Problem 1(a): Mean Difference Permutation Test

```
# Data
> exp  <- c(11, 33, 48, 34, 112, 369, 64, 44)
> ctrl <- c(177, 80, 141, 332)
> all  <- c(exp, ctrl)
> n_exp   <- length(exp)
> n_perm  <- 10000

> # Observed
> obs_md <- mean(exp) - mean(ctrl)

> # Build null distribution
> perm_md <- numeric(n_perm)
> for(i in seq_len(n_perm)) {
+    p        <- sample(all)
+    perm_md[i] <- mean(p[1:n_exp]) - mean(p[-(1:n_exp)])
+ }

> # One-sided p-value (Ha: μ1 < μ2)
> p_md <- mean(perm_md <= obs_md)

> # Results
> obs_md
[1] -93.125
> p_md
[1] 0.1046
```

Conclusion ($\alpha = 0.05$):
Observed mean difference = –93.125
Permutation p-value = 0.1046 > 0.05 ⇒ **fail to reject** H_0.

Problem 1(b): Sum of Experimental-Group Permutation Test

```
experimental <- c(11, 33, 48, 34, 112, 369, 64, 44)
sum_experimental <- sum(experimental)

715
```

```
control <- c(177, 80, 141, 332)
sum_control <- sum(control)
print(sum_control)

730

# Data
> exp   <- c(11, 33, 48, 34, 112, 369, 64, 44)
> ctrl  <- c(177, 80, 141, 332)
> all   <- c(exp, ctrl)
> n_exp <- length(exp)
> n_perm <- 10000

> # Observed sum
> obs_sum <- sum(exp)   # 715

> # Build null distribution of sums
> perm_sum <- numeric(n_perm)
> for(i in seq_len(n_perm)) {
+    p          <- sample(all)       # shuffle all 12 values
+    perm_sum[i] <- sum(p[1:n_exp])   # take first 8 as "experimental"
+ }

> # One-sided p-value (Ha: exp < ctrl)
> p_sum <- mean(perm_sum <= obs_sum)

> # Output
> obs_sum
[1] 715
> p_sum
[1] 0.1048
```

Conclusion (α = 0.05):
Observed experimental‑group sum = 715
Permutation p‑value = 0.1048 > 0.05 ⇒ **fail to reject** H_0.

Problem 1(c): Median‑Difference Permutation Test

```
# 1) Input
> exp   <- c(11,33,48,34,112,369,64,44)
```

```
> ctrl  <- c(177,80,141,332)
> all   <- c(exp, ctrl)
> n_exp <- length(exp)

> # 2) Observed statistic
> obs_md <- median(exp) - median(ctrl)

> # 3) Enumerate all 495 splits
> idxs    <- combn(12, n_exp)           # 8×495 matrix of indices
> mdiffs  <- apply(idxs, 2, function(i) {
+    median(all[i]) - median(all[-i])
+ })

> # 4) Exact one-sided p-value
> p_exact <- mean(mdiffs <= obs_md)

> # 5) Results
> obs_md
[1] -113
> p_exact
[1] 0.0969697
```

Conclusion ($\alpha = 0.05$):
Observed median difference = –113
Exact p-value = 0.09697 > 0.05 ⇒ **fail to reject** H_0.

Problem 1(d): Mean‑Rank‑Difference Permutation Test

```
# input
> exp  <- c(11,33,48,34,112,369,64,44)
> ctrl <- c(177,80,141,332)
> all  <- c(exp, ctrl)

> # 1) rank the pooled data
> ranks <- rank(all)          # vector of length 12

> # 2) observed statistic
> obs_md_rank <- mean(ranks[1:8]) - mean(ranks[9:12])

> # 3) enumerate all splits
> idxs <- combn(12, 8)        # 8×495 matrix
```

```
> mdiffs <- apply(idxs, 2, function(ix) {
+    mean(ranks[ix]) - mean(ranks[-ix])
+ })

> # 4) exact one-sided p-value
> p_exact_d <- mean(mdiffs <= obs_md_rank)

> # output
> obs_md_rank
[1] -4.125
> p_exact_d
[1] 0.03636364
```

Conclusion ($\alpha = 0.05$):
Observed mean-rank difference = –4.125
Exact p-value = 0.03636 < 0.05 ⇒ **reject** H_0.

Problem 1(e): Sum-of-Ranks Permutation Test

```
# observed sum of ranks
> obs_sum_rank <- sum(ranks[1:8])   # 41

> # compute all permuted sums
> rank_sums <- apply(idxs, 2, function(ix) sum(ranks[ix]))

> # exact one-sided p-value
> p_exact_e <- mean(rank_sums <= obs_sum_rank)  # also 18/495

> # output
> obs_sum_rank
[1] 41
> p_exact_e
[1] 0.03636364
```

Conclusion ($\alpha = 0.05$):
Observed sum of experimental ranks = 41
Exact p-value = 0.03636 < 0.05 ⇒ **reject** H_0.

Problem 3: Rejection Regions for (m,n)∈{5,6,7}

```
# Prepare a data.frame to store results
results <- expand.grid(m=5:7, n=5:7)
results[, c("N","total","c05","p05","c01","p01")] <- NA

for(i in seq_len(nrow(results))) {
  m <- results$m[i]
  n <- results$n[i]
  N <- m + n
  idxs      <- combn(N, m)
  rank_sums <- apply(idxs, 2, sum)

  candidates <- sort(unique(rank_sums))
  tail_prob  <- sapply(candidates, function(t) mean(rank_sums >= t))

  # find cutoffs
  w05 <- which(tail_prob <= 0.05)[1]
  w01 <- which(tail_prob <= 0.01)[1]

  results$N[i]     <- N
  results$total[i] <- choose(N, m)
  results$c05[i]   <- candidates[w05]
  results$p05[i]   <- tail_prob[w05]
  results$c01[i]   <- candidates[w01]
  results$p01[i]   <- tail_prob[w01]
}

# Display neatly
results
```

Conclusions for Problem 3:

m	n	N	$c_{0.05}$	$c_{0.01}$
5	5	10	36	39
5	6	11	40	43
5	7	12	44	47
6	5	11	46	49
6	6	12	50	54
6	7	13	55	59
7	5	12	57	60
7	6	13	62	66
7	7	14	66	71

- **At α=0.05:** reject H_0 if S1R≥c0.05S_{1R}\ge c_{0.05}.
- **At α=0.01:** reject H_0 if S1R≥c0.01S_{1R}\ge c_{0.01}.