Japanese Calligraphy Effect: Complete Mathematical Model

Image Effects Lab

1 Complete Brush Stroke Function

The Japanese calligraphy effect can be expressed as a composite function that maps an input image $I(x, y)$ to a rendered brush stroke at position (x, y) with various parameters. Here is the complete mathematical model:

1.1 The Master Equation

For each stroke seed position (c_x, c_y) and parameter $t \in [0, 1]$ along the stroke:

$$\boxed{\mathbf{S}(c_x, c_y, t) = \begin{bmatrix} x(t) \\ y(t) \\ W(t) \\ \mathbf{C}(c_x, c_y) \\ \alpha(t) \end{bmatrix}} \tag{1}$$

where each component is defined as follows:

1.2 Position Along Stroke Path

$$x(t) = c_x + \cos(\Theta) \cdot L \cdot (t - 0.5) + \cos\left(\Theta + \frac{\pi}{2}\right) \cdot \mathcal{C}(t) \tag{2}$$

$$y(t) = c_y + \sin(\Theta) \cdot L \cdot (t - 0.5) + \sin\left(\Theta + \frac{\pi}{2}\right) \cdot \mathcal{C}(t) \tag{3}$$

with curve offset:

$$\mathcal{C}(t) = \sin(t\pi) \cdot c_{\text{random}} \cdot \frac{L}{4}, \quad c_{\text{random}} \sim \mathcal{U}(-0.15, 0.15) \tag{4}$$

1.3 Stroke Angle Θ

The stroke angle depends on the stroke type classification:

$$\Theta(c_x, c_y) = \begin{cases} \theta_{\text{user}}(c_x, c_y) + \mathcal{U}(-0.15, 0.15) & \text{if directional region} \\ \arctan 2(G_y, G_x) & \text{if bark region and } ||\nabla I|| > 10 \\ \frac{\pi}{2} + \mathcal{U}(-0.3, 0.3) & \text{if bark region and } ||\nabla I|| \leq 10 \\ \arctan 2(G_y, G_x) + \frac{\pi}{2} + \mathcal{U}(-0.2, 0.2) & \text{if edge pixel} \\ \mathcal{U}(0, 2\pi) & \text{otherwise (random)} \end{cases} \tag{5}$$

where the gradient magnitude is:

$$||\nabla I|| = \sqrt{G_x^2 + G_y^2}, \quad G_x = \frac{\partial I}{\partial x}, \quad G_y = \frac{\partial I}{\partial y} \tag{6}$$

1.4 Stroke Length L

$$L(c_x, c_y) = \begin{cases} L_{\text{base}} \cdot \mathcal{U}(1.5, 2.2) \cdot m_w & \text{if directional region} \\ L_{\text{base}} \cdot \mathcal{U}(2.0, 2.8) \cdot m_w & \text{if bark region} \\ L_{\text{base}} \cdot 1.3 \cdot m_w & \text{if edge pixel} \\ L_{\text{base}} \cdot m_w & \text{otherwise} \end{cases} \tag{7}$$

where m_w is the width multiplier for each stroke type.

1.5 Brush Size (Variable Mode)

When variable brush size is enabled:

$$d(c_x, c_y) = \frac{E(c_x, c_y)}{255} \tag{8}$$

$$b_{\text{base}} = \frac{b_{\min} + b_{\max}}{2} \tag{9}$$

$$r_{\text{size}} = \frac{b_{\max} - b_{\min}}{2} \tag{10}$$

$$b(c_x, c_y) = b_{\text{base}} + (1 - d) \cdot r_{\text{size}} \cdot v_{\text{size}} \cdot \mathcal{U}(0.9, 1.1) \tag{11}$$

$$B(c_x, c_y) = \text{clip}(b(c_x, c_y), b_{\min}, b_{\max}) \tag{12}$$

where $E(c_x, c_y)$ is the Canny edge strength, and v_{size} is the size variation parameter.

1.6 Width Taper Function

The width along the stroke varies with parameter t:

$$w(t) = \begin{cases} \frac{t}{0.2} \cdot 0.9 & \text{if } t < 0.2 \quad \text{(gradual start)} \\ 0.9 + 0.1 \sin\left(\frac{(t-0.2)\pi}{0.4}\right) & \text{if } 0.2 \leq t \leq 0.6 \quad \text{(middle section)} \\ 1.0 - \frac{t-0.6}{0.4} \cdot \tau & \text{if } t > 0.6 \quad \text{(tapered tail)} \end{cases} \tag{13}$$

Final width at position t:

$$W(t) = \max\left(1, \lfloor B(c_x, c_y) \cdot w(t) \rfloor\right) \tag{14}$$

1.7 Color Sampling

Color is sampled from the original image region:

$$\mathbf{C}(c_x, c_y) = \frac{1}{|R|} \sum_{(i,j)\in R} I(i,j) + \mathcal{U}(-5,6)^3 \tag{15}$$

where R is the local region around (c_x, c_y), and $\mathcal{U}(-5,6)^3$ represents a 3D uniform random vector for color variation.

1.8 Ink Bleed Alpha Transparency

$$\alpha(t) = \begin{cases} 220 + \mathcal{U}(-10,10) & \text{if } t \le 0.7 \\ (220 + \mathcal{U}(-10,10)) \cdot \left(1 - \frac{t-0.7}{0.3} \cdot 0.4\right) & \text{if } t > 0.7 \text{ and ink bleed enabled} \end{cases} \tag{16}$$

2 Complete Unified Form

Combining all components, the complete brush stroke rendering function is:

$$\boxed{\begin{aligned} \mathbf{S}(c_x, c_y, t) = \Bigg[& c_x + \cos(\Theta(c_x, c_y)) \cdot L(c_x, c_y) \cdot (t - 0.5) \\ & \quad + \cos\left(\Theta(c_x, c_y) + \frac{\pi}{2}\right) \cdot \sin(t\pi) \cdot \mathcal{U}(-0.15, 0.15) \cdot \frac{L(c_x, c_y)}{4}, \\ & c_y + \sin(\Theta(c_x, c_y)) \cdot L(c_x, c_y) \cdot (t - 0.5) \\ & \quad + \sin\left(\Theta(c_x, c_y) + \frac{\pi}{2}\right) \cdot \sin(t\pi) \cdot \mathcal{U}(-0.15, 0.15) \cdot \frac{L(c_x, c_y)}{4}, \\ & \max\left(1, \lfloor B(c_x, c_y) \cdot w(t) \rfloor\right), \\ & \frac{1}{|R|} \sum_{(i,j)\in R} I(i,j) + \mathcal{U}(-5,6)^3, \\ & \alpha(t) \Bigg] \end{aligned}} \tag{17}$$

where all sub-functions Θ, L, B, w, and α are as defined above.

3 Edge Detection (Preprocessing)

Before stroke generation, edge detection is performed:

$$E(x,y) = \text{Canny}(I(x,y), \theta_{\text{low}} = 50, \theta_{\text{high}} = 150) \tag{18}$$

$$G_x(x,y) = \text{Sobel}_x(I(x,y)) = I * K_x \tag{19}$$

$$G_y(x,y) = \text{Sobel}_y(I(x,y)) = I * K_y \tag{20}$$

$$\theta_{\text{edge}}(x,y) = \arctan 2(G_y(x,y), G_x(x,y)) \tag{21}$$

where K_x and K_y are the Sobel kernels:

$$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad K_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \tag{22}$$

4 Parameters

- $b_{\min}, b_{\max}$: Minimum and maximum brush sizes
- L_{base}: Base stroke length
- τ: Tail taper amount $\in [0, 1]$
- v_{size}: Size variation sensitivity $\in [0, 1]$
- $\mathcal{U}(a, b)$: Uniform random distribution on interval $[a, b]$
- $I(x, y)$: Input image at pixel (x, y)