Q1

Wednesday, February 21, 2024 8:09 PM

Given:

$$\vec{r} = \begin{bmatrix} 3634.1 \\ 5926 \\ 1206.6 \end{bmatrix} km \qquad \text{and if } \mu_E = 398600.435507 \ \frac{km^3}{s^2}$$

↑ from JPL

$$\vec{v} = \begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix} km/s$$

· find ${}^I\vec{h}$: ${}^I\vec{h} = {}^I\vec{r} \times {}^I\vec{v}$

$$= \begin{bmatrix} 3634.1 \\ 5926 \\ 1206.6 \end{bmatrix} \times \begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix}$$

$$= \begin{vmatrix} i & j & z \\ 3634.1 & 5926 & 1206.6 \\ -6.9049 & 4.3136 & 2.6163 \end{vmatrix} =$$

$$\left((5926)(2.6163) - (1206.6)(4.3136)\right)\hat{i}$$

$$+ \left((1206.6)(-6.9049) - (3634.1)(2.6163)\right)\hat{j}$$

$$+ \left((3634.1)(4.3136) - (5926)(-6.9049)\right)\hat{k}$$

$${}^I\vec{h} = \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}$$

$$h = \sqrt{(10229.40)^2 + (-17839.35)^2 + (56594.49)^2} = 60226.71 \ \frac{kg\,km^2}{s}$$

$$\hat{h} = \frac{\vec{h}}{h} = \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix} \frac{1}{\sqrt{(10229.40)^2 + (-17839.35)^2 + (56594.49)^2}}$$

$$a = \frac{-\mu}{2\varepsilon}, \quad \varepsilon = \frac{V^2}{2} - \frac{\mu}{r}, \qquad r = 7055.49732 \ km, \ V = 8.551597 \ km/s$$

$$a = \frac{-(398600.436)}{2(-19.9301)} \qquad \varepsilon = \frac{(8.552)^2}{2} - \frac{398600.436}{7055.497}$$

$a = 9999.956067$

$$\boxed{a = 9999.956} \qquad \boxed{\varepsilon = -19.9301 \ \frac{kJ}{kg}}$$

$$\vec{n} = \hat{k} \times \vec{h}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 10229.40 & -17839.35 & 56594.49 \end{vmatrix}$$

$$= \left(0 - (1)(-17839.35)\right)$$

$$+ \left((1)(10229.40) - (0)\right)$$

$$+ \left((0) - 0\right)$$

$$\vec{n} = \begin{bmatrix} 17839.35 \\ 10229.40 \\ 0 \end{bmatrix}$$

$$\hat{n} = \frac{\vec{n}}{n} = \begin{bmatrix} 17839.35 \\ 10229.40 \\ 0 \end{bmatrix} \frac{1}{\sqrt{(17839.35)^2 + (10229.40)^2}} = \begin{bmatrix} 0.86603 \\ 0.5000 \\ 0 \end{bmatrix}$$

This might be wrong + useless + irrelevant to the actual problem?

* later note, turns out it was!

$$\vec{e} = \frac{1}{\mu_E}\left(\vec{v} \times \vec{h}\right) - \frac{\vec{r}}{r}$$

$$\vec{e} = \frac{1}{398600.44}\left(\underbrace{\begin{bmatrix} -6.9049 \\ 4.3136 \\ 2.6163 \end{bmatrix} \times \begin{bmatrix} 10229.40 \\ -17839.35 \\ 56594.49 \end{bmatrix}}\right) - \underbrace{\begin{bmatrix} 3634.1 \\ 5926.0 \\ 1206.6 \end{bmatrix} \frac{1}{\sqrt{(3634.1)^2 + (5926.0)^2 + (1206.6)^2}}}$$

$$\frac{1}{398600.44}\begin{bmatrix} 290799.084 \\ 417725.633 \\ 78751.406 \end{bmatrix} - \begin{bmatrix} 0.515074 \\ 0.839912 \\ 0.171016 \end{bmatrix}$$

$$\vec{e} = \begin{bmatrix} 0.21447679 \\ 0.20806842 \\ 0.02655421 \end{bmatrix}$$

$$\boxed{e = 0.29994 \approx 0.3}$$

since $e > 0$, $n > 0$, then:

$$i = \cos^{-1}\left(\frac{\vec{h} \cdot \hat{k}}{h}\right)$$

$$= \cos^{-1}\left(\frac{56594.49}{60226.71}\right)$$

$$\boxed{i = 0.34907085 \ rad}$$

$$\Omega = \cos^{-1}\left(\frac{\vec{n} \cdot \hat{i}}{n}\right)$$

$$= \cos^{-1}\left(\frac{17839.35}{20599.03}\right)$$

$$\boxed{\Omega = 0.523593 \ rads}$$

orbital elements:

$i = 0.34907085$ rad

$\Omega = 0.523593$ rads

$\omega = 0.2617767$ rads

$\nu = .2618327$ rads

$\varepsilon = -19.9301 \ \frac{kJ}{kg}$

$a = 9999.956$

$e = 0.29994 \approx 0.3$

$$\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{ne}\right)$$

$$= \cos^{-1}\left(\frac{5969.10683}{6179.6366}\right)$$

$$\boxed{\omega = 0.2617767 \text{ rads}}$$

$$\nu = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right)$$

$$\cos^{-1}\left(\frac{2044.48385}{2116.6243}\right)$$

$$\boxed{\nu = .2618327 \text{ rads}}$$

Q2

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2) My plots make sense because the orbital elements remain constant during the entire duration of our plot. Orbital elements *don't* change with time considering that the orbital elements are used to describe the overall behavior of an orbit!

   In reference to my 3d plots, the orbital elements seem correct because my values for "a" seem to align with the semi-latus rectum, the eccentricity, and the specific energy as well!