

M Earth = 0.39860-106

b)
$$F: na$$
, $\frac{F}{\Omega} = a$

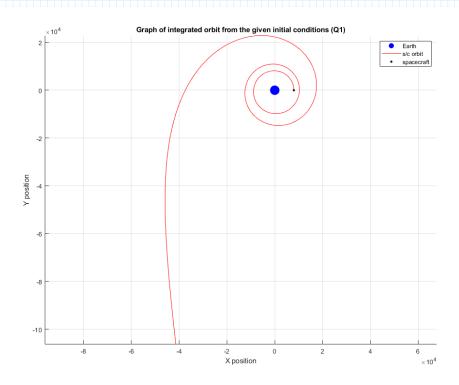
C) Given The equation:

$$\psi_{esc} = \psi_{o} = \frac{V_{o}}{\alpha_{T}} \left(\left[-\left(\frac{20 \, \alpha_{T}^{2} \, \Gamma_{o}^{2}}{V_{o}^{4}} \right)^{1/3} \right) \right]$$

$$\frac{1}{t_{esc}} = \frac{\left\| \frac{s/c_{vel_{int}}}{a_{T}} \right\|}{a_{T}} \left(1 - \left(\frac{20 a_{T}^{2} \left\| \frac{s/c_{per_{int}}}{a_{T}} \right\|^{2}}{\left\| \frac{s/c_{per_{int}}}{a_{T}} \right\|^{4}} \right)^{\frac{1}{2}} \right)$$

$$= \frac{7.0587}{0.0001} \left(1 - \left(\frac{20(0.0001)^2 (3000)^2}{(7.0587)^4} \right)^{1/8} \right)$$

7)

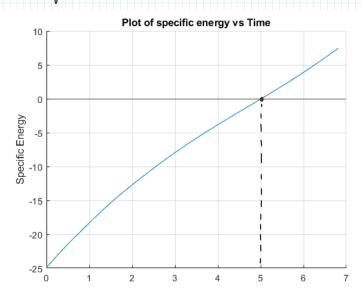


from inspection of integrated data, Cesc ls readed at 36413 seconds

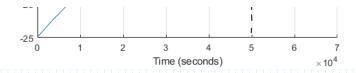
different from our analytical 34047 seconds by 2366 seconds! a little less lime I have

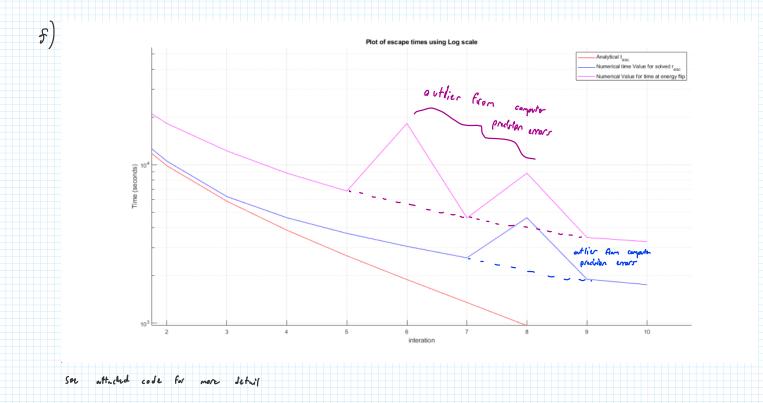
- · However, inspection of the change in energy sign, i.e. when & (specific energy) good from regative to partitle, occur roughly at 50130 seconds!
- Both numerically found escape times are greater than the analytical answer. I think the analytical answer is conservative with it's output because it really doesn't take into account the force of gravity on the spacecraft the same way that the numerical integrated does. Since it's only going off of

$$U_{sing} \quad e = \frac{V^{2}}{2} - \frac{\mu}{r} = \frac{||\vec{v}||^{2}}{2} - \frac{\mu}{||\vec{r}||}$$

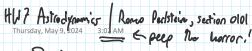


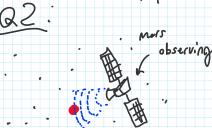
output because it really doesn't take into account the force of gravity on the spacecraft the same way that the numerical integrated does. Since it's only going off of the kinematics of the problem (other than the acceleration due to thrust being a nonkinematic term since it is a force), it does not account for the extra time it might take for the spacecraft to break from the planets gravity well, giving us a conservative value. TL:DR - The exclusion of the gravity term from the t_esc equation leads to a faster escape time being found. Since the t_esc equation is only influenced by a single force (and not the accounting for the gravitiational force being applied to the craft), it will find a escape time.





From the plot, it is clear that the analytical method is lacking in it's accuracy. As the acceleration due to thrust is increased, we see a larger separation of numerical vs analytical values of the escape time. However, there are two key things to note: the analytical method produces a smooth, continuous curve of solutions, because it gives you exactly one solution. The integration method requires that you find the value at which either the energy changes or the time at which you reach your analytically solved for r_esc. Disregarding the outliers of data in the provided plot, and assuming they are smooth curves, we can still see the growing gap between the analytical solution and the numerical integration soltuion. The main limitation, I believe of the analytical method is that it does not incorporate gravity into the equation at all, and only relies on the input thrust as it's only force. I think this limitation is what causes the analytical method to undershoot, as it requires less time to escape if there is no modeled force of gravity "hold you back" (resisting the spacecrafts power to escape orbit)





Gives :

Using the approximation:

· first, find DV to circularize

M mors = 0. 0 42828.106

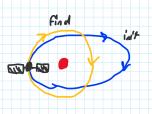
since we are going from eclipse to circular,

$$\left(\frac{DV}{x_{5p}},\right) = \frac{0.7725}{(250)(0.00981)}$$

$$\frac{m_i}{m_g} = 1.3702$$

$$\begin{array}{c}
\Delta n = n; \left(1 - e^{\left(\frac{-\Delta V}{2s_{1}}\right)}\right) \\
\Delta n = 1500 \left(1 - e^{\left(\frac{-(0.7725)}{(250)(0.00921)}\right)}\right)
\end{array}$$

So, 405. 2781 kg of propellent mass
was used suring the circularization maneure!



give :

Structural ratio
$$eq = \frac{M_E}{m. - n_{pl}} = 0.15$$

M_p = 0 after managerer

: first, we can find
$$n (mass (adlo))$$
:
$$n = \frac{m_1}{m_e} = \frac{1500}{1094.2} = 1.3072 (sum)$$

$$N = \underbrace{\frac{1 + \lambda}{\epsilon + \lambda}}, \quad N(\epsilon + \lambda) = \underbrace{1 + \lambda}$$

$$\lambda = \underbrace{\frac{n \epsilon - 1}{1 - m}}, \quad n \in + n\lambda = \underbrace{1 + \lambda}, \quad n \in + n\lambda = \underbrace{$$

· paylead mass :

$$\lambda m_{i} = m_{pl} = m_{pl}$$

$$\lambda m_{i} = m_{pl} (1+\lambda)$$

$$\frac{\lambda m_{i}}{1+\lambda} = m_{pl}$$

$$\frac{2.460(1500)}{1+2.1610} = 1023.2 \text{ hg}$$

for Q3:

I could not get

rry pock chop plot

to match with the

provided outcome. It

believe it is Jue to

my TOF motrix,

as the lambert solver

is the same as given

in the lecture.

Howaver, my solver down

get stock at a

few inputs, so I'm

not sure where three

real issue lies :(

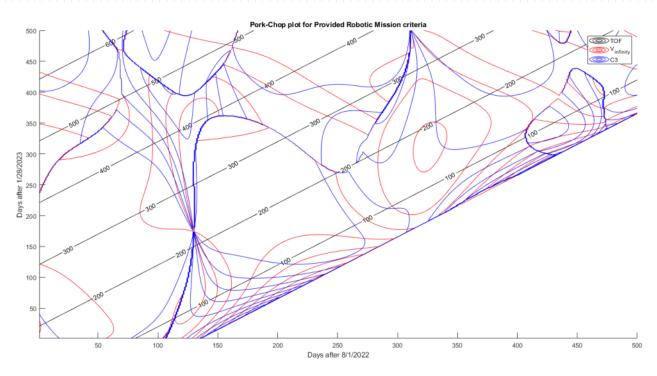


Table of Contents

```
%----- HW 7 MATLAB code -----%
% Romeo Perlstein, section 0101
% Chat it's over... I'm cooked!!! %
% Hey Eric (I think), or I mean Dr. Frizzell! Hope things have been good,
% PLEASE wish me luck in this class as I am STRUGGLING (Idk why it's been
% so hard to do school work I think I'm just cringe. Also I think Prof.
% Barbee doesn't like me heheh. OH WELL I guess the semester is over
close all
mew_earth = 0.39860*10^6; % km^3/s^2
% Equitorial, spherical orbit - velocity is constant
spacecraft_position_initial_1 = [8000;0;0]; % km
а
spacecraft_velocity_initial_1 = [0;sqrt(mew_earth/
spacecraft_position_initial_1(1));0];
accel_thrust1 = 1*10^-4; % kN/kg
b
see attached PDF
C
t01 = 0;
term1_1 = (norm(spacecraft_velocity_initial_1)/accel_thrust1);
term2 1 = (20*((accel thrust1)^2)*(norm(spacecraft position initial 1)^2))/
(norm(spacecraft_velocity_initial_1)^4);
tesc1 = term1_1*( 1-term2_1^(1/8) );
```

```
fprintf("Time to escape, in seconds (analytical):\n");
tesc1
fprintf("Time to escape, in minutes (analytical):\n");
tesc1/60
d
make a da graph-a
%--- ODE func values from HW1---%
tall er ant = (10^{-13}); % Tolerance
step_size = 1; % step size
max time = 2*tesc1; % max time (0->max time)
t = [0:step_size:max_time]; % timestep
% ODE options
ODE_options = odeset("RelTol", tall_er_ant, "AbsTol", tall_er_ant);
initial_state = [spacecraft_position_initial_1;spacecraft_velocity_initial_1];
[T1, Y1] = ode45(@myodefun, t, initial_state, ODE_options, mew_earth,
 accel_thrust1);
hold on
plot(0,0, ".b", "MarkerSize", 50, "DisplayName", "Earth")
plot(Y1(:,1), Y1(:,2), "-r", "DisplayName", "s/c orbit")
plot(spacecraft_position_initial_1(1),
 spacecraft position initial 1(2), ".k", "MarkerSize",
 10, "DisplayName", "spacecraft")
title("Graph of integrated orbit from the given initial conditions (Q1)")
xlabel("X position");
ylabel("Y position")
legend
grid on
axis equal
fprintf("The final velocity of the propegated orbit is:\n")
len1 = length(Y1(:, 4:6));
Y1(len1,4:6)
e
find r_esc, then find time using prop'd data
term1 1 1 =
norm(spacecraft_position_initial_1)*norm(spacecraft_velocity_initial_1);
term2_1_1 = 20*(accel_thrust1^2)*(norm(spacecraft_position_initial_1)^2);
resc1 = term1_1_1/(term2_1_1^(1/4));
fprintf("Radius of escape:\n")
resc1
tall_er_ant2 = 1;
```

for i=1:1:max_time+1

```
spef_energy(i) = (norm(Y1(i,4:6))^2)/2 - mew_earth/norm(Y1(i,1:3));
    if(spef energy(i) > 0-0.0001 \&\& spef energy(i) < 0+0.0001)
        time_of_energy_switch = i;
    end
    if ((norm(Y1(i,1:3)) < resc1+1) && (norm(Y1(i,1:3)) > resc1-1))
        time at esc radius = i;
    end
end
figure
hold on
plot(T1, spef_energy)
yline(0, "-k")
grid on
title("Plot of specific energy vs Time")
xlabel("Time (seconds)")
ylabel("Specific Energy")
fprintf("Time to reach calculated escape radius (seconds):\n")
time at esc radius
fprintf("Time of escape calculated (seconds):\n")
tesc1
fprintf("Difference in times (seconds):\n")
time at esc radius - tesc1
fprintf("Time when energy flips from negative to positive (time when it is 0)
 (seconds):\n")
time_of_energy_switch
% From inspection of the integrated data, r esc is reached at 36413
% seconds, different from our analytical answer of 34047 seconds, differing
% by roughly 2366 seconds (almost an hour!). However, inspection of the
% change in specific energy's sign value (i.e., when specific energy's
% value goes from negative to positive, or when it cross the x-axis) occurs
% at roughly 50130 seconds, which is about 20,000 seconds more than our
% calculated escape time!
% Both numerically found escape times are greater than the analytical
% answer. I think the analytical answer is conservative with it's output
% because it really doesn't take into account the force of gravity on the
% spacecraft the same way that the numerical integrated does. Since it's
% only going off of the kinematics of the problem (other than the
% acceleration due to thrust being a non-kinematic term since it is a
% force), it does not account for the extra time it might take for the
% spacecraft to break from the planets gravity well, giving us a
% conservative value.
% TL:DR - The exclusion of the gravity term from the t esc equation leads
% to a faster escape time being found. Since the t_esc equation is only
% influenced by a single force (and not the accounting for the
% gravitiational force being applied to the craft), it will find a faster
% escape time.
```

Do everything like 10 times.... hurray... FOR LOOP TIME BAYBE

```
accel_thrusts1 =
 [0.00015; 0.00025; 0.00035; 0.00045; 0.00055; 0.00065; 0.00075; 0.00085; 0.00095; 0.001];
loading time = "Loading: [";
for i=1:1:10
    accel_thrust = accel_thrusts1(i);
    term1_1 = (norm(spacecraft_velocity_initial_1)/accel_thrust);
    term2_1 = (20*((accel_thrust)^2)*(norm(spacecraft_position_initial_1)^2))/
(norm(spacecraft velocity initial 1)^4);
    tesc1_multi(i) = term1_1*( 1-term2_1^(1/8) );
    max_time = 10*tesc1; % max time (0->max_time)
    t = [0:step_size:max_time]; % timestep
    [T1, Y1] = ode45(@myodefun, t, initial_state, ODE_options, mew_earth,
 accel thrust);
    term1 1 1 =
 norm(spacecraft_position_initial_1)*norm(spacecraft_velocity_initial_1);
    term2 1 1 = 20*(accel thrust^2)*(norm(spacecraft position initial 1)^2);
    resc1 = term1_1_1/(term2_1_1^(1/4));
    for ii=1:1:max time+1
        spef_energy_multi(ii) = (norm(Y1(ii,4:6))^2)/2 - mew_earth/
norm(Y1(ii,1:3));
        if(spef_energy_multi(ii) > 0-0.001 && spef_energy_multi(ii) < 0+0.001)</pre>
            time_of_energy_switch_multi(i) = ii;
        end
        if ((norm(Y1(ii,1:3)) < resc1+1) && (norm(Y1(ii,1:3)) > resc1-1))
            time_at_esc_radius_multi(i) = ii;
        end
    end
    loading_time = loading_time + "=";
    fprintf(loading_time + "]\n")
end
if(length(time_at_esc_radius_multi) ~= 10)
    fprintf("You effed up!")
if(length(time_of_energy_switch_multi) ~= 10)
    fprintf("You effed up boy!")
end
figure
hold on
title("Plot of escape times using Log scale")
plot([1:1:10], tesc1_multi, "-r", DisplayName="Analytical t_e_s_c")
plot([1:1:10], time at esc radius multi, "-b", DisplayName="Numerical time
Value for solved r_e_s_c")
plot([1:1:10], time of energy switch multi, "-m", DisplayName="Numerical Value
 for time at energy flip")
xlabel("interation")
ylabel("Time (seconds)")
grid on
set(gca, "yscale", "log")
legend
```

g

From the plot, it is clear that the analytical method is lacking in it's accuracy. As the acceleration due to thrust is increased, we see a larger separation of numerical vs analytical values of the escape time. However, there are two key things to note: the analytical method produces a smooth, continuous curve of solutions, because it gives you exactly one solution. The integration method requires that you find the value at which either the energy changes or the time at which you reach your analytically solved for r_esc. Disregarding the outliers of data in the provided plot, and assuming they are smooth curves, we can still see the growing gap between the analytical solution and the numerical integration solution. The main limitation, I believe of the analytical method is that it does not incorporate gravity into the equation at all, and only relies on the input thrust as it's only force. I think this limitation is what causes the analytical method to undershoot, as it requires less time to escape if there is no modeled force of gravity "hold you back" (resisting the spacecrafts power to escape orbit)

```
Time to escape, in seconds (analytical):
tesc1 =
   3.4047e+04
Time to escape, in minutes (analytical):
ans =
  567.4548
The final velocity of the propegated orbit is:
ans =
    0.5396
             -4.6487
                              0
Radius of escape:
resc1 =
   2.9855e+04
Time to reach calculated escape radius (seconds):
time_at_esc_radius =
       36413
Time of escape calculated (seconds):
tesc1 =
   3.4047e+04
Difference in times (seconds):
ans =
```

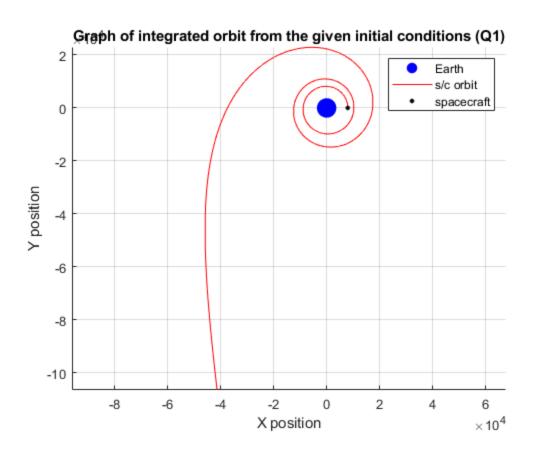
2.3657e+03

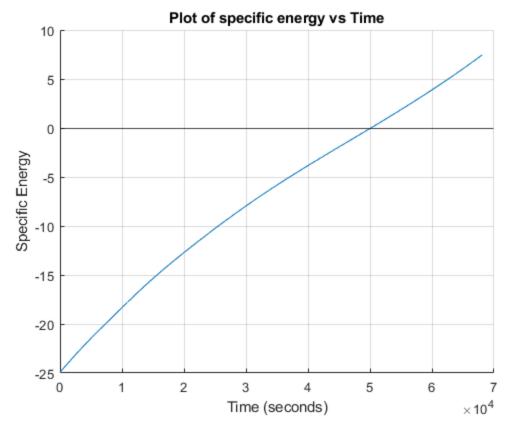
Time when energy flips from negative to positive (time when it is 0) (seconds):

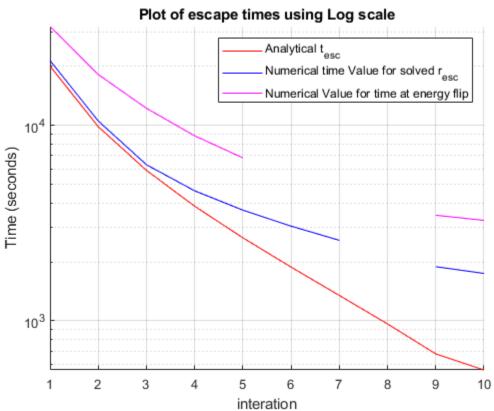
 $time_of_energy_switch =$

50130

Loading: [=]
Loading: [==]
Loading: [===]
Loading: [====]
Loading: [====]
Loading: [=====]
Loading: [=====]
Loading: [======]
Loading: [======]
Loading: [======]







Q2

```
Find stuff with things
```

```
mew_mars = 0.042828*10^6;
% Given
rp = 1000;
e = 0.25;
a = rp/(1-e);
ra = a*(1+e);
% Get the periapsis speed
v_peri = sqrt((2*mew_mars)/rp - (2*mew_mars)/(rp+ra));
v_circ = sqrt(mew_mars/rp);
deltaV = v_peri-v_circ;
% now solve for propellent change
Isp = 250;
g = 9.81*(10^{-3});
mi = 1500;
mi_over_mf = exp(deltaV/(Isp*g));
mf = mi/mi_over_mf;
deltaM = mi*(1-exp(-deltaV/(Isp*g)));
mf-mi;
part 2
given:
struct_ratio = 0.15;
% Find n (mass ratio)
n = mi/mf;
% now, find lambda (payload ratio)
lambda = (n*struct_ratio-1)/(1-n);
```

O3

```
mew_sun = 132712*10^6; % km^3/s^2
% Create a 2D matrix containing all of our TOF values
starting_depart_date = ymdhms2jd(2022, 8, 1, 12, 0, 0);
starting_arrival_date = ymdhms2jd(2023, 1, 28, 12, 0, 0);
```

% For shits and giggles, get payload mass

mpl = (lambda*mi)/(1+lambda);
me = struct_ratio*(mi-mpl);

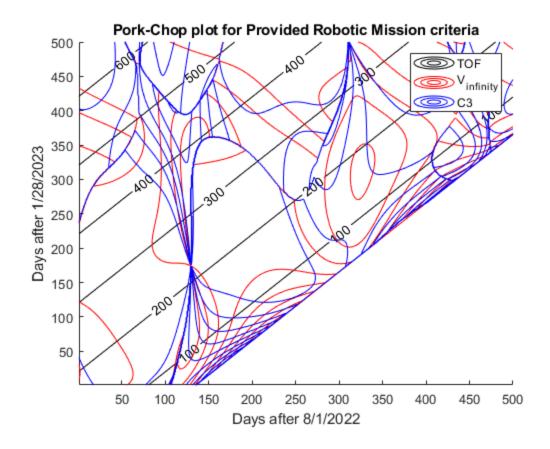
```
TOF_{matrix}(500,500) = 0;
TOF matrix check(500,500) = 0;
Julian matrix(500,500) = 0;
for i=0:1:499
    for ii=0:1:499
        TOF_matrix(ii+1,i+1) = ((starting_arrival_date+ii) -
 (starting_depart_date+i))*24*60*60;
        TOF matrix check(ii+1,i+1) = (starting arrival date+ii) -
 (starting depart date+i);
        if(TOF_matrix_check(ii+1,i+1) <= 0)</pre>
            TOF_{matrix}(ii+1,i+1) = 0;
            TOF_{matrix\_check(ii+1,i+1)} = 0;
        end
    end
end
% LAUNCH IS X AXIS, ARRIVAL IS Y
V_{infinity_matrix(500,500)} = 0;
C3 martix(500,500) = 0;
for i=0:1:499
    for ii=0:1:499
        % Find the position vectors of earth and mars
        [r1_vec, vel_earth] = findEarth(starting_depart_date+i);
        [r2 vec, vel mars] = findMars(starting arrival date+ii);
        TOF = TOF_matrix(ii+1, i+1);
        % If our number is less than 45 degrees, we're close to an
        % impossible TOF (and out of bounds of the problem statement
        % anyway), so skip it and set our velocities to 0
        if(TOF_matrix_check(ii+1,i+1) <= 45)</pre>
읒
              fprintf("skipping\n")
            V_{infinity_matrix(ii+1,i+1)} = 0;
            C3_{martix(ii+1,i+1)} = 0;
          elseif(TOF_matrix_check(i+1,ii+1) == 0)
읒
              fprintf("skipping2\n")
응
              V infinity matrix(ii+1,i+1) = 0;
응
              C3 martix(ii+1,i+1) = 0;
        else % If we're all good in the hood, actually find everything
            % Fist case, short way
            [v1_vec, v2_vec, stuck1] = romeosEpicLambartSolvor(r1_vec, r2_vec,
 TOF, "short", mew_sun);
            V infinity earth1 = v1 vec - vel earth;
            V_infinity_mars1 = v2_vec - vel_mars;
            % Get the magnitude:
            val1 = norm(V_infinity_earth1) + norm(V_infinity_mars1);
            % Second case, long way
            [v1_vec, v2_vec, stuck2] = romeosEpicLambartSolvor(r1_vec, r2_vec,
 TOF, "long", mew_sun);
            V_infinity_earth2 = v1_vec - vel_earth;
            V_infinity_mars2 = v2_vec - vel_mars;
            % Get the magnitude:
            val2 = norm(V_infinity_earth2) + norm(V_infinity_mars2);
            if(stuck1 == true && stuck2 == true)
```

```
응
                  fprintf("skipping1\n")
                V infinity matrix(ii+1,i+1) = 0;
                C3 martix(ii+1,i+1) = 0;
            end
            % now find which mag sum is smaller, and keep it:
            if (val1 < val2)
                C3 REAL = (norm(V infinity earth1))^2;
                V_infinity_REAL = norm(V_infinity_mars1);
            elseif (val2 < val1)</pre>
                C3_REAL = (norm(V_infinity_earth2))^2;
                V_infinity_REAL = norm(V_infinity_mars2);
            elseif (val1 == val2)
                C3_REAL = (norm(V_infinity_earth1))^2;
                V infinity REAL = norm(V infinity mars1);
            end
            % Save the value into our matricies for plotting purposes!
            V_infinity_matrix(ii+1,i+1) = V_infinity_REAL;
            C3_{martix(ii+1,i+1)} = C3_{REAL};
            % If we got stuck, just skip and set the value to 0, it's
            % probably not meant to be (I have no idea why it's getting
            % stuck, end me please!
            if(stuck1 == true && stuck2 == true)
응
                  fprintf("skipping1\n")
                V infinity matrix(ii+1,i+1) = 0;
                C3_{martix(ii+1,i+1)} = 0;
            end
        end
          fprintf("Loading, please wait...\nIterations complete: " +
 int2str(i) +", " + int2str(ii) + "\n")
    end
end
figure
hold on
% contour(TOF matrix, "-k", DisplayName="TOF");
[C, h] = contour(TOF_matrix_check, "-k", DisplayName="TOF");
clabel(C, h)
contour(V_infinity_matrix, "-r", DisplayName="V_i_n_f_i_n_i_t_y")
contour(C3_martix, "-b", DisplayName="C3")
legend
title("Pork-Chop plot for Provided Robotic Mission criteria")
xlabel("Days after 8/1/2022")
ylabel("Days after 1/28/2023")
% Contours are not like the example, but after hours of debugging and I
% can't seem to find out why. OH WELL I GUESS... I FUCKING HATE LAMBERT
% AHHHHH I HATE LAMBERT (its probably my TOF and not lambert)
```

```
% 2-body prop solver, pulled from ENAE301! Ah good times... good times.
function ydot = myodefun(t, y, mew, thrust)
    r_mag = norm(y(1:3));
    ydot(1,1) = y(4);
    ydot(2,1) = y(5);
    ydot(3,1) = y(6);
    % Get the unit vector of velocity vector
    unit_vec = [y(4);y(5);y(6)]/(norm([y(4);y(5);y(6)]));
    % Get the thrust vector
    thrust_vec = unit_vec*thrust;
    % Add it to accel
    ydot(4,1) = (-mew/r_mag^3)*y(1) + thrust_vec(1);
    ydot(5,1) = (-mew/r_mag^3)*y(2) + thrust_vec(2);
    ydot(6,1) = (-mew/r_mag^3)*y(3) + thrust_vec(3);
end
Got stuck in a loop, returning 0's
```

```
Got stuck in a loop, returning 0's
```

```
Got stuck in a loop, returning 0's
```



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