T11 $H_0: \xi \sim p_0(x) = 1\{(0,1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \{(0,1)\}$

а) $n=1$; α

$l = \frac{L_1}{L_0} = \frac{\frac{e}{e-1}e^{-x}}{1} \geq C \to e^{x} \geq B \to x \leq A$

$P(x \leq A | H_0) = \alpha \qquad \alpha = \int_0^A 1 \cdot dx = A$

G: $x \leq \alpha$; $\alpha_1 = \alpha$

$W = P(x \leq A | H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1-e^{-\alpha})$

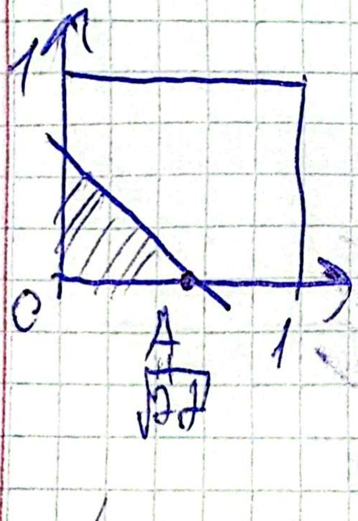
$\alpha_2 = 1 - W$

б) $n=2$

$l = \left(\frac{e}{e-1}\right)^2 e^{-x_1} e^{-x_2} \geq C \to e^{-(x_1+x_2)} \geq B$

$x_1 + x_2 \leq A$

$P(x_1 + x_2 \leq A | H_0) = \alpha$



$\iint\limits_{x_1+x_2 \leq A} 1 \cdot dx_1 dx_2 = \frac{A^2}{2} = \alpha \qquad A = \sqrt{2\alpha}$

G: $x_1 + x_2 \leq \sqrt{2\alpha} \qquad \alpha_1 = \alpha$

Не рассматриваем второй вариант, когда область не треуг. т.к. α мало

$$W = P(X_1 + X_2 \le A \mid H_1) = \iint \left(\frac{e}{e-1}\right)^2 e^{-x_1 - x_2}\, dx_1 dx_2$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A dx_1 \int_0^{A-x_1} e^{-x_1 - x_2}\, dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A \left(e^{-x_1} - e^{-A}\right) dx_1 = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-A} - Ae^{-A}\right)$$

$$\alpha_2 = 1 - W$$

c) $$l = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \ge c$$

$$\ln l = \sum \ln \frac{p_1(x_i)}{p_0(x_i)} \ge \ln c$$

$$\frac{\sum \eta_i - nM[\eta_i]}{\sqrt{n D\eta_1}} \sim N(0,1)$$

$$P(\ln l \ge \ln c \mid H_0) = \alpha$$

$$\eta = \ln\left(\frac{e}{e-1} e^{-x}\right) = \ln\frac{e}{e-1} - x$$

Сканировано с CamScanner

$$\ln l = \sum \ln \frac{e}{e-1} - \sum x_i \geq \ln c$$

$$G: \sum x_i \leq A$$

$$P(\sum x_i \leq A \mid H_0) = \alpha$$

$$P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \leq \frac{A - nMx}{\sqrt{nDx}} \middle| H_0\right) = \alpha$$

$$Mx = \frac{1}{2} \quad Dx = 1/12$$

$$\frac{A - \frac{n}{2}}{\sqrt{n/12}} = u_\alpha \quad G: \sum x_i \leq \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$\alpha_1 = \alpha$$

$$W = P(\sum x_i \leq A \mid H_1) = P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \leq \frac{A - nMx}{\sqrt{nDx}} \middle| H_1\right)$$

$$Mx = \frac{e}{e-1}\left(-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx\right) = \frac{e}{e-1}\left(-\frac{1}{e} - \right.$$

$$\left.-\left(\frac{1}{e}+1\right)\right) = \frac{e-2}{e-1}$$

$$Dx = \frac{e^2-3e+1}{(e-1)^2} \qquad W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$B = \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{2}} - n\frac{e-2}{e-1}}{\sqrt{n \cdot \frac{e^2-3e+1}{(e-1)^2}}} \to +\infty, \; n \to \infty$$

$$\Rightarrow W \to 1$$

$$n \to +\infty$$

$\Rightarrow$ критерий сост.

d)

$G: x_{min} < c \qquad P(\vec{x}_n \in G | H_0) = \alpha$

$P(x_{min} \leq c | H_0) = \alpha \qquad H_0: \xi \sim R(0,1)$

$\cancel{P(\vec{x}) \in} \quad P(x_{min} \leq c) = 1 - (1 - \underset{c}{F(c)})^n = \alpha$

$c = 1 - (1-\alpha)^{\frac{1}{n}}$

$G: x_{min} \leq 1 - (1-\alpha)^{\frac{1}{n}}$

$\alpha_1 = \alpha$

$W = P(\vec{x}_n \in G | H_1) = P(x_{min} \leq c | H_1) =$

$= 1 - \left(1 - \frac{e}{e-1}(1 - e^{-c})\right)^n \qquad \alpha_2 = 1 - W$

$$e^{-c} = e^{-1+(1-\alpha)^{\frac{1}{n}}} = e^{-1}\cdot e^{(e^{\frac{1}{n}\ln(1-\alpha)})} =$$

$$= e^{-1}\cdot e^{(1+\frac{1}{n}\ln(1-\alpha)+o(\frac{1}{n}))} = e^{(\frac{1}{n}\ln(1-\alpha)+o(\frac{1}{n}))}$$

$$1-\left[1-\frac{e}{e-1}\left(1-e^{(\frac{1}{n}\ln(1-\alpha)+o(\frac{1}{n}))}\right)\right]^n =$$

$$= 1-\left[1-\frac{e}{e-1}\left(1-1-\left(\frac{1}{n}\ln(1-\alpha)+o(\frac{1}{n})\right)\right)\right]^n =$$

$$= 1-\left[1+\frac{e}{e-1}\cdot\ln(1-\alpha)\cdot\frac{1}{n}+o(\frac{1}{n})\right]^n \to$$

$$\to 1-e^{\frac{e}{e-1}\ln(1-\alpha)} = 1-(1-\alpha)^{\frac{e}{e-1}} \neq 1$$

$\Rightarrow$ не состоят