\limits_{x\to 1\pm 0} \left(\dfrac{3x-1}{x+2}\right)^{\frac{1}{\sqrt[3]{x}-1}}$ $\quad \dfrac{3-1}{1+2} = \dfrac{2}{3} < 0$

показатель бл. вблизи $x=1$ имеет разные знаки, причем:

$\lim\limits_{x\to 1-0} \dfrac{1}{\sqrt[3]{x}-1} = -\infty \qquad \lim\limits_{x\to 1+0} \dfrac{1}{\sqrt[3]{x}-1} = +\infty$

$\Rightarrow \lim\limits_{x\to 1-0} \left(\dfrac{3x-1}{x+2}\right)^{\frac{1}{\sqrt[3]{x}-1}} = +\infty$

$\lim\limits_{x\to 1+0} \left(\dfrac{3x-1}{x+2}\right)^{\frac{1}{\sqrt[3]{x}-1}} = 0$

7) $$\lim_{x\to+\infty} \frac{\sqrt[4]{2+x^5} - \sqrt{2x^3+3}}{(x+\sin x)\sqrt{7x}} =$$

$$\lim_{x\to+\infty} \frac{x^{\frac{5}{4}}\sqrt[4]{\frac{2}{\infty}+1} - x^{\frac{3}{2}}\sqrt{2+\frac{3}{\infty}}}{(x\pm 1)\sqrt{7x}} =$$

$$\lim_{x\to\infty} \frac{x^{\frac{3}{2}}\left(x^{-\frac{1}{4}} - \sqrt{2}\right)}{x^{\frac{3}{2}}\sqrt{7}} = -\sqrt{\frac{2}{7}}$$

8) $$\lim_{x\to\frac{\pi}{2}-0} \frac{\ln(1+\operatorname{tg}x)}{\ln(1+3\operatorname{tg}x)} =$$

При $\frac{\pi}{2}-0$ $\operatorname{tg}x \to \infty$

$t = \operatorname{tg}x$

$$= \lim_{t\to\infty} \frac{\ln(1+t)}{\ln(1+3t)} = \lim_{t\to\infty} \frac{\ln t}{\ln 3t}$$

$$= \lim_{t\to\infty} \frac{\ln t}{\ln t + \ln 3} = \lim_{t'\to\infty} \frac{t'}{t'(1+\frac{\ln 3}{t'})} \frac{1}{}$$

$t' = \ln t$

$= 1$ Ответ: 1.

(5) a) $x = \frac{1}{t(t+1)}$

$y = \frac{(t+1)^2}{t}$

~~Наклонная~~ ~~Наклонная~~ асимптоты.

$x \to \infty = 1$ $t \to 0$ и $t \to -1$,

$k = \lim\limits_{t \to 0} \frac{y(t)}{x(t)} = \lim\limits_{t \to 0} \frac{(t+1)^2 t(t+1)}{t} = 1$

$b = \lim\limits_{t \to 0} (y(t) - 1x(t)) =$

$\lim\limits_{t \to 0} \frac{t^2+2t+1}{t} - \frac{1}{t(t+1)} = \lim\limits_{t \to 0} \frac{(t+1)^3 - 1}{t(t+1)}$

$= \lim\limits_{t \to 0} \frac{t((t+1)^2 + t + 1 + 1)}{t(t+1)} = \lim\limits_{t \to 0} \frac{3}{1} = 3$

$y = x + 3$ — наклонная асимпт.

При $t \to -1$ $k = 0$ $b = 0$

$y = 0$ — гор. ас.

~~При $t \to 0$ $y \to \infty$~~

$x'(t) = -\frac{1}{t^2(t+1)} - \frac{1}{t(t+1)^2}$

$y'(t) = \frac{2(t+1)t - (t+1)^2}{t^2} = \frac{(t+1)(2t - t - 1)}{t^2}$

$= \frac{(t+1)(t-1)}{t^2}$

$x'(t)$ + 0 + 0 +

$x(t)$ ↗ -1 ↗ 0 ↗

$$x'(t) = -\left(\frac{t-t-1}{t^2(t+1)^2}\right) = \frac{1}{t^2(t+1)^2}$$

$t=0 \quad t=-1$

$y'(t)$ + − 0 − +

$y(t)$ ↗ -1 ↘ 0 ↘ 1 ↗

$$y'_x = \frac{y'_t}{x'_t} = \frac{(t+1)(t-1)\, t^2(t+1)^2}{t^2}$$

$$= (t^2-1)(t+1)^2 = 0$$

$$(t+1)^3(t-1) = 0$$

$t=1 \quad t=-1$

+ max − min +

↗ -1 ↘ 0 ↘ 1 ↗

$$y(t) - 0 = y(t) = \frac{(t+1)^2}{t} = 0$$

$t=-1$

$t=0$

− 0 − 0 +

$-1 \quad 0$

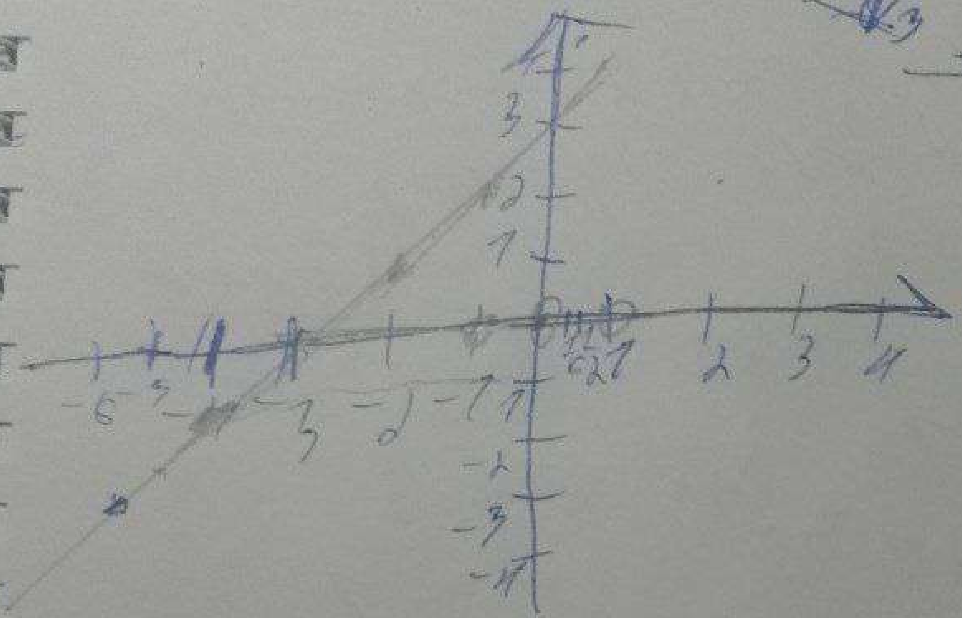
$$y(t) - x \cdot 3 = \frac{(t+1)^2}{t} - \frac{1}{t(t+1)} \cdot 3 =$$

$$\frac{(t+1)^3 - 1 \cdot 3\, t(t+1)}{t(t+1)} = \frac{t((t+1)^2 + (t+1) + 1) \cdot}{3\, t(t+1)}$$

$$\frac{3 \cdot (t-1)}{} \quad \frac{(t-1)^2 - t + 2 = 3t + 3}{} =$$

$$\frac{(t-1)^2 - 2t + 5 \quad t-1}{t+1} = \frac{t^2 - \cancel{6t} + 6}{(t+1)}$$

[illegible]



$$y(t) = \frac{(t-1)^2}{t}$$

$$x(t) = \frac{1}{t(t-1)}$$

t	x(t)	y(t)
$(-\infty; -1)$	возрастает от $-\infty$ до $+\infty$	↗
$(-1; 0)$	↗	↘
$(0; 1)$	↗	↘
$(1; +\infty)$	↗	↗

б) $y = 2 \operatorname{arctg} x - x$

$y(-x) = 2 \operatorname{arctg}(-x) - x =$

$-2 \operatorname{arctg} x - x = -y(x)$

— ФУНКЦ. НЕЧЕТ.

$y' = \frac{2}{1+x^2} + 1 = \frac{2+1+x^2}{1+x^2} =$

$\frac{x^2+3}{x^2+1} = 0 \quad x^2 \neq -3$

$x^2 \neq -1$

ЭКС. Т. ОТСУТСТВ[illegible]

$y'' = \frac{0(1+x^2) + 2x}{(1+x^2)^2} =$

$y'' = -\frac{4x}{(1+x^2)^2}$

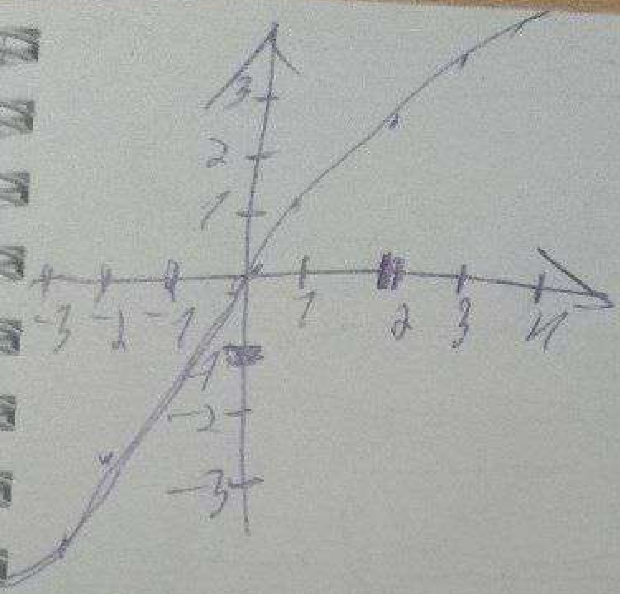
$D = 16 - 4 = 12$

$x_{1,2} = \frac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$

+ −

∪ 0 ∩



в) $y = -\arcsin\sqrt{1-4x^2} + 2\sqrt{1-4x^2}$

$D(y): 1-4x^2 \geq 0$

$4x^2 \leq 1$

$x^2 \leq \frac{1}{4}$

$x \leq \frac{1}{2}$

$x \geq -\frac{1}{2}$

$y(-x) = y(x)$ ч.т.д.

$y' = \frac{1}{4\sqrt{1-4x^2}} * -\frac{2}{2\sqrt{1-4x^2}} =$

$\frac{4\sqrt{1-4x^2}-1}{2\sqrt{1-4x^2}} = 0 \quad x = \pm\frac{1}{2}$

$-\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$y'' =$ ~~$\frac{1}{4} \cdot \ldots$~~

$$y'' = \frac{\frac{1 - \sqrt[4]{1-4x^2}}{4\sqrt[4]{1-4x^2}} - \frac{1}{2}\,\frac{\sqrt[4]{1-4x^2}-1}{\sqrt{1-4x^2}}}{(1-4x^2)} =$$

$$\frac{1}{4}\left(\frac{\sqrt[4]{1-4x^2} - 2\sqrt[4]{1-4x^2} + 2}{\sqrt{1-4x^2}}\right)$$

$$= \frac{1}{4}\left(\frac{2 - \sqrt[4]{1-4x^2}}{\sqrt{1-4x^2}}\right)\frac{1}{1-4x^2}$$

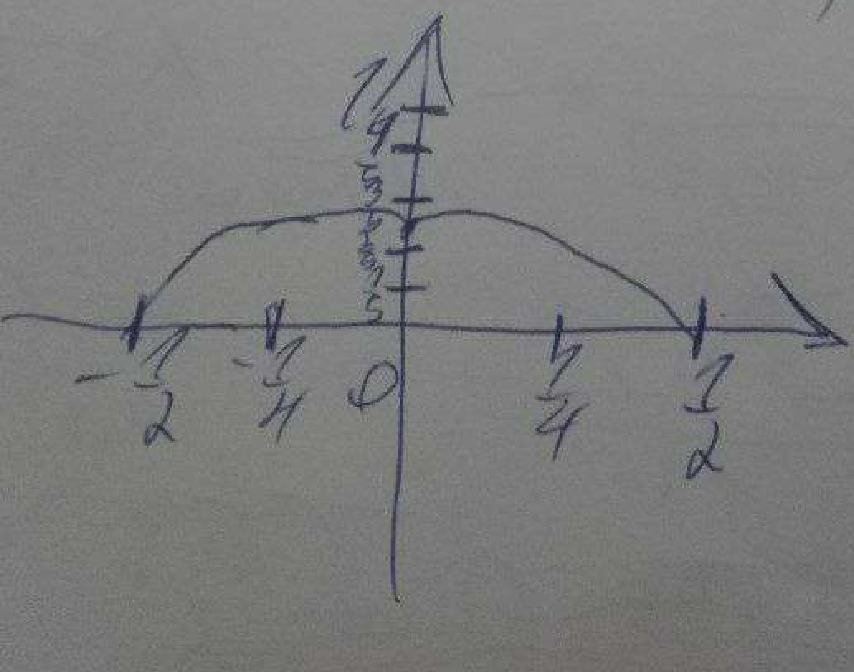
$$= \frac{2 - \sqrt[4]{1-4x^2}}{4\,(1-4x^2)^{1{,}5}}$$

$$1 - 4x^2 = 15 \cap \quad -\frac{1}{2} \quad \cup \quad \frac{1}{2} \cap$$

$$4x^2 = -15$$

$$x^2 = -\frac{15}{4}$$

$y(0) = 2 - \arcsin 1$



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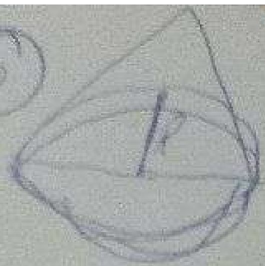
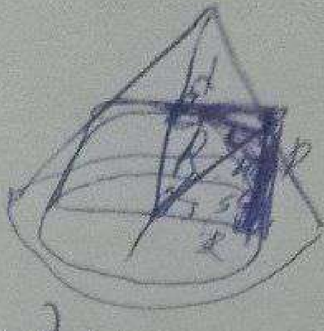




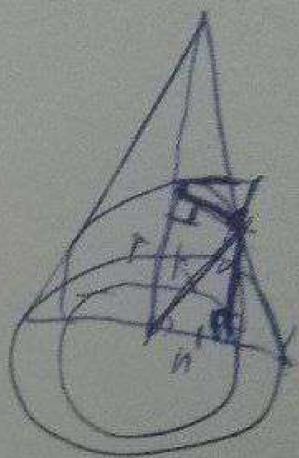
$V_k = \frac{1}{3}\pi r^2 h$

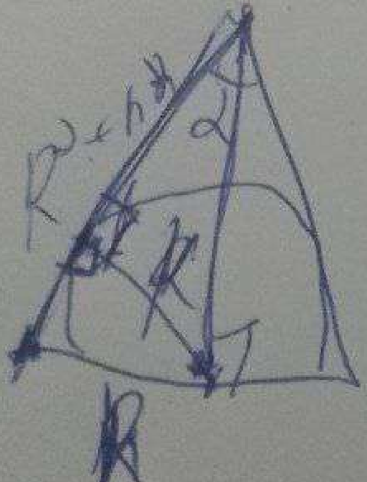
R – рад. полушара

r – рад. осн. кон.

h – высота кон.

$h = R + d$





$R^2 + h^2 = *^2$

$R^2 - r^2 + h^2 - r^2$

$R \quad = \frac{R}{h} \quad \frac{R}{h}$

$h \sin\alpha = R = r\cos\alpha$

$V_k = \pi\left(\frac{r}{h}\right)^2 \cdot \frac{h^3}{3} = \frac{1}{3}\pi r^2 h =$

$V_{min} = \frac{1}{3}\pi \frac{R^2}{\cos^2\alpha} \cdot \frac{R}{\sin\alpha} = \frac{\pi R^3}{3\cos^2\alpha\sin\alpha}$

$V_{min} \to min$

$\cos^2\alpha \sin\alpha \to max$

$f(x) = \cos^2\alpha \sin\alpha$

$f'(x) = -2\cos\alpha\sin^2\alpha + \cos^3\alpha =$

$\cos\alpha(-2\sin^2\alpha + \cos^2\alpha) = 0$

$\cos\alpha = 0$ $\qquad$ $\cos^2\alpha = 2\sin^2\alpha$

$\alpha = \frac{\pi}{2} + \pi n$ $\qquad$ $\cos\alpha = \sqrt{2}\sin\alpha$

$1 = \sqrt{2}\,tg\alpha$

$tg\alpha = \frac{1}{\sqrt{2}}$

$\alpha = arctg\left(\frac{1}{\sqrt{2}}\right) + \pi n$

max

$+ \quad - \quad +$

$arctg\frac{1}{\sqrt{2}}$ $\qquad$ $\frac{\pi}{2}$

Ответ: $arctg\frac{1}{\sqrt{2}}$