Q1 :

given :
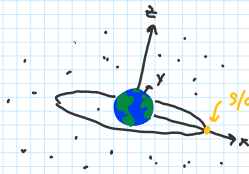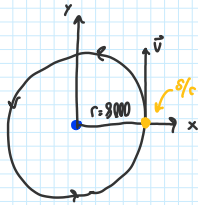
$$s/c_{pos_{init}} = \begin{bmatrix} 8000 \\ 0 \\ 0 \end{bmatrix} km$$

$$\mu_{Earth} = 0.39860 \cdot 10^6$$

$$\|\vec{v}\| = \sqrt{\frac{\mu_E}{r_{circ}}} = \sqrt{\frac{0.39860 \cdot 10^6}{8000}} = \underline{7.0587 \ km/s}$$

$$\cdot \ T/m = 1 \cdot 10^{-7} \ kN/kg$$

a)

$$s/c_{vel_{init}} = \begin{bmatrix} 0 \\ 7.0587 \\ 0 \end{bmatrix} km/s$$

b) $F = ma, \quad \frac{F}{m} = a$

$$\vec{a} = \underbrace{\frac{-\mu_E}{\|\vec{r}\|^3} \cdot \vec{r}}_{\text{Force o' Gravity}} + \frac{T}{m} \underbrace{\frac{\vec{v}}{\|\vec{v}\|}}_{\substack{\text{direction} \\ \text{of vel}}}$$

also from lecture:

$$a_T = \underbrace{\frac{1}{2} \frac{d(v^2)}{ds}}_{\substack{2v \frac{dv}{ds} \\ \frac{dv}{dt}}} + \underbrace{\frac{\mu_E}{r^2} \frac{dr}{ds}}_{\sin \gamma} \Big\} \text{(acceleration due to thrust)}$$

} unusable in our problem because not fully in terms of t (not all $\frac{d}{dt}$)

c) Given the equation:

$$t_{esc} - \underset{\downarrow}{t_0} = \frac{V_0}{a_T} \left(1 - \left(\frac{20 \ a_T^2 \ r_0^2}{V_0^4} \right)^{1/8} \right)$$

recall: $\vec{a} = \frac{\mu_E}{\|\vec{r}\|^3} \vec{r} + \frac{T}{m} \frac{\vec{v}}{\|\vec{v}\|}$

a_T = acceleration due to thrust, so
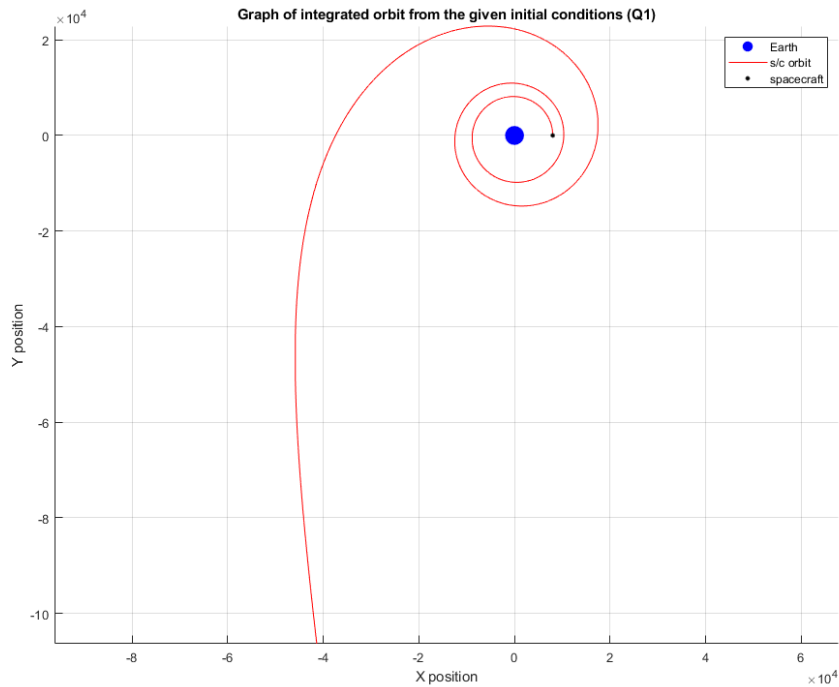
$a_T = 1 \cdot 10^{-7} \ kN/kg$

Then:

$$t_{esc} = \frac{\| s/c_{vel_{init}} \|}{a_T} \left(1 - \left(\frac{20 \ a_T^2 \ \| s/c_{pos_{init}} \|^2}{\| s/c_{vel_{init}} \|^4} \right)^{1/8} \right)$$

$$= \frac{7.0587}{0.0001} \left(1 - \left(\frac{20(0.0001)^2 (8000)^2}{(7.0587)^4} \right)^{1/8} \right)$$

$$= 3.4047728 6187 \cdot 10^4 \ seconds$$

$$t_{esc} = \boxed{3.4047 \ seconds}$$

d)

Graph of integrated orbit from the given initial conditions (Q1)

e) given that r_{esc} can be found

$$r_{esc} = \frac{\| s/c_{pos\ init} \| \| s/c_{vel\ init} \|}{\left(20\ \sigma_r^2 \| s/c_{pos\ init} \|^2 \right)^{1/4}}$$

$$r_{esc} = \frac{(8000)(7.0587)}{\left(20(0.0001)^2 (8000)^2 \right)^{1/4}}$$

$$\boxed{r_{esc} = 29855\ km}$$

from inspection of integrated
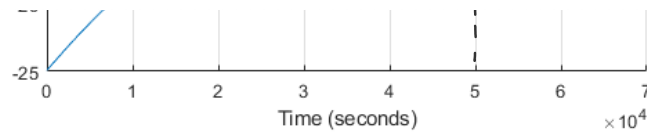data, r_{esc} is reached
at 36413 seconds

different from our analytical
34047 seconds by 2366 seconds!
a little less than 1 hour

- However, inspection of the
change in energy sign,
i.e. when ε (specific energy)
goes from negative to positive,
occurs roughly at 50130 seconds!

Using $\varepsilon = \dfrac{v^2}{2} - \dfrac{\mu}{r} = \dfrac{\| \vec{v} \|^2}{2} - \dfrac{\mu}{\| \vec{r} \|}$

Plot of specific energy vs Time

Both numerically found escape times are
greater than the analytical answer. I think the
analytical answer is conservative with it's
output because it really doesn't take into
account the force of gravity on the
spacecraft the same way that the numerical
integrated does. Since it's only going off of
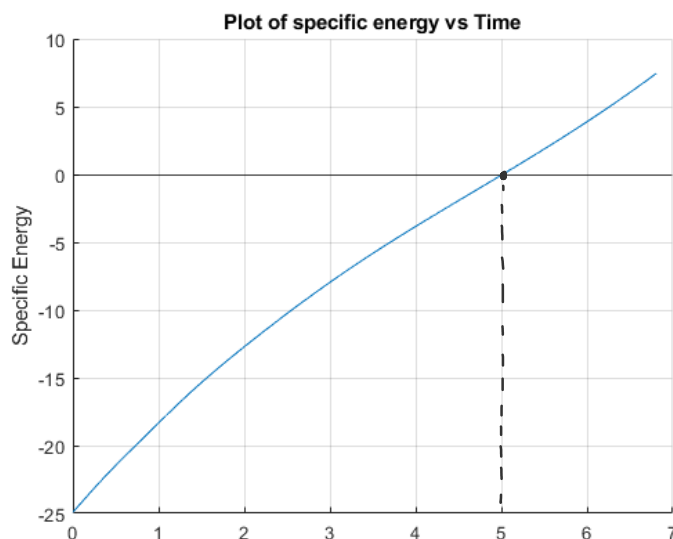
output because it really doesn't take into account the force of gravity on the spacecraft the same way that the numerical integrated does. S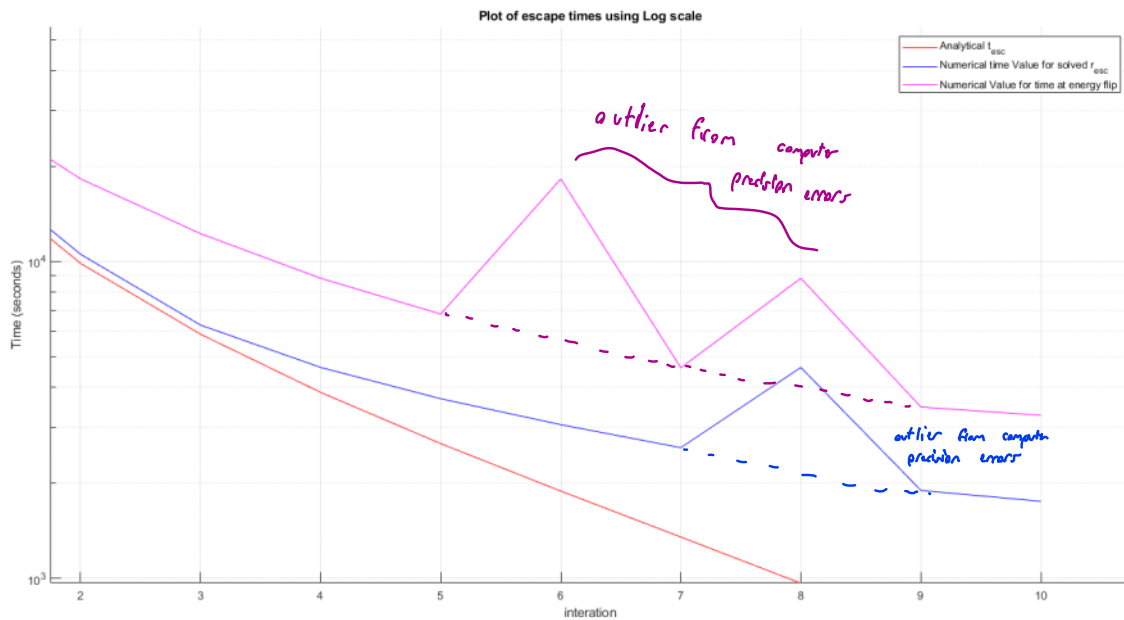ince it's only going off of the kinematics of the problem (other than the acceleration due to thrust being a non-kinematic term since it is a force), it does not account for the extra time it might take for the spacecraft to break from the planets gravity well, giving us a conservative value.
TL:DR - The exclusion of the gravity term from the t_esc equation leads to a faster escape time being found. Since the t_esc equation is only influenced by a single force (and not the accounting for the gravitiational force being applied to the craft), it will find a faster
escape time.

f)

Plot of escape times using Log scale

outlier from computer precision errors (handwritten)

outlier from computer precision errors (handwritten)

Legend:
- Analytical t_{esc}
- Numerical time Value for solved r_{esc}
- Numerical Value for time at energy flip

See attached code for more detail

g) From the plot, it is clear that the analytical method is lacking in it's accuracy. As the acceleration due to thrust is increased, we see a larger separation of numerical vs analytical values of the escape time. However, there are two key things to note: the analytical method produces a smooth, continuous curve of solutions, because it gives you exactly one solution. The integration method requires that you find the value at which either the energy changes or the time at which you reach your analytically solved for r_esc. Disregarding the outliers of data in the provided plot, and assuming they are smooth curves, we can still see the growing gap between the analytical solution and the numerical integration soltuion. The main limitation, I believe of the analytical method is that it does not incorporate gravity into the equation at all, and only relies on the input thrust as it's only force. I think this limitation is what causes the analytical method to undershoot, as it requires less time to escape if there is no modeled force of gravity "hold you back" (resisting the spacecrafts power to escape orbit)