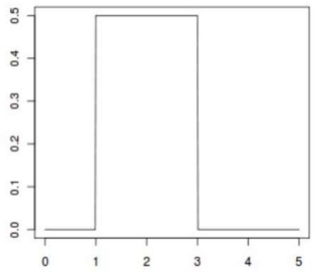
Uniform distribution

- Also called rectangular distribution describes experiments where there is an arbitrary (equal probability) outcome that lies between certain bounds.
- $f(x) = \begin{cases} \frac{1}{b-a} & for\ a \le x \le b \\ 0 & for\ \ a > x\ or\ x > b \end{cases}$
- $F(x) = \int_{-\infty}^{x} f(x)dx$

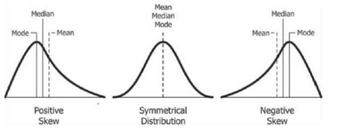
$F(x) = \begin{cases} 0 & for\ a > x \\ \frac{x-a}{b-a} & for\ a \le x \le b \\ 1 & for\ x > b \end{cases}$



1

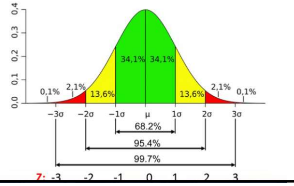
Moments

- Mathematically, the *n*-th moment of a continuous function about value c is:
  $\mu_n = \int_{-\infty}^{\infty} (x-c)^n f(x)dx$
- If $f(x)$ is a probability density function:
  - The first raw moment (c=0) is
  - The second central moment (c=mean) is
  - The third standardized moment (central moment divided by σ) is
    - Measure of asymmetry



Normal distribution

- A function whose value at any given interval represents the relative likelihood of the samples in that interval.
- $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
- $P(x > \sigma) = ?$
- standard normal distribution: μ=0 and σ=1
  - $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$



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Vector operations:

- Dot product: $\begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} . \begin{bmatrix} 0 \\ -2 \\ 14 \end{bmatrix} = 8*0+5*(-2)+4*14 = 46$
  - The result of vector dot product (aka inner product) is a scalar (just a number!)
  - The dot product of a vector with itself is the square of its magnitude: $\begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} . \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} =$
    $8*8+5*5+4*4 = 105$
  - The dot product is also related to the angle between the two vectors: $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ where θ is the angle btw the two vectors.
  - If $\vec{A}.\vec{B}$ =0, it means that the 2 vectors are perpendicular to each other.

Vector operations:

- Cross product: $\begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} \times \begin{bmatrix} 0 \\ -2 \\ 14 \end{bmatrix} = \begin{bmatrix} 5*14-(-2)*4 \\ -(8*14-4*0) \\ 8*(-2)-0*5 \end{bmatrix} = \begin{bmatrix} 78 \\ -112 \\ -16 \end{bmatrix}$
  - The result of vector cross product is another vector (**NOT** just a number!)
  - The resultant vector is perpendicular to both original vectors.
  - $\begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} \times \begin{bmatrix} 78 \\ -112 \\ -16 \end{bmatrix}$=0 and $\begin{bmatrix} 0 \\ -2 \\ 14 \end{bmatrix} \times \begin{bmatrix} 78 \\ -112 \\ -16 \end{bmatrix}$=0
  - The cross product is also related to the angle between the two vectors: $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\vec{n}$ where θ is the angle btw the two vectors and $\vec{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.
  - If $\vec{A}\times\vec{B}$ =0, it means that the 2 vectors are colinear, either in the same direction or exact opposite direction.

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Matrix operations:

- Element-wise addition: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$
- Multiplication: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} . \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$
- Is AB=BA? Try with A= $\begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix}$
- $\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}$ can be used to scale a vector by α: $\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha x \\ \alpha y \end{bmatrix} = \alpha \begin{bmatrix} x \\ y \end{bmatrix}$

Inverse of a 2-by-2 matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Inverse of a 3-by-3 matrix:

$$A^{-1} = \frac{1}{\det(A)} \mathrm{adj(A)}$$
$$\mathrm{adj(A)} = C^T$$
$$C = (-1)^{i+j} M_{ij}$$

Adjugate of a matrix is the transpose of the cofactor matrix.
M_{ij}, the (i, j) minor, is the determinant of the submatrix formed by deleting the ith row and jth column.

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TF = The number of times a word appears in a document divided by the total number of words in the document

- e.g.: when a 30-word document contains the term "student" 5 times, the TF for the word 'student' is 5/30=1/6.

IDF = $log\left(\frac{N}{df_x}\right)$

df_x: number of documents containing x

N: total number of documents

- e.g.: Let's assume the size of the corpus is 100 documents. If there are 20 documents that contain the term "student", then the IDF is:

  $log\left(\frac{100}{20}\right) = log5 = 0.70$

Mathematically, TF-IDF ($W_{x,y}$) of a word x in a document y is obtained from:

$$W_{x,y} = TF_{x,y} \times IDF_x$$

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What is Singular Value Decomposition (SVD)?

the singular value decomposition of a complex m-by-n matrix M is given by:

- $\boldsymbol{M = U\Sigma V^T}$

where m-by-m $\boldsymbol{U}$ and n-by-n $\boldsymbol{V}$ are orthogonal matrices, and $\boldsymbol{\Sigma}$ is a m-by-n rectangular diagonal matrix with non-negative real numbers on the diagonal.

orthogonal matrix:

- a square matrix whose columns and rows are orthonormal vectors
- In simple terms: $\boldsymbol{V^T V = V V^T = I}$ or $\boldsymbol{V^T = V^{-1}}$

		Predicted Class		
		Positive	**Negative**	
Actual Class	**Positive**	True Positive (TP)	False Negative (FN) **Type II Error**	**Sensitivity** $\frac{TP}{(TP+FN)}$
	Negative	False Positive (FP) **Type I Error**	True Negative (TN)	**Specificity** $\frac{TN}{(TN+FP)}$
		Precision $\frac{TP}{(TP+FP)}$	**Negative Predictive Value** $\frac{TN}{(TN+FN)}$	**Accuracy** $\frac{TP+TN}{(TP+TN+FP+FN)}$

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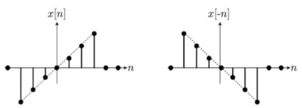
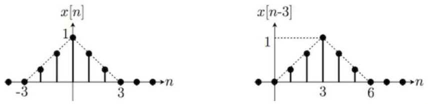
Flipping (time reversal)

- It flips the signal over the y (vertical) axis.
- A technique to focus wave energy to a selected point in space and time, localize and characterize a source of wave propagation, and/or communicate information between two points.



Time Shifting

- Time shifting is, as the name suggests, the shifting of a signal in time. This is done by adding or subtracting an integer quantity of the shift to the time variable in the function. Subtracting a fixed positive quantity from the time variable will shift the signal to the right (delay) by the subtracted quantity, while adding a fixed positive amount to the time variable will shift the signal to the left (advance) by the added quantity.

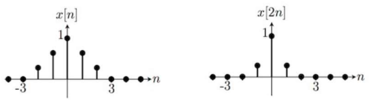


Time Scaling

- Time scaling compresses or dilates a signal by multiplying the time variable by some quantity. If that quantity is greater than one, the signal becomes narrower and the operation is called decimation. In contrast, if the quantity is less than one, the signal becomes wider and the operation is called expansion or interpolation, depending on how the gaps between values are filled.

Decimation

- In decimation, the input of the signal is changed to be $f(cn)$. The quantity used for decimation c must be an integer so that the input takes values for which a discrete function is properly defined. The decimated signal $f(cn)$ corresponds to the original signal $f(n)$ where only each n sample is preserved (including $f(0)$), and so we are throwing away samples of the signal.
- It is the process of reducing the sampling rate. In practice, this usually implies lowpass-filtering a signal, then throwing away some of its samples. (Also downsampling or compaction)



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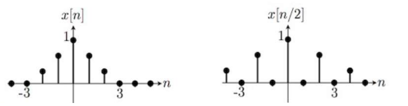
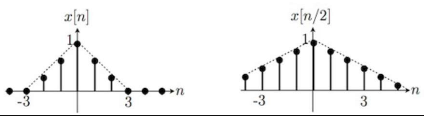
Expansion

- In expansion, the input of the signal is changed to be $f\left(\frac{n}{c}\right)$. We know that the signal $f(n)$ is defined only for integer values of the input n. Thus, in the expanded signal we can only place the entries of the original signal f at values of n that are multiples of c. In other words, we are spacing the values of the discrete-time signal $c-1$ entries away from each other. Since the signal is undefined elsewhere, the standard convention in expansion is to fill in the undetermined values with zeros.
- It produces an approximation of the sequence that would have been obtained by sampling the signal at a higher rate (upsampling).



Interpolation

- In practice, we may know specific information that allows us to provide good estimates of the entries of $f\left(\frac{n}{c}\right)$ that are missing after expansion. This process of inferring the undefined values is known as interpolation.



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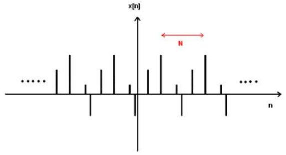
Periodic signals:

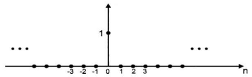
- $x[n] = x[n+N]$



Special signals

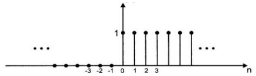
Unit sample:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Unit step:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

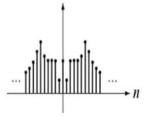


$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots = \sum_{k=0}^{\infty} \delta[n-k]$$

Even and Odd signals

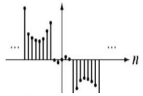
Even signal:

- Signal is flipped about the y-axis
- $x[n] = x[-n]$



Odd signal:

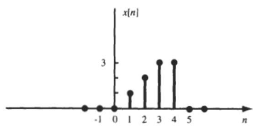
- Signal is flipped around the origin
- $x[n] = -x[-n]$
- At n=0, $x[0] = -x[0] = 0$.



Therefore, a generic signal can be represented as:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

e.g.:



$$x[n] = 0\ \delta[n] + 1\ \delta[n-1] + 2\ \delta[n-2] + 3\ \delta[n-3] + 4\ \delta[n-4]$$

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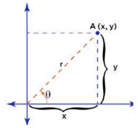
Cartesian form of a complex number: $z = x + jy$

- x is the real part, Re(z)
- y is the imaginary part, Im(z)
- Imaginary unit: $j = \sqrt{-1}$

Polar form of a complex number: $z = r(\cos\theta + j\sin\theta)$

- r is the magnitude of z, $|z|$
- θ is the angle (phase) of z
- $r = \sqrt{x^2 + y^2}$ $\quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

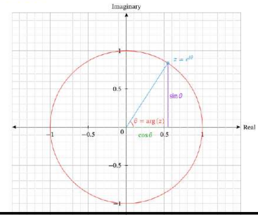


Exponential form of a complex number: $z = re^{j\theta}$

- θ must be strictly in radians (NOT degrees)

Euler's formula:

- $e^{j\theta} = \cos\theta + j\sin\theta$
- $z = r(\cos\theta + j\sin\theta) = re^{j\theta}$



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Conjugate of a complex number $(a + bi)$:

- $conj(a + bi) = a - bi$

Addition/subtraction of complex numbers:

- $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$

Multiplication of complex numbers:

- $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Division of complex numbers:

- $\frac{(a+bi)}{(c+di)} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$
- Prove the above relationship as a practice.

Multiplication of complex numbers and its conjugate:

- $(a + bi)(a - bi) = a^2 + b^2$

In polar form, for $z_1 = r_1(\cos\theta_1 + j\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + j\sin\theta_2)$:

- $z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2))$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2))$

In exponential form, for $z_1 = r_1 e^{j\theta_1}$ and $z_2 = r_2 e^{j\theta_2}$:

- $z_1 z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)}$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$

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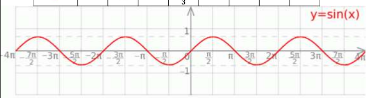
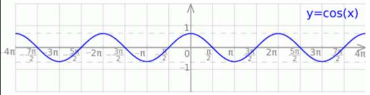
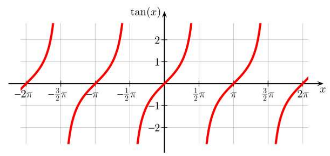
	α							
Градусы:	0°	30°	45°	60°	90°	180°	270°	360°
Радианы:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
$\mathrm{tg}\,\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—	0	—	0
$\mathrm{ctg}\,\alpha$	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	—	0	—

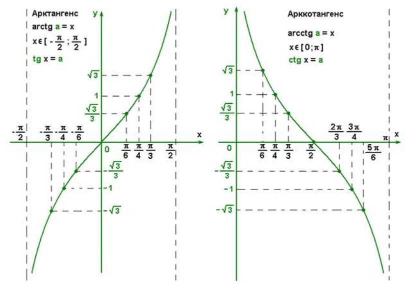






x	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$
$\mathrm{arctg}\,x$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\mathrm{arcctg}\,x$	$\frac{5\pi}{6}$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

Арктангенс и арккотангенс



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