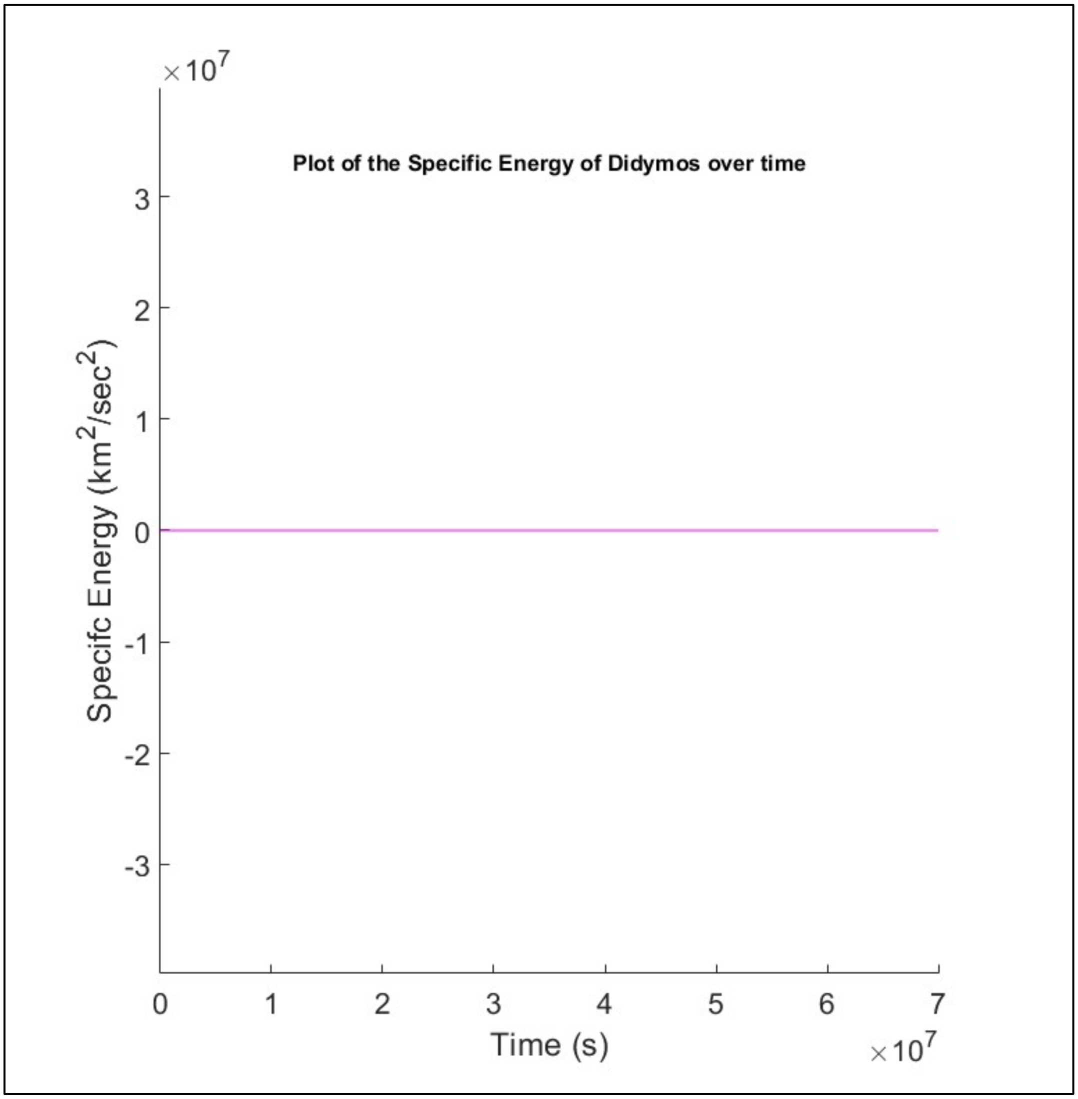
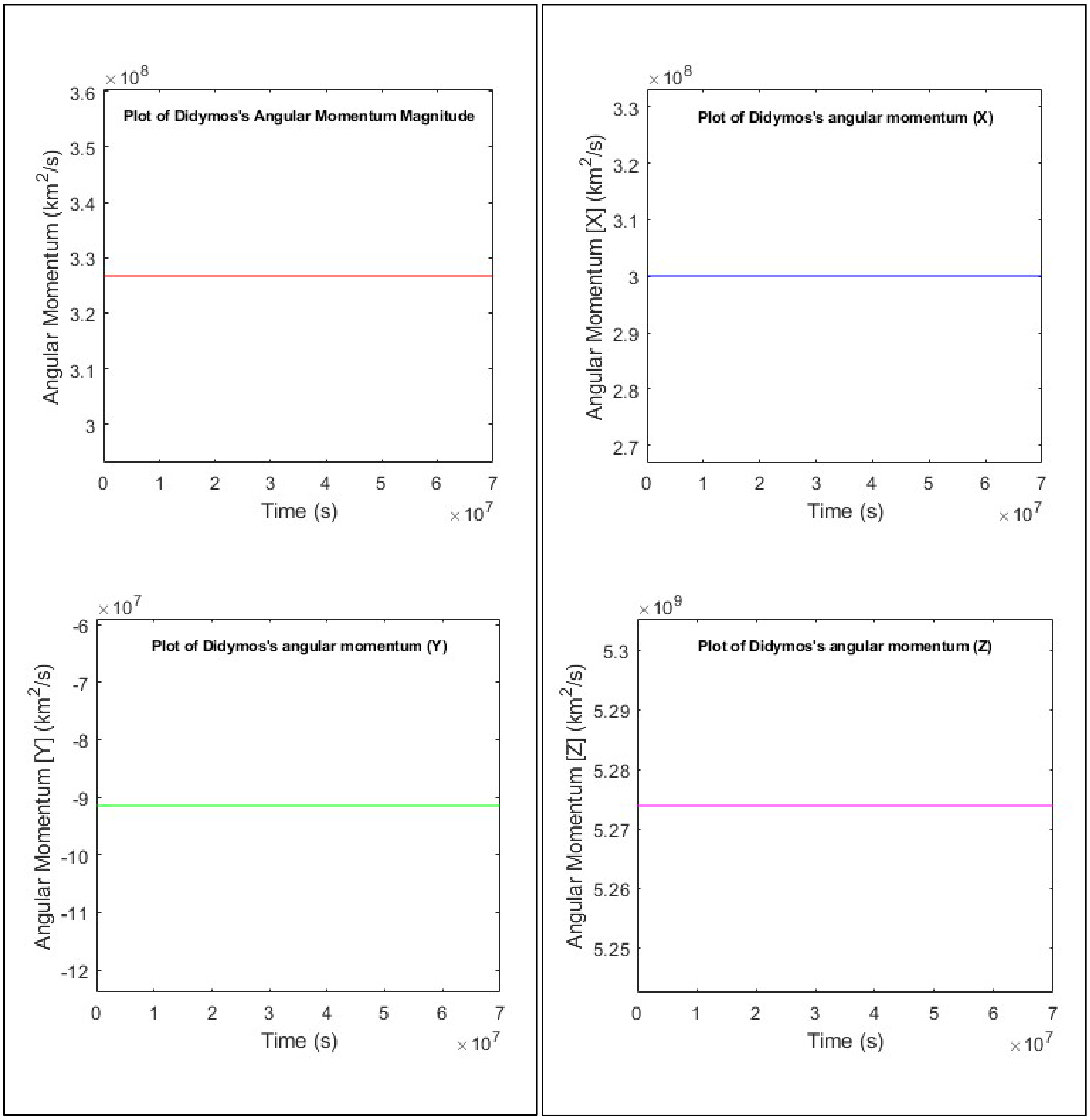
Question 1:

a)


Plot of the Specific Energy of Didymos over time
Specifc Energy (km^2/sec^2)
×10^7
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
Time (s)
×10^7

b)



c)

From my understanding, the previous two plots indicate that my two-body-problem propagator is working properly because the momentum and energy is "conserved" (i.e., they are pretty much a straight line!). This makes sense due to the constraints we're applying. If energy or angular momentum was not conserved, the planet would eventually begin spiraling in or out or in some way, shape, or form and not form a perfect orbit.

Question 2:

a)

All values of υ

b)

0^o and 180^o

c)

0^o

d)

0^o

Question 3:

a)

The eccentricity came out to be **1.2436**, meaning that e > 1. With that case, Luke's orbit should be a **hyperbolic** conic section!

b) Using energy: $E = v^2/2 + \mu/r = -\mu/2a$

$a = -\mu/2E = (3.986*10^5)/(2* 3.92270560672) = -50806.7696078$ km

The semi-major axis of this orbit is at **−50806.7696078 km** (which checks out since it is a hyperbola, and the semi-major axis is located on the outside of it!)

c) Using specific angular momentum: h = r*v*cos(flight angle)

$h = 12378 * 8.5 * \cos(0.5^o) = 105208.993811$ km^2/s

The **specific** angular momentum of Luke's orbit is **105208.993811 km^2/s**

d) $$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}} = \sqrt{1 + \frac{2(3.92270560672)(105208.993811)^2}{(3.986*10^5)^2}} = 1.2436$$

As stated in part a), the eccentricity of this orbit is 1.2436

e) $R_P = a(1-e) = -50806.7696078(1-1.2436) = 12377.1497095$ km

The radius of the periapsis is **12377.1497095 km**

Question 4:

a) $p = a(1-e^2) = 20,000*(1-(0.4)^2) = 16800$ km

$r = p/(1 - e*\cos(\upsilon)) = 16800/(1 - (0.4)*\cos(30^0)) = 12477.62418928247$ km

The radius at a true anomaly of 30^0 is **12477.6242 km**

b) $r = p/(1 - e*\cos(\upsilon)) = 16800/(1 - (0.4)*\cos(330^0)) = 12477.62418928247$

The radius at a true anomaly of 330^0 is also **12477.6242 km** (makes sense, its basically a mirror image)

c) v = $\sqrt{2*(-\frac{\mu}{2a}+\frac{\mu}{r})}$ = $\sqrt{2*(-\frac{3.986*10^5}{2(20,000)}+\frac{3.986*10^5}{12477.62418928247})}$ = 6.630261525936095 km/s

The velocity at an anomaly of 30^0 is **6.6303 km/s**

d) v = $\sqrt{2*(-\frac{\mu}{2a}+\frac{\mu}{r})}$ = $\sqrt{2*(-\frac{3.986*10^5}{2(20,000)}+\frac{3.986*10^5}{12477.62418928247})}$ = 6.630261525936095 km/s

The velocity at an anomaly of 330^0 is **6.6303 km/s** (which still makes sense, mirror image

e) h = $\sqrt{\mu p}$ = $\sqrt{(3.986*10^5)(16800)}$ = 81832.02307165576
flight angle = arccos(h/rv) = arccos(81832.0231/(12477.62418928247 * 6.630261525936095))
flight_angle = 8.449113362178327^0

The flight angle at a true anomaly of 30^0 is **8.4491^0**

f)

The flight angle at a true anomaly of 330^0 is *also* **8.4491^0**

g) apoapsis = a(1+e) = 20,000(1 + (0.4) = 28000 km

The apoapsis of this orbit is **28000 km**

h) $\sqrt{2*(-\frac{\mu}{2a}+\frac{\mu}{apoapsis})}$ = $\sqrt{2*(-\frac{3.986*10^5}{2(20,000)}+\frac{3.986*10^5}{\mathbf{28000}})}$ = 2.922572252559134 km/s

The velocity at the apoapsis is **2.923 km/s**, which checks out! That is when it is the slowest in orbit

Question 5:

a) $\sqrt{(2\mu)/periapsis}$ = $\sqrt{(2(3.986*10^5)/10000}$ = 8.928605714219886 km/s

The velocity at the periapsis is **8.9286 km/s**

b) Since e=1, we have a parabolic trajectory, so no apoapsis

Since e=1, our apoapsis is at **infinity** (since we are in a parabolic trajectory)

c) D

Again, the conic section of this orbit is a **parabola**

Table of Contents


```
%-------- HW 1 MATLAB code --------%
% Romeo Perlstein, section 0101 %

% Constants
mew_sun = 1.32712 * (10^11);
tall_er_ant = (10^-13);
step_size = 10000;
max_time = 70000000;

% Time step
t = [0:step_size:max_time];

% ODE options
ODE_options = odeset("RelTol", tall_er_ant, "AbsTol", tall_er_ant);

% format the data for 64 bit numeros
format long
```

Question 1

a)

From HW0

```
didymos_initial_x = -2.39573*10^8;
didymos_initial_y = -2.35661*10^8;
didymos_initial_z = 9.54384*10^6;
didymos_initial_vx = 1.24732*10^1;
didymos_initial_vy = -9.74427*10^0;
didymos_initial_vz = -8.78661*10^-1;
didymos_initial_state = [didymos_initial_x; didymos_initial_y;
 didymos_initial_z; didymos_initial_vx; didymos_initial_vy;
 didymos_initial_vz];

% Get didymos data
[T2,Y2] = ode45(@myodefun, t, didymos_initial_state, ODE_options, mew_sun);

% Get the specific energy
for i=1:1:(max_time/step_size+1) % I forget how to do it in one line my bad
    spef_energy(i) = (((sqrt(Y2(i,4)^2 + Y2(i,5)^2 + Y2(i,6)^2))^2)/2) -
 (mew_sun/sqrt(Y2(i,1)^2+Y2(i,2)^2+Y2(i,3)^2));
end
axes('FontSize', 16, 'NextPlot', 'add')
plot(T2, spef_energy, "-m")
title("Plot of the Specific Energy of Didymos over time", Units="normalize",
 Position=[.5,.9], FontSize=12)
xlabel("Time (s)")
ylabel("Specifc Energy (km^2/sec^2)")
axis equal
```

b)

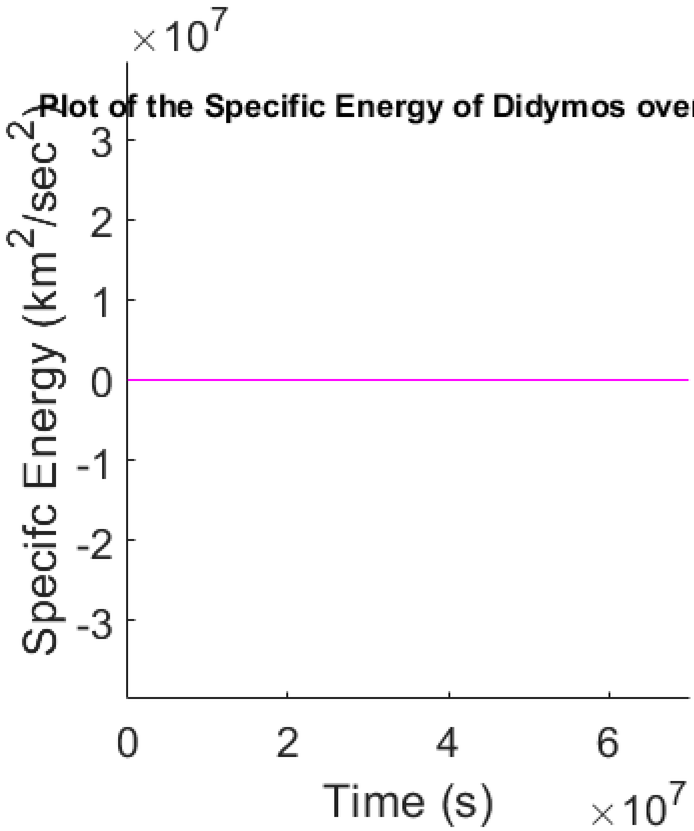
Get position and velocity vectors

```
didymos_r = [Y2(:,1),Y2(:,2),Y2(:,3)];
didymos_v = [Y2(:,4),Y2(:,5),Y2(:,6)];

% Cross them bad boys (real)
didymos_h = cross(didymos_r,didymos_v);

% Get the magnitude
for i=1:1:(max_time/step_size+1) % I forget how to do it in one line my bad
    didymos_h_mag(i) = sqrt(didymos_h(i,1)^2 + didymos_h(i,2)^2 +
 didymos_h(i,2)^2);
end

figure

tiledlayout(2,2);
```

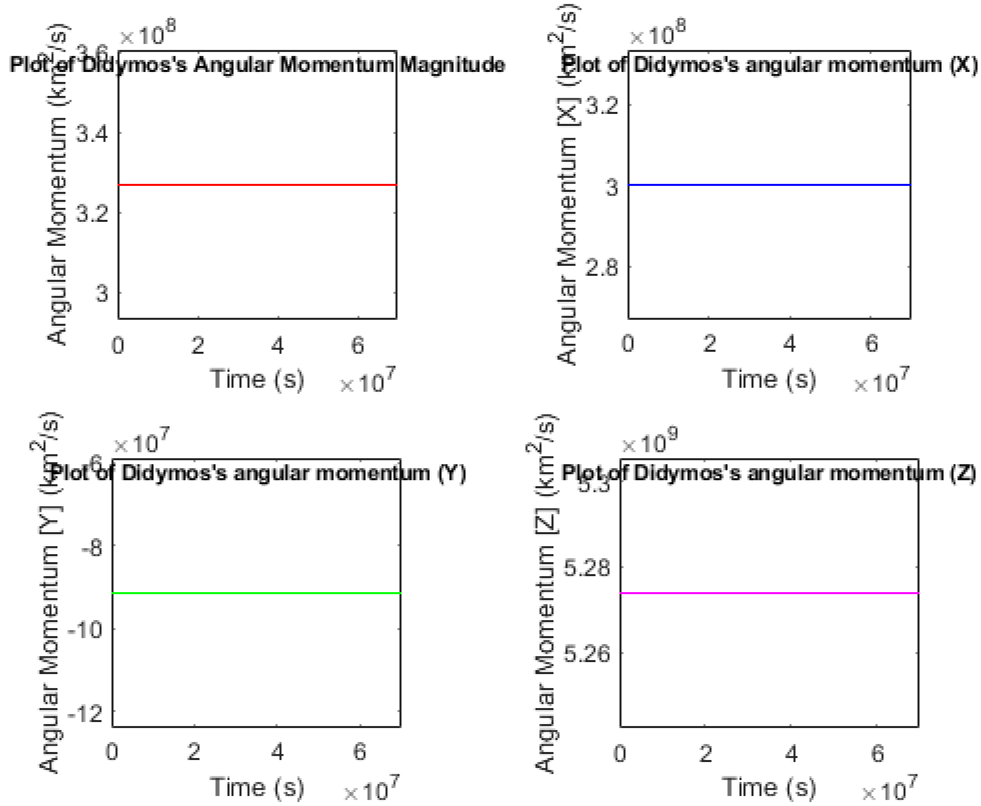
```
nexttile % tile 1
plot(T2,didymos_h_mag, "-r")
title("Plot of Didymos's Angular Momentum Magnitude", Units="normalize",
 Position=[.5,.9], FontSize=8)
xlabel("Time (s)")
ylabel("Angular Momentum (km^2/s)")
axis equal

nexttile % tile 2
plot(T2, didymos_h(:,1), "-b")
title("Plot of Didymos's angular momentum (X)", Units="normalize",
 Position=[.5,.9], FontSize=8)
xlabel("Time (s)")
ylabel("Angular Momentum [X] (km^2/s)")
axis equal

nexttile % tile 3
plot(T2, didymos_h(:,2), "-g")
title("Plot of Didymos's angular momentum (Y)", Units="normalize",
 Position=[.5,.9], FontSize=8)
xlabel("Time (s)")
ylabel("Angular Momentum [Y] (km^2/s)")
axis equal

nexttile % tile 4
plot(T2, didymos_h(:,3), "-m")
title("Plot of Didymos's angular momentum (Z)", Units="normalize",
 Position=[.5,.9], FontSize=8)
xlabel("Time (s)")
ylabel("Angular Momentum [Z] (km^2/s)")
axis equal
```

c)

From my understanding, the previous two plots indicate that my two-body-problem propagator is working properly because the momentum and energy are "conserved" (i.e., they are pretty much a straight line!). This makes sense due to the constraints we're applying. If energy or angular momentum were not conserved, the planet would eventually begin spiraling in or out or in some way, shape, or form and not form a perfect orbit.



Plot of the Specific Energy of Didymos over time
Specifc Energy (km²/sec²)
×10⁷
3
2
1
0
-1
-2
-3
0
2
4
6
Time (s)
×10⁷



Plot of Didymos's Angular Momentum Magnitude
Angular Momentum (km²/s)
×10⁸
Time (s)
Plot of Didymos's angular momentum (X)
Angular Momentum [X] (km²/s)
Plot of Didymos's angular momentum (Y)
Angular Momentum [Y] (km²/s)
Plot of Didymos's angular momentum (Z)
Angular Momentum [Z] (km²/s)
×10⁹

2

a)

all angles of true anomaly

b)

180 deg and 0 deg

c)

0 deg

d)

0 deg

3

```
alt = 6000;
v = 8.5; % km/s
flight_path_ang = 0.5; % degrees
mew = 3.986*10^5; % km3/s2
earth_radius = 6378; % Radius of Earth
r = earth_radius + alt; % total radius value

spef_energy3 = (v^2)/2 - mew/r; % specific energy
spef_ang_momentum3 = r*v*cosd(flight_path_ang); % specific angular momentum
eccen = sqrt(1+(2*spef_energy3*(spef_ang_momentum3)^2)/(mew^2)); %
 eccentricity

semi_major_axis = -mew/(2*spef_energy3); % semi-major axis

rp = semi_major_axis*(1-eccen);
```

a)

The eceentricity came out to be 1.2436, meaning that e > 1. With that case, Luke's orbit should be a hyperbolic conic section!

b)

```
semi_major_axis
% The semi-major axis of this orbit is at #50806.7696078 km (which checks
% out since it is a hyperbola, and the semi-major axis is located on the
```

```
% outside of it!)
```

c)

```
spef_ang_momentum3
% The specific angular momentum of Luke's orbit is 105208.993811 km2/s
```

d)

```
eccen
% As stated in part a), the eccentricity of this orbit is 1.2436
```

e)

```
rp
% The radius of the periapsis is 12377.1497095 km
```

```
semi_major_axis =

    -5.080676960781794e+04


spef_ang_momentum3 =

     1.052089938113507e+05


eccen =

   1.243612215557545


rp =

     1.237714970948228e+04
```

4

```
a4 = 20000;
e4 = 0.4;
mew = 3.986*10^5; % km3/s2
```

a)

```
anom = 30; % in degrees

p4 = a4*(1-(e4^2)); % get P value
r4 = p4/(1+e4*cosd(anom)) % Find radius at specified anomaly
```

b)

```
anom2 = 330;
r4_2 = p4/(1+e4*cosd(anom2)) % Find radius at specified anomaly
```

c)

using r4 for anom = 30 degrees

```
v4 = sqrt(2*(-mew/(2*a4) + mew/r4)) % km/s
```

d)

using r4_2 for anom = 330 degrees

```
v4_2 = sqrt(2*(-mew/(2*a4) + mew/r4_2)) % km/s
```

e)

```
h4 = sqrt(mew*p4);
flight_ang = acosd(h4/(r4*v4)) % flight angle at anom = 30 degrees
```

f)

```
flight_ang_2 = acosd(h4/(r4_2*v4_2)) % flight angle at anom = 330 degrees
```

g)

```
apoapsis = a4*(1+e4)
```

h)

```
v4_3 = sqrt(2*(-mew/(2*a4) + mew/apoapsis))
```

```
r4 =

     1.247762418928247e+04


r4_2 =

     1.247762418928247e+04


v4 =

   6.630261525936095
```

```
v4_2 =

   6.630261525936095


flight_ang =

   8.449113362178327


flight_ang_2 =

   8.449113362178327


apoapsis =

       28000


v4_3 =

   2.922572252559134
```

5

```
rp5 = 10000;
e5 = 1;
mew = 3.986*10^5; % km3/s2
```

a)

```
v5 = sqrt((2*mew)/rp5)
```

b)

The apoapsis of a parabolic orbit is infinity!

c)

This orbit's conic is parabolic!

```
% figure
% % Plot Sun
% surf(X_2,Y_2,Z_2)
% hold on
```

```
v5 =
```

8.928605714219886

ODE function handles

From ENAE301

```
function ydot = myodefun(t, y, mew)
    r_mag = norm(y(1:3));
    ydot(1,1) = y(4);
    ydot(2,1) = y(5);
    ydot(3,1) = y(6);
    ydot(4,1) = (-mew/r_mag^3)*y(1);
    ydot(5,1) = (-mew/r_mag^3)*y(2);
    ydot(6,1) = (-mew/r_mag^3)*y(3);
end
```

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