HW5 Astrodynamics | Romeo Perlstein, section 0101

Thursday, April 11, 2024 10:02 AM

Q1

a) find total ΔV for transfer

given:

$\mu_{sun} = 1.3271 \cdot 10^{11}\ km^3/s^2$ radius of mercury: 2440.5 km

$\mu_{mer} = 2.2032 \cdot 10^{4}\ km^3/s^2$ radius of saturn: 60,268 km

$\mu_{sat} = 1.2671 \cdot 10^{8}\ km^3/s^2$

$r_{mer_{sun}} = 57.909 \cdot 10^6$ km

$m_{sun} = 1.9985 \cdot 10^{30}$ kg

$m_{mer} = 3.3010 \cdot 10^{23}$ kg

$r_{sat_{sun}} = 1432.041 \cdot 10^6$ km

$m_{sat} = 5.6832 \cdot 10^{26}$ kg

} from NASA fact sheet NSSDCA

$r_{craft_{mer}} = 400$ km + planet radius = 2,840.5 km

$r_{craft_{sat}} = 10000$ km + planet radius = 70,268 km

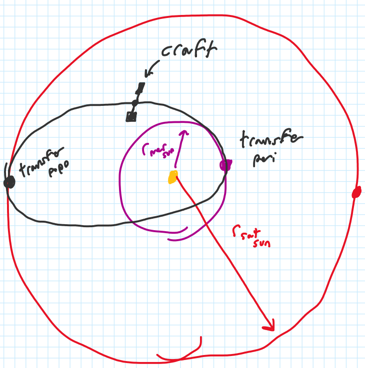
• start solving:

$$V_{mer} = \sqrt{\frac{\mu_{sun}}{r_{mer_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^6}} = 47.8721\ km/s$$

$$V_{sat} = \sqrt{\frac{\mu_{sun}}{r_{sat_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^6}} = 9.6267\ km/s$$

$$V_{craft_{initial_{mer}}} = \sqrt{\frac{\mu_{mer}}{r_{craft_{mer}}}} = \sqrt{\frac{2.2032 \cdot 10^4}{2,840.5}} = 2.785\ km/s$$

$$V_{craft_{final_{sat}}} = \sqrt{\frac{\mu_{sat}}{r_{craft_{sat}}}} = \sqrt{\frac{1.2671 \cdot 10^8}{70\,268}} = 42.4650\ km/s$$



$$V_{transfer_{peri}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{mer_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^6} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^6 + 1432.041 \cdot 10^6)}\right)} = 66.3726\ km/s$$

$$V_{transfer_{apo}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{sat_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^6} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^6 + 1432.041 \cdot 10^6)}\right)} = 2.6840\ km/s$$

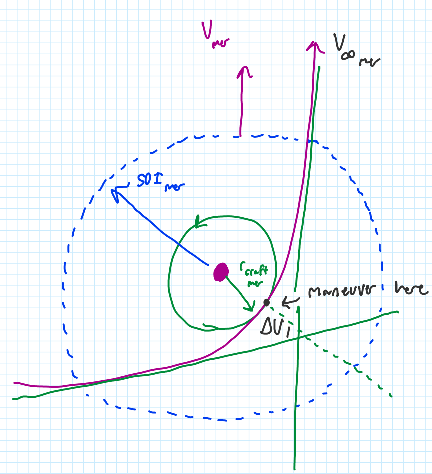
• now zoom in on mercury:

$$V_{\infty_{mer}} = V_{transfer_{peri}} - V_{mer} = 66.3726 - 47.8721 = \boxed{18.5006\ km/s}$$

$$V_{hyperbola_{peri_{mer}}} = \sqrt{2\left(\frac{\mu_{mer}}{r_{craft_{mer}}} + \frac{V_{\infty_{mer}}^2}{2}\right)} = \sqrt{2\left(\frac{2.2032 \cdot 10^4}{2840.5} + \frac{(18.5006)^2}{2}\right)} = 18.9152\ km/s$$

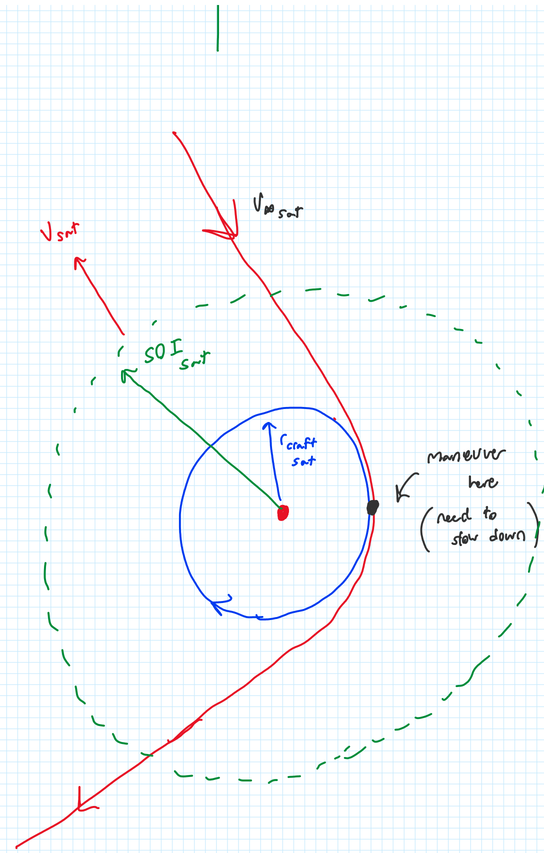
$$\Delta V_1 = V_{hyperbola_{peri_{mer}}} - V_{craft_{initial_{mer}}} = 18.9152 - 2.785 = 16.1301\ km/s$$

$$SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909 \cdot 10^6 \left(\frac{3.3010 \cdot 10^{23}}{1.9985 \cdot 10^{30}}\right)^{2/5} = \boxed{1.1218 \cdot 10^5\ km}$$



$$\cdot\ SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909\cdot 10^{6}\left(\frac{3.3010\cdot 10^{23}}{1.9985\cdot 10^{30}}\right)^{2/5} = \boxed{1.1218\cdot 10^{5}\ \text{km}}$$

since assuming circ orbit, $a_{mer} = r_{mer}$



• now zoom in on saturn :

* ASSUMING periapsis of hyperbola is SAME AS PARKING ORBIT!!!

$$\cdot\ V_{\infty\, sat} = V_{sat} - V_{transfer\ Apo} = 9.6267 - 2.6840 = \boxed{6.9427\ \text{km/s}}$$

$$\cdot\ V_{hyperbola\ peri\ sat} = \sqrt{2\left(\frac{\mu_{sat}}{r_{craft\, sat}} + \frac{V_{\infty\, sat}^{2}}{2}\right)} = \sqrt{2\left(\frac{1.2671\cdot 10^{8}}{70268} + \frac{(6.9427)^{2}}{2}\right)} = 60.4546\ \text{km/s}$$

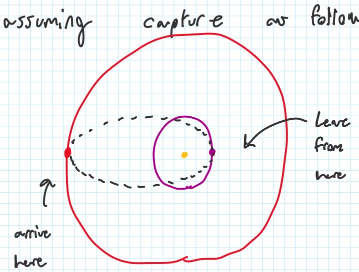
$$\cdot\ \Delta V_2 = V_{hyperbola\ peri\ sat} - V_{craft\ final\ sat} = 60.4546 - 42.4650 = 17.9896\ \text{km/s}$$

$$\cdot\ SOI_{sat} = r_{sat}\left(\frac{M_{sat}}{M_{sun}}\right)^{2/5} = (1432.041\cdot 10^{6})\left(\frac{5.6832\cdot 10^{26}}{1.9985\cdot 10^{30}}\right) = \boxed{5.4640\cdot 10^{7}\ \text{km}}$$

$$\cdot\ \Delta V_{tot} = \Delta V_1 + \Delta V_2 = 16.1301 + 17.9896 = \boxed{34.1197\ \text{km/s}}$$

• FIND TOF :

assuming capture as follows:



then we can assume an ellipse during heliocentric transfer:

$$\left.\begin{aligned} r_p &= r_{mer} \\ r_a &= r_{sat} \end{aligned}\right\}\ a = \frac{r_{mer} + r_{sat}}{2} = 7.44975\cdot 10^{8}$$

$$\cdot\ r_p = a(1-e) \quad \frac{r_p}{a} = 1-e \quad e = 1 - \frac{r_p}{a} = \frac{r_{mer}}{a} = 0.9223$$

$E = 180^{\circ}$ * assuming instant capture

$$t - T = \sqrt{\frac{a^3}{\mu_{sun}}}\left(E - e\sin(E)\right)$$

$$t = \sqrt{\frac{(7.44975\cdot 10^{8})^{3}}{1.3271\cdot 10^{11}}}\left(\pi - 0.9223\sin(\pi)\right)$$

$$\boxed{t = 1.7535\cdot 10^{8}\ \text{seconds}}$$

HW5 Astrodynamics | Ramon Perlstein, Section 0101

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Q2 :

Given:

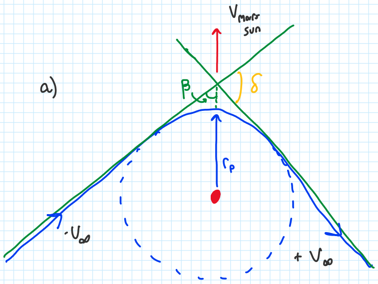
$e = 1.2 \quad \mu_{mars} = 0.042828 \cdot 10^6$

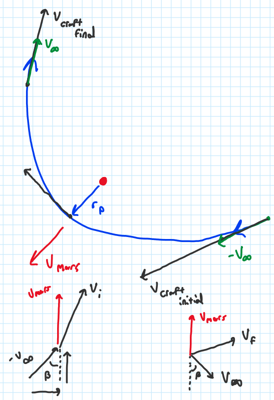
$r_p = 5380$

* assuming circ orbit

$$V_{mars} = \sqrt{\frac{\mu_{sun}}{r_{mars}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{227.956 \cdot 10^6}} = 24.1285 \text{ km/s}$$

a)





$$e = 1 + \frac{r_p V_\infty^2}{\mu}$$

$$e - 1 = \frac{r_p V_\infty^2}{\mu}$$

$$\frac{(e-1)\mu}{r_p} = V_\infty^2$$

$$V_\infty = \sqrt{\frac{(e-1)\,\mu_{mars}}{r_p}}$$

$$V_\infty = \sqrt{\frac{(1.2-1)(42828)}{5380}}$$

$$V_\infty = 1.2618 \text{ km/s}$$

now do vector addition to find $V_{initial}$ & V_{final}

$$\vec{V}_{initial} = \vec{V}_{mars} + \vec{V}_\infty^- = \begin{bmatrix} 0 \\ V_{mars} \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} V_\infty \sin(\beta) \\ V_\infty \cos(\beta) + V_{mars} \end{bmatrix}, \quad \beta = \cos^{-1}(1/e) = \cos^{-1}(1/1.2) = \underline{\underline{33.5573^\circ}}$$

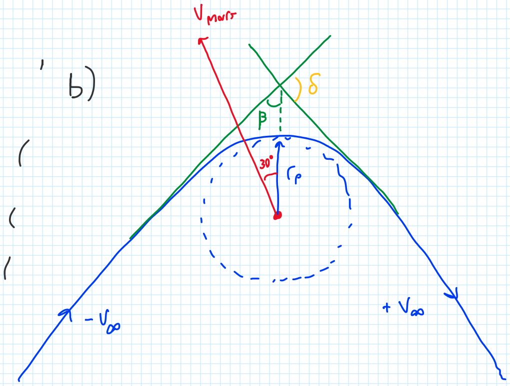
$$= \begin{bmatrix} 1.2618 \sin(33.5573^\circ) \\ \underbrace{1.2618 \cos(33.5573^\circ)}_{1.0515} + 24.1285 \end{bmatrix}$$

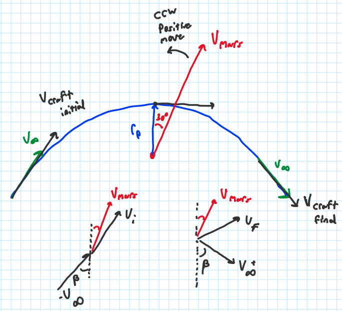
$$\vec{V}_{initial} = \begin{bmatrix} 0.6975 \\ 25.1800 \end{bmatrix} \text{ km/s}$$

$$\vec{V}_{final} = \vec{V}_{mars} + \vec{V}_\infty^+ = \begin{bmatrix} 0 \\ V_{mars} \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ -V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} V_\infty \sin(\beta) \\ V_{mars} - V_\infty \cos(\beta) \end{bmatrix}$$

b)





recall:

$$\underline{\underline{V_\infty = 3.0411 \text{ km/s}}}$$

$$\vec{V}_{initial} = \vec{V}_{mars} + \vec{V}_\infty^-$$

$$= \begin{bmatrix} V_{mars} \sin(30^\circ) \\ V_{mars} \cos(30^\circ) \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 12.0642 \\ 20.8959 \end{bmatrix} + \begin{bmatrix} 0.6975 \\ 1.0515 \end{bmatrix}$$

$$\vec{V}_{initial} = \begin{bmatrix} 12.7617 \\ 21.9474 \end{bmatrix}$$

$$\vec{V}_{final} = \vec{V}_{mars} + \vec{V}_\infty^+$$

$$= \begin{bmatrix} V_{mars} \sin(30^\circ) \\ V_{mars} \cos(30^\circ) \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ -V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 12.0642 \\ 20.8959 \end{bmatrix} + \begin{bmatrix} 0.6975 \\ -1.0515 \end{bmatrix}$$

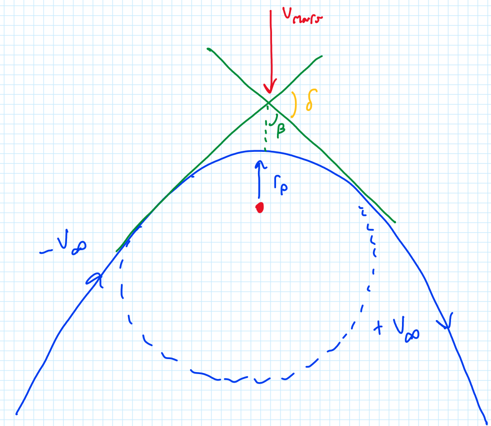
$$\vec{V}_{final} = \begin{bmatrix} 12.7617 \\ 19.8444 \end{bmatrix}$$

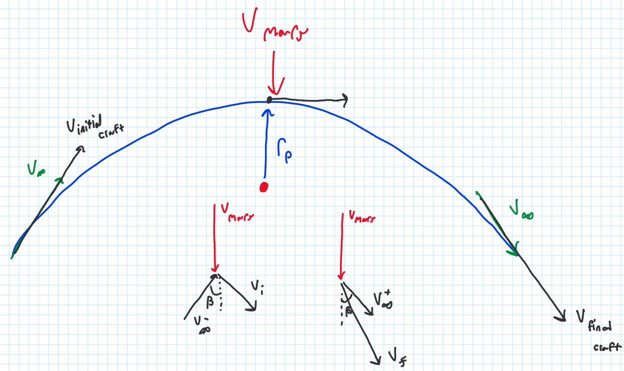
$$\boxed{\begin{aligned} \|\vec{V}_{initial}\| &= 25.3879 \text{ km/s} \\ \|\vec{V}_{final}\| &= 23.5936 \text{ km/s} \end{aligned} \Bigg\} \text{ magnitude decreases!}}$$

ΔV same as A?

$$\boxed{\Delta V = 1.3414 \text{ km/s}}$$

c)





$$\vec{V}_{initial} = \vec{V}_{mars} + \vec{V}_\infty^- = \begin{bmatrix} 0 \\ -V_{mars} \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6975 \\ -24.1285 + 1.0515 \end{bmatrix}$$

$$\vec{V}_{initial} = \begin{bmatrix} 0.6975 \\ -23.0770 \end{bmatrix} \text{ km/s}$$

$$\vec{V}_{final} = \begin{bmatrix} 0 \\ -V_{mars} \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\beta) \\ -V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6975 \\ -24.1285 - 1.0515 \end{bmatrix}$$

$$\vec{V}_{final} = \begin{bmatrix} 0.6975 \\ -25.1800 \end{bmatrix} \text{ km/s}$$

$$\boxed{\begin{aligned} \|\vec{V}_{initial}\| &= 23.0875 \text{ km/s} \\ \|\vec{V}_{final}\| &= 25.1896 \text{ km/s} \end{aligned} \Bigg\} \text{ magnitude increases!}}$$

$$\boxed{\Delta V = 1.5414 \text{ km/s}}$$

$$= \begin{bmatrix} V_\infty \sin(\beta) \\ V_{mars} - V_\infty \cos(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6975 \\ 24.1285 - 1.2618\cos(\beta) \end{bmatrix} = \begin{bmatrix} 0.6975 \\ 24.1285 - 1.0515 \end{bmatrix}$$

$$\vec{v}_{final} = \begin{bmatrix} 0.6975 \\ 23.0770 \end{bmatrix} \text{ km/s}$$

$$\Delta V^2 = 2V_\infty^2\left(1-\cos(\delta)\right), \quad \delta = 180 - 2\beta = 112.8854° = 1.9702 \text{ rads}$$

$$\Delta V^2 = 2V_\infty^2\left(1 - \cos(1.9702)\right)$$

$$\Delta V = \sqrt{2(1.2618)^2\left(1 - \cos(1.9702)\right)}$$

$$\boxed{\Delta V = 1.3414} \text{ km/s}$$

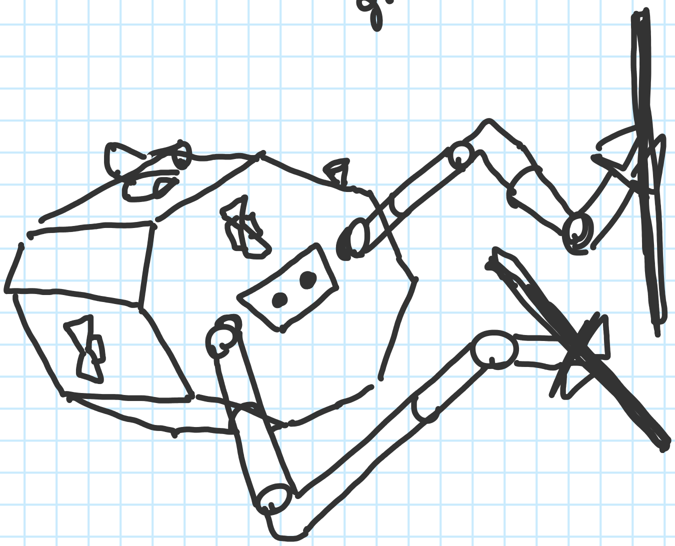
$\|V_{initial}\| = 25.1896$ km/s
$\|V_{final}\| = 23.0875$ km/s
} magnitude decreases!

HW 5 Astrodynamics | Romeo Perlstein, section 0101

Thursday, April 11, 2024 11:44 PM

← notice!

My bad :(
I'm really
trying to get
these in earlier, life's
just hard and I've
gotta suck it up!



enjoy
robot
:)

Also, see Attached code!