Q1

Wednesday, February 21, 2024 8:09 PM

Given:

$$\vec{r} = \begin{bmatrix} 3634.1 \\ 5926 \\ 1206.6 \end{bmatrix} \text{ km} \qquad \text{and if } \mu_E = 398600.435507 \ \frac{km^3}{s^2}$$

↑ from JPL

$$\vec{v} = \begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix} \text{ km/s}$$

- Find $^I\vec{h}$:

$$^I\vec{h} = {}^I\vec{r} \times {}^I\vec{v}$$

$$= \begin{bmatrix} 3634.1 \\ 5926 \\ 1206.6 \end{bmatrix} \times \begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 3634.1 & 5926 & 1206.6 \\ -6.9049 & 4.3136 & 2.6163 \end{vmatrix} =$$

$$\left((5926)(2.6163) - (1206.6)(4.3136)\right)\hat{i}$$

$$+ \left((1206.6)(-6.9049) - (3634.1)(2.6163)\right)\hat{j}$$

$$+ \left((3634.1)(4.3136) - (5926)(-6.9049)\right)\hat{k}$$

$$^I\vec{h} = \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}$$

$$h = \sqrt{(10229.40)^2 + (-17839.35)^2 + (56594.49)^2} = 60226.71 \ \frac{kg\,km^2}{s}$$

$$\hat{h} = \frac{\vec{h}}{h} = \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix} \frac{1}{\sqrt{(10229.40)^2 + (-17839.35)^2 + (56594.49)^2}}$$

$$\vec{n} = \hat{k} \times \vec{h}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 10229.40 & -17839.35 & 56594.49 \end{vmatrix}$$

$$= \left(0 - (1)(-17839.35)\right) + \left((1)(10229.40) - (0)\right) + \left((0) - 0\right)$$

$$\vec{n} = \begin{bmatrix} 17839.35 \\ 10229.40 \\ 0 \end{bmatrix}$$

$$\hat{n} = \frac{\vec{n}}{n} = \begin{bmatrix} 17839.35 \\ 10229.40 \\ 0 \end{bmatrix} \frac{1}{\sqrt{(17839.35)^2 + (10229.40)^2}} = \begin{bmatrix} 0.86603 \\ 0.5000 \\ 0 \end{bmatrix}$$

this might be wrong + useless + irrelevant to the actual problem? * later note, turns out it was!

$$\vec{e} = \frac{1}{\mu_E}\left(\vec{v} \times \vec{h}\right) - \frac{\vec{r}}{r}$$

$$\vec{e} = \frac{1}{398600.44}\left(\begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix} \times \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}\right) - \begin{bmatrix} 3634.1 \\ 5926.0 \\ 1206.6 \end{bmatrix} \frac{1}{\sqrt{(3634.1)^2 + (5926.0)^2 + (1206.6)^2}}$$

$$\frac{1}{398600.44}\begin{bmatrix} 290799.084 \\ 417725.633 \\ 78751.406 \end{bmatrix} - \begin{bmatrix} 0.515074 \\ 0.839912 \\ 0.171016 \end{bmatrix}$$

$$\vec{e} = \begin{bmatrix} 0.21447679 \\ 0.20600842 \\ 0.02655421 \end{bmatrix}$$

$e = 0.29999 \approx 0.3$

$$a = \frac{-\mu}{2\mathcal{E}} \qquad \mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r}$$

$r = 7055.49732$ km

$V = 8.551597$ km/s

$$a = \frac{-(398600.436)}{2(-19.9301)} \qquad \mathcal{E} = \frac{(8.552)^2}{2} - \frac{398600.436}{7055.497}$$

$a = 9999.956067$

$\mathcal{E} = -19.9301 \ \frac{kJ}{kg}$

$a = 9999.956$

Since $e > 0$, $n = 0$, given:

$$i = \cos^{-1}\left(\frac{\vec{h} \cdot \hat{k}}{h}\right)$$

$$= \cos^{-1}\left(\frac{56594.49}{60226.71}\right)$$

$i = 0.34907085$ rad

$$\Omega = \cos^{-1}\left(\frac{\vec{n} \cdot \hat{i}}{n}\right)$$

$$= \cos^{-1}\left(\frac{17839.35}{20599.03}\right)$$

$\Omega = 0.523593$ rads

orbital elements:

- $i = 0.34907085$ rad
- $\Omega = 0.523593$ rads
- $\omega = 0.261769$ rads
- $\nu = .2618327$ rads
- $\mathcal{E} = -19.9301 \ \frac{kJ}{kg}$
- $a = 9999.956$
- $e = 0.29999 \approx 0.3$

$$\omega = \cos^{-1}\left(\frac{\hat{n}\cdot\vec{e}}{ne}\right)$$

$$= \cos^{-1}\left(\frac{59691.0683}{61796366}\right)$$

$\omega = 0.2617769$ rads

$$\nu = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right)$$

$$\cos^{-1}\left(\frac{20419.8365}{21166.243}\right)$$

$\nu = .2618327$ rads

Q2

Wednesday, February 21, 2024 9:45 PM

2) My plots make sense because the orbital elements remain constant during the entire duration of our plot. Orbital elements *don't* change with time considering that the orbital elements are used to describe the overall behavior of an orbit!

   In reference to my 3d plots, the orbital elements seem correct because my values for "a" seem to align with the semi-latus rectum, the eccentricity, and the specific energy as well!

Table of Contents


```
%-------- HW 2 MATLAB code --------%
% Romeo Perlstein, section 0101 %
```

Q2

Use matlab func to find orbital elements:

```
% Given:

r = [3634.1 ; 5926 ; 1206.6];
v = [-6.9049 ; 4.3136 ; 2.6163];
mew_Earth = 398600.44;

[i_param, omega_param, w_param, true_anom, ex, ey, ez, a, spef_energy] =
 cartToOrbitalElements(r, v, mew_Earth)
```

Q2-1

Plot the changing orbital elements as subplots

```
% Plotting constants
tall_er_ant = (10^-13);
step_size = 10000;
max_time = 70000000;

% Time step
t = [0:step_size:max_time];

% ODE options
ODE_options = odeset("RelTol", tall_er_ant, "AbsTol", tall_er_ant);

% Didymos Orbit information
didymos_initial_x = -2.39573*10^8;
didymos_initial_y = -2.35661*10^8;
didymos_initial_z = 9.54384*10^6;
didymos_initial_vx = 1.24732*10^1;
didymos_initial_vy = -9.74427*10^0;
didymos_initial_vz = -8.78661*10^-1;
didymos_initial_state = [didymos_initial_x; didymos_initial_y;
 didymos_initial_z; didymos_initial_vx; didymos_initial_vy;
 didymos_initial_vz;0;0;0];
```

```
for i=1:length(t)
    [i_param(i), omega_param(i), w_param(i), true_anom(i), ex(i), ey(i),
 ez(i), a(i), spef_energy(i)] = cartToOrbitalElements(r, v, mew_Earth);
    e(i) = sqrt(ex(i)^2 + ey(i)^2 + ez(i)^2);
end
tiledlayout(2, 4)
nexttile
plot(0:length(t)-1, i_param)
title("Plot of i Value Over Time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, omega_param)
title("Plot of Omega Value Over Time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, w_param)
title("Plot of Omega Value Over Time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, w_param)
title("Plot of w Value Over Time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, true_anom)
title("Plot of True Anomaly Over Time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, e)
title("Plot of Magnitude of e Over time")
xlabel("Time (s)")
ylabel("Radians (rads/sec")

nexttile
plot(0:length(t)-1, a)
title("Plot of 'a' Over time")
xlabel("Time (s)")
ylabel("")

nexttile
plot(0:length(t)-1, spef_energy)
title("Plot of a spef_energy over time")
```

```
xlabel("Time (s)")
ylabel("")
```

```
i_param =

   0.349070851292407
```

```
omega_param =

   0.523592535376530
```

```
w_param =

   0.261776744891492
```

```
true_anom =

   0.261832711153479
```

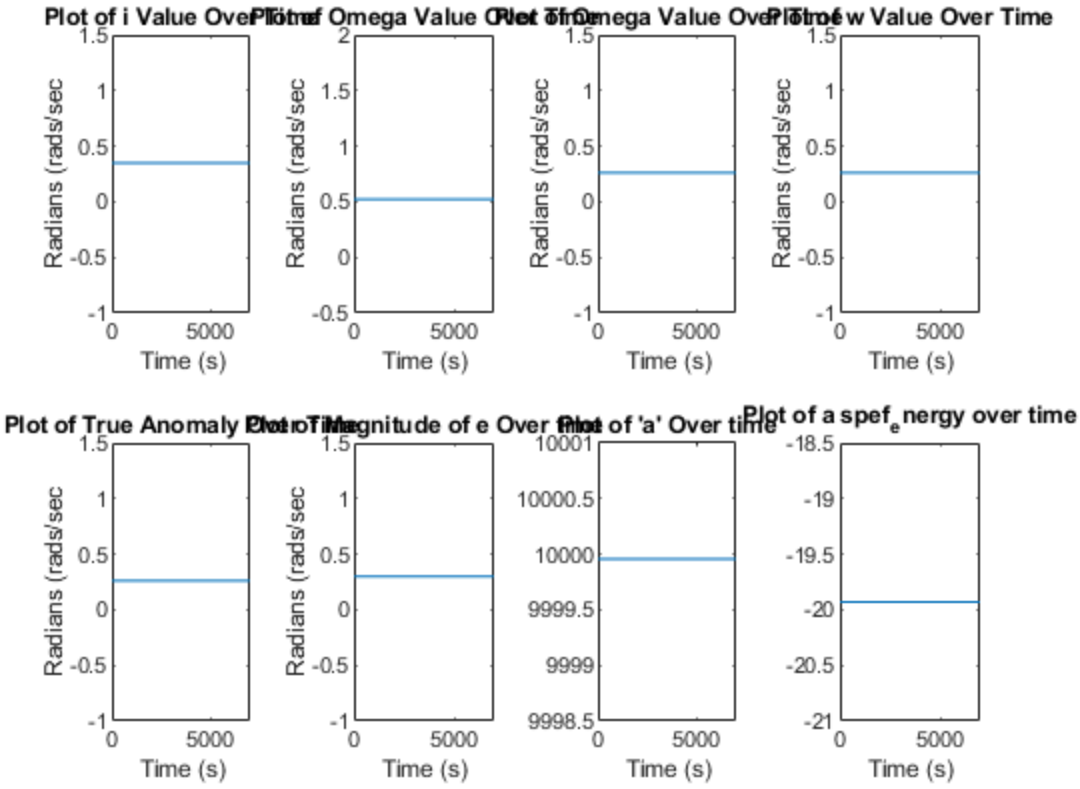
```
ex =

   0.214476789865968
```

```
ey =

   0.208068417383442
```

```
ez =

   0.026554206850339
```

```
a =

     9.999956067213976e+03
```

```
spef_energy =

 -19.930109558523867
```



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