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Brainy Business: Utilizing Bayes' Rule and Decision Trees for Brainet Launch

APM1134 Summative Assessment 2

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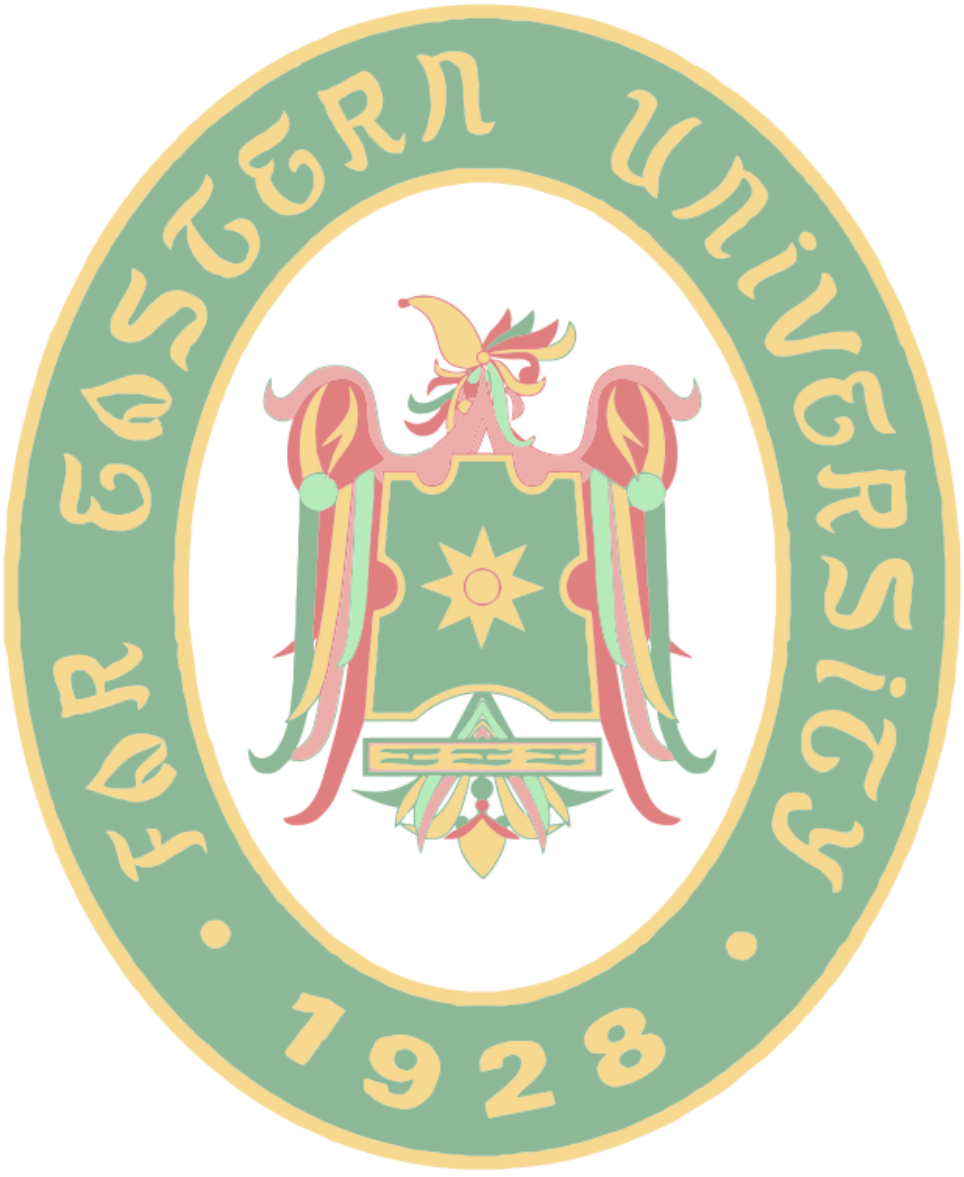
3rd Year Applied Mathematics (Data Science)



Problem Motivation

Charlotte Rothstein and her company *Cerebrosoft* is facing a difficult decision in the face of launching of their new product, Brainret. Although the product is promising, Charlotte is confounded with the competitive market of Silicon Valley, implementing competitive price for their product, and how many Brainret will sell in the first place. The following are the variables to consider:

1. Price – what price should the Brainret be sold?
    a. Low Price (lP) – $30 (for better market share)
    b. Medium Price (mP) – $40 (for balanced between market share and revenues)
    c. High Price (hP) – $50 (for maximizing revenues)
2. Development (Cost) – $800,000
3. Support and Shipping – $50,000 / yr
4. Prior Probabilities
    a. Low Competition (lC) – 0.1
    b. Medium Competion (mC) – 0.70
    c. High Competition (hC) – 0.2
5. sold:
    a. Low Sales (lS) – 50,0000 units
    b. Medium Sales (mS) – 30,000 units
    c. High Sales (hS) – 20,000 units

TABLE 1 Probability distribution of unit sales, given a high price ($50)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.2	0.25	0.3
30,000 units	0.25	0.3	0.35
20,000 units	0.55	0.45	0.35

TABLE 2 Probability distribution of unit sales, given a medium price ($40)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.25	0.30	0.40
30,000 units	0.35	0.40	0.50
20,000 units	0.40	0.30	0.10

TABLE 3 Probability distribution of unit sales, given a low price ($30)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.35	0.40	0.50
30,000 units	0.40	0.50	0.45
20,000 units	0.25	0.10	0.05

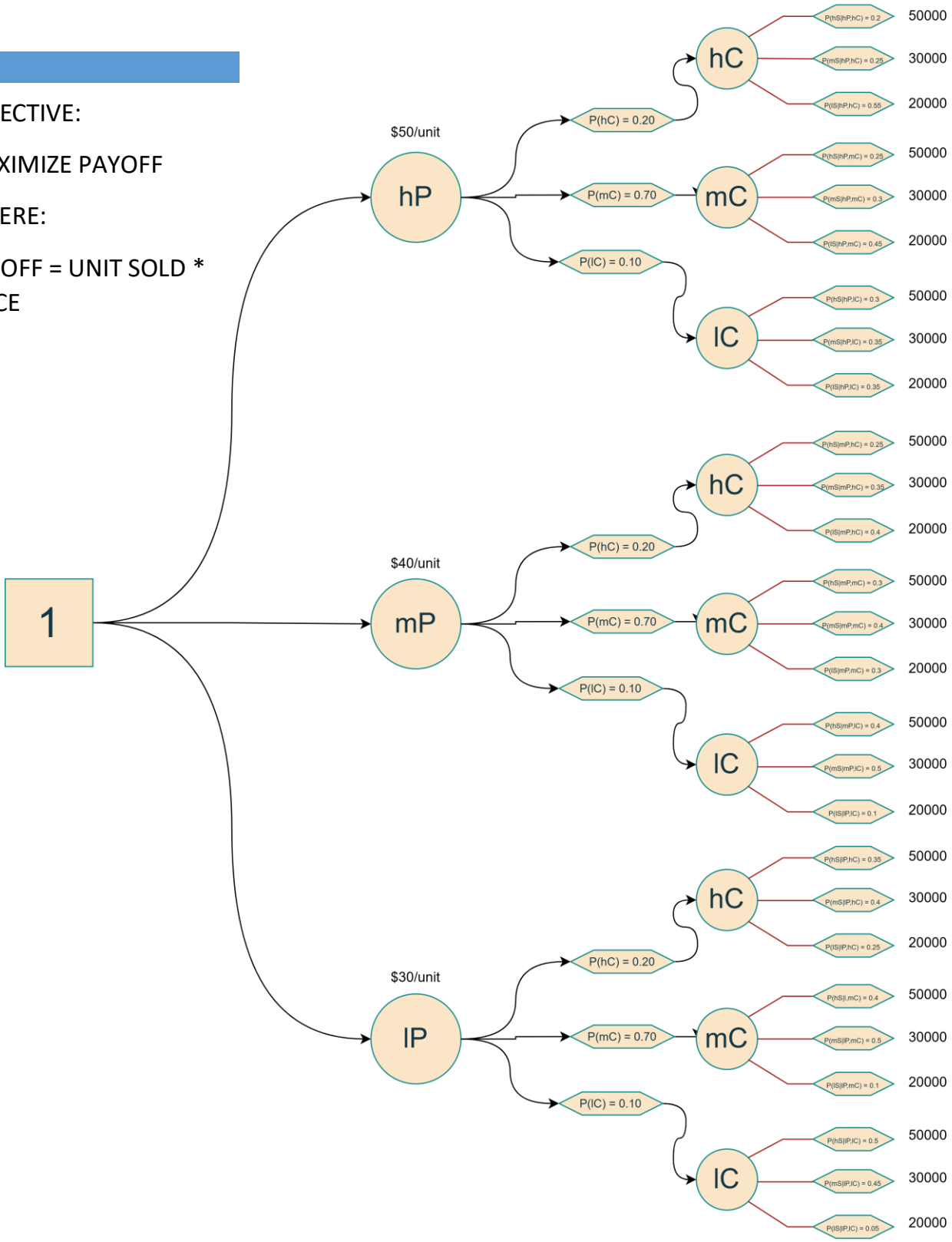
Initial Decision Tree

OBJECTIVE:

MAXIMIZE PAYOFF

WHERE:

PAYOFF = UNIT SOLD * PRICE



First Fold

CALCULATED AS FOLLOWS:

Let $s \in \{x|\forall x \in Sales\}$,

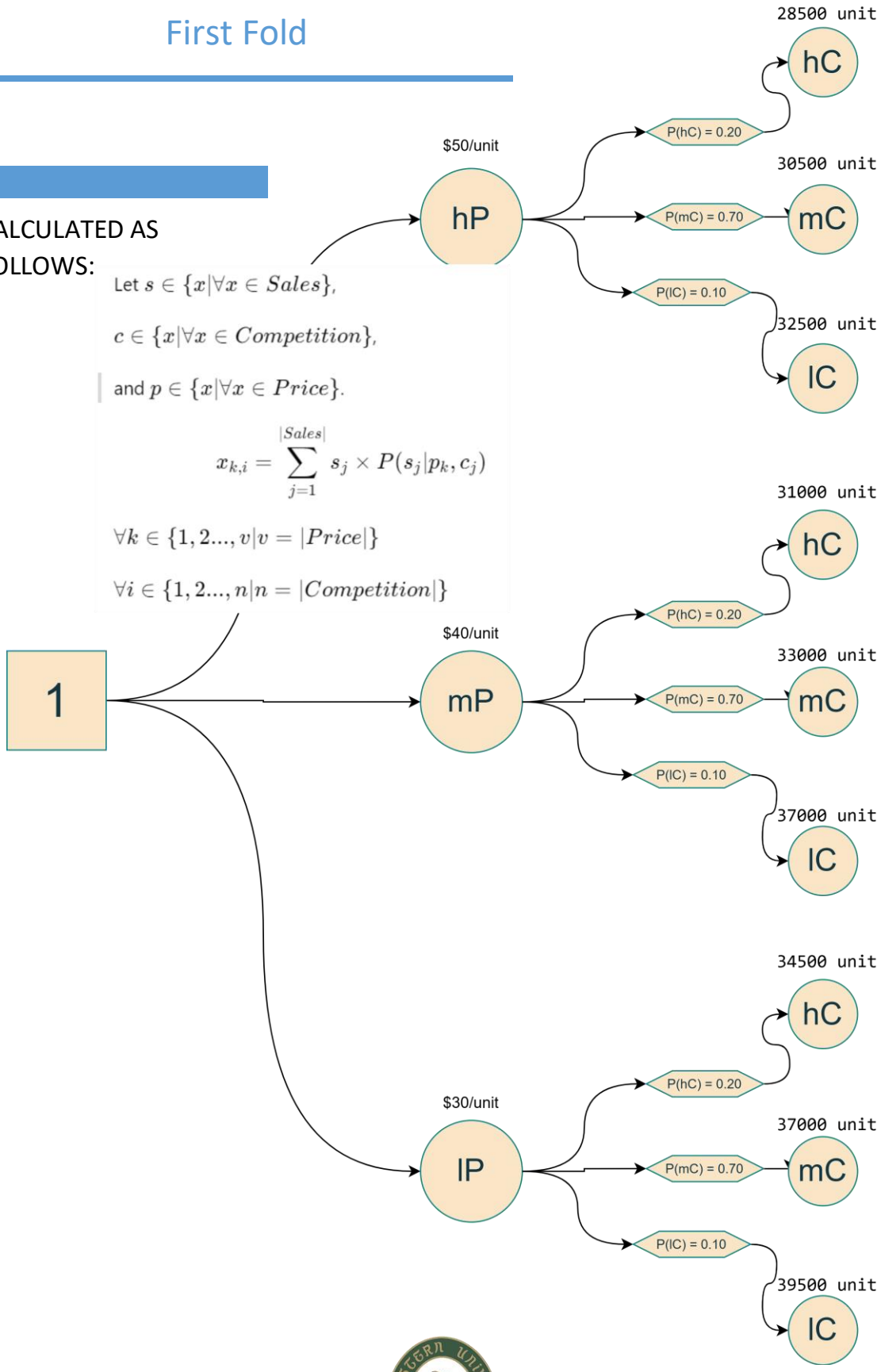
$c \in \{x|\forall x \in Competition\}$,

and $p \in \{x|\forall x \in Price\}$.

$$x_{k,i} = \sum_{j=1}^{|Sales|} s_j \times P(s_j|p_k, c_j)$$

$\forall k \in \{1, 2..., v|v = |Price|\}$

$\forall i \in \{1, 2..., n|n = |Competition|\}$



Second Fold

CALCULATED AS FOLLOWS:

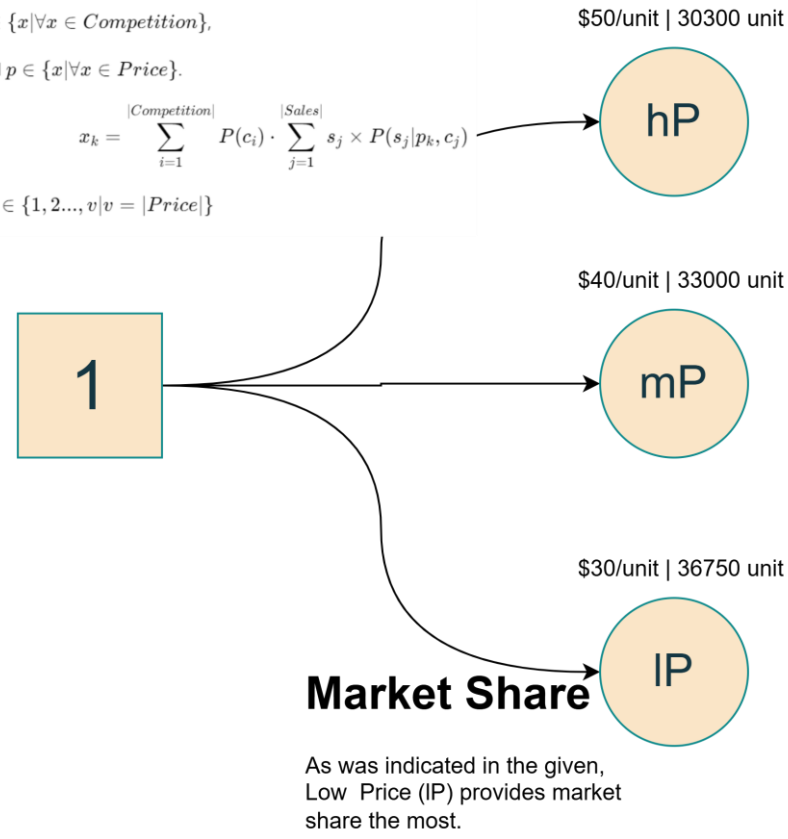
Let $s \in \{x | \forall x \in Sales\}$,

$c \in \{x | \forall x \in Competition\}$,

and $p \in \{x | \forall x \in Price\}$.

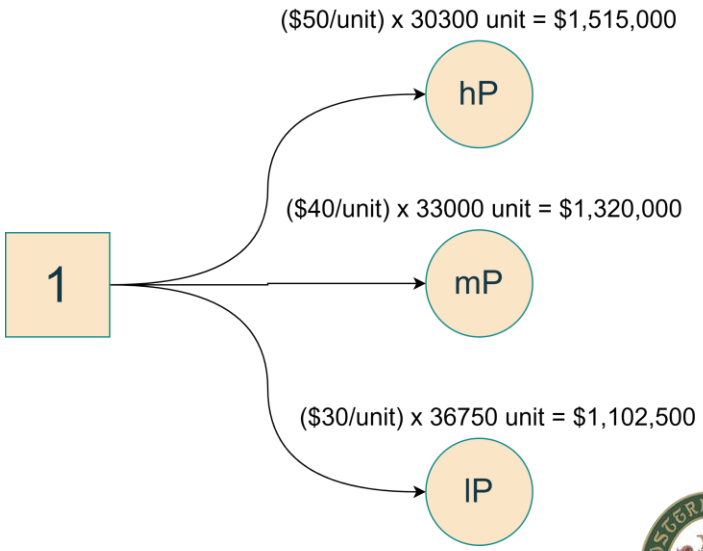
$$x_k = \sum_{i=1}^{|Competition|} P(c_i) \cdot \sum_{j=1}^{|Sales|} s_j \times P(s_j | p_k, c_j)$$

$\forall k \in \{1, 2..., v | v = |Price|\}$



Market Share

As was indicated in the given, Low Price (lP) provides market share the most.



Decision

Since hP have the maximum payoff.

Charlotte should Choose to sell BrainRet at $50 per unit (hP).

Market Research

The marketing research company offered their services for **$10,000**. They have the following track record:

Table 4: Probability Distribution of Prediction Given High Price ($50)

Predicted \ **Given**	**hC**	**mC**	**lC**
P-hC	0.80	0.15	0.03
P-mC	0.15	0.80	0.07
P-lC	0.05	0.05	0.90

$$P(Predicted|Competition, Price)$$

Table 5: Probability Distribution for each Competition.

hC	**mC**	**lC**
0.20	0.70	0.10

$$P(Competition)$$

Table 6: Joint Probability Distribution of Prediction and Competition Given High Price ($50)

Predicted \ **Given**	**hC**	**mC**	**lC**
P-hC	0.16	0.105	0.003
P-mC	0.03	0.560	0.007
P-lC	0.01	0.035	0.090

$$P(Predicted|Competition, Price) \times P(Competition)$$

Table 7: Marginal Probability Distribution of Predictions Given High Price ($50)

P-hC	**P-mC**	**P-lC**
0.268	0.597	0.135

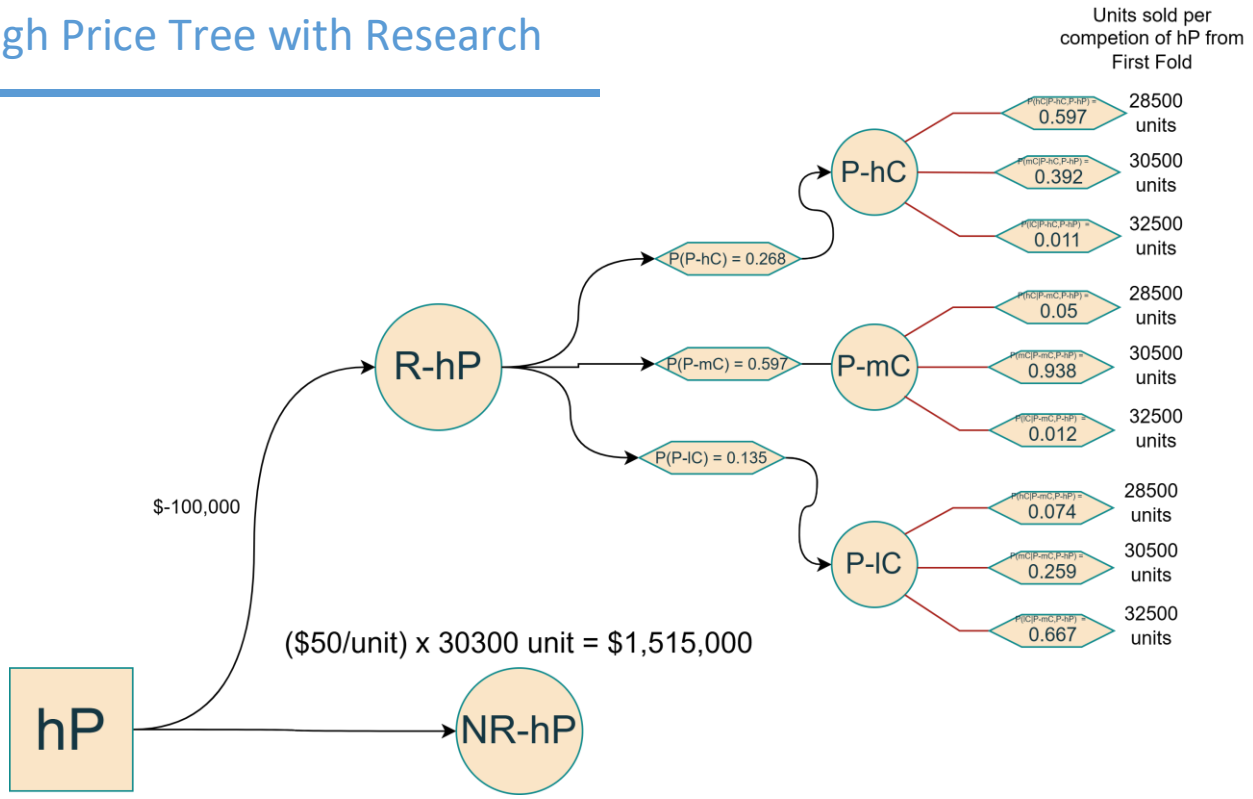
$$P(Predicted)$$

Table 8: Probability Distribution of Comptetion Given High Price ($50)

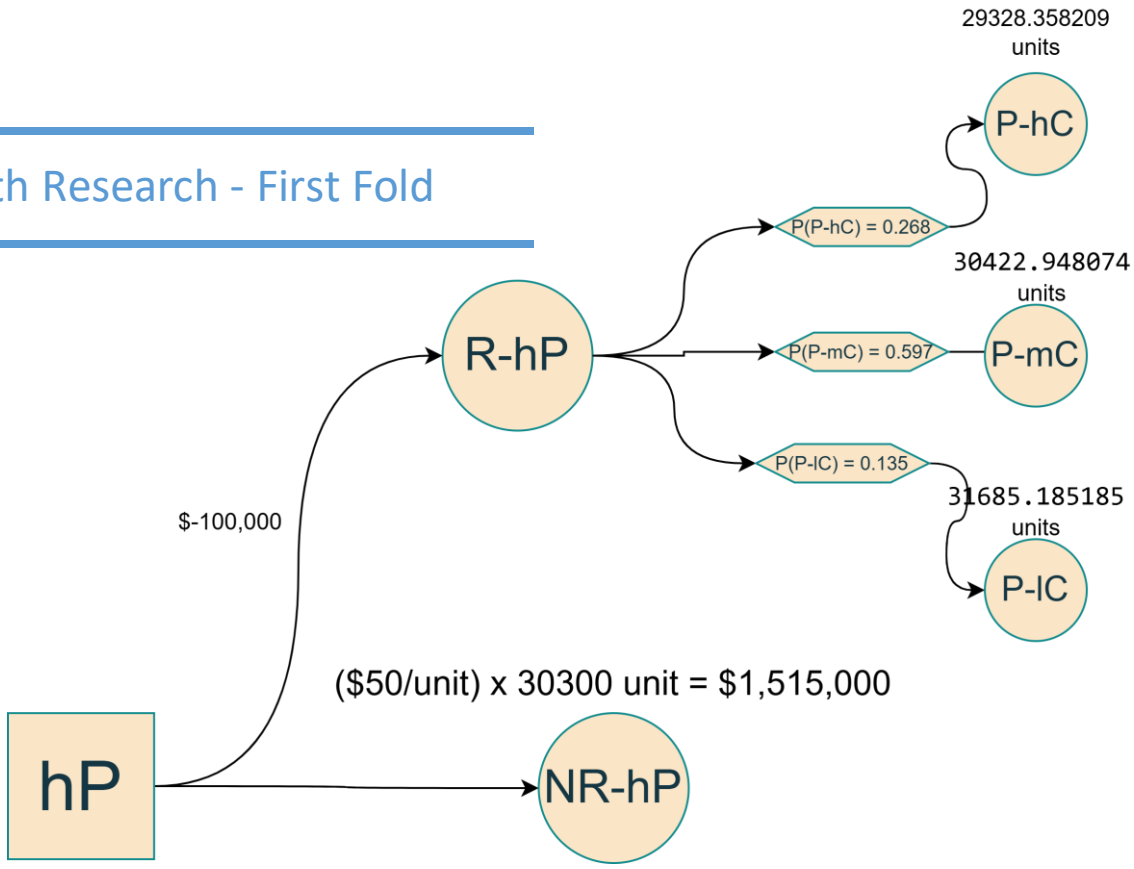
Competition \ **Given**	**P-hC**	**P-mC**	**P-lC**
hC	0.597015	0.050251	0.074074
mC	0.391791	0.938023	0.259259
lC	0.011194	0.011725	0.666667

$$P(Competition|Predicted, Price) = \frac{P(Predicted|Competition, Price) \times P(Competition)}{P(Predicted)}$$

High Price Tree with Research



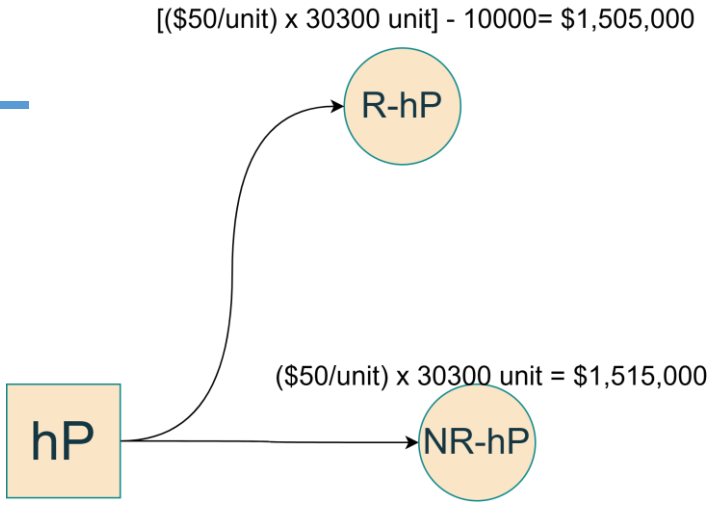
With Research - First Fold



Decision

Since NR-hP have the maximum payoff.

Charlotte should Choose to sell BrainRet at $50 per unit (hP) but NOT conduct market research.



Conclusion

Based on the the Decision Tree conducted with Bayes' Rule, the most optimal decision for Charlotte is **to sell Brainret at $50 (hP) and NOT conduct market research**. Doing so will brought forth $1,515,000 payoff. Based on the payoffs, the following should be some of the financial statements of the company:

$$P(Predicted)$$

$$\text{Payoff}_{optimal} = \$1,515,000$$

$$\text{Development Cost} = \text{Cost} = \$800,000$$

$$\text{Payoff}_{optimal,\text{w/o cost}} = \$1,515,000 - \$800,000 = 715,000$$

$$\text{Support and Shipping} = \text{Expense} = \$715,000$$

$$\text{Payoff}_{\text{optimal, w/o expenses \& cost}} = \$715,000 - \$50,000 = \$665,000$$

Assuming that payoff is on a per year basis.

Notebook (Code link):

https://github.com/RomandRapido/Operations_Research/blob/main/SA2_Lansangan/SA2_Lansangan.ipynb