Part 1:
Neuron and Neural Network

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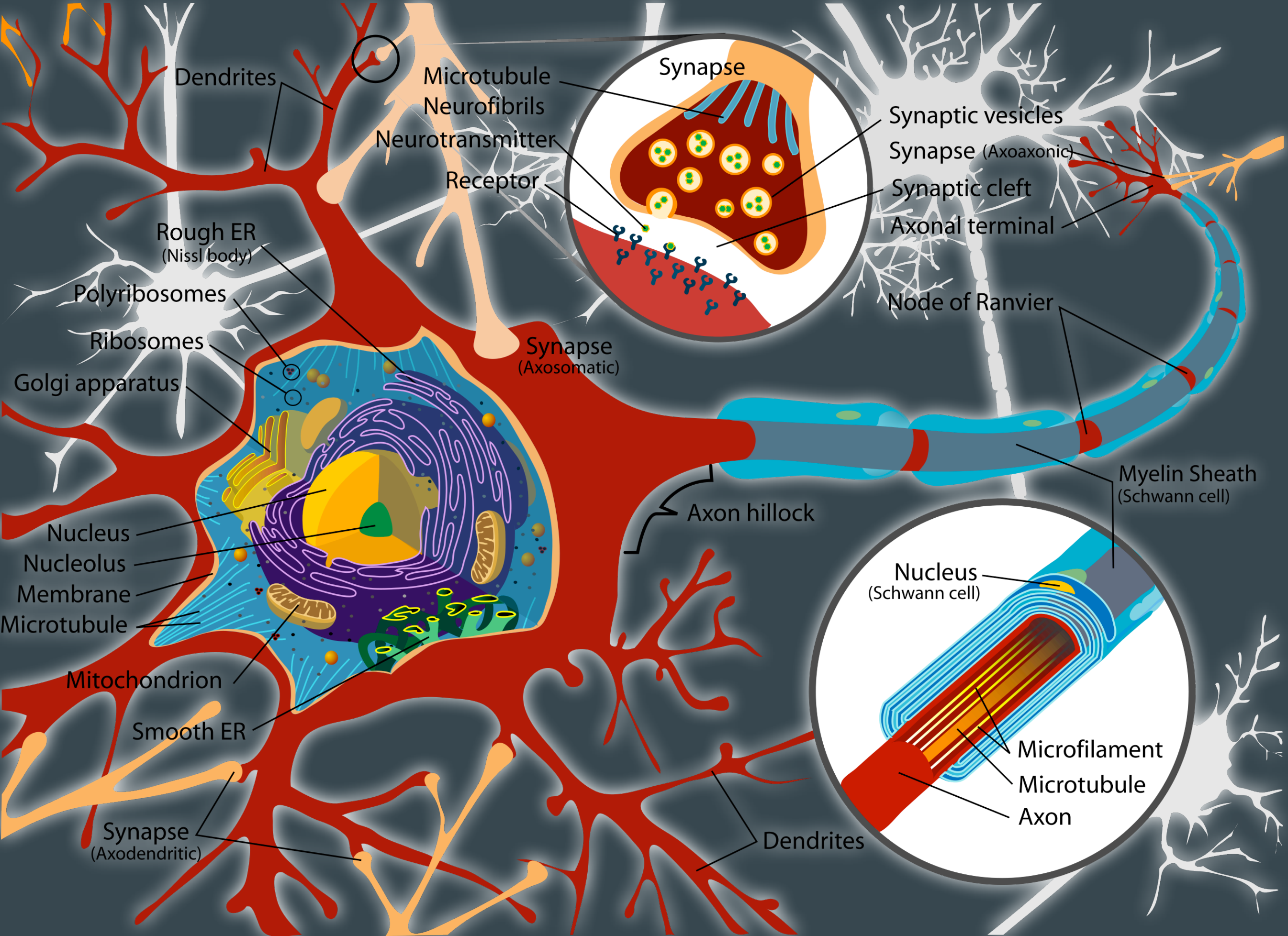
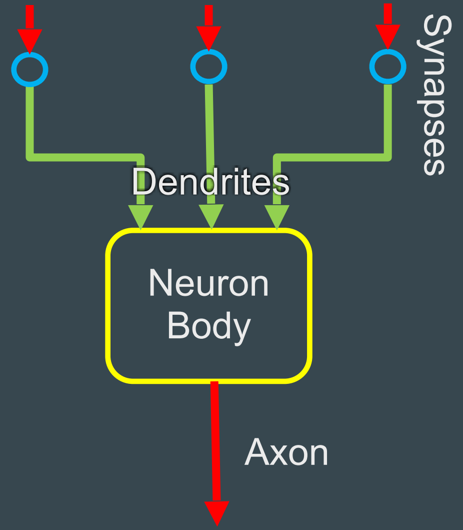
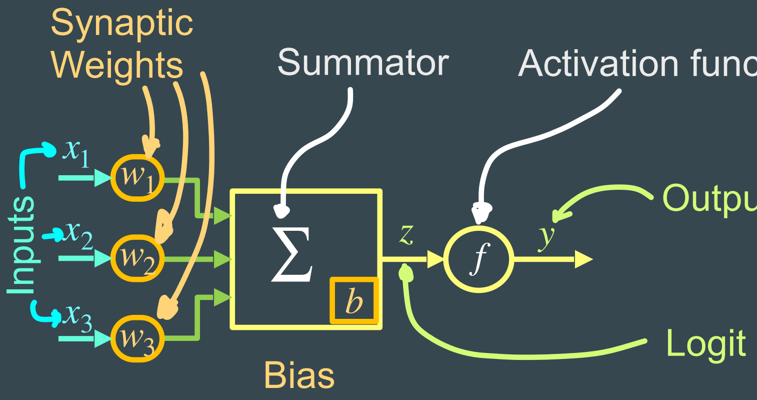
Biological Neuron





Mathematical Model of Neuron



Tunable parameters

Weights Inputs Bias

$$y = f(z) = f(w_1x_1 + w_2x_2 + w_3x_3 + b) =$$

$$f\left(\sum_{i=1}^{N} w_i x_i + b\right) =$$

$$f\left(\langle \mathbf{w}, \mathbf{x} \rangle + b\right)$$

Scalar Product $\longrightarrow$ $\langle \mathbf{w}, \mathbf{x} \rangle = \sum_{i=1}^{N} w_i x_i$

$$\theta(z) = \begin{cases} 0 & if\ z \geq 0 \\ 1 & if\ z < 0 \end{cases}$$

Threshold activation function

Separator

$$y = \theta(z) = \theta\left(\langle \mathbf{w}, \mathbf{x}\rangle + b\right)$$

Separator for a single neutron

$\theta(z)$ Changes when

$$z = 0$$

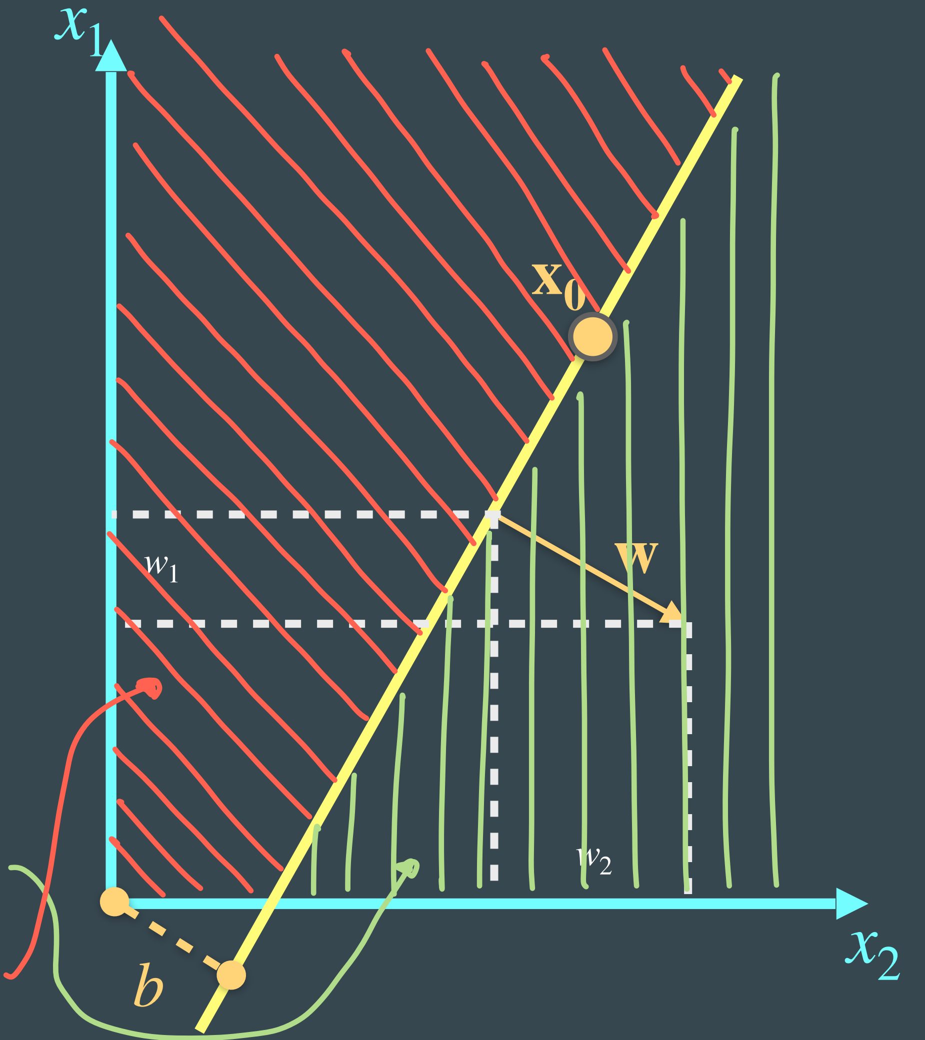
$$\langle \mathbf{w}, \mathbf{x}\rangle + b = 0 \qquad \langle \mathbf{w}, \mathbf{x}\rangle = -b$$

$$\left\langle \mathbf{w}, \mathbf{x} - \mathbf{x_0}\right\rangle = 0$$

$$\theta(z) = \begin{cases} 1 & if\ z \geq 0 \\ 0 & if\ z < 0 \end{cases}$$

$\langle \mathbf{w}, \mathbf{x}\rangle + b \geq 0$

$\langle \mathbf{w}, \mathbf{x}\rangle + b < 0$



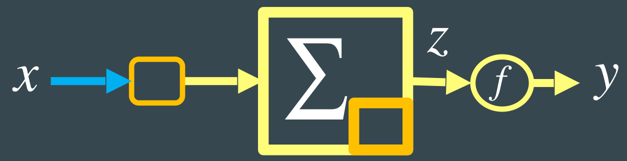
Negation Operator

$\bar{x}$

X	Not X
0	1
1	0

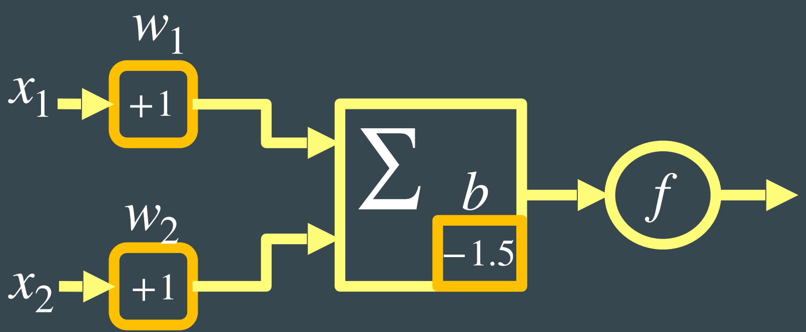
<u>Task</u>:

Design a neuron that performs negation



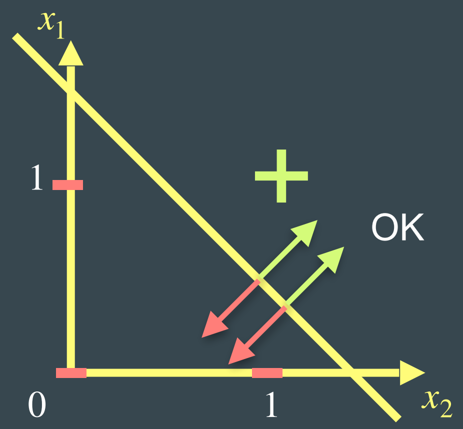
Operator AND

$$x_1 \ \& \ x_2$$



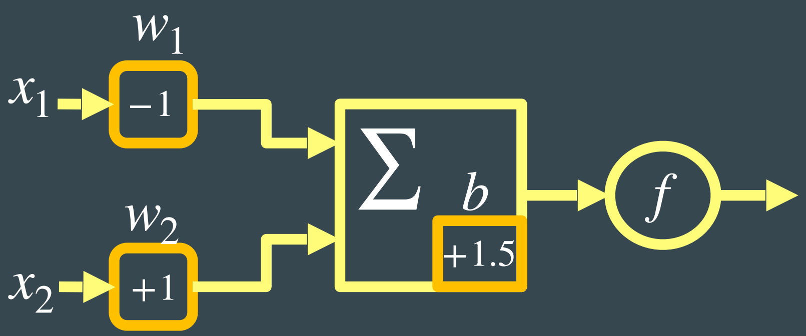
X1	X2	X1 & X2
1	1	1
1	0	0
0	1	0
0	0	0

$$\begin{cases} 1w_1 + 1w_2 + b \geq 0 \\ 0w_1 + 1w_2 + b < 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



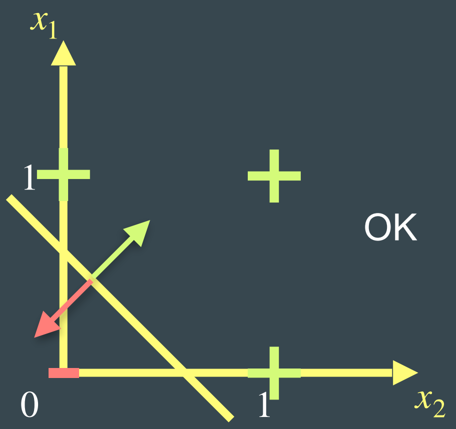
Operator OR

$$x_1 \| x_2$$



X1	X2	X1 & X2
1	1	1
1	0	1
0	1	1
0	0	0

$$\begin{cases} 1w_1 + 1w_2 + b \geq 0 \\ 0w_1 + 1w_2 + b \geq 0 \\ 1w_1 + 0w_2 + b \geq 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

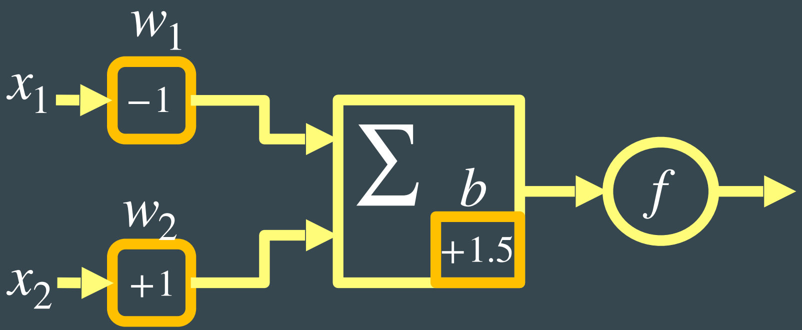


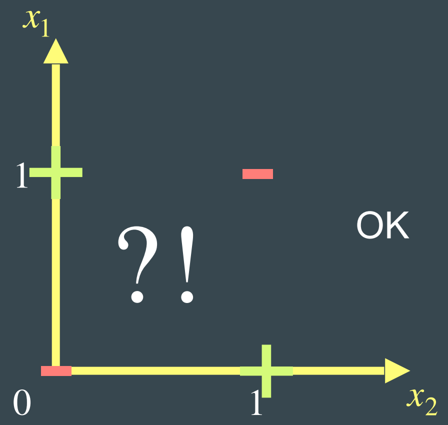
OK

Operator XOR

$$x_1 \oplus x_2$$

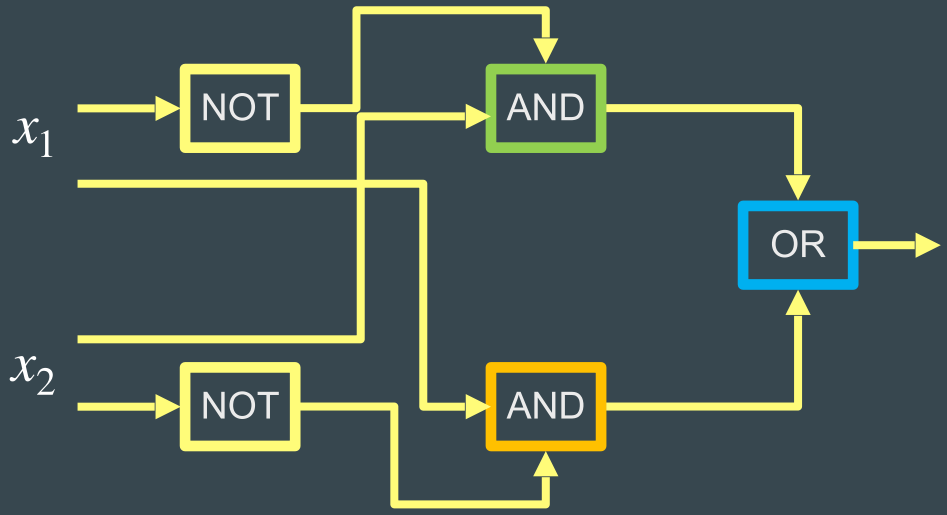
X1	X2	X1 xOr X2
1	1	0
1	0	1
0	1	1
0	0	0





Solution

$$x_1 \oplus x_2 = (\bar{x}_1 \,\&\, x_2) || (x_1 \,\&\, \bar{x}_2)$$



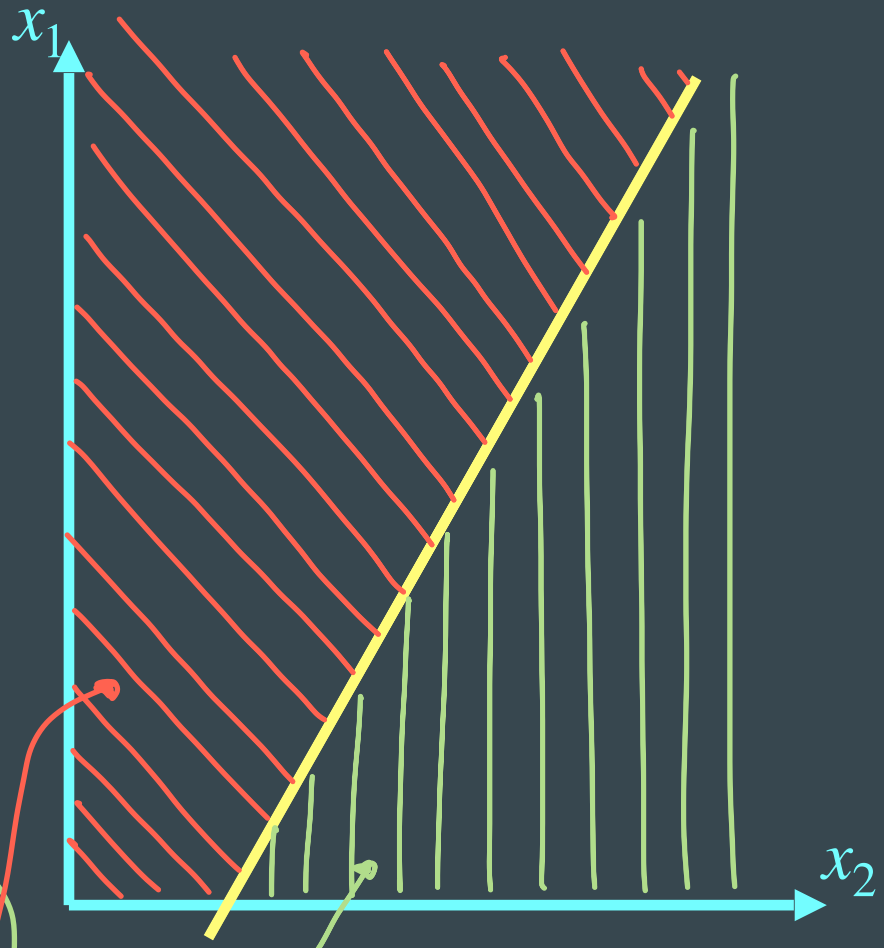
Network allows to perform a non-linear operation

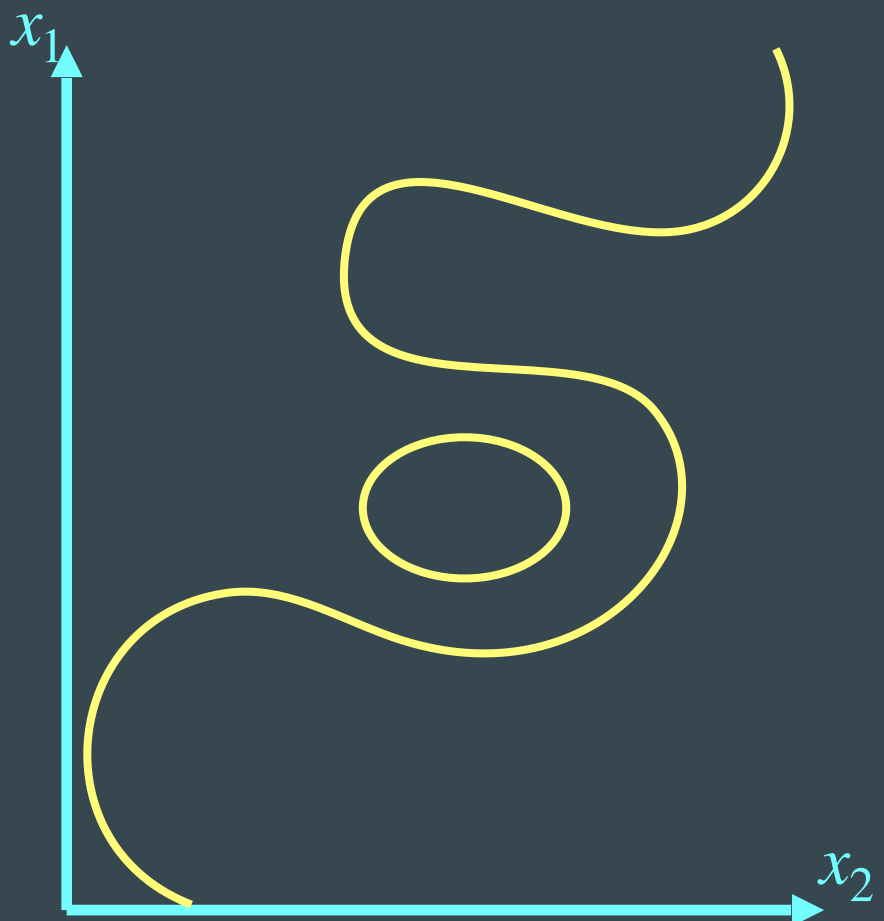
5 neurons in this network.

Theoretically you may use 3.

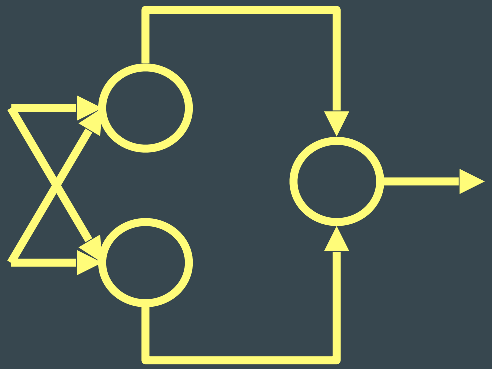
Task: design that solution.

Neuron VS Network





Network of Linear Neurons



$$y_3 = f(z_3) = z_3 = w_1^3 y_1 + w_2^3 y_2 + b^3$$

$$y_1 = f(z_1) = z_1 = w_1^1 x_1 + w_2^1 x_2 + b^1$$

$$y_2 = f(z_2) = z_1 = w_1^2 x_1 + w_2^2 x_2 + b^2$$

$$y_3 = w_1^3 \left[w_1^1 x_1 + w_2^1 x_2 + b^1\right] + w_2^3 \left[w_1^2 x_1 + w_2^2 x_2 + b^2\right] + b^3$$

$$y_3 = x_1 \left[w_1^3 w_1^1 + w_2^3 w_1^2\right] + x_2 \left[w_1^3 w_2^1 + w_2^3 w_2^2\right] + \left[w_1^3 b^1 + w_2^3 b^2 + b^3\right]$$

$$y_3 = x_1 \tilde{w}_1 + x_2 \tilde{w}_2 + \tilde{b}$$

Sigmoid

Threshold function

$$\theta(z) = \begin{cases} 1 \;\; if \;\; z \geq 0 \\ 0 \;\; if \;\; z < 0 \end{cases}$$

Sigmoid

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

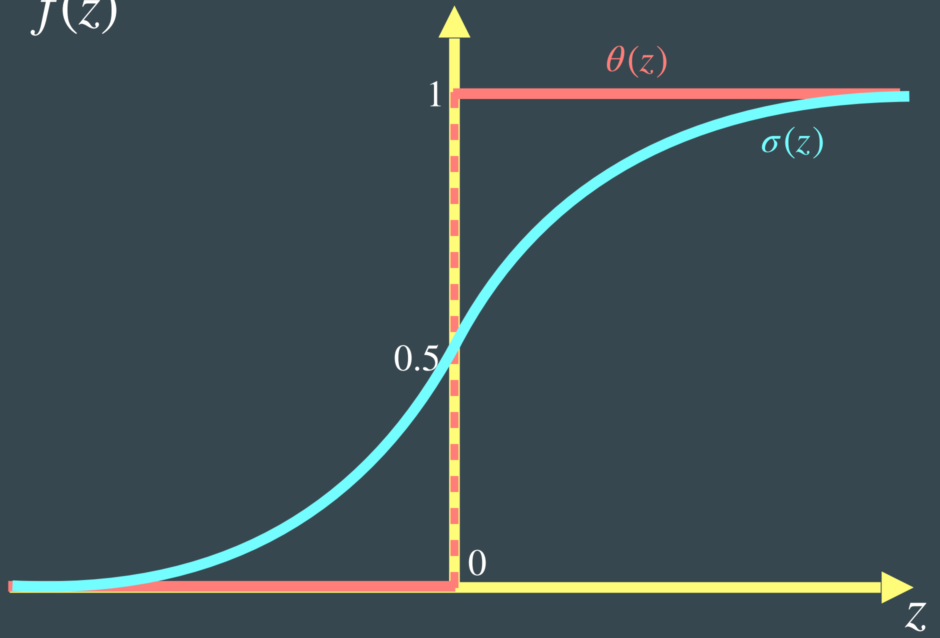
$z \to +\infty \qquad \sigma(z) \to 1$

$z \to -\infty \qquad \sigma(z) \to 0$

$\sigma(z) \in (0,1)$

$\sigma(0) = 0.5$

$f(z)$



Sigmoid

Threshold function

$$\theta(z) = \begin{cases} 1 \ \ if \ z \geq 0 \\ 0 \ \ if \ z < 0 \end{cases}$$

Sigmoid

$$\sigma(z) = \frac{1}{1 + \exp\left(-\frac{z}{T}\right)}$$

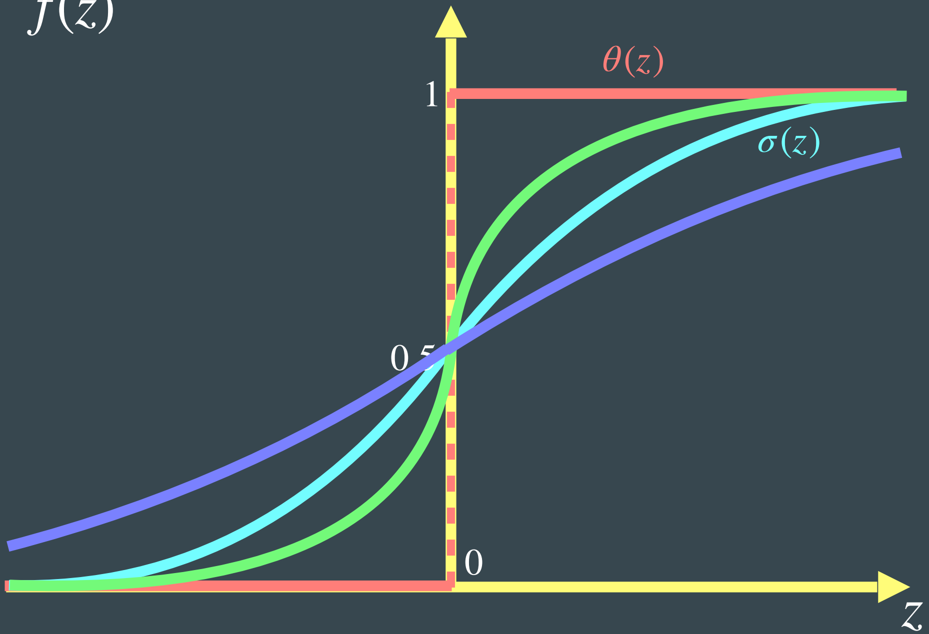
$z \to +\infty \qquad \sigma(z) \to 1$

$z \to -\infty \qquad \sigma(z) \to 0$

$\sigma(z) \in (0,1)$

$\sigma(0) = 0.5$

$f(z)$



$T \to 0$

$\sigma(z) \to \theta(z)$

Summary

- Biological neuron
- Mathematical model of neuron
- Logical operations as neurons
- Linearity of neuron and non-linearity of network
- Pointlessness of combining of linear neurons