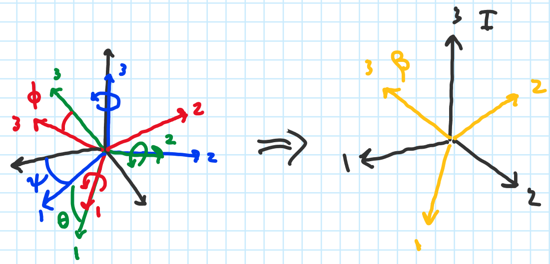
HW8 Astrodynamics | Romeo Perlstein section 0101

Thursday, May 9, 2024 9:05 AM

Q1 :



Given a spacecraft's orientation:

as (30, 40, 10) degrees:

3-2-1 Euler: $T_z \, T_y \, T_x$

$$T_z = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

$$R = \begin{bmatrix} c(30^\circ) & -s(30^\circ) & 0 \\ s(30^\circ) & c(30^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(40^\circ) & 0 & \sin(40^\circ) \\ 0 & 1 & 0 \\ -\sin(40^\circ) & 0 & \cos(40^\circ) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(10^\circ) & -s(10^\circ) \\ 0 & s(10^\circ) & c(10^\circ) \end{bmatrix}$$

$$\boxed{R = \begin{bmatrix} 0.6634 & -0.3957 & 0.6350 \\ 0.3830 & 0.9087 & 0.1661 \\ -0.6428 & 0.1330 & 0.7544 \end{bmatrix}}$$

B) $$\cos(\phi) = \frac{1}{2}\left(R_{11} + R_{22} + R_{33} - 1\right)$$

$$\cos(\phi) = \frac{1}{2}\left(0.6634 + 0.9087 + 0.7544 - 1\right)$$

$$\phi = \cos^{-1}\left(\frac{1}{2}\left(0.6634 + 0.9087 + 0.7544 - 1\right)\right)$$

$$\boxed{\phi = 48.4519^{\circ}}$$

$$\vec{e} = \frac{1}{2\sin(\phi)}\begin{bmatrix} C_{23} - C_{32} \\ C_{31} - C_{13} \\ C_{12} - C_{21} \end{bmatrix} = \frac{1}{2\sin(48.4519^{\circ})}\begin{bmatrix} 0.1661 - 0.1330 \\ -0.6428 - 0.6350 \\ -0.3957 - 0.3830 \end{bmatrix}$$

$$\boxed{\vec{e} = \begin{bmatrix} 0.0221 \\ -0.8537 \\ -0.5203 \end{bmatrix}}$$

C) using the formula for Euler to quaternion.

$$q_1 = e_1 \sin(\phi/2) = 0.0221 \sin\left(\frac{48.4519^{\circ}}{2}\right) = 0.0091$$

$$q_2 = e_2 \sin(\phi/2) = -0.8537 \sin\left(\frac{48.4519^{\circ}}{2}\right) = -0.3503$$

$$q_3 = e_3 \sin(\phi/2) = -0.5203 \sin\left(\frac{48.4519^{\circ}}{2}\right) = -0.2135$$

$$q_4 = \cos(\phi/2) = \cos\left(\frac{48.4519^{\circ}}{2}\right) = 0.9119$$

$$\boxed{\begin{aligned} q_1 &= 0.0091 \\ q_2 &= -0.3503 \\ q_3 &= -0.2135 \\ q_4 &= 0.9119 \end{aligned}}$$

* check:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 =$$

$$(0.0091)^2 + (-0.3503)^2 + (-0.2135)^2 + (0.9119)^2 = 1$$

D) given angular velocity of frame B,

$${}^{B}\vec{\omega} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix} \text{ rads/sec}$$

We can find the rate of change of the quaternion vector $\dot{\vec{\beta}}$:

$$\vec{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0.9119 \\ 0.0091 \\ -0.3503 \\ -0.2135 \end{bmatrix}$$

$$\dot{\vec{\beta}} = \begin{bmatrix} \dot{\beta}_0 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$\left(\frac{1}{2}\right) \begin{bmatrix} 0.9119 & -0.0091 & 0.3503 & 0.2135 \\ 0.0091 & 0.9119 & 0.2135 & -0.3503 \\ -0.3503 & -0.2135 & 0.9119 & -0.0091 \\ -0.2135 & 0.3503 & 0.0091 & 0.9119 \end{bmatrix} \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \dot{\beta}_0 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{bmatrix} = \begin{bmatrix} 0.034577 \\ 0.066946 \\ 0.080519 \\ 0.018423 \end{bmatrix} s^{-1}}$$

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Q2 :

Given :

$$I = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{bmatrix} \text{ kg}\cdot\text{m}^2$$

a) since we are assuming torque - free rigid body :

$${}^{B}\vec{h}_c = \begin{bmatrix} I_{11} w_1 \\ I_{22} w_2 \\ I_{33} w_3 \end{bmatrix}, \text{ where } \vec{w} \text{ given as } \begin{bmatrix} 10 \\ 0 \\ 30 \end{bmatrix} \text{deg/sec}$$

- need $\vec{w}$ in rad/sec, so do conversion

$$\vec{w} = \begin{bmatrix} 10 \\ 0 \\ 30 \end{bmatrix} \cdot \frac{\pi}{180} = \begin{bmatrix} 0.174533 \\ 0 \\ 0.523599 \end{bmatrix}$$

- now find ${}^{B}\vec{h}_c$:

$$\begin{bmatrix} 10 \cdot 0.1745 \\ 20 \cdot 0 \\ 30 \cdot 0.5236 \end{bmatrix} = \boxed{\begin{bmatrix} 1.74533 \\ 0 \\ 15.70796 \end{bmatrix}}$$

- now find kinetic energy :

rotational kinetic energy :

$$T = \frac{1}{2} I_{11} w_1^2 + \frac{1}{2} I_{22} w_2^2 + \frac{1}{2} I_{33} w_3^2$$

$$= \frac{1}{2}(10)(0.1745)^2 + \frac{1}{2}(20)(0)^2 + \frac{1}{2}(30)(0.5236)^2$$

$$\boxed{T = 4.2646 \text{ J}}$$

b) in Torque-free motion,

$\vec{L} = 0$! so our EOM's

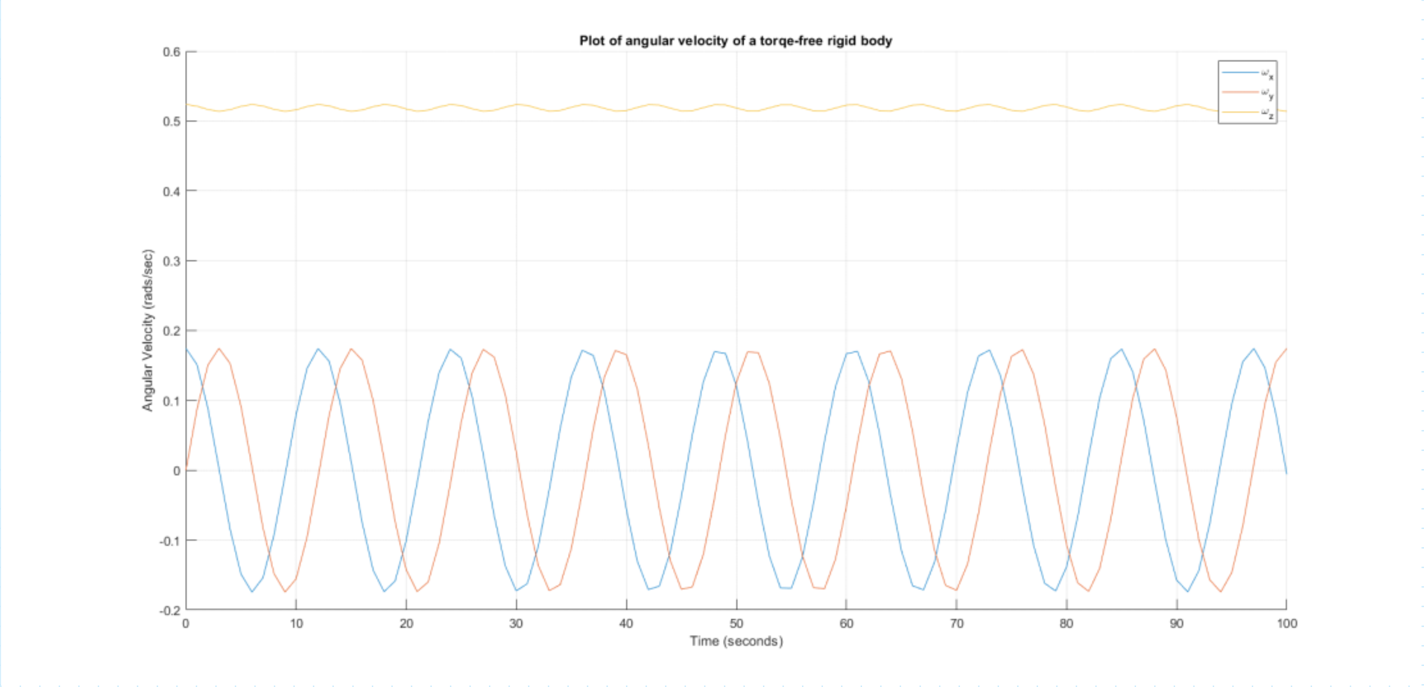
are:

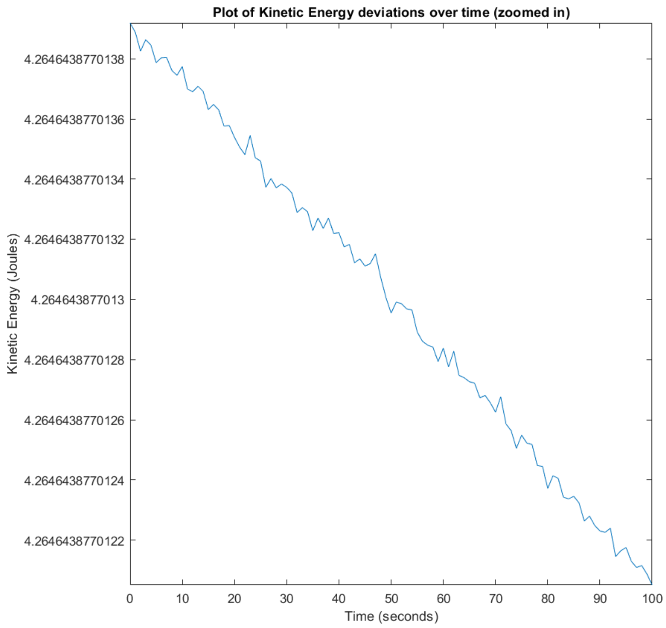
$$\begin{cases} I_{11}\dot{\omega}_1 = -(I_{33}-I_{22})\,\omega_2\omega_3 \\ I_{22}\dot{\omega}_2 = -(I_{11}-I_{33})\,\omega_3\omega_1 \\ I_{33}\dot{\omega}_3 = -(I_{22}-I_{11})\,\omega_1\omega_2 \end{cases}$$

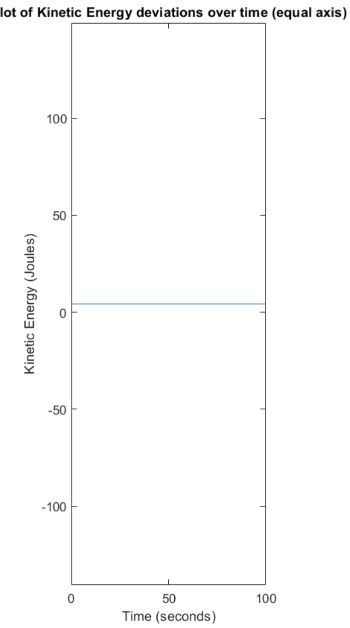
$$\begin{cases} \dot{\omega}_1 = \dfrac{-(I_{33}-I_{22})\,\omega_2\omega_3}{I_{11}} \\ \dot{\omega}_2 = \dfrac{-(I_{11}-I_{33})\,\omega_3\omega_1}{I_{22}} \\ \dot{\omega}_3 = \dfrac{-(I_{22}-I_{11})\,\omega_1\omega_2}{I_{33}} \end{cases}$$

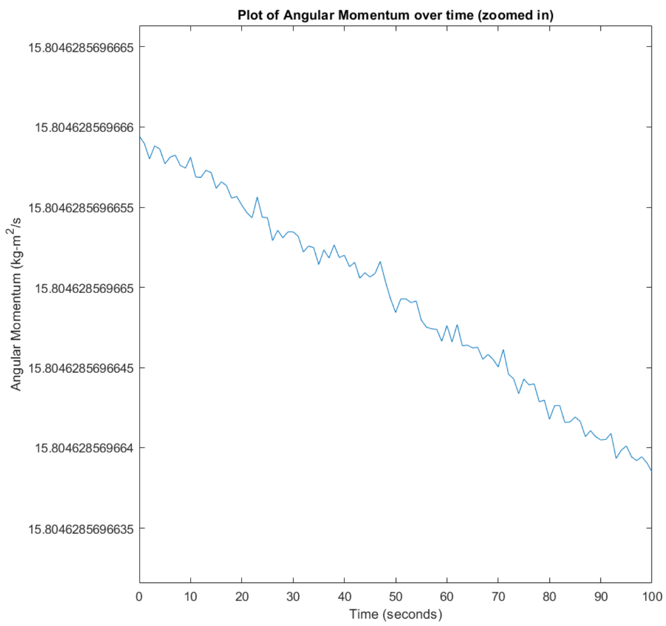
$\hookrightarrow$ implement in matlab

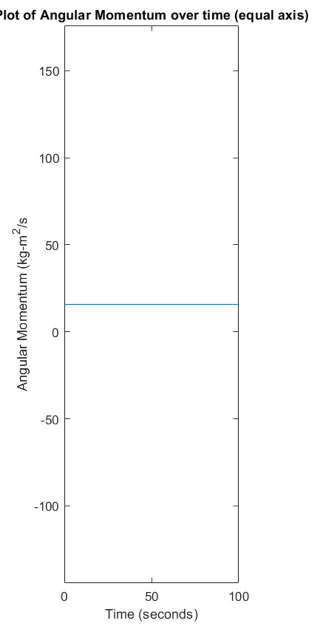
PLOTS:

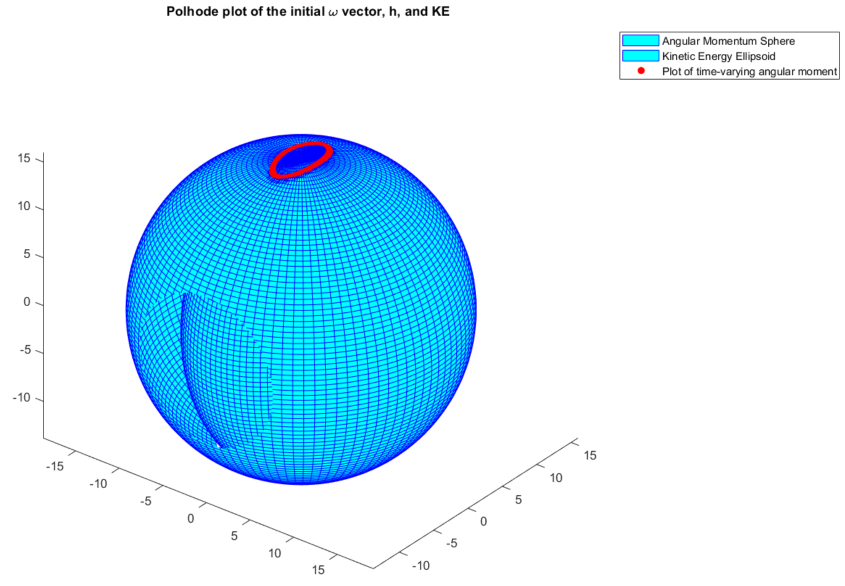


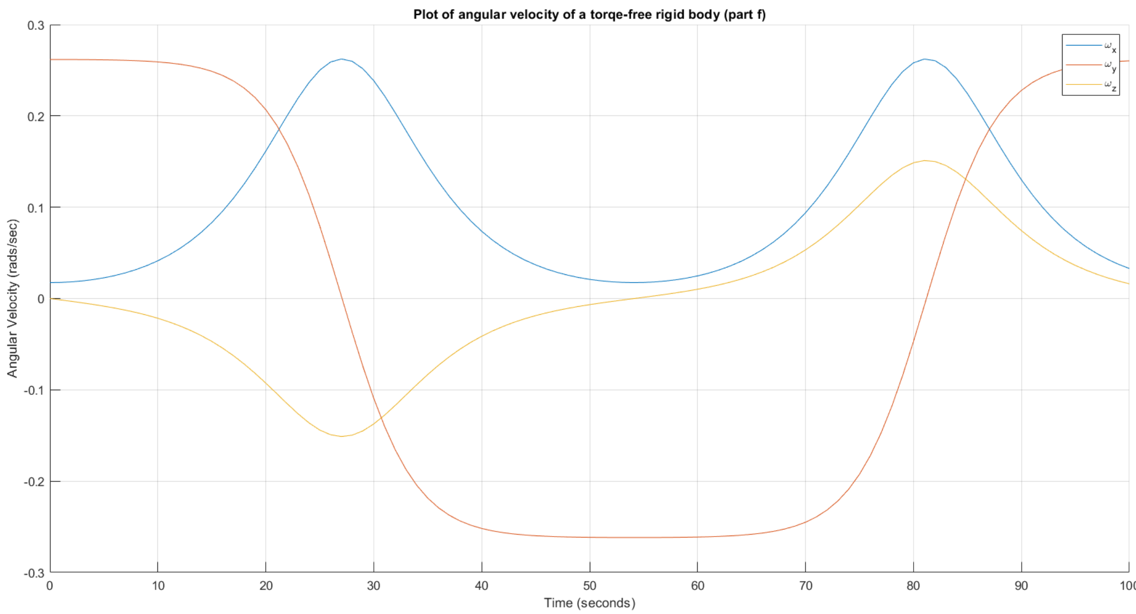

Plot of Kinetic Energy deviations over time (zoomed in)
Kinetic Energy (Joules)
Time (seconds)
4.2646438770138
4.2646438770136
4.2646438770134
4.2646438770132
4.264643877013
4.2646438770128
4.2646438770126
4.2646438770124
4.2646438770122
0 10 20 30 40 50 60 70 80 90 100


Plot of Kinetic Energy deviations over time (equal axis)
Kinetic Energy (Joules)
Time (seconds)
100
50
0
-50
-100
0 50 100


Plot of Angular Momentum over time (zoomed in)
Angular Momentum (kg-m^2/s)
Time (seconds)
15.8046285696665
15.804628569666
15.8046285696655
15.804628569665
15.8046285696645
15.804628569664
15.8046285696635
0 10 20 30 40 50 60 70 80 90 100


Plot of Angular Momentum over time (equal axis)
Angular Momentum (kg-m^2/s)
Time (seconds)
150
100
50
0
-50
-100
0 50 100


Polhode plot of the initial ω vector, h, and KE
Angular Momentum Sphere
Kinetic Energy Ellipsoid
Plot of time-varying angular moment


Plot of angular velocity of a torqe-free rigid body (part f)
Angular Velocity (rads/sec)
Time (seconds)
ω_x
ω_y
ω_z

Polhode plot of the ω vector from part f), h, and KE