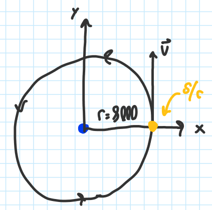
HW7 Astrodynamics | Ramos Perlstein 0101

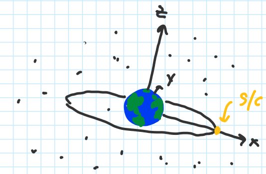
Monday, May 6, 2024 9:43 PM

Q1 :

given :

$$s/c_{pos_{init}} = \begin{bmatrix} 8000 \\ 0 \\ 0 \end{bmatrix} km$$





$\mu_{Earth} = 0.39860 \cdot 10^6$

$$\|\vec{v}\| = \sqrt{\frac{\mu_E}{r_{circ}}} = \sqrt{\frac{0.39860 \cdot 10^6}{8000}} = \underline{\underline{7.0587 \ km/s}}$$

• $T/m = 1 \cdot 10^{-4} \ kN/kg$

a) $$\boxed{s/c_{vel_{init}} = \begin{bmatrix} 0 \\ 7.0587 \\ 0 \end{bmatrix} km/s}$$

b) $F = ma$, $\frac{F}{m} = a$

$$\vec{a} = \underbrace{\frac{-\mu_E}{\|\vec{r}\|^3}}_{\text{Force o' Gravity}} \cdot \vec{r} + \frac{T}{m} \underbrace{\frac{\vec{v}}{\|\vec{v}\|}}_{\text{direction of vel}}$$

also from lecture:

$$a_T = \frac{1}{2} \underbrace{\frac{d(v^2)}{ds}}_{2v\frac{dv}{ds} \rightsquigarrow \frac{dv}{dt}} + \frac{\mu_E}{r^2} \underbrace{\frac{dr}{ds}}_{\sin\gamma} \quad \text{(acceleration due to thrust)}$$

unusable in our problem because not fully in terms of t (not all $\frac{d}{dt}$)

c) Given the equation:

$$t_{esc} - \underset{\downarrow \, 0}{t_0} = \frac{V_0}{a_T}\left(1 - \left(\frac{20 \, a_T^2 \, r_0^2}{V_0^4}\right)^{1/8}\right)$$

recall: $\vec{a} = \frac{\mu_E}{\|\vec{r}\|^3}\vec{r} + \frac{T}{m}\frac{\vec{v}}{\|\vec{v}\|}$

a_T = acceleration due to thrust, so

$a_T = 1 \cdot 10^{-4} \ kN/kg$

Then:

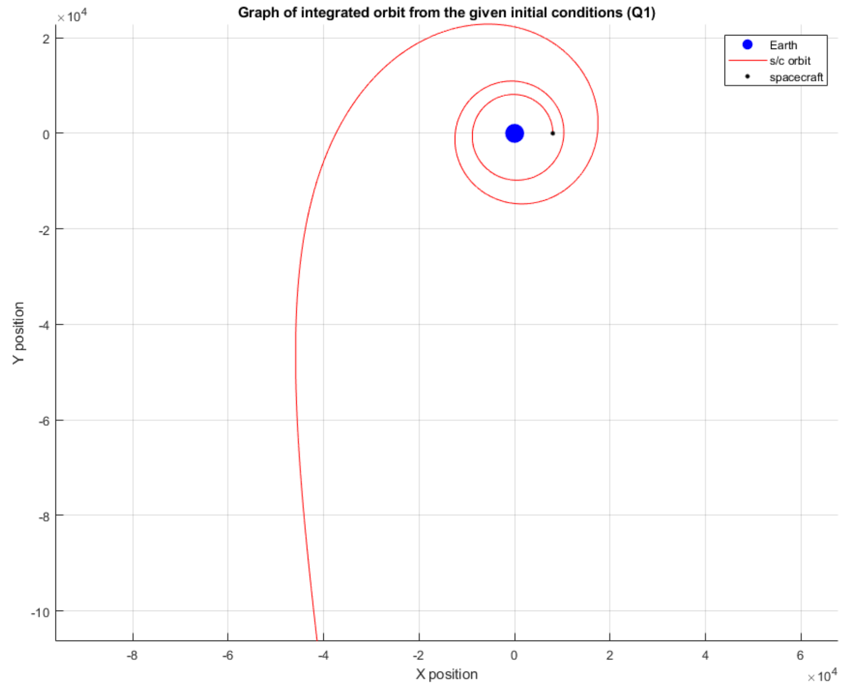
$$t_{esc} = \frac{\|s/c_{vel_{init}}\|}{a_T}\left(1 - \left(\frac{20 \, a_T^2 \, \|s/c_{pos_{init}}\|^2}{\|s/c_{vel_{init}}\|^4}\right)^{1/8}\right)$$

$$= \frac{7.0587}{0.0001}\left(1 - \left(\frac{20(0.0001)^2(8000)^2}{(7.0587)^4}\right)^{1/8}\right)$$

$$= 3.404728 \, 6187 \cdot 10^4 \text{ seconds}$$

$$t_{esc} = \boxed{34047 \text{ seconds}}$$

d)



e) given that r_{esc} can be found

$$r_{esc} = \frac{\|s/c_{pos\,init}\| \, \|s/c_{vel\,init}\|}{(20\, a_T^2 \|s/c_{pos\,init}\|^2)^{1/4}}$$

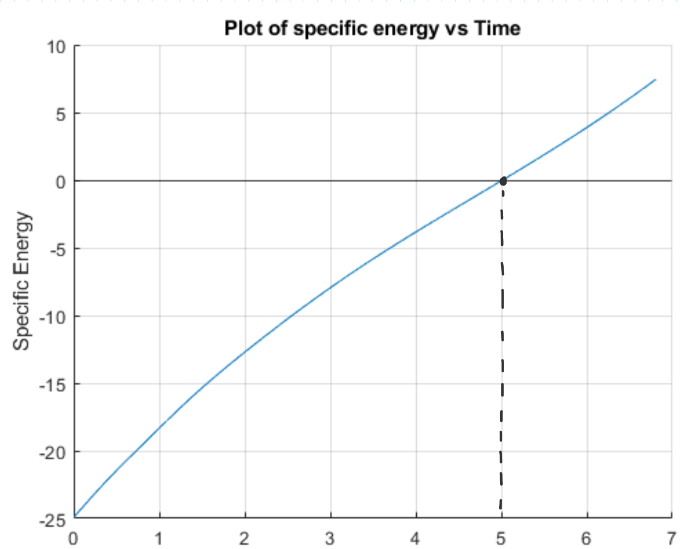
$$r_{esc} = \frac{(8000)(7.0587)}{(20(0.0001)^2(8000)^2)^{1/4}}$$

$$\boxed{r_{esc} = 29855 \text{ km}}$$

From inspection of integrated data, r_{esc} is reached at 36413 seconds

different from our analytical 34047 seconds by 2366 seconds! a little less than 1 hour

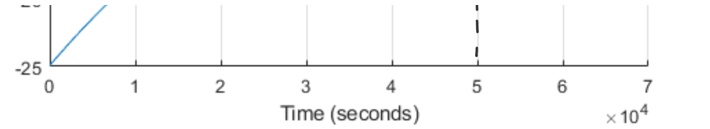
Using $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{\|\vec{v}\|^2}{2} - \frac{\mu}{\|\vec{r}\|}$



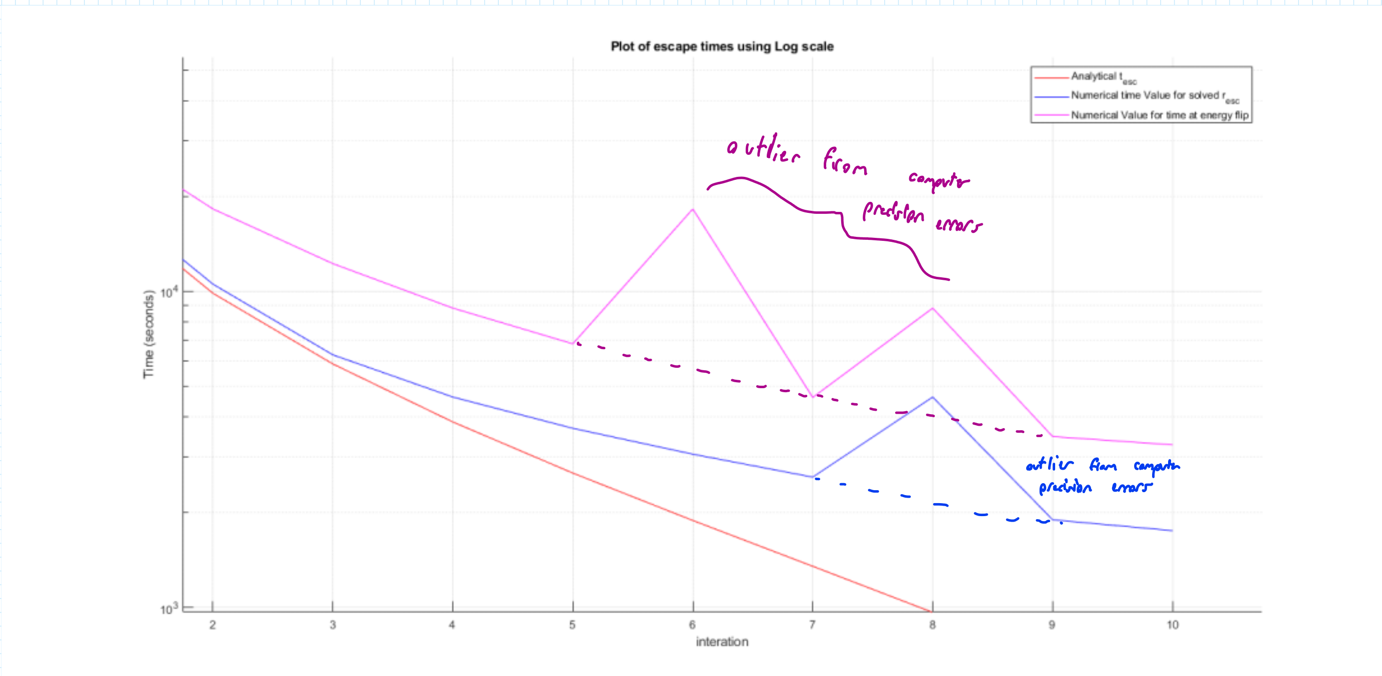
- However, inspection of the change in energy sign, i.e. when $\mathcal{E}$ (specific energy) goes from negative to positive, occurs roughly at 50130 seconds!

- Both numerically found escape times are greater than the analytical answer. I think the analytical answer is conservative with it's output because it really doesn't take into account the force of gravity on the spacecraft the same way that the numerical integrated does. Since it's only going off of

output because it really doesn't take into account the force of gravity on the spacecraft the same way that the numerical integrated does. Since it's only going off of the kinematics of the problem (other than the acceleration due to thrust being a non-kinematic term since it is a force), it does not account for the extra time it might take for the spacecraft to break from the planets gravity well, giving us a conservative value. TL:DR - The exclusion of the gravity term from the t_esc equation leads to a faster escape time being found. Since the t_esc equation is only influenced by a single force (and not the accounting for the gravitiational force being applied to the craft), it will find a faster
escape time.



f)

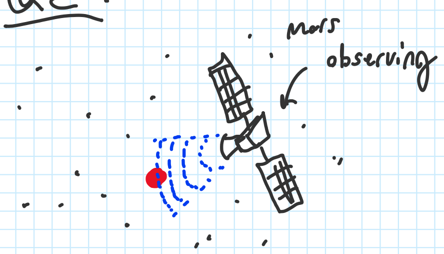


See attached code for more detail

g) From the plot, it is clear that the analytical method is lacking in it's accuracy. As the acceleration due to thrust is increased, we see a larger separation of numerical vs analytical values of the escape time. However, there are two key things to note: the analytical method produces a smooth, continuous curve of solutions, because it gives you exactly one solution. The integration method requires that you find the value at which either the energy changes or the time at which you reach your analytically solved for r_esc. Disregarding the outliers of data in the provided plot, and assuming they are smooth curves, we can still see the growing gap between the analytical solution and the numerical integration soltuion. The main limitation, I believe of the analytical method is that it does not incorporate gravity into the equation at all, and only relies on the input thrust as it's only force. I think this limitation is what causes the analytical method to undershoot, as it requires less time to escape if there is no modeled force of gravity "hold you back" (resisting the spacecrafts power to escape orbit)

HW7 Astrodynamics | Romeo Perlstein, section 0101
Thursday, May 9, 2024 3:03 AM ← peep the horror!

Q2:



Given:

- $r_p = 1000$ km
- $e = 0.25$
- $r_p = a(1-e)$
- $a = \frac{r_p}{(1-e)} = \frac{1000}{(1-e)}$

$= 1333.3333$ km

- $r_a = a(1+e) = 1666.6667$ km
- $I_{sp} = 250$ sec
- $m_{initial} = 1500$ kg

Using the approximation:

$\Delta V = I_{sp}\, g_0 \ln\left(\frac{m_i}{m_f}\right)$

- first, find ΔV to circularize

$V_{peri} = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_p + r_a}}$

$\mu_{mars} = 0.042828 \cdot 10^6$

$V_{peri} = \sqrt{\frac{2(0.042828 \cdot 10^6)}{1000} - \frac{2(0.042828 \cdot 10^6)}{1000 + 1666.6667}}$

$= 7.3168$ km/s

$V_{circ} = \sqrt{\frac{\mu}{r_p}} = 6.5443$ km/s

since we are going from eclipse to circular, s/c needs to slow down!

rearranging, we get:

- $\frac{m_i}{m_f} = e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)}$

or

- $\frac{\Delta m}{m_i} = 1 - e^{\left(\frac{-\Delta V}{I_{sp} g_0}\right)}$

must convert g_0 from m/s² to km/s² since ΔV is km/s

$e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)} = \frac{0.7725}{(250)(0.00981)}$

$\frac{m_i}{m_f} = 1.3702$

$m_f = \frac{m_i}{1.3702} = 1094.7$ kg

or

$\Delta m = m_i \left(1 - e^{\left(\frac{-\Delta V}{I_{sp} g_0}\right)}\right)$

$\Delta m = 1500 \left(1 - e^{\left(\frac{-(0.7725)}{(250)(0.00981)}\right)}\right)$

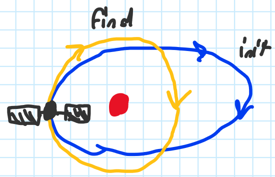
$\Delta m = 405.2781$ kg

- So, 405.2781 kg of propellant mass was used during the circularization maneuver!

$$\Delta V = |V_{circ} - V_{peri}|$$

$$= |6.5443 - 7.3168|$$

$$= 0.7725 \text{ km/s}$$ ← total ΔV needed to circularize at 1000 km



• Part 2 type thing:

given:

- Structural ratio $\epsilon = \dfrac{m_E}{m_i - m_{pL}} = 0.15$
- $M_p = 0$ after maneuver

· first, we can find n (mass ratio):

$$n = \frac{m_i}{m_f} = \frac{1500}{1147.2} = 1.3072 \quad \text{(soln)}$$

· now, find λ (payload ratio):

$$n = \frac{1+\lambda}{\epsilon+\lambda}, \qquad n(\epsilon+\lambda) = 1+\lambda$$

$$n\epsilon + n\lambda = 1+\lambda$$

$$n\epsilon = 1+\lambda - n\lambda$$

$$n\epsilon = 1 + (1-n)\lambda$$

$$\frac{n\epsilon - 1}{1-n} = \lambda$$

$$\lambda = \frac{n\epsilon - 1}{1-n}$$

$$= \frac{(1.3072)(0.15)-1}{1-(1.3072)}$$

$$\boxed{\lambda = 2.1460}$$

· payload fraction (or ratio) $\boxed{\lambda = 2.1460}$

· payload mass:

$$\lambda = \frac{m_{pL}}{m_i - m_{pL}}$$

$$\lambda m_i - \lambda m_{pL} = m_{pL}$$

$$\lambda m_i = m_{pL}(1+\lambda)$$

$$\frac{\lambda m_i}{1+\lambda} = m_{pL}$$

$$m_{pL} = \frac{2.1460(1500)}{1+2.1460} = \boxed{1023.2 \text{ kg}}$$

$$m_{pL} = \frac{2.1460(1500)}{1+2.1460} = \boxed{\underline{1023.2 \text{ kg}}}$$

$$m_L = \epsilon(m_i - m_{pL}) = \underline{71.5197 \text{ kg}}$$

hmmm...
seems light!

HW 7 Astrodynamics | Ronco Pedstdn section 0101

Thursday, May 9, 2024 4:15 AM

For Q3:

I could not get my pork chop plot to match with the provided outcome. I believe it is due to my TOF matrix, as the lambert solver is the same as given in the lecture. However, my solver does get stuck at a few inputs, so I'm not sure where the real issue lies :(

pork chop plots are so painful!