Q1

Tuesday, March 26, 2024 7:27 PM

Given:

Mars $\mu = 0.042828 \cdot 10^6 \ \frac{km^3}{s^2}$

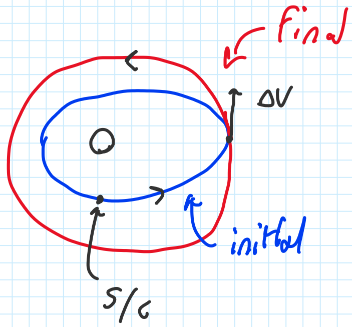
$r_p = 4000\ km$

$e = 0.3$

$i = 45°$

$\Omega = 270°$

$\omega = 30°$



a) **final:**

$e = 0$

$i = 45°$

$\Omega = 270°$

$\omega = 30°$

$r = r_a = ?$

$$r_p = a_i(1 - e_i),$$

$$a_i = (r_p / 1 - e_i)$$

$$a_i = 4000 / (1 - 0.3) = \underline{5714.3\ km}$$

$$r_a = a_i(1 + e_i)$$

$$r_a = 5714.3(1 + .3) = 7428.6\ km$$

$$r_f = r_a = 7428.6\ km$$

$$V_a = \sqrt{\frac{2\mu_m}{r_a} - \frac{\mu_m}{a}}$$

$$V_a = \sqrt{\frac{2(4.28 \cdot 10^4)}{7428.6} - \frac{4.28 \cdot 10^4}{5714.3}}$$

$$= 2.0083\ km/s$$

$$V_f = \sqrt{\frac{2(4.28 \cdot 10^4)}{r_f} - \frac{4.28 \cdot 10^4}{a_f}}, \quad a_f = r_f \quad \text{for } e = 0,$$

$$V_f = \sqrt{\frac{\mu}{r_f}} = \sqrt{\frac{4.28 \cdot 10^4}{7428.6}} = 2.4003\ km/s$$

$$\boxed{\Delta V = V_f - V_a = 0.3921\ km/s}$$

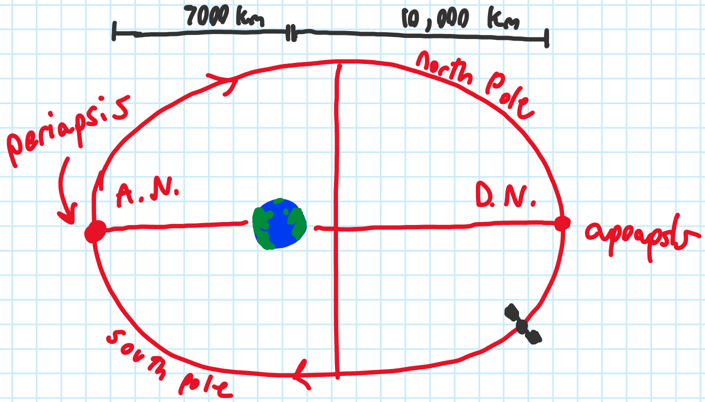
b) The overall magnitude of the velocity will increase since we need to move faster to change to a bigger orbit, and then become constant once we are moving in a circular orbit. However, since the velocity of an ellipse is not uniform throughout, our final velocity compared to the velocity of the original orbit at the periapsis results in a decrease of magnitude. This is because the maximum velocity that an orbit achieves is at the periapsis, but since we are conducting the maneuver at the apoapsis which is the slowest point of an orbit, changing our orbit to a larger circular, while increasing the velocity in magnitude, requires less deltaV

Q2

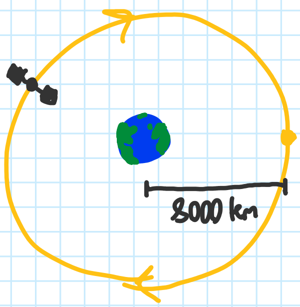
Thursday, March 28, 2024 12:18 AM

a)

initial orbit:



Find orbit:



* orbits move in same direction

$r_p = 7000$ km

$r_a = 10{,}000$ km

$e = 0.1765$

$a = r_p/(1-e)$

$= 7000/(1-0.1765)$

$a = 8500$ km

$$\left.\begin{matrix} r_p \\ r_a \end{matrix}\right\} \quad \frac{r_a}{1+e} = \frac{r_p}{1-e}$$

$r_p + r_p e = r_a - r_a e$

$r_p - r_a = -r_a e - r_p e$

$r_p - r_a = (-r_a - r_p)e$

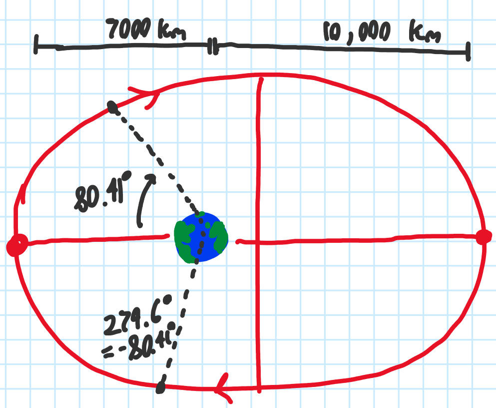
$$e = \frac{r_p - r_a}{-r_a - r_p}$$

$$e = \frac{7000 - 10000}{-10000 - 7000} = 0.1765$$

Using $r = \dfrac{a(1-e^2)}{1+e^2\cos(\nu)}$, we find

$$\nu = \cos^{-1}\left(\frac{a(1-e^2) - r}{re}\right),$$

for $r = 8000$, $\boxed{\begin{matrix}\nu = 80.4059^\circ \\ 279.5949^\circ\end{matrix}}$



Possible maneuver locations!

b) going from ellipse to circular:

$$V_e = \sqrt{\frac{2\mu}{r} - \frac{2\mu}{r_a + r_p}}$$

$$V_e = \sqrt{\frac{2(3.986 \cdot 10^5)}{8000} - \frac{3.986 \cdot 10^5}{r_p + r_a}} = 7.2633 \text{ km/s}$$

$$V_c = \sqrt{\frac{\mu}{r_c}} = \sqrt{\frac{3.986 \cdot 10^5}{8000}} = 7.0587 \text{ km/s}$$

<u>find flight path angle γ_e:</u>

$$\gamma_e = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_c V_e}\right), \quad p = a(1 - e^2) = 8235.3 \text{ km}$$

$$\gamma_e = \cos^{-1} \frac{\sqrt{(3.986 \cdot 10^5)(8235.3)}}{(8000)(7.2633)}$$

$$\underline{\gamma_e = 9.9541^\circ}$$

$$\Delta V = \sqrt{V_e^2 + V_c^2 - 2 V_e V_c \cos(-\gamma)}$$

$$= \sqrt{(7.2633)^2 + (7.0587)^2 - 2(7.2633)(7.0587)\cos(-9.9541^\circ)}$$

$$\boxed{\Delta V = 1.2149 \text{ km/s}}$$

c) If the burn was conducted at the other location, roughly -80 degrees, then the delta V required would be roughly the same! This is because all of the orbital parameters in use are not only constant, but also the flight path angle would be the same, so none of the parameters would be different from the first burn location

Q3

Thursday, March 28, 2024 8:04 AM

EARTh:

$\mu = 3.986 \cdot 10^5$

initial:

$a = 20{,}000$ km

$e = 0.3$

$i = 5°$

$\Omega = 30°$

$\omega = 45°$

final:

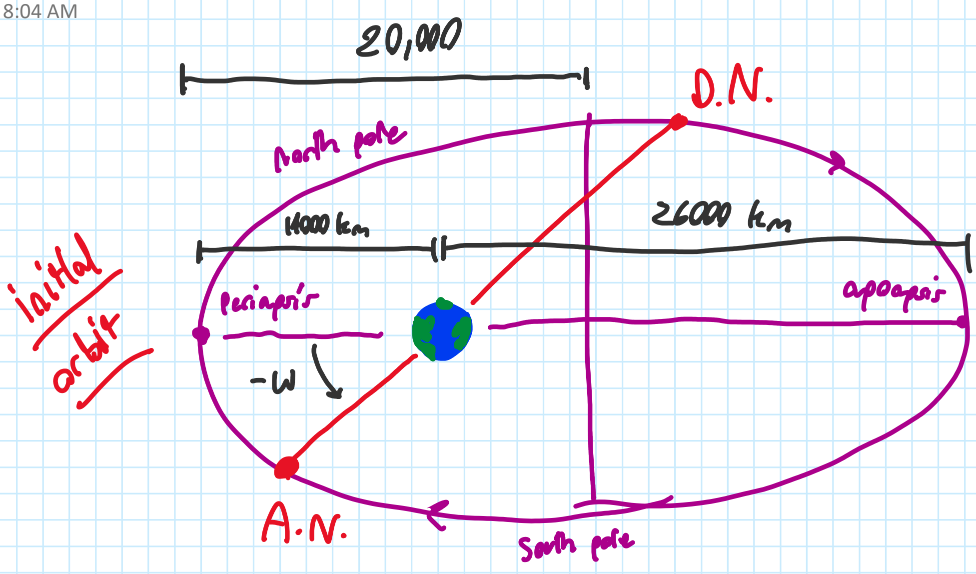
$a = 20{,}000$ km

$e = 0.3$

$i = 5°$

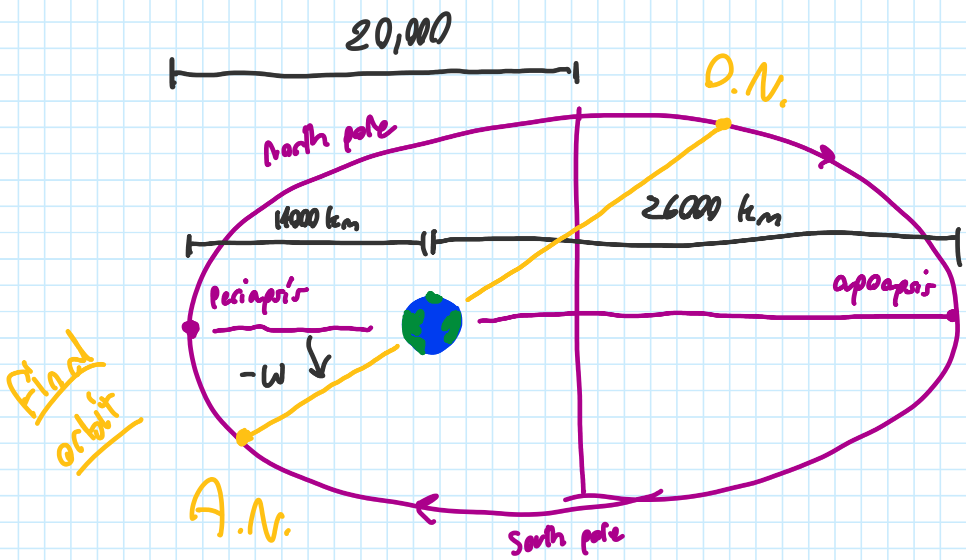
$\Omega = 30°$

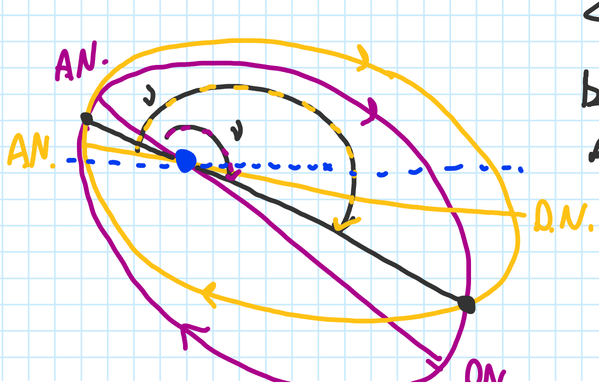
$\omega = 30°$



$r_p = a(1-e) = 20{,}000(1-0.3) = 14000$

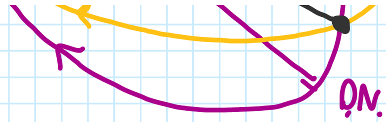
$r_a = a(1+e) = 20000(1+0.3) = 26000$





2 intersection points, between each of the Ascending & descending nodes!

at $\nu_{f_{close}} = 7.5°$ or



$\nu_{f_{close}} = 7.5$ or

$\nu_{f_{far}} = 187.5°$ $\left(180 + (45-30)/2\right)$

* verified with orbit plotter

b) first need r @ ν_i

$$r = \frac{a(1-e^2)}{1+e\cos(\nu)} = \frac{20000(1-0.3^2)}{1+(0.3)\cos(7.5°)}$$

← closest to periapsis (using find orbit true anom

$$= \underline{14028 \text{ km}}$$

$$V = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.896 \cdot 10^5}{16486}} = \underline{5.3306 \text{ km/s}}$$

$$\Delta V = \sqrt{2v^2(1-\cos(45°-30°))} = \boxed{1.3916 \text{ km/s}}$$

Q4

Thursday, March 28, 2024 9:18 AM

a) given $\mu_{saturn} = 3.7931187 \cdot 10^7 \ km^3/s^2$

$r = 60000$

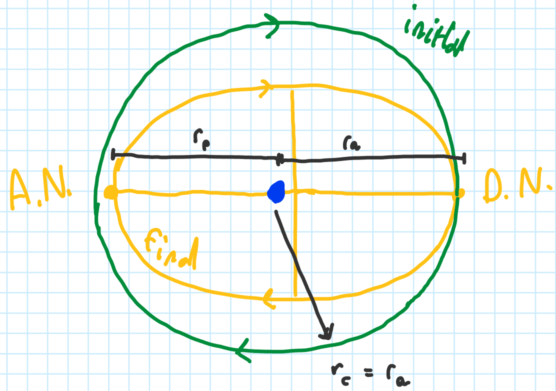
$i = 10°$

$$\Delta V = \sqrt{2V^2(1-\cos(\Delta i))}$$

$$\Delta V = \sqrt{2\left(\sqrt{\frac{3.793 \cdot 10^7}{60,000}}\right)^2 \left(1-\cos(15° - 10°)\right)}$$

$$\boxed{\Delta V = 2.1935}$$

b) Since the initial and final orbits are circular, in theory the Maneuver can occur anywhere on the orbit, HOWEVER **in order to ensure only inclination has changed, the maneuver would need to be executed at either the ascending or descending nodes of the orbit!**



c) Given $r_p = 59000$ km

$r_a = 60000$ km,

$$e = \frac{r_p - r_a}{(-r_a - r_p)} \rightarrow r_p = a(1-e)$$

$$r_a = a(1+e)$$

$$\frac{r_p}{1-e} = \frac{r_a}{1+e}$$

$$r_p + r_p e = r_a - r_a e$$

$$r_p - r_a = (-r_p - r_a)e$$

$$\frac{r_p - r_a}{-r_a - r_p} = e$$

$$e = \frac{59000 - 60000}{-60000 - 59000} = 0.0084$$

$$a = r_p / 1-e = \frac{59000}{1 - 0.0084} = \underline{59500 \text{ km}}$$

circular orbit vel:

$$V_c = \sqrt{\frac{\mu}{r_c}} = \sqrt{\frac{3.7931187 \cdot 10^7}{60,000}} = 25.1433 \text{ km/s}$$

$$V_{f_{apop}} = \sqrt{\frac{2\mu}{r_a} - \frac{\mu}{a}} = 25.0375 \text{ km/s}$$

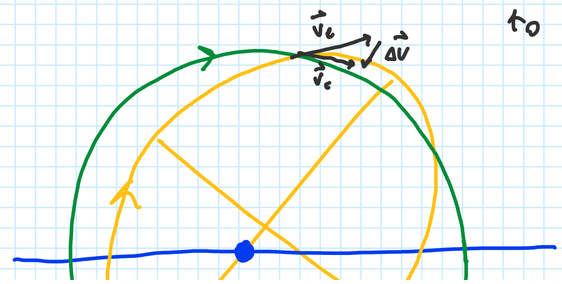
at descending node (apoapsis),

$\gamma_1 = \gamma_2 = 0$;

$$\Delta V^2 = V_c^2 + V_{f_a}^2 - (2 V_c V_{f_a} \cos(i_f - i_i))$$

d) If we were not only changing our orbital inclination but also our argument of periapsis, we would need to first find out our new orbit intersection points! This would first change our final velocity value for the ellipse, as we would need to find the velocity at one of the intersections points rather than the descending node as we have done here. As well, we would need to re-evaluate our delta V calculations:

similar to this!



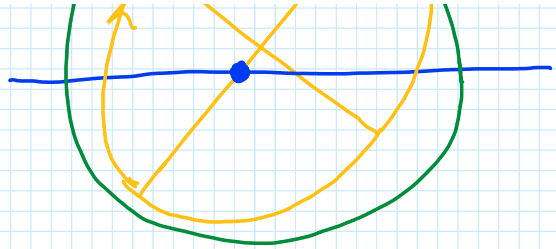
$$\Delta V^2 = V_c^2 + V_{f_a}^2 - \left(2 V_c V_{f_a} \cos(i_f - i_i)\right)$$

$$\Delta V = \sqrt{V_c^2 + V_{f_a}^2 - \left(2 V_c V_{f_a} \cos(i_f - i_i)\right)}$$

$$= 2.1414 \text{ km/s !}$$

however, since lowering orbit, need to burn at apoapsis AND in retrograde. I.e., our actual ΔV is

$$\underline{\underline{-2.1414 \text{ km/s}}}$$



In this case, we would not need to worry about necessarily changing up our current formula, but we need to make sure that our orbit intercept location is correct so we are working with the correct final velocity and as such correct deltaV. This being the case, we can use the same formula:

$$\Delta V = V_c^2 + V_{e_f}^2 - \left(2 V_c V_{e_f} \cos(\Delta i)\right)$$

Except our v_ellipse_final (vef) would be found at the new point of intersection between the original circular orbit, and the new elliptical orbit. Again, since we are going from large circular to smaller ellipse, our deltaV should be negative