HW5 Astrodynamics | Romeo Perlstein, section 0101

Thursday, April 11, 2024 10:02 AM

Q1

a) find total ΔV for transfer

given:

$\mu_{sun} = 1.3271 \cdot 10^{11}\ km^3/s^2$ radius of mercury: 2440.5 km

$\mu_{mer} = 2.2032 \cdot 10^{4}\ km^3/s^2$ radius of saturn: 60,268 km

$\mu_{sat} = 1.2671 \cdot 10^{8}\ km^3/s^2$

$m_{sun} = 1.9985 \cdot 10^{30}\ kg$ $r_{mer_{sun}} = 57.909 \cdot 10^{6}\ km$

$m_{mer} = 3.3010 \cdot 10^{23}\ kg$ $r_{sat_{sun}} = 1432.041 \cdot 10^{6}\ km$

$m_{sat} = 5.6832 \cdot 10^{26}\ kg$

from NASA fact sheet NSSDCA

$r_{craft_{mer}} = 400\ km + \text{planet radius} = 2{,}840.5\ km$

$r_{craft_{sat}} = 10000\ km + \text{planet radius} = 70{,}268\ km$

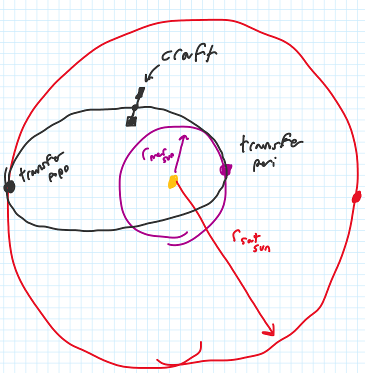
• start solving:

$$V_{mer} = \sqrt{\frac{\mu_{sun}}{r_{mer_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^{6}}} = 47.8721\ km/s$$

$$V_{sat} = \sqrt{\frac{\mu_{sun}}{r_{sat_{sun}}}} = \sqrt{\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^{6}}} = 9.6267\ km/s$$

$$V_{craft_{initial_{mer}}} = \sqrt{\frac{\mu_{mer}}{r_{craft_{mer}}}} = \sqrt{\frac{2.2032 \cdot 10^{4}}{2{,}840.5}} = 2.785\ km/s$$

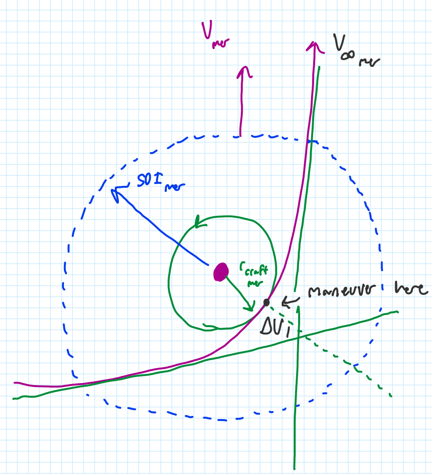
$$V_{craft_{final_{sat}}} = \sqrt{\frac{\mu_{sat}}{r_{craft_{sat}}}} = \sqrt{\frac{1.2671 \cdot 10^{8}}{70\,268}} = 42.4650\ km/s$$



$$V_{transfer_{peri}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{mer_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{57.909 \cdot 10^{6}} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^{6} + 1432.041 \cdot 10^{6})}\right)} = 66.3726\ km/s$$

$$V_{transfer_{apo}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{sat_{sun}}} - \frac{\mu_{sun}}{(r_{mer_{sun}} + r_{sat_{sun}})}\right)} = \sqrt{2\left(\frac{1.3271 \cdot 10^{11}}{1432.041 \cdot 10^{6}} - \frac{1.3271 \cdot 10^{11}}{(57.909 \cdot 10^{6} + 1432.041 \cdot 10^{6})}\right)} = 2.6840\ km/s$$

• now zoom in on mercury:



$$V_{\infty_{mer}} = V_{transfer_{peri}} - V_{mer} = 66.3726 - 47.8721 = 18.5006\ km/s$$

$$V_{hyperbola_{esc_{mer}}} = \sqrt{2\left(\frac{\mu_{mer}}{r_{craft_{mer}}} + \frac{V_{\infty_{mer}}^{2}}{2}\right)} = \sqrt{2\left(\frac{2.2032 \cdot 10^{4}}{2840.5} + \frac{(18.5006)^2}{2}\right)} = \boxed{18.9152\ km/s}$$

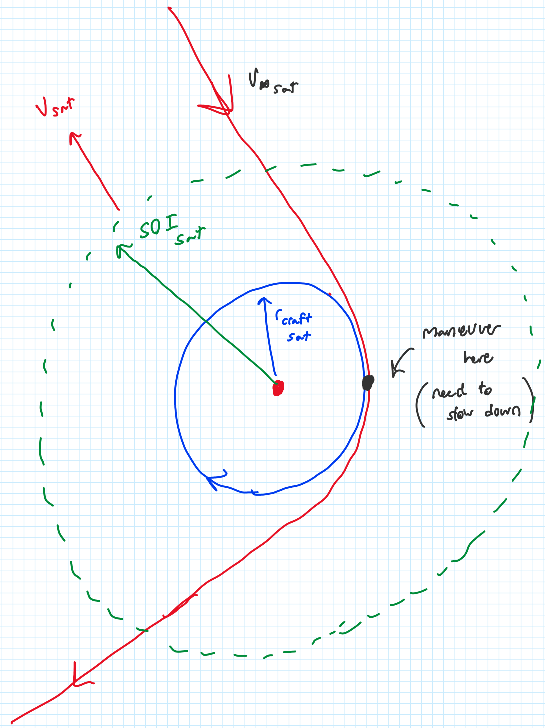
$$\Delta V_1 = V_{hyperbola_{esc_{mer}}} - V_{craft_{initial_{mer}}} = 18.9152 - 2.785 = 16.1301\ km/s$$

$$SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909 \cdot 10^{6}\left(\frac{3.3010 \cdot 10^{23}}{1.9985 \cdot 10^{30}}\right)^{2/5} = \boxed{1.1218 \cdot 10^{5}\ km}$$

$$\cdot\ SOI_{mer} = r_{mer}\left(\frac{m_{mer}}{m_{sun}}\right)^{2/5} = 57.909\cdot10^6\left(\frac{3.3010\cdot10^{23}}{1.9985\cdot10^{30}}\right)^{2/5} = \boxed{1.1218\cdot10^5\ km}$$

since assuming circ orbit, $a_{mer} = r_{mer}$

• now zoom in on saturn :



* ASSUMING periapsis of hyperbola is SAME AS PARKING ORBIT!!!

$$\cdot\ V_{\infty\,sat} = V_{sat} - V_{transfer\,Apo} = 9.6267 - 2.6840 = 6.9427\ km/s$$

$$\cdot\ V_{hyperbola\,esc\,sat} = \sqrt{2\left(\frac{\mu_{sat}}{r_{craft\,sat}} + \frac{V_{\infty\,sat}^{\ 2}}{2}\right)} = \sqrt{2\left(\frac{1.2671\cdot10^8}{70268} + \frac{(6.9427)^2}{2}\right)} = \boxed{60.4546\ km/s}$$

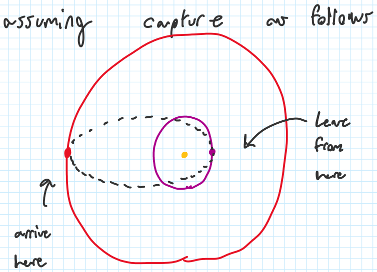
$$\cdot\ \Delta V_2 = V_{hyperbola\,esc\,sat} - V_{craft\,final\,sat} = 60.4546 - 42.4650 = 17.9896\ km/s$$

$$\cdot\ SOI_{sat} = r_{sat}\left(\frac{M_{sat}}{M_{sun}}\right)^{2/5} = (1432.041\cdot10^6)\left(\frac{5.6832\cdot10^{26}}{1.9985\cdot10^{30}}\right) = \boxed{5.4640\cdot10^7\ km}$$

$$\cdot\ \Delta V_{tot} = \Delta V_1 + \Delta V_2 = 16.1301 + 17.9896 = \boxed{34.1197\ km/s}$$

• FIND TOF :

assuming capture as follows:



then we can assume an ellipse during heliocentric transfer:

$$\left.\begin{array}{l} r_p = r_{mer} \\ r_a = r_{sat} \end{array}\right\} a = \frac{r_{mer} + r_{sat}}{2} = 7.44975\cdot10^8$$

$$\cdot\ r_p = a(1-e) = \frac{r_p}{a} = 1-e \quad e = 1 - \frac{r_p}{a} = \frac{r_{mer}}{a} = 0.9223$$

$E = 180^\circ$ * assuming instant capture

$$t - T = \sqrt{\frac{a^3}{\mu_{sun}}}\left(E - e\sin(E)\right)$$

$$t = \sqrt{\frac{(7.44975\cdot10^8)^3}{1.3271\cdot10^{11}}}\left(\pi - 0.9223\sin(\pi)\right)$$

$$\boxed{t = 1.7535\cdot10^8\ \text{seconds}}$$

Q2 :

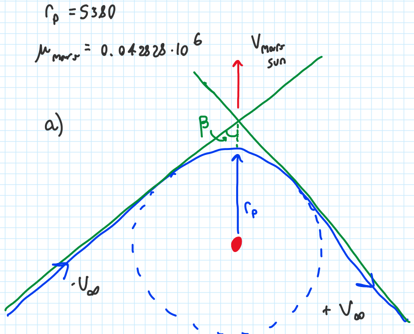
· Given:

$e = 1.2$

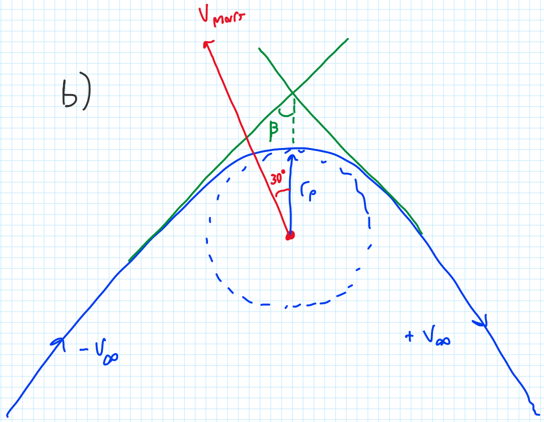
$r_p = 5380$

$\mu_{mars} = 0.042828 \cdot 10^6$

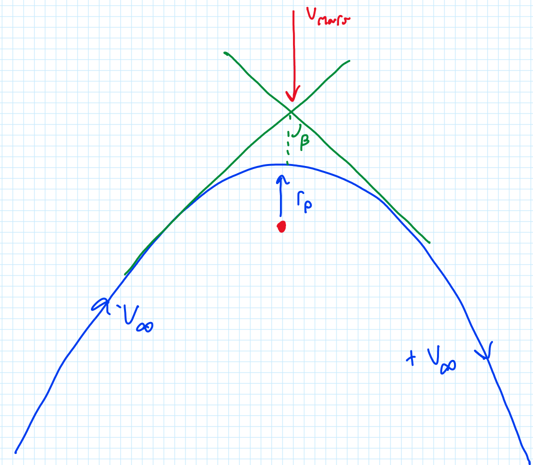
a)

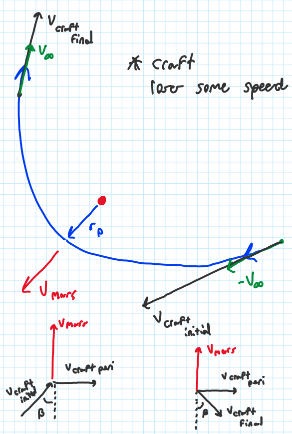


b)



c)





* craft lose some speed!