HW 5 Astrodynamics | Romeo Perlstein, section 0101

Thursday, April 11, 2024 10:02 AM

Q1

a) find total ΔV for transfer

given:

$\mu_{sun} = 1.3271 \cdot 10^{11}\ km^3/s^2$ radius of mercury: 2440.5 km

$\mu_{mer} = 2.2032 \cdot 10^{4}\ km^3/s^2$ radius of saturn: 60,268 km

$\mu_{sat} = 1.2671 \cdot 10^{8}\ km^3/s^2$

$r_{mer_{sun}} = 57.909 \cdot 10^6$ km

$m_{sun} = 1.9985 \cdot 10^{30}$ kg

$m_{mer} = 3.3010 \cdot 10^{23}$ kg

$r_{sat_{sun}} = 1432.041 \cdot 10^6$ km

$m_{sat} = 5.6832 \cdot 10^{26}$ kg

} from NASA fact sheet NSSDCA

$r_{craft_{mer}}$ = 400 km + planet radius = 2,840.5 km

$r_{craft_{sat}}$ = 10000 km + planet radius = 70,268 km

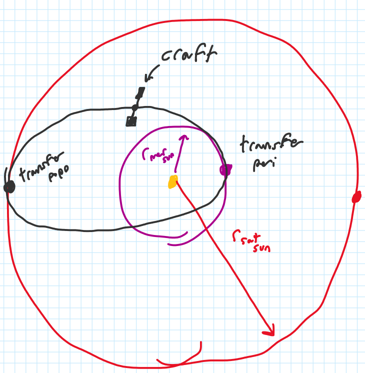
- start solving:

$$V_{mer} = \sqrt{\frac{\mu_{sun}}{r_{mer_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^6}} = \underline{47.8721}\ km/s$$

$$V_{sat} = \sqrt{\frac{\mu_{sun}}{r_{sat_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^6}} = \underline{9.6267}\ km/s$$

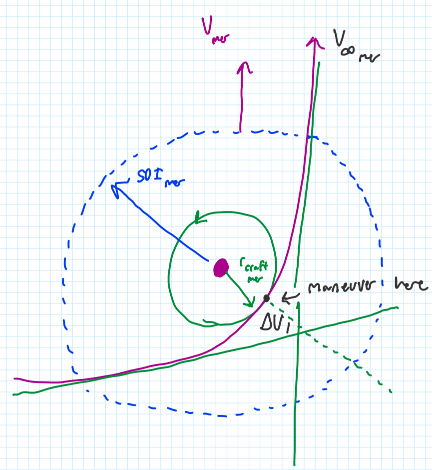
$$V_{craft_{initial_{mer}}} = \sqrt{\frac{\mu_{mer}}{r_{craft_{mer}}}} = \sqrt{\frac{2.2032 \cdot 10^4}{2,840.5}} = \underline{2.785\ km/s}$$

$$V_{craft_{final_{sat}}} = \sqrt{\frac{\mu_{sat}}{r_{craft_{sat}}}} = \sqrt{\frac{1.2671 \cdot 10^8}{70\,268}} = \underline{42.4650\ km/s}$$



$$V_{transfer_{peri}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{mer_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^6} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^6 + 1432.041 \cdot 10^6)}\right)} = \underline{66.3726}\ km/s$$

$$V_{transfer_{apo}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{sat_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^6} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^6 + 1432.041 \cdot 10^6)}\right)} = \underline{2.6840\ km/s}$$



- now zoom in on mercury:

$$V_{\infty_{mer}} = V_{transfer_{peri}} - V_{mer} = 66.3726 - 47.8721 = \underline{18.5006}\ km/s$$

$$V_{hyperbola_{esc_{mer}}} = \sqrt{2\left(\frac{\mu_{mer}}{r_{craft_{mer}}} + \frac{V_{\infty_{mer}}^2}{2}\right)} = \sqrt{2\left(\frac{2.2032 \cdot 10^4}{2840.5} + \frac{(18.5006)^2}{2}\right)} = \boxed{18.9152\ km/s}$$

$$\Delta V_1 = V_{hyperbola_{esc_{mer}}} - V_{craft_{initial_{mer}}} = 18.9152 - 2.785 = \underline{16.1301}\ km/s$$

$$SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909 \cdot 10^6 \left(\frac{3.3010 \cdot 10^{23}}{1.9985 \cdot 10^{30}}\right)^{2/5} = \boxed{1.1218 \cdot 10^5\ km}$$

- $SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909\cdot10^{6}\left(\frac{3.3010\cdot10^{23}}{1.9985\cdot10^{30}}\right)^{2/5} = \boxed{1.1218\cdot10^{5}\ \text{km}}$

since assuming circ orbit, $a_{mer} = r_{mer}$

- now zoom in on saturn :

* Assuming periapsis of hyperbola is SAME AS PARKING ORBIT !!!

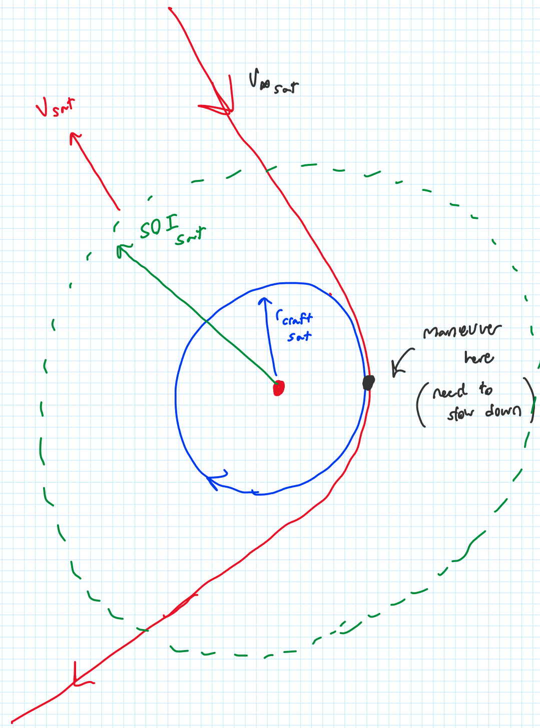
- $V_{\infty_{sat}} = V_{sat} - V_{transfer_{Apo}} = 9.6267 - 2.6840 = 6.9427$ km/s

- $V_{hyperbola_{esc_{sat}}} = \sqrt{2\left(\frac{\mu_{sat}}{r_{craft_{sat}}} + \frac{V_{\infty_{sat}}^{2}}{2}\right)} = \sqrt{2\left(\frac{1.2671\cdot10^{8}}{70268} + \frac{(6.9427)^2}{2}\right)} = \boxed{60.4546\ \text{km/s}}$

- $\Delta V_2 = V_{hyperbola_{esc_{sat}}} - V_{craft_{final_{sat}}} = 60.4546 - 42.4650 = 17.9896$ km/s

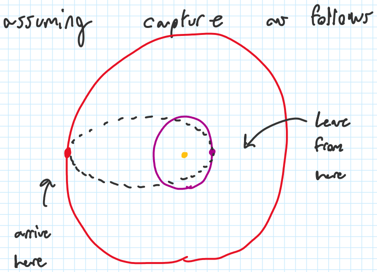
- $SOI_{sat} = r_{sat}\left(\frac{M_{sat}}{M_{sun}}\right)^{2/5} = (1432.041\cdot10^{6})\left(\frac{5.6832\cdot10^{26}}{1.9985\cdot10^{30}}\right) = \boxed{5.4640\cdot10^{7}\ \text{km}}$

- $\Delta V_{tot} = \Delta V_1 + \Delta V_2 = 16.1301 + 17.9896 = \boxed{34.1197\ \text{km/s}}$



- FIND TOF :

assuming capture as follows:



then we can assume an ellipse during heliocentric transfer:

$$\left.\begin{array}{l} r_p = r_{mer} \\ r_a = r_{sat} \end{array}\right\} a = \frac{r_{mer} + r_{sat}}{2} = 7.44975\cdot10^{8}$$

- $r_p = a(1-e) = \frac{r_p}{a} = 1-e \quad e = 1 - \frac{r_p}{a} = \frac{r_{mer}}{a} = 0.9223$

$E = 180^{\circ}$ * assuming instant capture

$$t - T = \sqrt{\frac{a^3}{\mu_{sun}}}\left(E - e\sin(E)\right)$$

$$t = \sqrt{\frac{(7.44975\cdot10^{8})^3}{1.3271\cdot10^{11}}}\left(\pi - 0.9223\sin(\pi)\right)$$

$$\boxed{t = 1.7535\cdot10^{6}\ \text{seconds}}$$

Q2 :

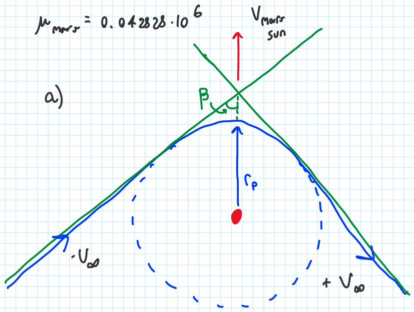
· Given:

$e = 1.2$

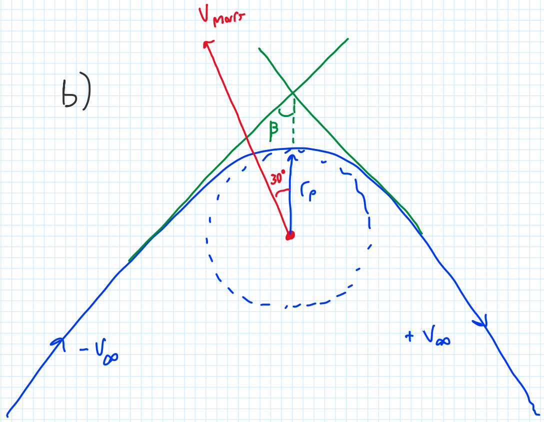
$r_p = 5380$

$\mu_{mars} = 0.042828 \cdot 10^6$

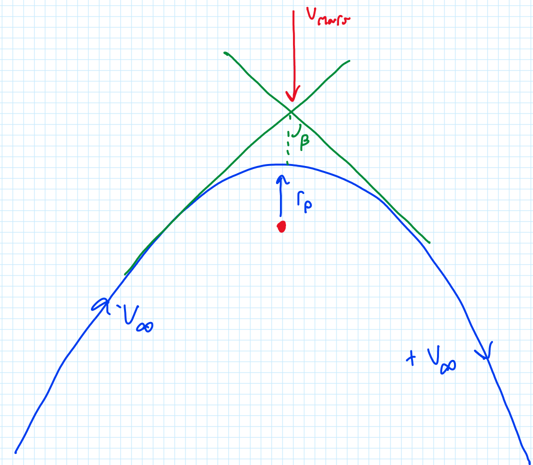
a)

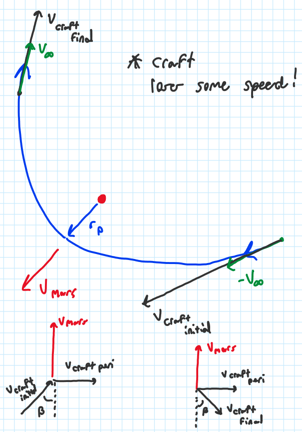


b)



c)





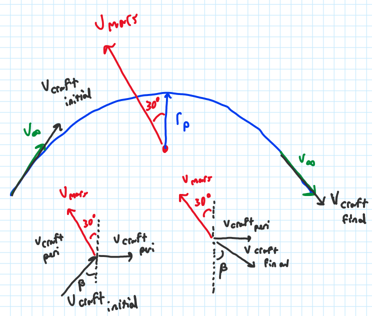


Table of Contents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%-------- HW 5 MATLAB code --------%
% Romeo Perlstein, section 0101 %

% IT'S A NEW WEEK - HOPEFULLY I CAN KEEP UP THE WORK AND NOT FALL BEHIND!
```

Q1

Find deltaV for Mercury to Jupiter transfer using conic sections

```
% Givens:
mew_mercury = 22031.868551; % km^3/s^2 - FROM JPL
mew_saturn = 126712764.1; % km^3/s^2 - FROM JPL
radius_planet_mercury = 2440.5; % km - from NASA fact sheet
r_craft_mercury = 400+radius_planet_mercury; % km
r_mercury = 57.909*10^6; % semi-major axis - from NASA fact sheet

radius_planet_saturn = 60268; % km - from NASA fact sheet
r_craft_saturn = 10000+radius_planet_saturn; % km
r_saturn = 1432.041*10^6; % semi-major axis - from NASA fact sheet

% since we are assuming circular orbits, we need to find the orbit
% velocity of both planets!
mew_sun = 132712*10^6; % from NASA fact sheet
v_mercury = sqrt(mew_sun/r_mercury);
v_saturn = sqrt(mew_sun/r_saturn);

% find velocities of orbits of spacecraft
v_initial_craft_mercury = sqrt(mew_mercury/r_craft_mercury);
v_final_craft_saturn = sqrt(mew_saturn/r_craft_saturn);

% Find the velocity to transfer from mercury to saturn
v_transfer_peri = sqrt(2*((mew_sun/r_mercury) - (mew_sun/(r_mercury
+r_saturn))));
v_transfer_apo = sqrt(2*((mew_sun/r_saturn) - (mew_sun/(r_mercury
+r_saturn))));

% Now, lets get the escape velocity from mercury
v_escape_mercury = v_transfer_peri - v_mercury;
v_escape_hyperbola_mercury = sqrt(2*((mew_mercury/r_craft_mercury) +
 ((v_escape_mercury^2)/2)));

% Now we can find the delta V to get to Saturn
deltaV1 = v_escape_hyperbola_mercury - v_initial_craft_mercury;
```

```
% Now do saturn
v_escape_saturn = v_saturn - v_transfer_apo;

% MAKING ASSUMPTION THAT HYPERBOLA PERIAPSIS IS SAME AS PARKING ORBIT
% PERIAPSIS, A. BECAUSE THE PROBLEM DOESN'T SAY WE CAN'T AND B. BECAUSE I
% WOULD NOT BE ABLE TO SUBMIT THE HW ON TIME
v_escape_hyperbola_saturn = sqrt(2*((mew_saturn/r_craft_saturn) +
 ((v_escape_saturn^2)/2)));
deltaV2 = v_escape_hyperbola_saturn - v_final_craft_saturn;

% now get the supplementary info
mass_mercury = .3301*10^24; % kg
mass_saturn = 568.32*10^24; % kg
mass_sun = 1998500*10^24; % kg
SOI_mercury = r_mercury*(mass_mercury/mass_sun)^(2/5); % km - Matches with
 Wikipedia!
SOI_saturn = r_saturn*(mass_saturn/mass_sun)^(2/5); % km - Matches with
 Wikipedia!
deltaV_total = deltaV1+deltaV2;

% Find TOF, assuming ellipse:
a = (r_mercury+r_saturn)/2;
e = 1-(r_mercury/a);
E = pi;
t = sqrt((a^3)/mew_sun)*(pi-0.9223*sin(pi)); % seconds!!!
```

Q2

do a bunch of stuff I don't have time to finish :/ given:

```
e2 = 1.2;
rp2 = 5380;
a2 = 1-(rp2/e2);
mew_mars = 0.042828 *10^6; % km^3/s^2 - from NASA fact sheet
% assuming circular orbit
r_mars = 227.956 * 10^6;
v_mars = sqrt(mew_sun/r_mars);
```

a

assuming velocity of planet is same direction as flyby

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