Part 2: Trainable Networks

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Mikhail Romanov

Training Philosophy

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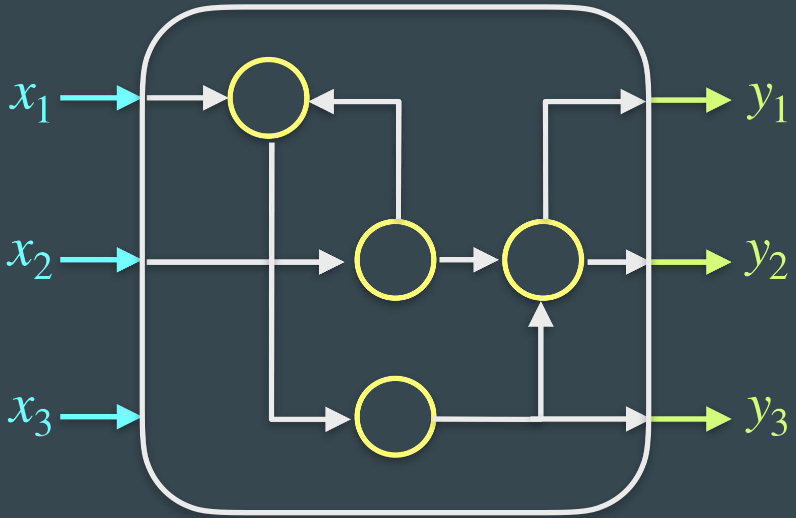

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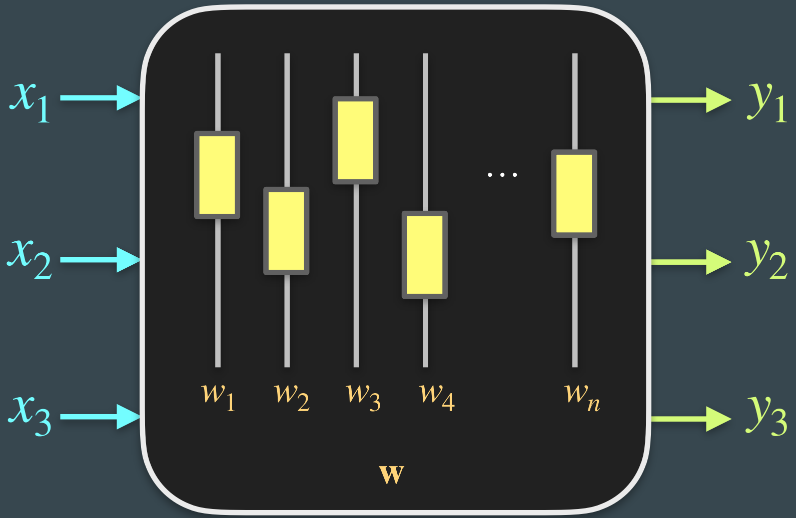

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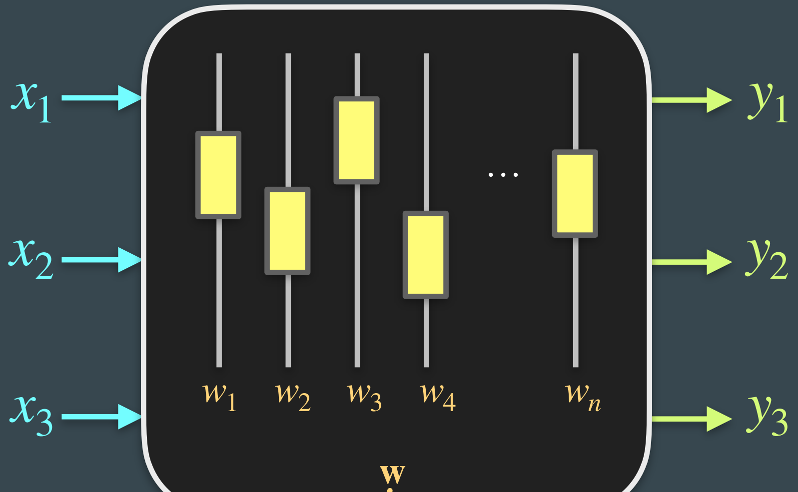

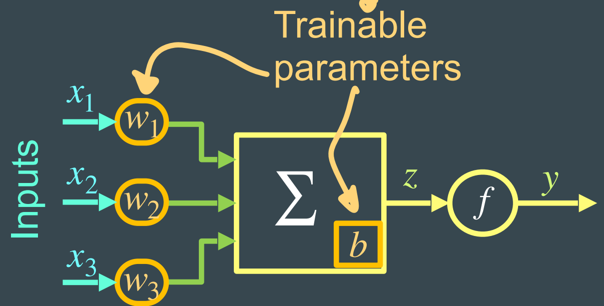

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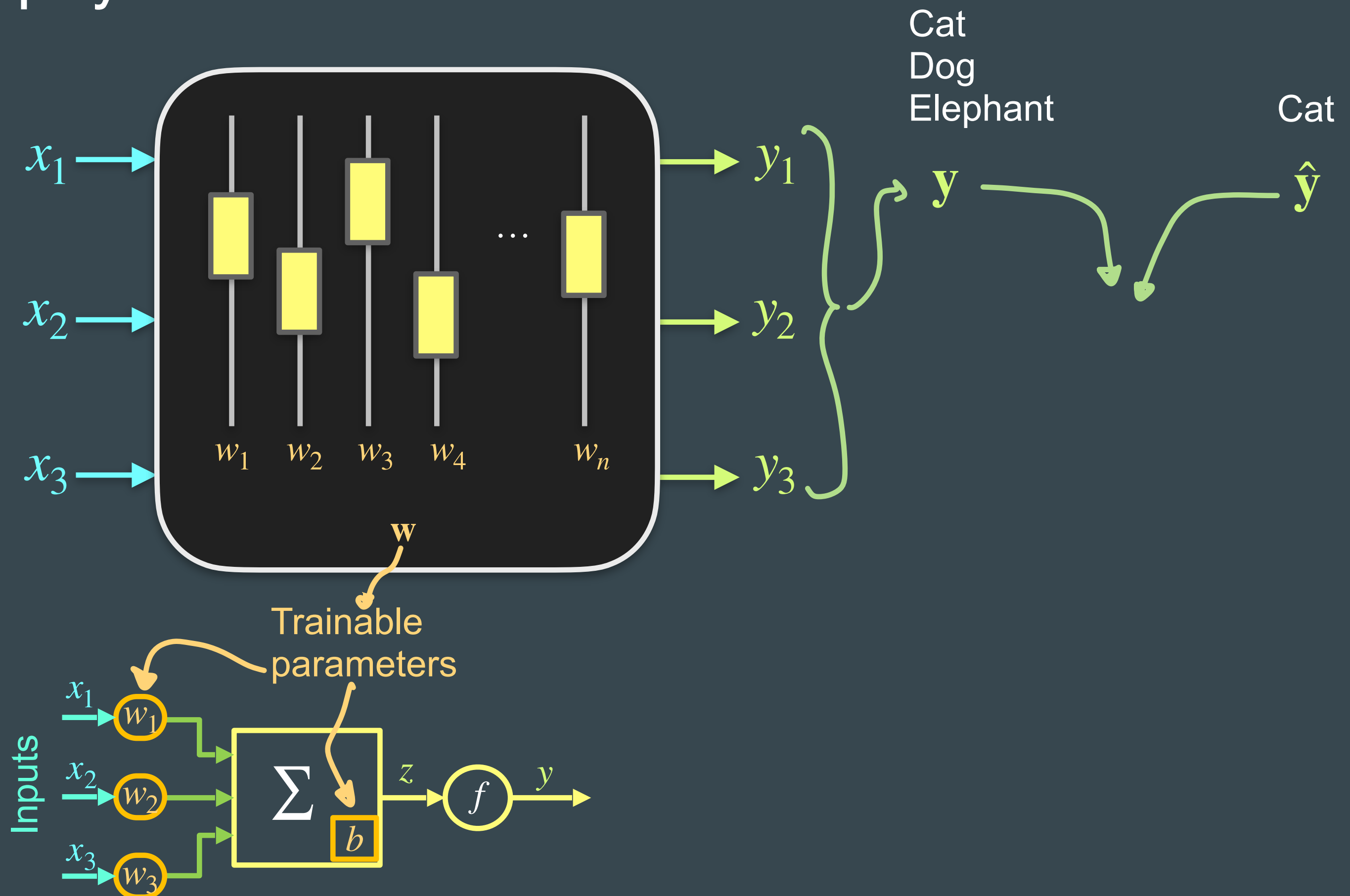

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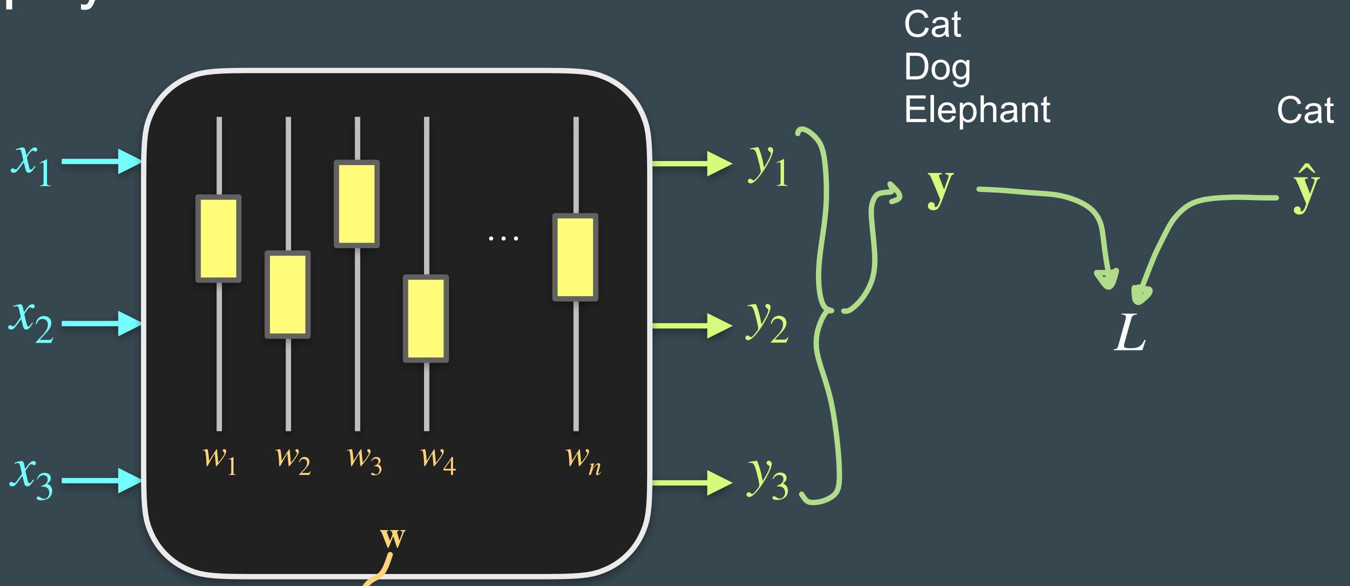
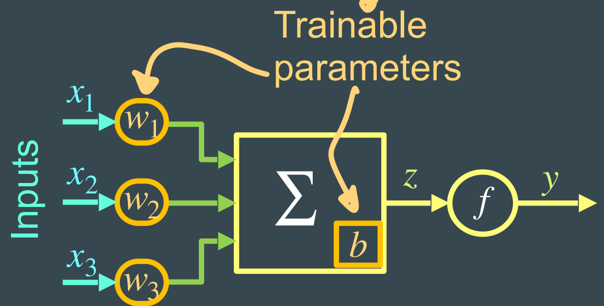
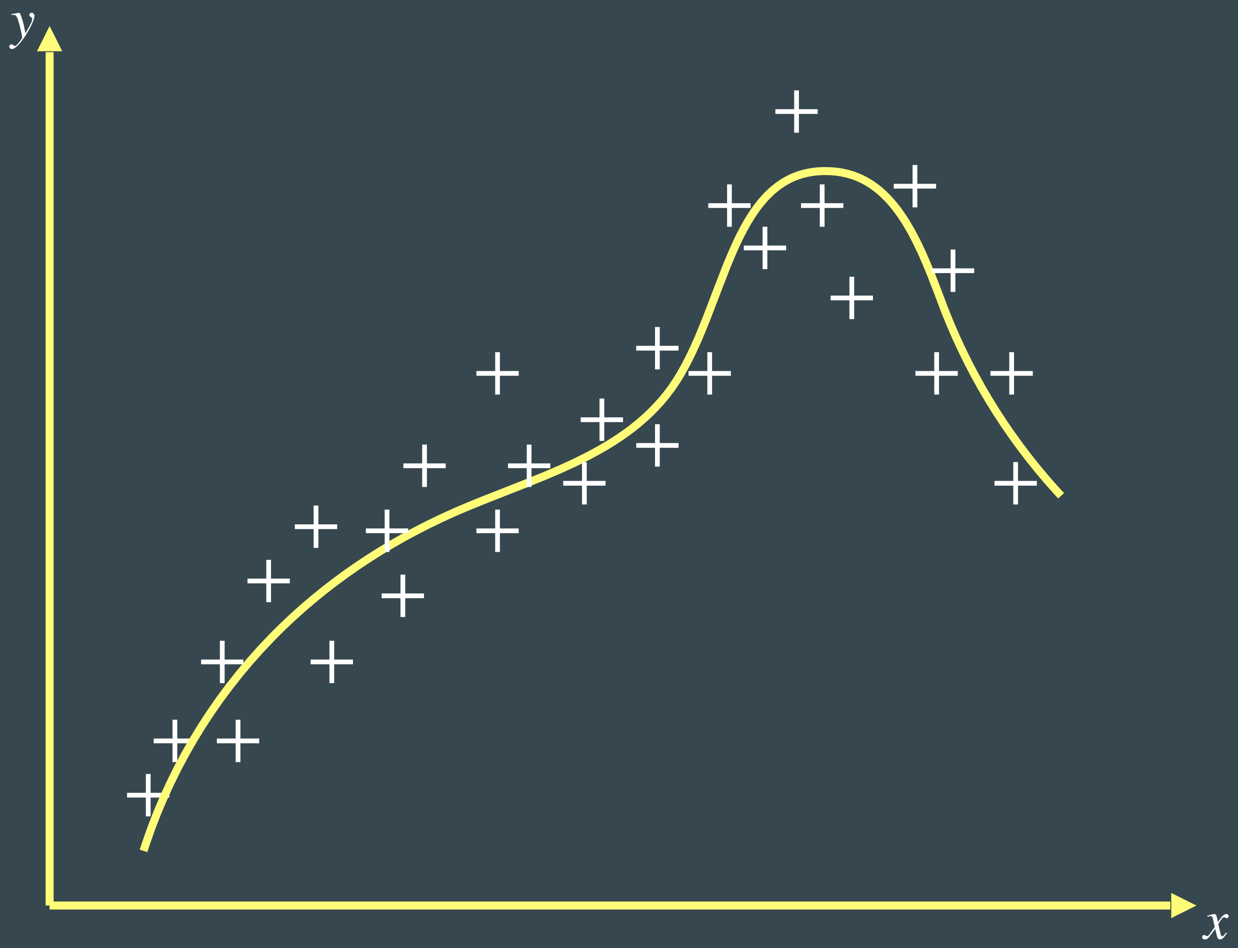


$$\mathbf{w} = \underset{\mathbf{w}}{\text{argmin}}\, L(Net_{\mathbf{w}}(x), t)$$



Dependency Reconstruction



$$y_i = f(x_i) + \epsilon_i$$

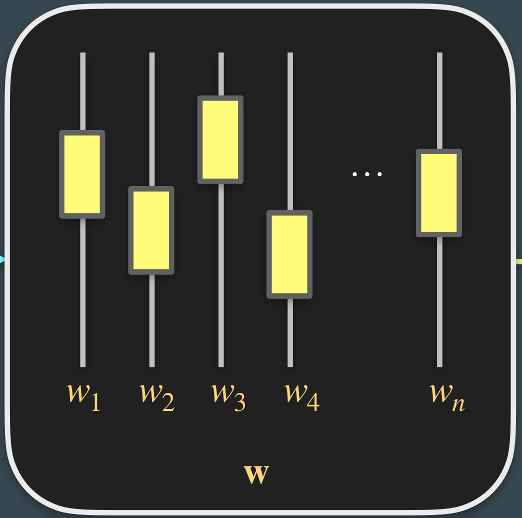
ϵ_i	Noise
$f(x_i)$	Hidden Dependency
x_i	Circumstances of Measurement
y_i	Result of measurement

Tuning the network

Features	Labels
$\mathbf{x}_1$	$\mathbf{y}_1$
$\mathbf{x}_2$	$\mathbf{y}_2$
$\mathbf{x}_3$	$\mathbf{y}_3$
$\mathbf{x}_4$	$\mathbf{y}_4$
$\mathbf{x}_5$	$\mathbf{y}_5$
$\mathbf{x}_6$	$\mathbf{y}_6$
$\dots$	$\dots$
$\mathbf{x}_S$	$\mathbf{y}_S$

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \dots \\ x_N \end{bmatrix}$$

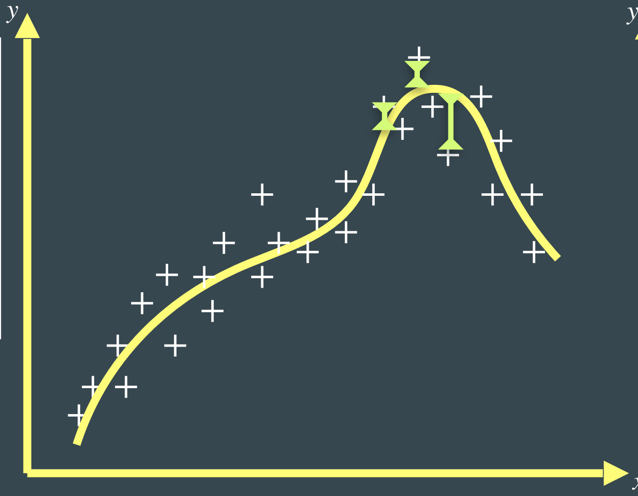
$\mathbf{x}$

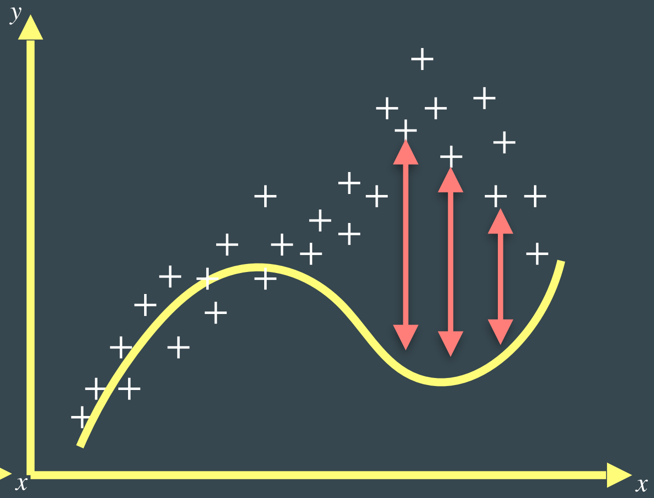


$\hat{\mathbf{y}}$

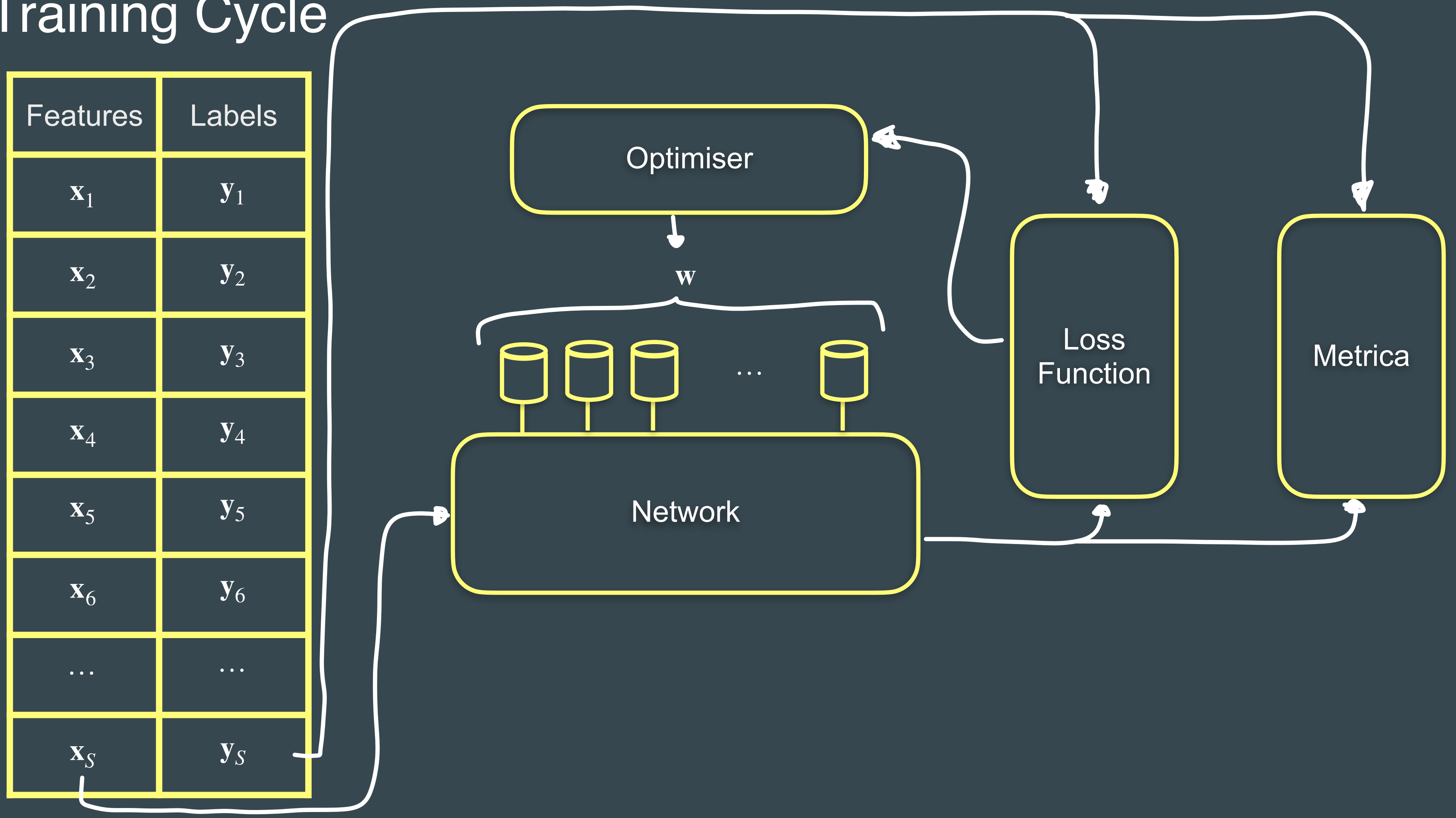
$$L_{total} = \sum_{s=1}^{S} L\left(Net_{\mathbf{w}}(x_s), y_s\right)$$

$$\mathbf{y} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_N \end{bmatrix}$$





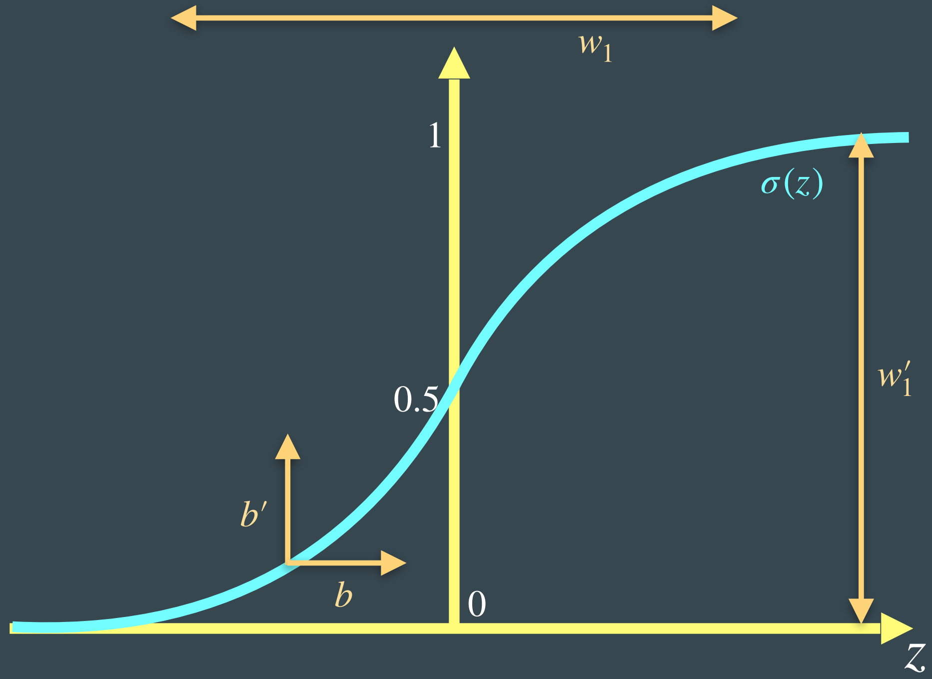
Training Cycle



Shaping the sigmoid

Sigmoid

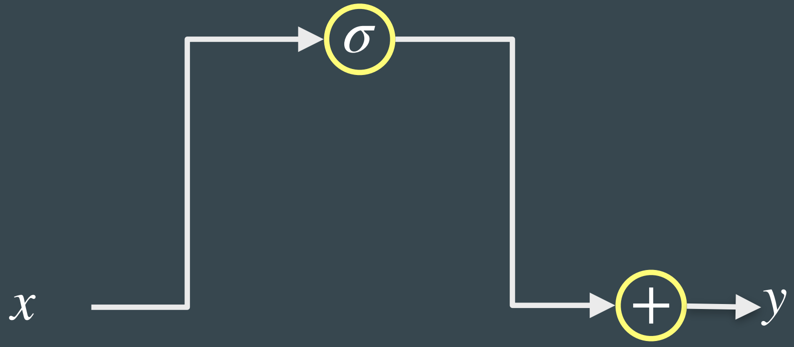
$$\sigma(z) = \frac{1}{1 + \exp(-z)} \qquad y = w'\sigma(wx+b)+b'$$

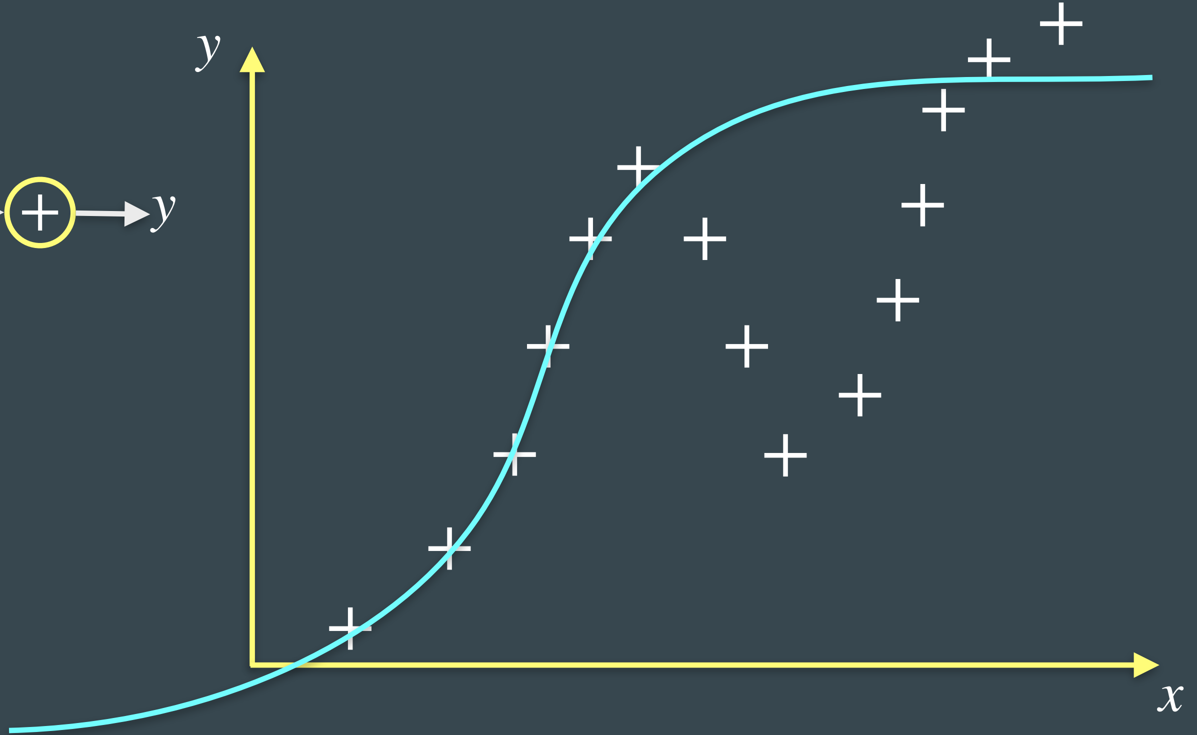


Two Layer Neural Net

$$y = w_1'\sigma(w_1 x+b_1)+b_1'$$

$$\hat{y} = b^{out}+\sum_{i=1}^{N} w_i^{out}\sigma(w^i x+b^i)$$



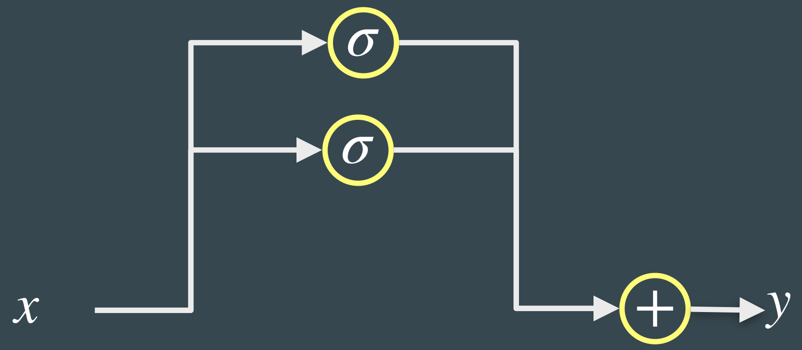


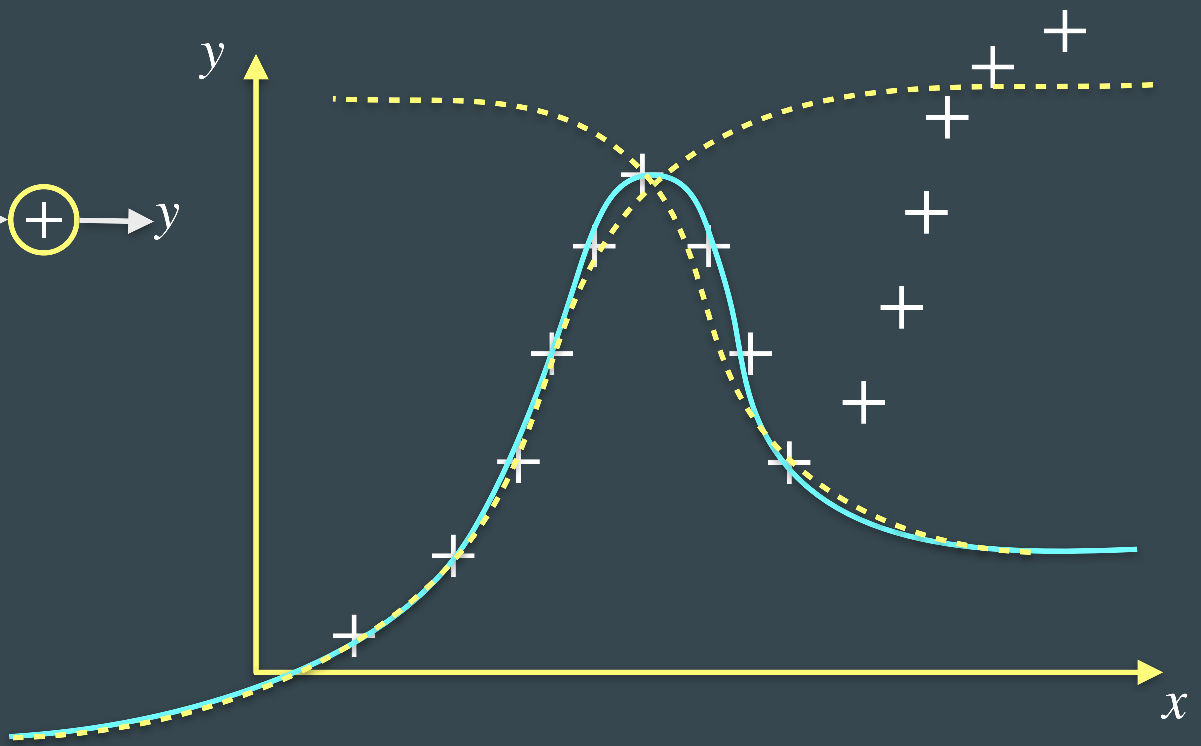
Two Layer Neural Net

$$y = w_1'\sigma(w_1 x + b_1) + b_1'$$

$$+ w_2'\sigma(w_2 x + b_2) + b_2'$$

$$\hat{y} = b^{out} + \sum_{i=1}^{N} w_i^{out}\sigma(w^i x + b^i)$$





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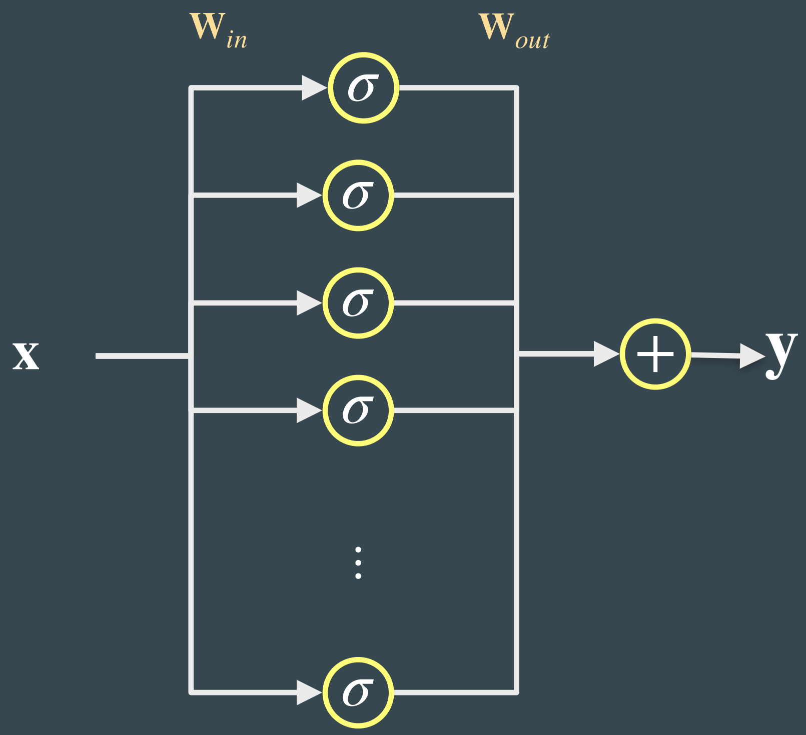


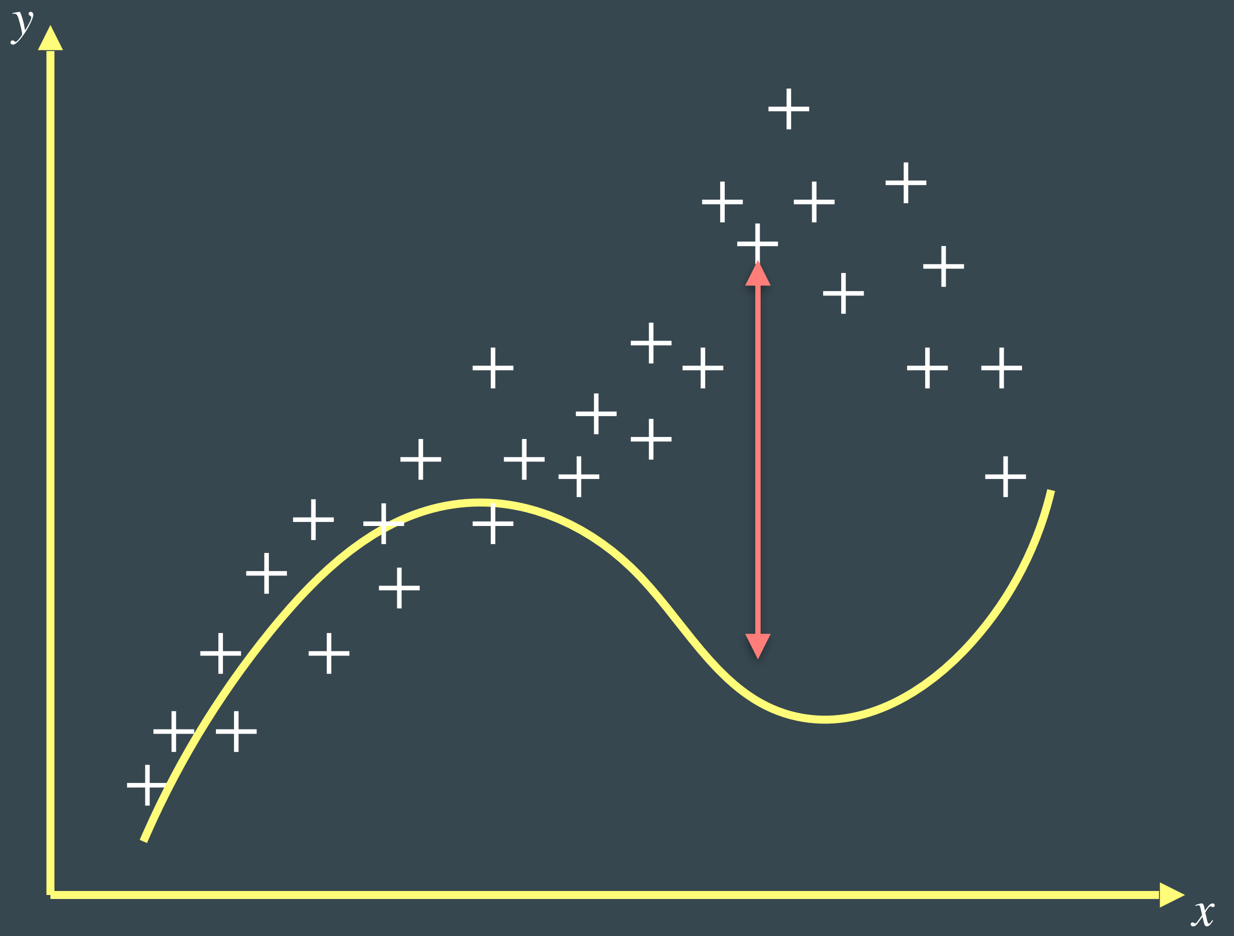
$$\hat{\mathbf{y}} = \mathbf{b}_{out} + \sum_{i=1}^{N} \mathbf{W}_{out}\sigma(\mathbf{W_{in}}\mathbf{X} + \mathbf{b}_{in})$$

Linear Combination of Sigmoids is Full System!

Loss function

The simplest loss function is
Mean Squared Error
(MSE)

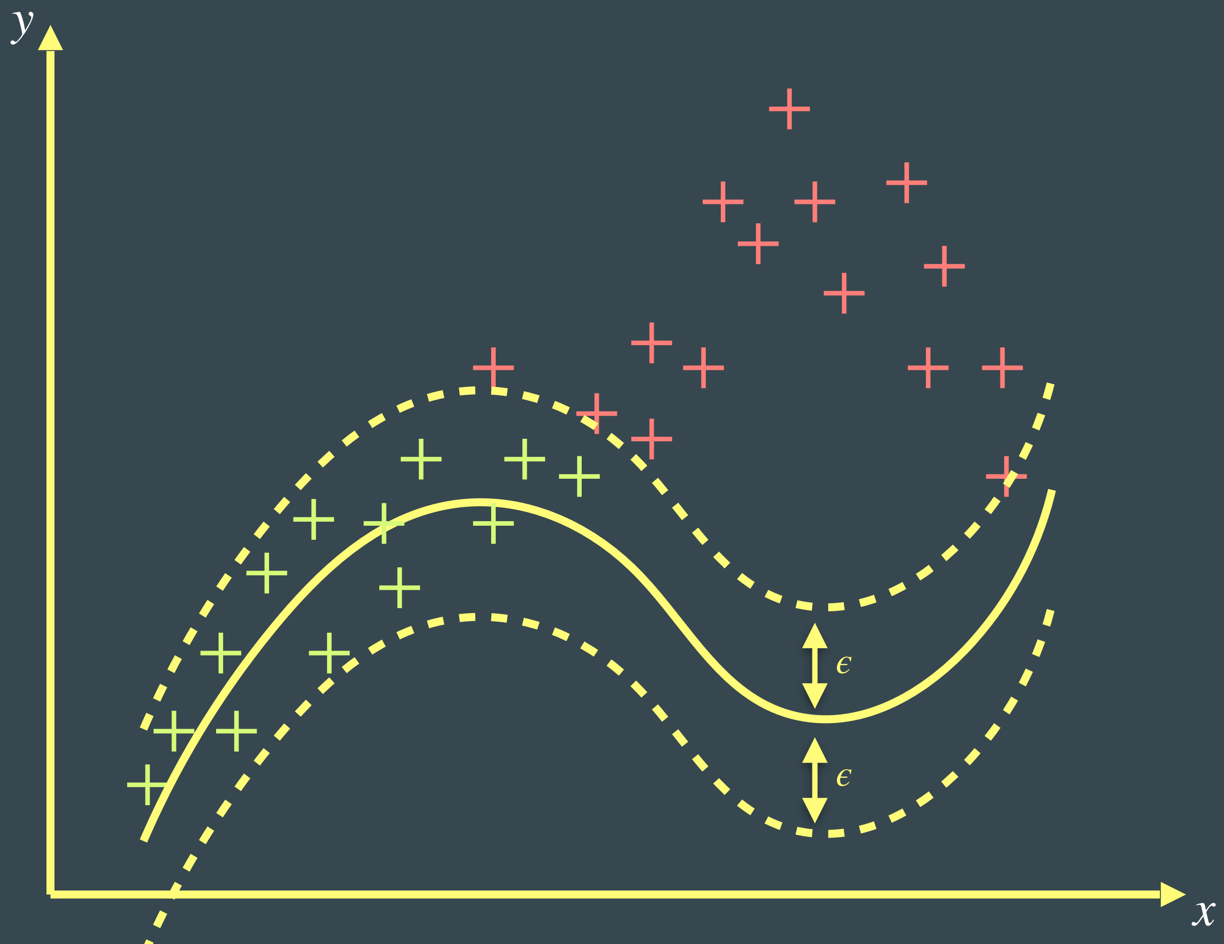
$$MSE(y, \hat{y}) = \frac{1}{N} \sum_{s=1}^{S} (y_s - \hat{y}_s)^2$$



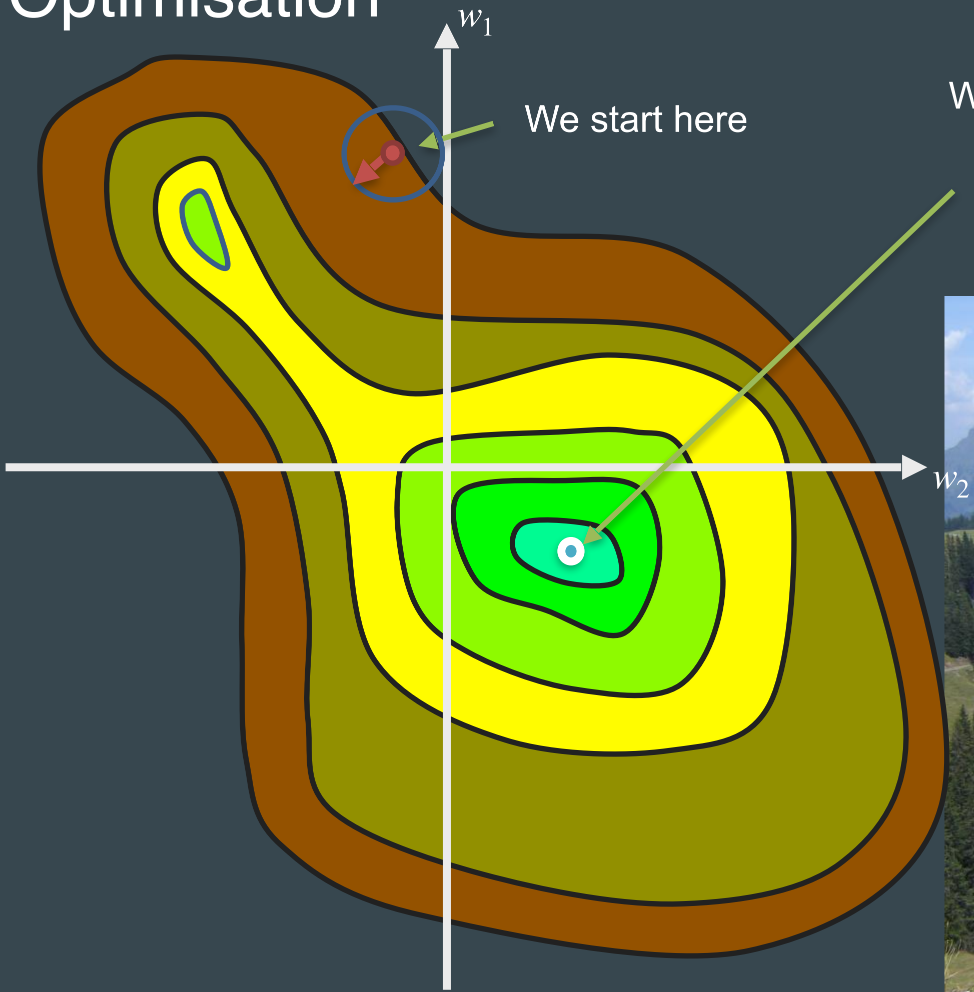
Metrica

Example:
Epsilon-precision

$$Acc_{\epsilon} = \frac{1}{N} \sum_{i=1}^{N} \left[\;\; y_i - \hat{y}_i \;\;\; < \epsilon \right]$$



Optimisation

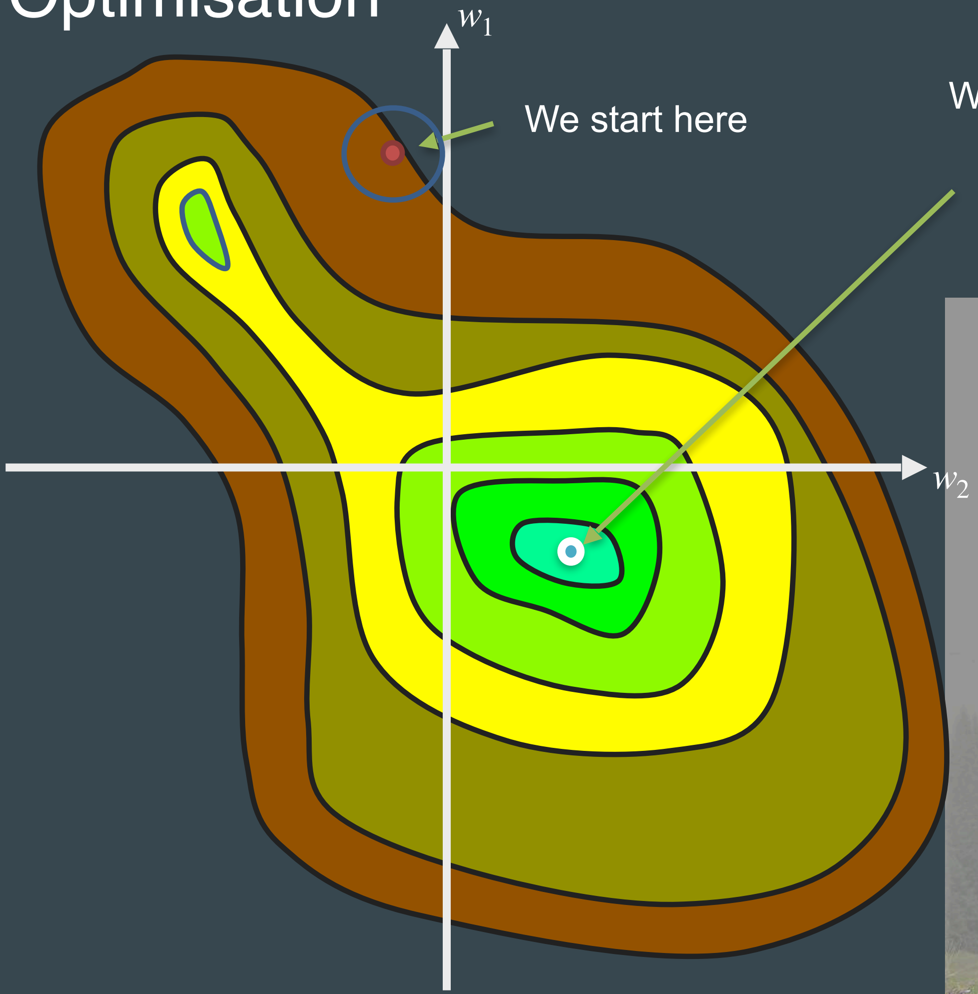


We start here

We want to arrive here

We are in “Switzerland”

Optimisation



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In the mist
What would you do?

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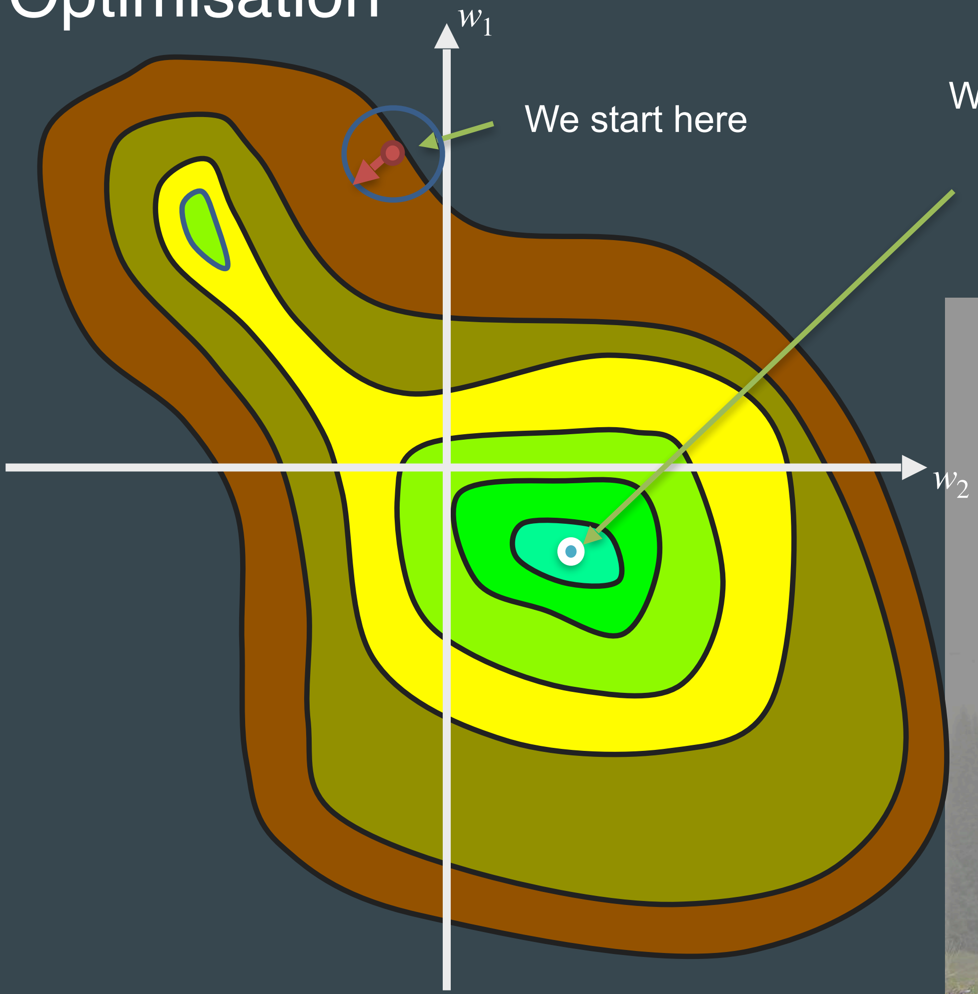

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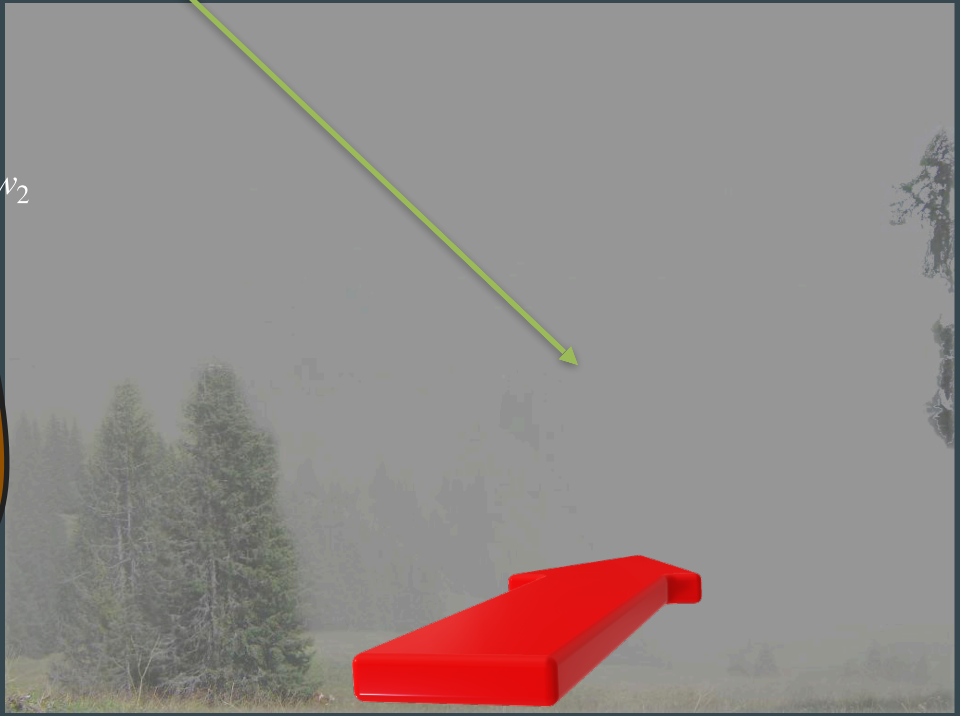
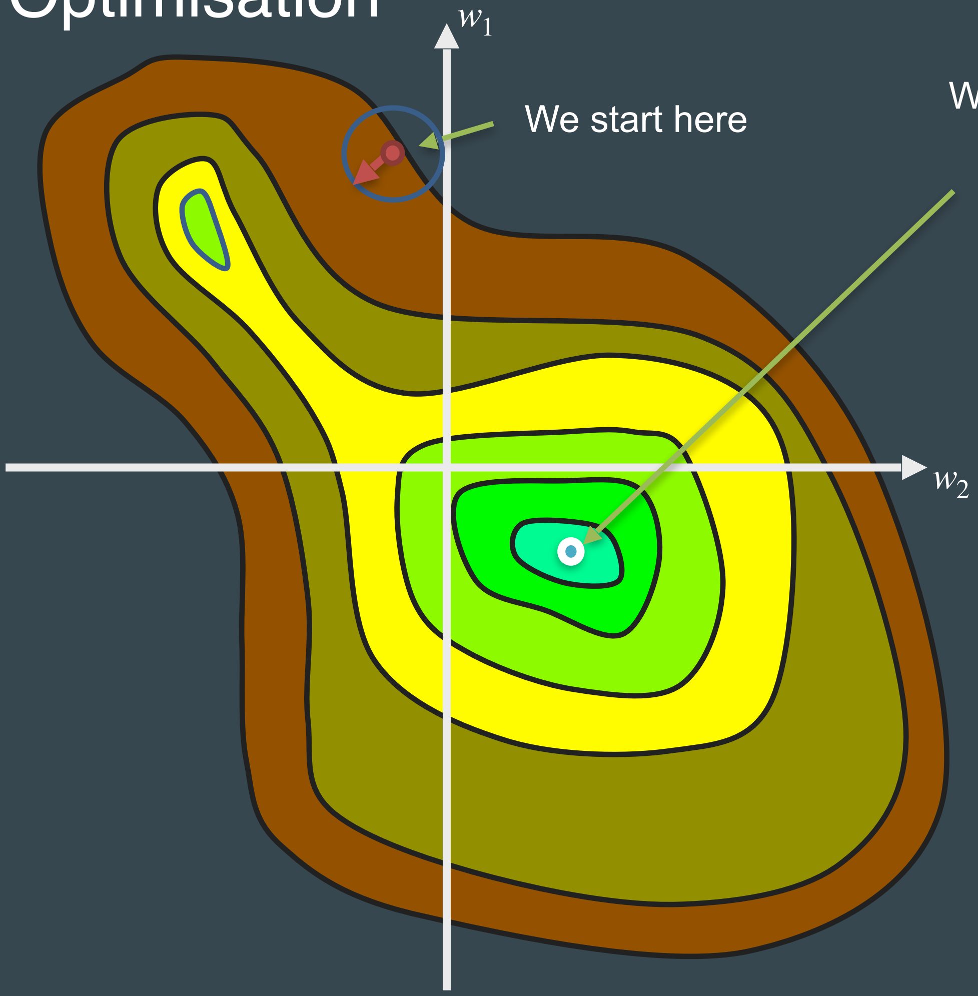


Optimisation

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_p \end{bmatrix} \qquad \nabla_{\mathbf{w}} L = \begin{bmatrix} \frac{\partial L}{\partial w_0} \\ \frac{\partial L}{\partial w_1} \\ \vdots \\ \frac{\partial L}{\partial w_p} \end{bmatrix}$$

Optimisation



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$$\mathbf{w}^{t=1} = \mathbf{w}^{t=0} - \alpha \nabla_{\mathbf{w}} L_{total}^{t=0}$$

$$L_{total}^{t=0} = \frac{1}{S} \sum_{s=1}^{S} L(Net_{\mathbf{w^{t=0}}}(x_s), y_s)$$

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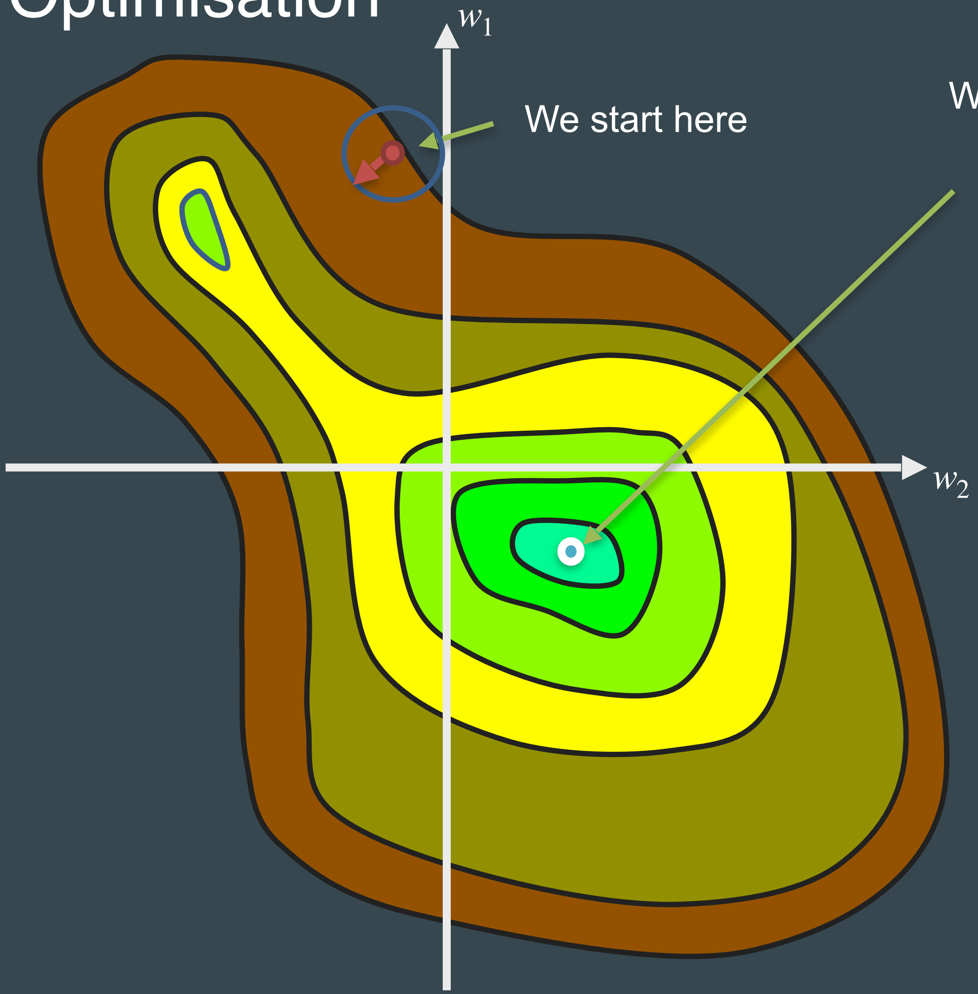

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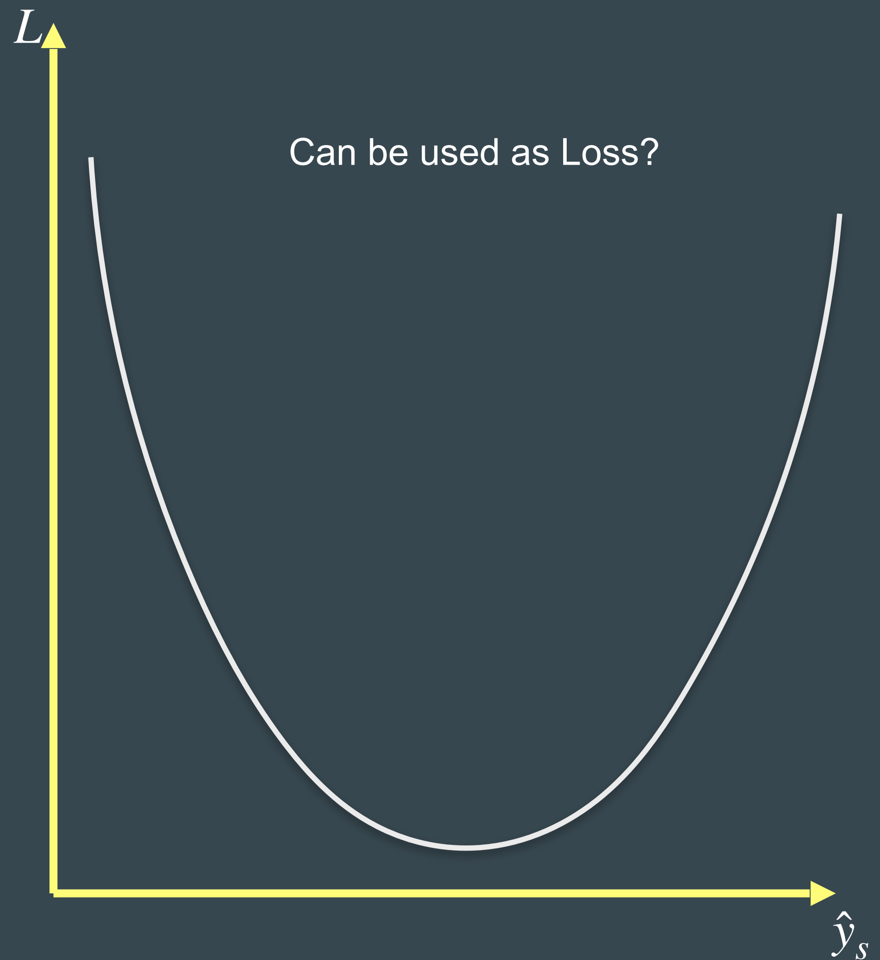
$$\mathbf{w}^{t=2} = \mathbf{w}^{t=1} - \alpha \nabla_{\mathbf{w}} L_{total}^{t=1}$$

$$\dots$$

$$\mathbf{w}^{t+1} = \mathbf{w}^{t} - \alpha \nabla_{\mathbf{w}} L_{total}^{t}$$

$$L_{total}^{t=0} = \frac{1}{S} \sum_{s=1}^{S} L(Net_{\mathbf{w^{t=0}}}(x_s), y_s)$$

What can be used as Loss Function



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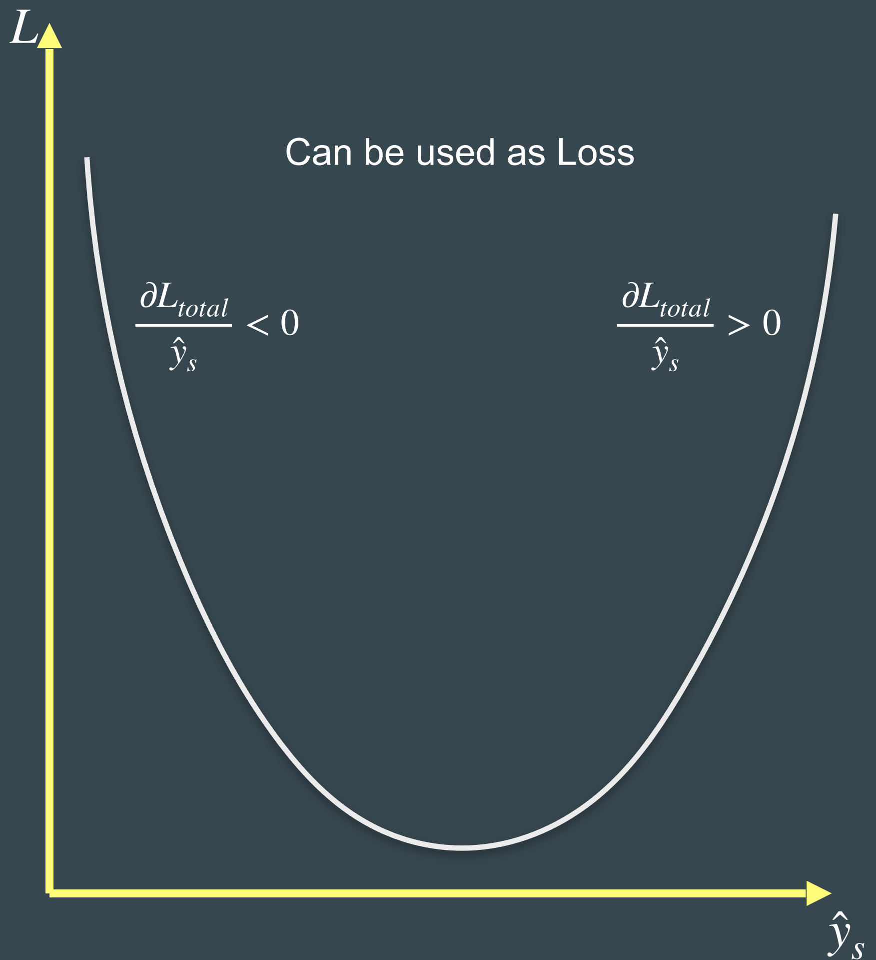

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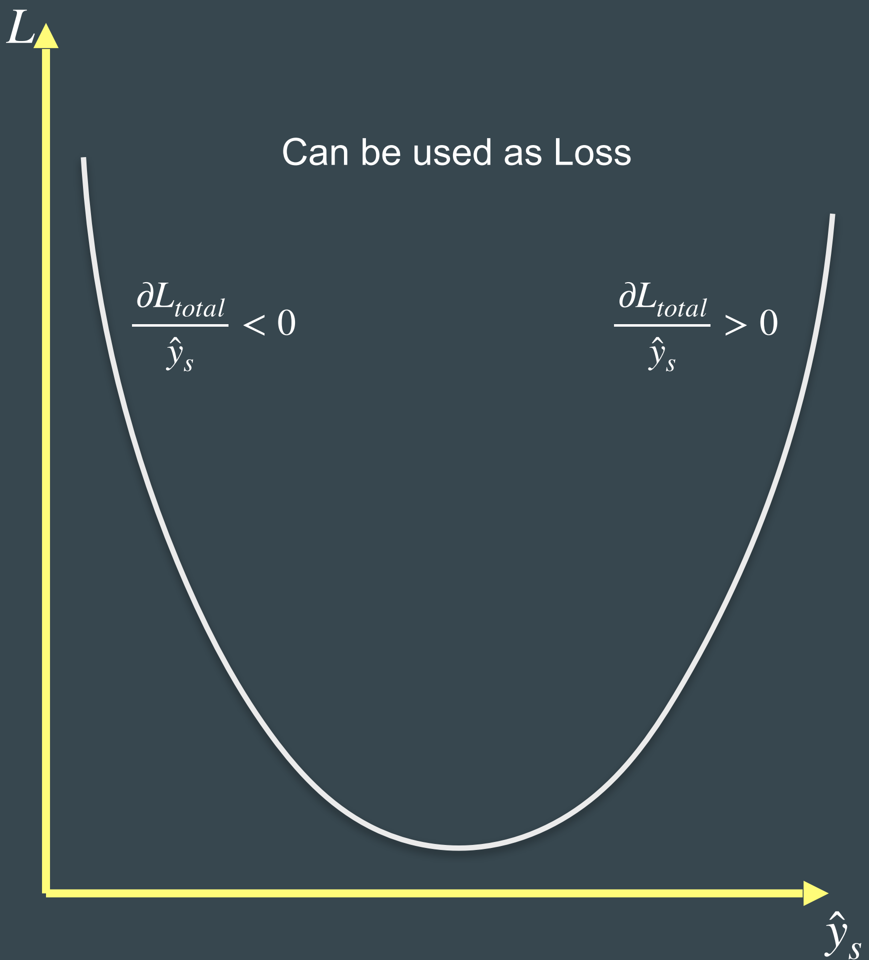

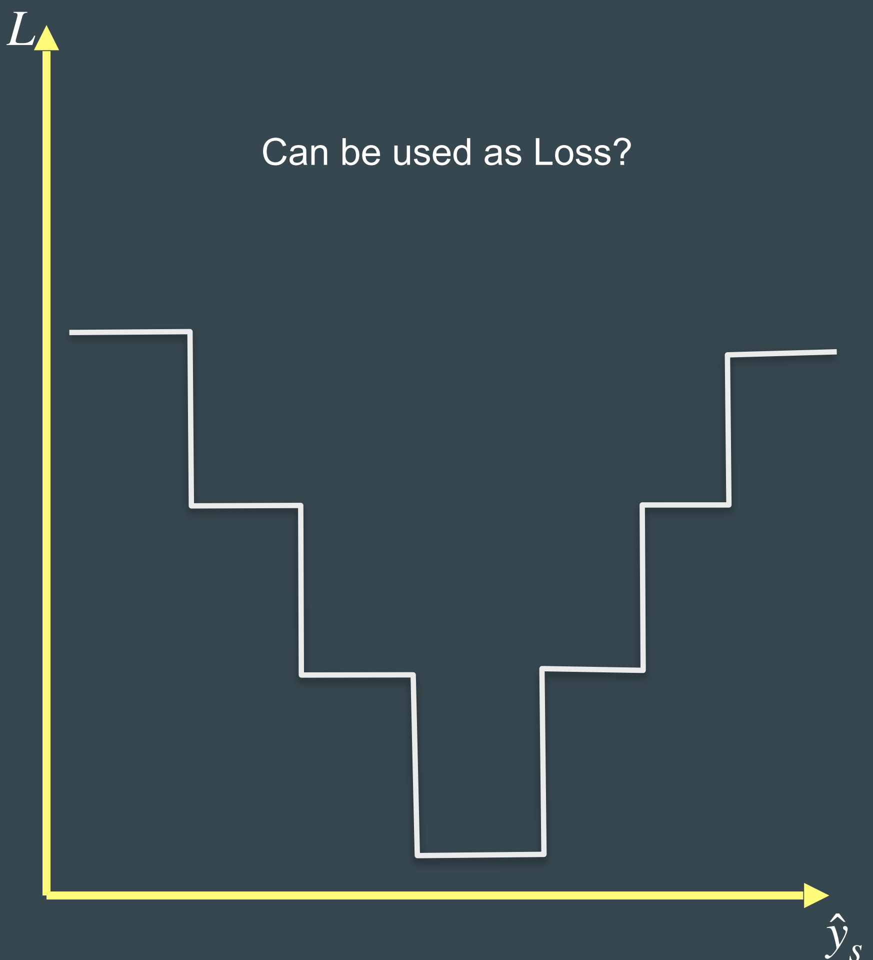

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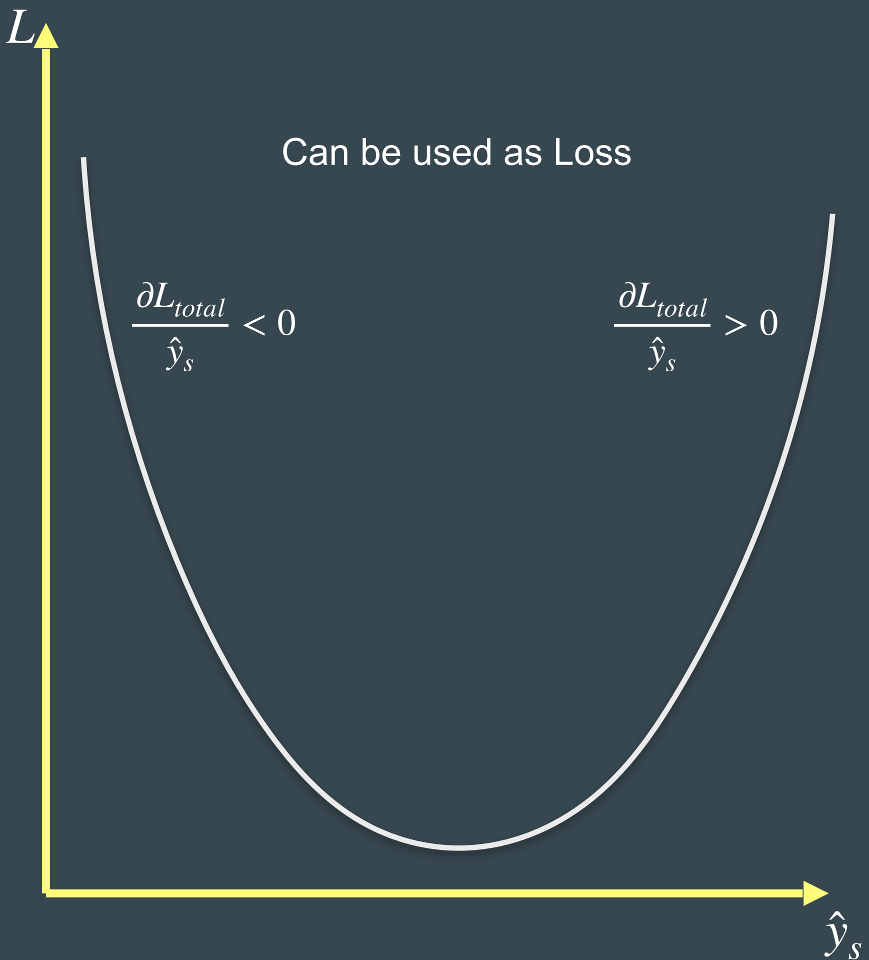

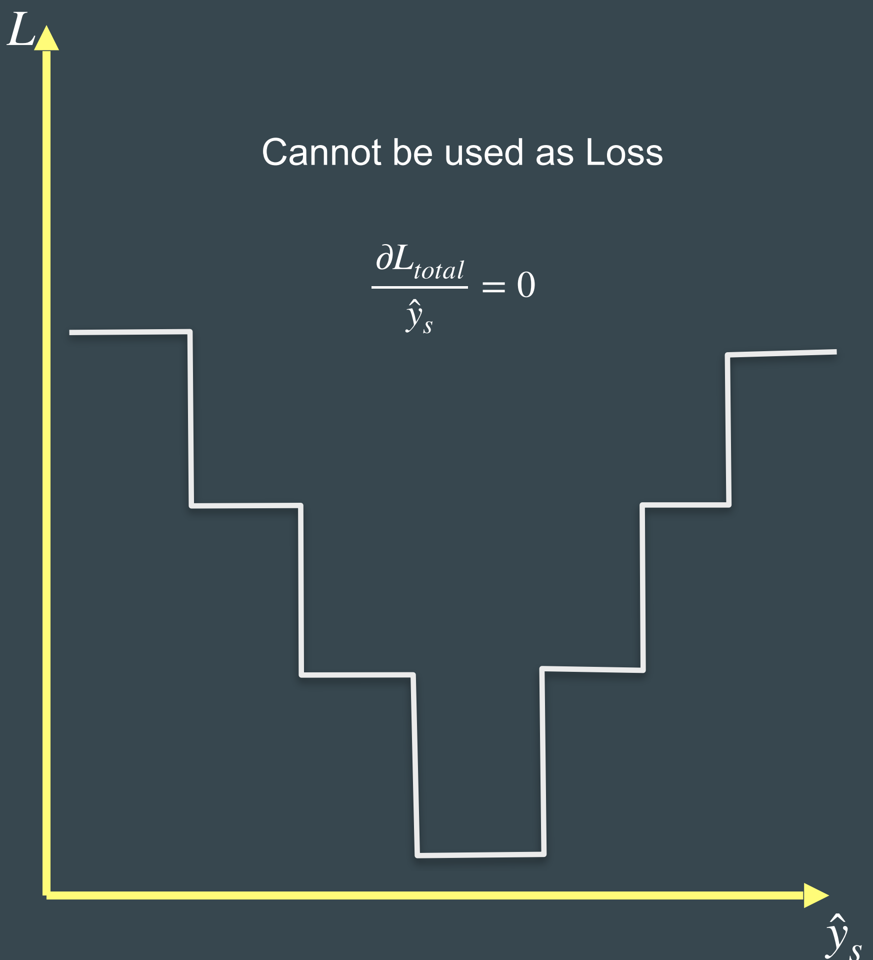

What is missing?

$$\frac{\partial L_{total}}{\hat{y}_s} \neq 0$$

$$\mathbf{w}^{t+1} = \mathbf{w}^t - \alpha \nabla_{\mathbf{w}} L_{total}^t$$

$$\nabla_{\mathbf{w}} L = \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial w_1} \\ \vdots \\ \frac{\partial L}{\partial w_p} \end{bmatrix}$$

Backpropagation

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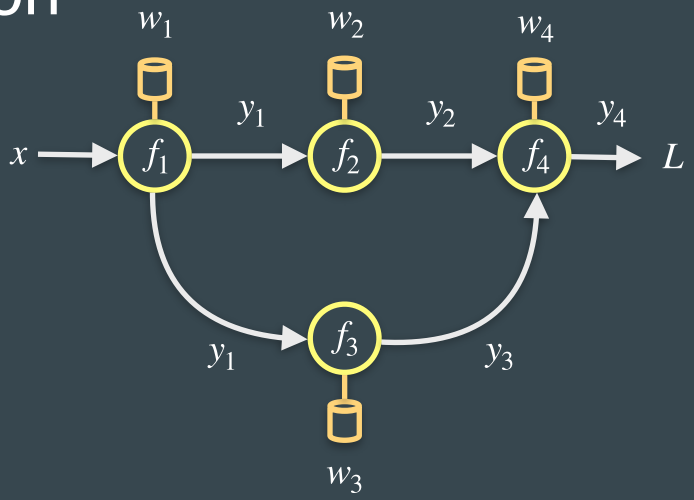

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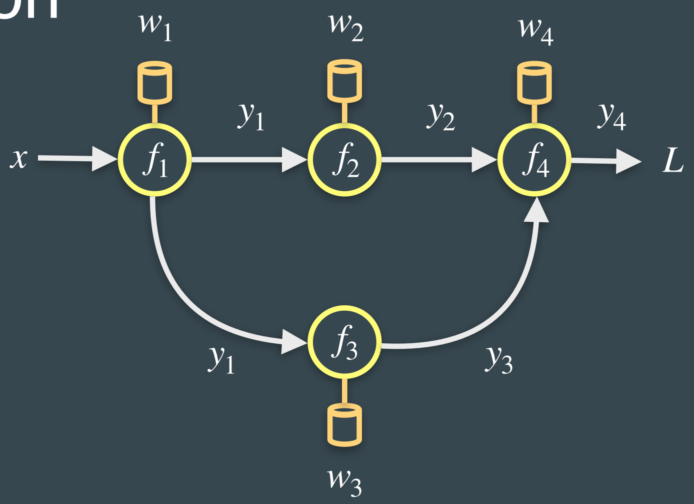

$$L\left(f_4\left(f_3\left(f_1(x)\right), f_2\left(f_1(x)\right)\right)\right)$$

Backpropagation

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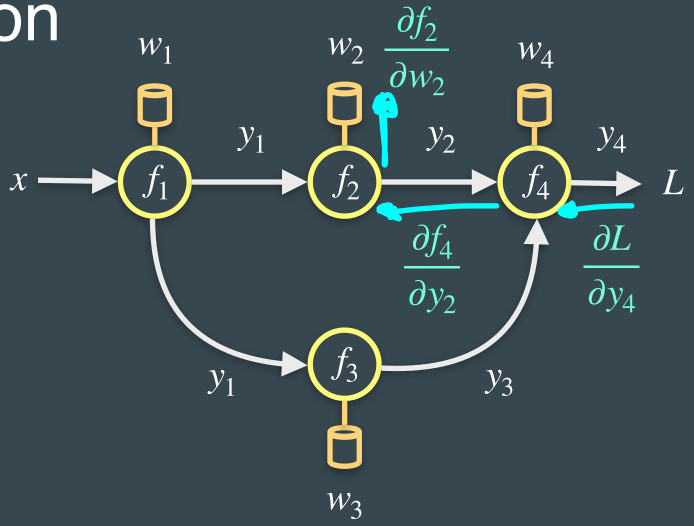

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$$\frac{\partial L}{\partial w_2}$$

Backpropagation

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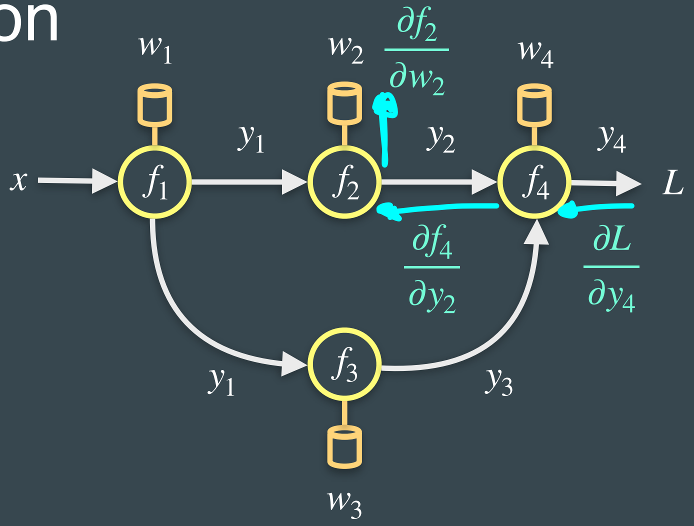

$$L\left(f_4\left(f_3\left(f_1(x)\right), f_2\left(f_1(x)\right)\right)\right)$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial y_4}\frac{\partial f_4}{\partial y_2}\frac{\partial f_2}{\partial w_2}$$

Backpropagation

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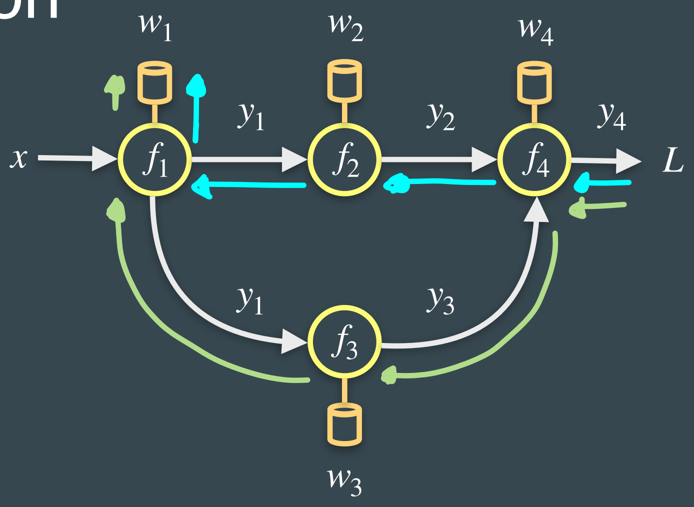

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$$\frac{\partial L}{\partial w_1}$$

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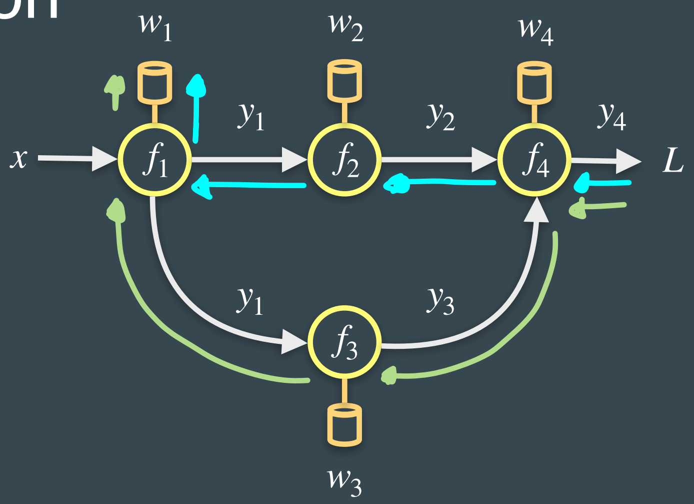

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$$\frac{\partial L}{\partial w_1} = \underbrace{\frac{\partial L}{\partial y_4}\frac{\partial f_4}{\partial y_2}\frac{\partial f_2}{\partial y_1}\frac{\partial f_1}{\partial w_1}} + \underbrace{\frac{\partial L}{\partial y_4}\frac{\partial f_4}{\partial y_3}\frac{\partial f_3}{\partial y_1}\frac{\partial f_1}{\partial w_1}}$$

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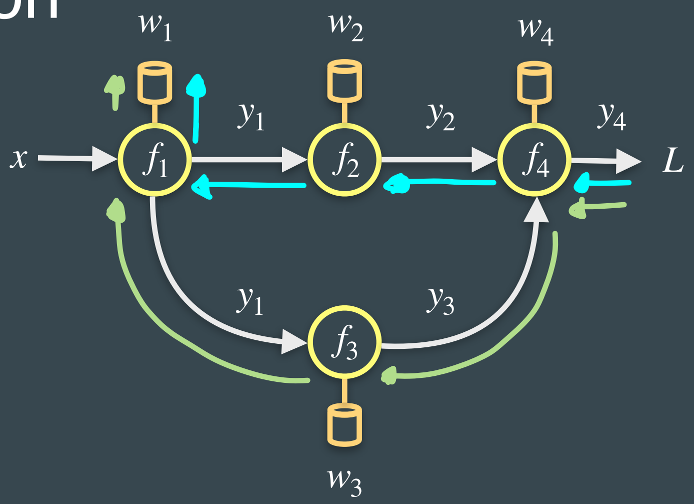

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial y_4}\left(\frac{\partial f_4}{\partial y_2}\frac{\partial f_2}{\partial y_1} + \frac{\partial f_4}{\partial y_3}\frac{\partial f_3}{\partial y_1}\right)\frac{\partial f_1}{\partial w_1}$$

Summary

- Example: dependency reconstruction
- Training philosophy
- Training cycle
- Two-layer FCNN and its awesomeness! — ARCHITECTURE
- MSE — LOSS
- Accuracy — METRICS
- Gradient Descent— OPTIMISER
- Backpropagation —Gradient calculation method