№4.

$\xi \sim \rho(x) = p\{(-1,1)\setminus\{0\}\} + q\{0\} + q\{2\}$

$\int\limits_{-\infty}^{+\infty} \rho(x) = 1 = \int\limits_{-1}^{1} p\,dx + 2q = 2p + 2q \Rightarrow q = \frac{1}{2} - p$

$\theta = p$

$\rho(x,\theta) = \theta\{(-1,1)\setminus\{0\}\} + \left(\frac{1}{2} - \theta\right)\{0\} + \left(\frac{1}{2} - \theta\right)\{2\}$

$\theta \in (0; \frac{1}{2})$

$\vec{X}_n$ – выборка

$\alpha_1 = M[\xi] = \int\limits_{-\infty}^{+\infty} x\,\rho(x)dx = \underbrace{\int\limits_{-1}^{1} x\cdot\theta\,dx}_{\to 0} +$

$+ 2\left(\frac{1}{2} - \theta\right) + \underbrace{0\cdot\left(\frac{1}{2} - \theta\right)}_{\to 0} = 1 - 2\theta$

$\alpha_2 = M[\xi^2] = \int\limits_{-1}^{1} x^2\theta\,dx + 2 - 4\theta =$

$= \frac{2\theta}{3} + 2 - \frac{12\theta}{3} = 2 - \frac{10}{3}\theta$

$D[\xi] = \alpha_2 - \alpha_1^2 = 2 - \frac{10}{3}\theta - 1 + 4\theta - 4\theta^2 =$

$= 1 + \frac{2}{3}\theta - 4\theta^2$

$$\alpha_1 = \tilde{\alpha}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$$

$$1 - 2\theta = \bar{x} \to \tilde{\theta}_1 = \frac{(1-\bar{x})}{2}$$ ОММ

$$M[\tilde{\theta}_1] = M\left[\frac{1}{2} - \frac{1}{2}\bar{x}\right] = \frac{1}{2} - \frac{1}{2}M[\xi] =$$

$$= \frac{1}{2} - \frac{1}{2}[1 - 2\theta] = \frac{1}{2} - \frac{1}{2} + \theta = \theta$$ несмещ.

$$D[\tilde{\theta}_1] = D\left[\frac{1}{2} - \frac{1}{2}\bar{x}\right] = \frac{1}{4}\cdot\frac{1}{n}D\xi =$$

$$= \frac{1}{4}\cdot\frac{1}{n}\left[1 + \frac{2}{3}\theta - 4\theta^2\right] \to 0,\ n \to \infty$$

$\Rightarrow$ сост. по дост. признаку

а) регулярность модели

$$\frac{\partial}{\partial\theta}\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{+\infty}\frac{\partial}{\partial\theta} f(x)\,dx \overset{?}{=} 0$$

$$\int_{-1}^{1}\frac{\partial}{\partial\theta}\cdot p(x,\theta)\,dx + \frac{\partial}{\partial\theta}\left(\frac{1}{2}-\theta\right) + \frac{\partial}{\partial\theta}\left(\frac{1}{2}-\theta\right) =$$

$$= 2 - 1 - 1 = 0$$ ~~модель рег~~

$$I(\theta) = \int_{-\infty}^{+\infty} \left(\frac{\partial \ln \rho(x,\theta)}{\partial \theta} \right)^2 \rho(x,\theta)\,dx =$$

$$= \int_{-1}^{1} \left(\frac{\partial \ln p(x,\theta)}{\partial \theta} \right)^2 p(x,\theta)\,dx + \left(\frac{\partial \ln p(\theta)}{\partial \theta} \right)^2 p(\theta) =$$

$$= \int_{-1}^{1} \frac{1}{\theta^2} \cdot \theta\,dx + \left(\frac{2}{\frac{1}{2} - \theta} \right)^2 \left(\frac{1}{2} - \theta \right) =$$

$$= \frac{2}{\theta} + \frac{2 \cdot 2}{1 - 2\theta} = \frac{2 - 4\theta + 4\theta}{\theta(1-2\theta)} = \frac{2}{\theta(1-2\theta)}$$

$I(\theta) > 0$, непр $\Rightarrow$ модель регулярна

д) рег. оценки

$D[\tilde{\theta}_1]$ огр на компакте из $(0, \frac{1}{2})$

$$D[\tilde{\theta}_1] \geq \frac{1}{nI(\theta)} = \frac{\theta(1-2\theta)}{2n}$$

$$\frac{1}{4n} \left(\frac{3 + 2\theta - 12\theta^2}{3} \right) > \frac{\theta(1-2\theta)}{2n}$$

$$\frac{1}{2} + \frac{1}{3}\theta - 2\theta^2 > \theta - 2\theta^2$$

не достигает нижней грани

ничего не можем сказать

ОМП Пусть значение из (-1, 0) ∪ {0} m раз

$$L(\theta)=\prod_{i=1}^{n} g(x_i,\theta)=\theta^m\left(\frac{1}{2}-\theta\right)^{n-m}$$

$$\ln L=m\ln\theta+(n-m)\ln\left(\frac{1}{2}-\theta\right)$$

$$(\ln L)'_\theta=\frac{m}{\theta}+\frac{m-n}{\frac{1}{2}-\theta}=0$$

$$\frac{1}{2}m-\cancel{m\theta}+\cancel{m\theta}-n\theta=0$$

$$\tilde{\theta}_2=\frac{1}{2}\frac{m}{n}=\frac{1}{2}\nu$$

$$\frac{\partial^2\ln L(\theta)}{\partial\theta^2}=-\frac{m}{\theta^2}+\frac{m-n}{\left(\frac{1}{2}-\theta\right)^2}=$$

$$=\frac{\cancel{m\theta^2}-n\theta^2-m\cdot\frac{1}{4}+m\cdot\theta-\cancel{m\theta^2}}{\theta^2\left(\frac{1}{2}-\theta\right)^2}=$$

$$=\frac{m\theta-\frac{m}{4}-n\theta^2}{\theta^2\left(\frac{1}{2}-\theta\right)^2}=\frac{m\left(\theta-\frac{1}{4}\right)-n\theta^2}{\theta^2\left(\frac{1}{2}-\theta\right)^2}=$$

$$\frac{m\left((\theta-\frac{1}{4})-\frac{n}{m}\theta^2\right)}{\theta^2(\frac{1}{2}-\theta)^2}$$

$\forall\theta\in(0;\frac{1}{2}) \hookrightarrow \theta-\frac{1}{4}<\theta^2;\ n\geq m \Rightarrow$

$$\frac{m\left((\theta-\frac{1}{4})-\frac{n}{m}\theta^2\right)}{\theta^2\cdot(\frac{1}{2}-\theta)^2}<0 \Rightarrow \tilde{\theta}_2=\frac{1}{2}\nu\text{ - max}$$

$p=2\theta$

$M[\tilde{\theta}_2]=\frac{1}{2}M[\nu]=\frac{1}{2}p=\frac{1}{2}2\theta$ несмещ.

~~$\tilde{\theta}=2\theta$ несмещ~~

~~$D[\tilde{\theta}]=4D[\tilde{\theta}]=D[\nu]=\frac{p(1-p)}{n}=\frac{\theta(1-\theta)}{n}\to 0,\ n\to\infty \Rightarrow$ состоятельно~~

~~$D[\tilde{\theta}]=\frac{p(1-p)}{n}=\frac{2\theta(1-2\theta)}{n}$~~

$D[\tilde{\theta}_2]=\frac{1}{4}D[\nu]=\frac{\theta(1-2\theta)}{2n}\to 0,\ n\to\infty$

$\Rightarrow$ состоят.

~~$\tilde{\theta}$ оценка несостоят.~~

~~$D[\tilde{\theta}]=$~~

модель регулярна и $\mathcal{D}$ огр на компакте $\Rightarrow \tilde{\theta}$ регулярная.

$$\mathcal{D}[\tilde{\theta}_2] \geq \frac{1}{n I(\theta)} = \frac{\theta(1-2\theta)}{2n}$$

$\Rightarrow \tilde{\theta}_2$ эффективна $\Rightarrow \tilde{\theta}_1$ не эффективна