Ряд Тейлора в $\cdot$ (0,0)

$$f(x,y) = f(0,0) + \frac{x}{1!}\frac{\partial f}{\partial x}(0,0) + \frac{y}{1!}\frac{\partial f}{\partial y}(0,0) + \frac{x^2}{2!}\frac{\partial^2 f}{\partial x^2}(0,0) +$$

$$+ \frac{xy}{1!\cdot 1!}\left(\frac{\partial f}{\partial x}(0;0)\right)\frac{1}{\partial y} + \frac{y^2}{2!}\frac{\partial^2 f}{\partial y^2}(0,0) + c$$

$f(x,y) = \ln(1+x+y)$; $f(0,0) = \ln(1) = 0$

$\frac{\partial f}{\partial x} = \frac{1}{x+y+1}$; $\frac{\partial f}{\partial x}(0,0) = 1$; $\frac{\partial f}{\partial y} = \frac{1}{x+y+1}$; $\frac{\partial f}{\partial y}(0,0) = 1$

$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{(x+y+1)^2}$ $\frac{\partial^2 f}{\partial x}(0,0) = -1$ $\frac{\partial^2 f}{\partial y^2} = \frac{-1}{(x+y+1)^2}$ $\frac{\partial^2 f}{\partial y^2} = -1$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{-1}{(x+y+1)^2}$ $\frac{\partial^2 f}{\partial x \partial y} = -1$

$f(x,y)$ в окрест $\cdot(0,0) = 0 + x + y - \frac{x^2}{2} - xy - \frac{y^2}{2}$

$f(x,y)$ в окр $\cdot(0,0) = \boxed{-\frac{x^2}{2} - \frac{y^2}{2} - xy + x + y}$

2. Задача Коши

$u'''(x) + u''(x) - u'(x) = x$; $u(0)=1$; $u'(0)=0$; $u''(0)=0$

Диффер. ур-ие III порядка.

$$F(p) = \int_0^\infty e^{-pt} f(t)\,dt \qquad F[u'''(x) + u''(x) - u'(x)](s) = F[x](s)$$

$$F[u'''(x)](s) = s^3\left(F[u(x)](s)\right) - s^2 u(0) - u''(0) - s u'(0) = s^3\left(F[u(x)](s)\right) - s^2$$

$$F[u''(x)](s) = s^2\left(F[u(x)](s)\right) - s u(0) - u'(0) = s^2\left(F[u(x)](s)\right) - s$$

$$F[u'(x)](s) = s\left(F[u(x)](s)\right) - u(0) = s\left(F[u(x)](s)\right) - 1$$

$$F[x](s) = \frac{1}{s^2}$$

$\Downarrow$

$$\cancel{s^3\left(F[u(x)](s)\right) + s^2\left(F[u(x)](s)\right) - s\left(F[u(x)](s)\right) + 1 - s^2 - s = \frac{1}{s^2}}$$

$$s^3\left(F[u(x)](s)\right) + s^2\left(F[u(x)](s)\right) - s\left(F[u(x)](s)\right) = \frac{1}{s^2}$$

$$(s^2+s-1)\left(s \cdot F[u(x)](s) - 1\right) = \frac{1}{s^2} \longrightarrow F[u(x)](s) = \frac{s^4+s^3-s^2+1}{s^3(s^2+s-1)}$$

$$u(x) = f\left[\frac{s^4+s^3-s^2+1}{s^3(s^2+s-1)}\right](x) \longrightarrow u(x) = f\left[\frac{-1}{s^3} - \frac{1}{s^2} - \frac{1}{s} + \underbrace{\frac{3}{(s+\frac12)^2 - \frac54} + \frac{2s}{(s+\frac12)^2-\frac54}}_{=\frac{2}{(s+\frac12)^2-\frac54} + \frac{2(s+\frac12)}{(s+\frac12)^2-\frac54}}\right](x)$$

$$f\left[\frac{-1}{s^3}\right](x) = \frac{-x^2}{2}; \quad f\left[\frac{-1}{s^2}\right](x) = x; \quad f\left[\frac{-1}{s}\right](x) = -1;$$

$$f\left[\frac{2}{(s+\frac12)^2 - \frac54}\right](x) = \frac{2e^{-\frac12(1+\sqrt5)x}\left(e^{\sqrt5 x} - 1\right)}{\sqrt5}$$

$$f\left[\frac{2(s+\frac12)}{\left((s+\frac12)^2 - \frac54\right)}\right](x) = e^{-\frac12(1+\sqrt5)x} \cdot \left(e^{\sqrt5 x} + 1\right)$$

$$u(x) = \frac{-x^2}{2} - x + e^{-\frac12 x(1+\sqrt5)} \frac{\left(e^{\sqrt5 x} + 1\right) - 1 + 2e^{-\frac12(1+\sqrt5)x}\left(e^{x\sqrt5} - 1\right)}{\sqrt5}$$

Диагонализировать матрицу

$A\cdot v = \lambda \cdot v \quad Av - \lambda v = (A - \lambda E)\cdot v = 0. \qquad A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

ненул. решение: собств знач. λ при $\det(A - \lambda E) = 0$

$$\begin{vmatrix} 4-\lambda & 1 & -2 \\ 1 & 0-\lambda & 0 \\ -2 & 0 & 0-\lambda \end{vmatrix} = -\lambda^3 + 4\lambda^2 + 5\lambda = \lambda(-\lambda^2 + 4\lambda + 5) = \lambda(5-\lambda)(\lambda+1) = 0$$

$\lambda = 0; -1; 5$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \qquad A = PDP^{-1}$$

$\lambda = 0 \quad (A - \lambda E)v = 0 \quad \left(\begin{array}{ccc|c} 4 & 1 & -2 & 0 \\ 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x_1 = 0$
$x_2 - 2x_3 = 0$ $\quad X = \begin{pmatrix} 0 \\ 2x_3 \\ x_3 \end{pmatrix}$ Пусть $x_3 = 1 \to X = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

$\lambda = -1 \quad A - \lambda E = \begin{pmatrix} 5 & 1 & -2 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \quad \left(\begin{array}{ccc|c} 5 & 1 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{2}{5} & 0 \\ 0 & \frac{4}{5} & \frac{2}{5} & 0 \\ 0 & \frac{2}{5} & \frac{1}{5} & 0 \end{array}\right) \to$

$\to \left(\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{2}{5} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x_1 - x_2 \cdot 0.5 = 0 \\ x_2 + x_3 \cdot 0.5 = 0 \end{array} \quad \begin{array}{l} x_2 = -\frac{1}{2}\cdot x_3 \\ x_1 = \frac{1}{2}\cdot x_3 \end{array}$

$X = \begin{pmatrix} +0.5x_3 \\ -0.5x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \to X = \begin{pmatrix} 0.5 \\ -0.5 \\ 1 \end{pmatrix}$

$\lambda = 5 \quad A - \lambda E = \begin{pmatrix} -1 & 1 & -2 \\ +1 & -5 & 0 \\ -2 & 0 & -5 \end{pmatrix} \quad \left(\begin{array}{ccc|c} -1 & 1 & -2 & 0 \\ 1 & -5 & 0 & 0 \\ -2 & 0 & -5 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & -4 & -2 & 0 \\ -2 & 0 & -5 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right) \to$

$\to \left(\begin{array}{ccc|c} 1 & 0 & 2.5 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x_1 + 2.5x_3 = 0 \\ x_2 + 0.5x_3 = 0 \end{array} \quad X = \begin{pmatrix} -2.5x_3 \\ -0.5x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \to X = \begin{pmatrix} -2.5 \\ -0.5 \\ 1 \end{pmatrix}$

$$P = \begin{pmatrix} 0 & 0.5 & -2.5 \\ 2 & -0.5 & -0.5 \\ 1 & 1 & 1 \end{pmatrix} \to P^{-1} = \begin{pmatrix} 0 & \frac{2}{5} & \frac{1}{5} \\ \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{15} & \frac{2}{15} \end{pmatrix} \qquad A = PDP^{-1}$$

Ответ: $P = \begin{pmatrix} 0, & 0.5, & -2.5 \\ 2, & -0.5, & -0.5 \\ 1, & 1, & 1 \end{pmatrix} \; D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix} \; P^{-1} = \begin{pmatrix} 0 & 0.4 & 0.2 \\ 1/3 & -1/3 & 2/3 \\ -1/3 & -1/15 & \frac{2}{15} \end{pmatrix}$