QI

a) find total DV for transfer

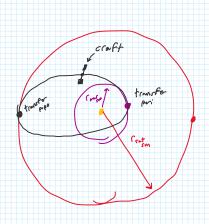
/ su = 1.3271.10" kn//s= ladius of mercury: 2440.5 km $\mu_{sun} = 1.3271 \cdot 10^{11} \text{ km}/s^{2}$ Condits of mercury: 2440.5 km $\mu_{sun} = 2.2032 \cdot 10^{4} \text{ km}^{2}/s^{2}$ Condits of satistic: 60,268 km $\mu_{sun} = 1.2671 \cdot 10^{8} \text{ km}^{2}/s^{2}$ $\mu_{sun} = 1.2671 \cdot 10^{8} \text{ km}^{2}/s^{2}$ $\mu_{sun} = 1.4985 \cdot 10^{20} \text{ kg}$ $\mu_{sun} = 3.3010 \cdot 10^{23} \text{ kg}$ $\mu_{sun} = 3.3010 \cdot 10^{23} \text{ kg}$ $\mu_{sun} = 1.32.041 \cdot 10^{6} \text{ km}$ $\mu_{sun} = 3.3010 \cdot 10^{23} \text{ kg}$ $\mu_{sun} = 1.32.041 \cdot 10^{6} \text{ km}$ $\mu_{sun} = 3.3010 \cdot 10^{23} \text{ kg}$ $\mu_{sun} = 1.32.041 \cdot 10^{6} \text{ kg}$ $\mu_{sun} = 1.32.041 \cdot 10^{6} \text{ kg}$ $\mu_{sun} = 1.32.041 \cdot 10^{6} \text{ kg}$ M sut = 5.6632.1026 kg

Confour = 400 km + planet codist = 2,840.5 km

Conft = 10000 km + planet radiur = 70,268 km

· Start solving: · Vpar = \[\int_{\text{four}}^{\text{from}} = \frac{1.3271.10^{11}}{57.909.10} = 47.872\] ***/s Vs. + 2 Jusus = [1.3271.10] = 9.6267 kg/s

· V coatt initial year = (conft = 2.2032.104 = 2.785 key/s · U craft = \(\frac{\mu_{sut}}{\cappa_{oraft}} = \) \(\frac{1.2671.10^8}{70268} = 42.4650 \) \(\frac{\mu_{oraft}}{\sigma_{oraft}} = \)



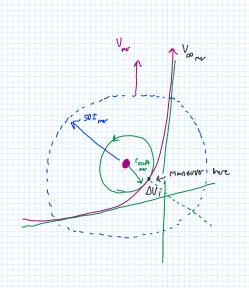
· V transfer = \[2\left(\frac{Msun}{corrsun}\) \left(\frac{corrsun}{corrsun}\right) = \left(2\left(\frac{1.3271.10"}{57.909.10"}\) \left(\frac{57.409.10"}{57.409.10"}\) \left(\frac{57.409.10"}{57.409.10"}\) = \left(6.3726 \text{ ten/s}\) · V timefor = \[2\left(\frac{\mu_{\sun}}{\sum_{\sun}} - \frac{\mu_{\sun}}{\sum_{\sun}} - \frac{\mu_{\sun}}{\sum_{\sun}} \right) = \left(2\left(\frac{1.3271\left(0)^4}{[432.041\left]06} - \frac{1.3271\left(0)^4}{(57.404\left(0)6 + [432.041\left(0)6])} = 2.6840 \text{ tem/5}

· NOW ZOOM in on mercury:

. Voo = V transferen - Vou = 66.3726 - 47.8721 = 18.5006 km/5

 $V_{\text{hyperbol}} = \sqrt{2\left(\frac{\mu_{\text{ner}}}{\sigma_{\text{ner}}} + \frac{V_{\text{ga}_{\text{ner}}}^2}{2}\right)} = \sqrt{2\left(\frac{2.2032 \cdot 10^4}{2840.5} + \frac{(18.5006)^2}{2}\right)} = 18.9152 \text{ tm/s}$

· DV = Vnyperbola - Vcrupt = 18.9152 - 2.785 = 16.130 km/s



• SOI mer =
$$\binom{r_{mr}}{r_{syn}}^{2/s} = 57.909.10^{6} \left(\frac{5.3010.10^{25}}{1.9485.10^{30}}\right)^{2/5} =$$

The assuming circ orbit, o-mr= r_{mr}

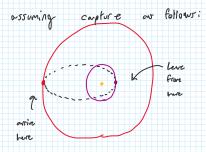
· now zoom in on saturn :

of ASSLMING perinpola of hyperbola Is SAME AS PARKENG ORBIT!!

$$V_{\text{hypothelia}} = \sqrt{2\left(\frac{\mu_{\text{Sh}}t}{c_{\text{conft}_{\text{Sh}}t}} + \frac{V_{\text{ab}}z}{z}\right)} = \sqrt{2\left(\frac{1.2671 \cdot 10^{2}}{70268} + \frac{(6.4427)^{2}}{Z}\right)} = 60.4546 \text{ km/s}$$

$$\cdot SOI_{sat} = \left(\frac{M_{sat}}{M_{sun}} \right)^{2/s} = \left(\frac{1}{1492.041.106} \right) \left(\frac{5.6832.10^{26}}{1.4925.10^{50}} \right) = \underbrace{\left(\frac{1}{14925.10^{50}} \right)^{2/s}}_{5.4640.10^{7} \text{ km}}$$

· FIND TOF :



Non we can assure an allipse during believering transfer:

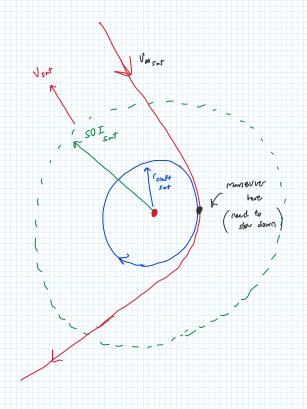
$$r_0 = r_{out}$$
 $g = r_{out} + r_{out} = 7.44975.108$

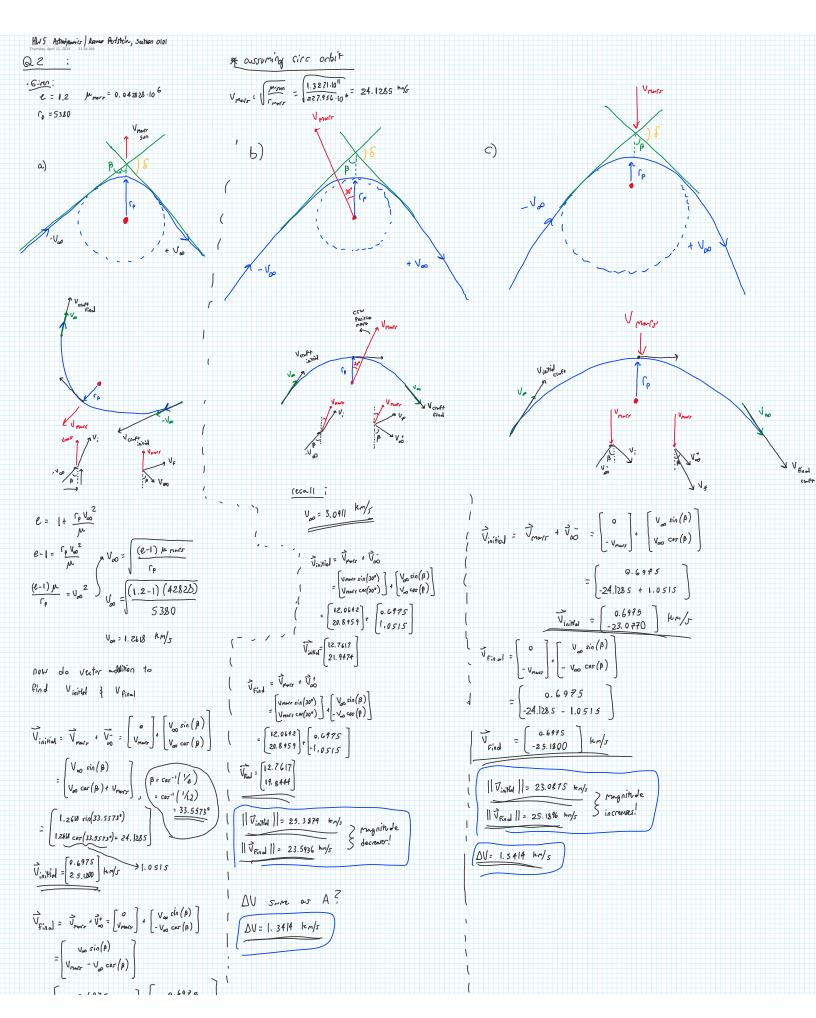
$$(r_p = a(1-e)) = \frac{r_p}{a} = 1-e$$
 $e = 1-\frac{r_p}{a} = \frac{r_{min}}{a} = 0.9223$

E=180° * assuming instant capture

$$t - T : \sqrt{\frac{\kappa^{3}}{\mu_{sm}}} \left(E - e \sin(E) \right)$$

$$+ = \sqrt{\frac{(7.44975 \cdot 10^{4})^{3}}{1.3271 \cdot 10^{9}}} \left(\pi - 0.422 \beta \sin(\pi) \right)$$





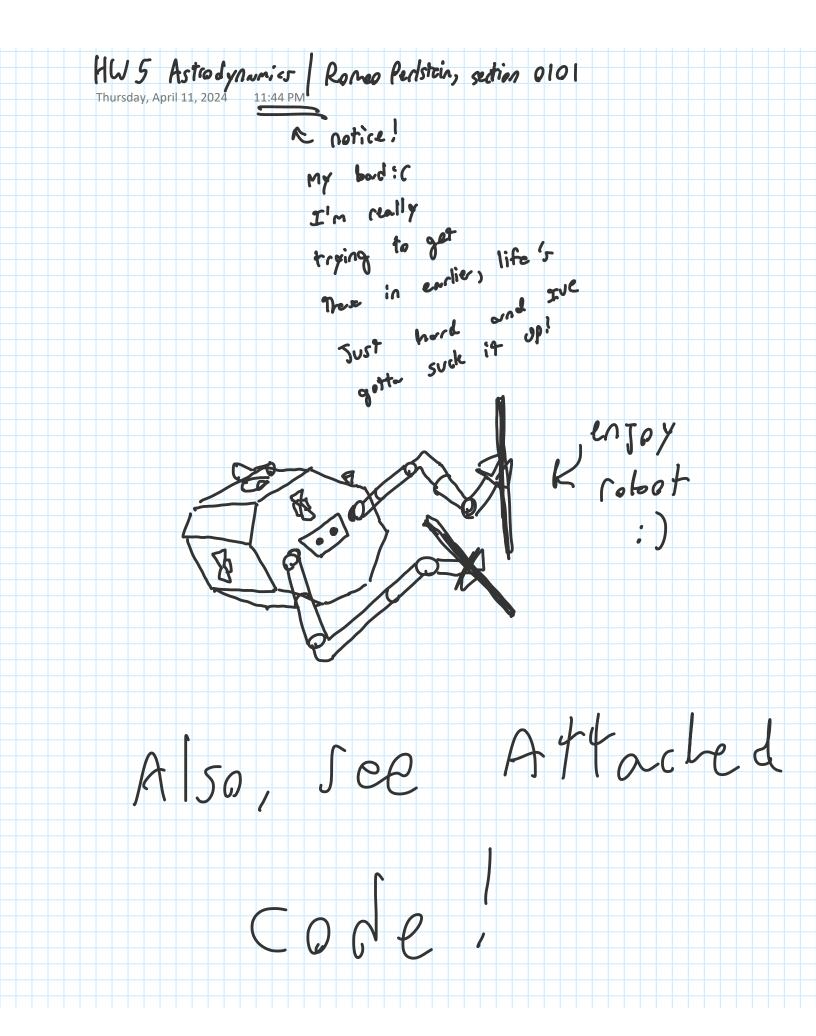


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Q1

Find deltaV for Mercury to Jupiter transfer using conic sections

```
% Givens:
mew_mercury = 22031.868551; % km<sup>3</sup>/s<sup>2</sup> - FROM JPL
mew saturn = 126712764.1; % km<sup>3</sup>/s<sup>2</sup> - FROM JPL
radius_planet_mercury = 2440.5; % km - from NASA fact sheet
r_craft_mercury = 400+radius_planet_mercury; % km
r_mercury = 57.909*10^6; % semi-major axis - from NASA fact sheet
radius planet saturn = 60268; % km - from NASA fact sheet
r_craft_saturn = 10000+radius_planet_saturn; % km
r_saturn = 1432.041*10^6; % semi-major axis - from NASA fact sheet
% since we are assuming circular orbits, we need to find the orbit
% velocity of both planets!
mew sun = 132712*10^6; % from NASA fact sheet
v_mercury = sqrt(mew_sun/r_mercury);
v_saturn = sqrt(mew_sun/r_saturn);
% find velocities of orbits of spacecraft
v_initial_craft_mercury = sqrt(mew_mercury/r_craft_mercury);
v_final_craft_saturn = sqrt(mew_saturn/r_craft_saturn);
% Find the velocity to transfer from mercury to saturn
v_transfer_peri = sqrt(2*((mew_sun/r_mercury) - (mew_sun/(r_mercury))
+r_saturn))));
v_transfer_apo = sqrt(2*((mew_sun/r_saturn) - (mew_sun/(r_mercury))
+r saturn)));
```

```
% Now, lets get the escape velocity from mercury
v escape mercury = v transfer peri - v mercury;
v_hyperbola_peri_mercury = sqrt(2*((mew_mercury/r_craft_mercury) +
 ((v escape mercury^2)/2)));
% Now we can find the delta V to get to Saturn
deltaV1 = v_hyperbola_peri_mercury - v_initial_craft_mercury;
% Now do saturn
v_escape_saturn = v_saturn - v_transfer_apo;
% MAKING ASSUMPTION THAT HYPERBOLA PERIAPSIS IS SAME AS PARKING ORBIT
% PERIAPSIS, A. BECAUSE THE PROBLEM DOESN'T SAY WE CAN'T AND B. BECAUSE I
% WOULD NOT BE ABLE TO SUBMIT THE HW ON TIME
v hyperbola peri saturn = sqrt(2*((mew saturn/r craft saturn) +
 ((v_escape_saturn^2)/2)));
deltaV2 = v_hyperbola_peri_saturn - v_final_craft_saturn;
% now get the supplementary info
mass_mercury = .3301*10^24; % kg
mass_saturn = 568.32*10^24; % kg
mass_sun = 1998500*10^24; % kg
SOI_mercury = r_mercury*(mass_mercury/mass_sun)^(2/5); % km - Matches with
Wikipedia!
SOI_saturn = r_saturn*(mass_saturn/mass_sun)^(2/5); % km - Matches with
Wikipedia!
deltaV_total = deltaV1+deltaV2;
% Find TOF, assuming ellipse:
a = (r mercury+r saturn)/2;
e = 1-(r_mercury/a);
E = pi;
t = sqrt((a^3)/mew_sun)*(pi-0.9223*sin(pi)); % seconds!!!
```

Q2

do a bunch of stuff I don't have time to finish:/ given:

```
e2 = 1.2;
rp2 = 5380;
mew_mars = 0.042828 *10^6; % km^3/s^2 - from NASA fact sheet
% assuming circular orbit
r_mars = 227.956 * 10^6;
v_mars = sqrt(mew_sun/r_mars);
% find escape velocity and velocity at periapsis
v_escape_mars2 = sqrt(((e2-1)*mew_mars)/rp2); % from notes? I guess it works
v_hyperbola_peri_mars2 = sqrt(2*((mew_mars/rp2) + ((v_escape_mars2^2)/2))); %
unused
% Get the hyperbola angle
Beta = acosd(1/e2); % get Beta angle
```

a

```
find v initial, v final, and then find deltaV
v_mars_vecA = [0;v_mars]; % Only acts in Y direction
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);v_escape_mars2*cosd(Beta)]; %
Get the vector components
v_initalA = v_mars_vecA + v_escape_mars2_vec; % Get the vector sum
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);-v_escape_mars2*cosd(Beta)]; %
Get the new escape vector
v_finalA = v_mars_vecA + v_escape_mars2_vec; % Get the vector sum
turn_angle = (180-(2*Beta))*(pi/180); % get the turn angle of the parabola
deltaV_A = sqrt(2*(v_escape_mars2^2)*cos(1-turn_angle)); % get the deltaV
using the deltaV eq. from notes
% Now get the norms
v_initalA_mag = norm(v_initalA);
v_finalA_mag = norm(v_finalA);
b
find v_initial, v_final, and then find deltaV
mars_vec_angle = 30; % Given angle away from velocity vector
% Get velocity vectors
v_mars_vecB = [v_mars*sind(mars_vec_angle);v_mars*cosd(mars_vec_angle)];
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);v_escape_mars2*cosd(Beta)];
v_initalB = v_mars_vecB + v_escape_mars2_vec; % Get vector sum
% get final stuff
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);-v_escape_mars2*cosd(Beta)];
v_finalB = v_mars_vecB + v_escape_mars2_vec; % Get vector sum
% Get deltaV (turn angle does not change!)
deltaV_B = sqrt(2*(v_escape_mars2^2)*cos(1-turn_angle));
% Get magnitudes
v_initalB_mag = norm(v_initalB);
v_finalB_mag = norm(v_finalB);
C
do pretty much exactly the same thing as above, again
v_mars_vecC = [0; -v_mars];
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);v_escape_mars2*cosd(Beta)];
v_initalC = v_mars_vecC + v_escape_mars2_vec;
v_escape_mars2_vec = [v_escape_mars2*sind(Beta);-v_escape_mars2*cosd(Beta)];
v_finalC = v_mars_vecC + v_escape_mars2_vec;
```

```
deltaV_C = sqrt(2*(v_escape_mars2^2)*cos(1-turn_angle));
v_initalC_mag = norm(v_initalC);
v_finalC_mag = norm(v_finalC);
```

Q3

solve keplers problamo

spacecraft 1

```
mew earth = 0.39860*10^6;
a1 = 15000;
e1 = 0.4;
i1 = 60;
Omega1 = 45;
w1 = 0;
true anom1 = 145;
deltaT = 3600; % seconds in an hour
% first, find E1_0 and mean motion
E1 0 = acos((e1+cosd(true anom1))/(1+e1*cosd(true anom1))*pi/180);
n = sqrt((a1^3)/mew_earth);
% now, solve the equation for E and get a constant
% Assuming k = 0 for simplicity? not sure how to solve for two unknowns
k = 0;
C1 = deltaT/n - 2*pi*k + (E1 0 - e1*sin(E1 0));
E1_guess = pi; % initial guess
fx1 = C1 - E1_guess + e1*sin(E1_guess); % begin iteration to solve for E
fx1_prime = -1 + e1*E1_guess*cos(E1_guess);
E1_prev = E1_guess - fx1/fx1_prime;
for ii=1:1:100 % 100 iterations should do the trick
    fx1 = C1 - E1_prev + e1*sin(E1_prev); % begin iteration to solve for E
    fx1 prime = -1 + e1*E1 prev*cos(E1 prev);
    E1_new = E1_prev - fx1/fx1_prime;
    E1_prev = E1_new;
end
E1 = E1 \text{ new};
% Now, find true anomaly
% through algebra, we find that:
true_anom1_final = acos((cos(E1)-e1)/(1-e1*cos(E1))); % rads
true anom1 final deg = true anom1 final*180/pi; % convert to degrees
% now put it through the orbital elements converter
[pos_vec1_initial, vel_vec1_initial, spef_energy1_intial] =
orbitalElementsToCart(a1, e1, i1, Omega1, w1, true_anom1, mew_earth, "deg");
[pos_vecl_final, vel_vecl_final, spef_energyl_final] =
 orbitalElementsToCart(al, el, il, Omegal, wl, true_anoml_final_deg,
mew_earth, "deg");
```

```
fprintf("Position and Velocity vectors for spacecraft_1 after 1 hour in flight
\n")
fprintf("Initial:\n")
pos_vecl_initial
vel_vecl_initial
fprintf("\nFinal:\n")
pos_vecl_final
vel_vecl_final
```

spacecraft 2

```
a2 = 21000;
e2 = 0.6;
i2 = 90;
Omega2 = 0;
w2 = 0;
true\_anom2 = 35;
% first, find E1_0 and mean motion
E2_0 = acos((e2+cosd(true\_anom2)))/(1+e2*cosd(true\_anom2))*pi/180);
n2 = sqrt((a2^3)/mew_earth);
% now, solve the equation for E and get a constant
% Assuming k = 0 for simplicity? not sure how to solve for two unknowns
k = 0;
C2 = deltaT/n2 - 2*pi*k + (E2_0 - e2*sin(E2_0));
E2_guess = pi; % initial guess
fx2 = C2 - E2_guess + e2*sin(E2_guess); % begin iteration to solve for E
fx2_prime = -1 + e2*E2_guess*cos(E2_guess);
E2_prev = E2_guess - fx2/fx2_prime;
for ii=1:1:100 % 100 iterations should do the trick
    fx2 = C2 - E2_prev + e2*sin(E2_prev); % begin iteration to solve for E
    fx2\_prime = -1 + e2*E2\_prev*cos(E2\_prev);
    E2_{new} = E2_{prev} - fx2/fx2_{prime};
   E2_prev = E2_new;
E2 = E2_{new};
% Now, find true anomaly
% through algebra, we find that:
true\_anom2\_final = acos((cos(E2)-e2)/(1-e2*cos(E2))); % rads
true_anom2_final_deg = true_anom2_final*180/pi; % convert to degrees
% now put it through the orbital elements converter
[pos_vec2_initial, vel_vec2_initial, spef_energy2_intial] =
orbitalElementsToCart(a2, e2, i2, Omega2, w2, true_anom2, mew_earth, "deg");
[pos_vec2_final, vel_vec2_final, spef_energy2_final] =
 orbitalElementsToCart(a2, e2, i2, Omega2, w2, true_anom2_final_deg,
 mew_earth, "deg");
```

```
fprintf("Position and Velocity vectors for spacecraft_2 after 1 hour in flight
\n")
fprintf("Initial:\n")
pos_vec2_initial
vel_vec2_initial
fprintf("\nFinal:\n")
pos_vec2_final
vel vec2 final
```

Conclusion

While (I believe) the above algorithm works, converting it to it's own specific function would be beneficial for general use. However, this current algorithm makes several assumptions. For one, it assumes that the orbits will always be ellipses (or circular), since it does not account for the TOF equation for a parabola or hyperbola. This is something that would need to be added! As well, we are assuming that the true_anom is always less that 180 degrees, and thus are not doing a half-plane check. finally and most important, we are only considering the possibility that our predictions are relative to one orbit, as in, we are not attempting to solve if the craft makes multiple orbits in the given time (1 hour), rather just where it is relative to the orbit. In order to figure out how many orbits the craft has made as well, we would need to include the K parameter into the equation, rather than just setting it to zero. There is much to be desired for, which I assume is why it's called "Keplers problem" and not "Keplers solution" !!! - Romeo

```
Position and Velocity vectors for spacecraft_1 after 1 hour in flight
Initial:

pos_vec1_initial =

1.0e+04 *

-1.8456
1.8456
0

vel_vec1_initial =

-0.6142
0.6142
0

Final:

pos_vec1_final =

1.0e+04 *

-1.8287
1.8287
0

vel_vec1_final =
```

```
0.7975
   -0.7975
Position and Velocity vectors for spacecraft_2 after 1 hour in flight
Initial:
pos_vec2_initial =
   1.0e+03 *
   7.3815
   5.1686
         0
vel\_vec2\_initial =
  -3.1236
   7.7285
Final:
pos_vec2_final =
  1.0e+04 *
  -2.4785
   1.3683
vel_vec2_final =
  -2.6320
  -1.5001
```

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