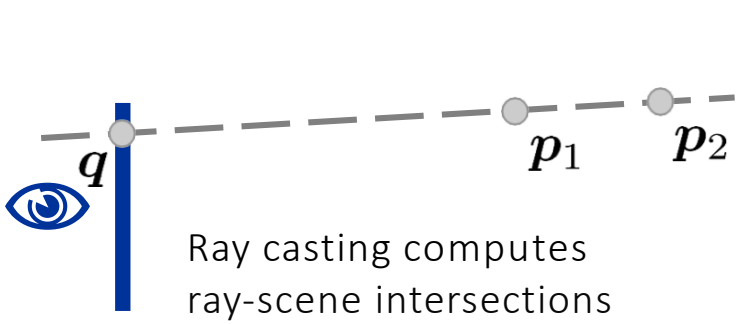
Lecture 19:

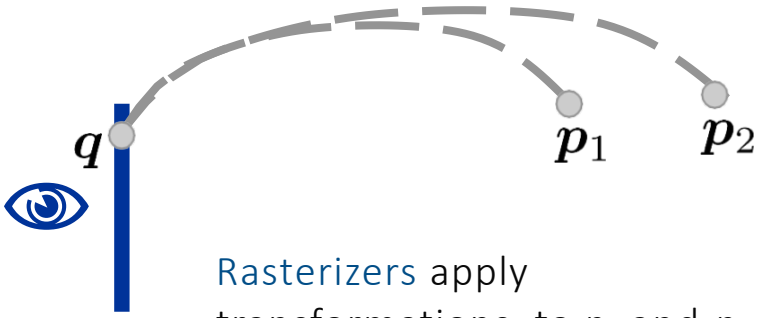
Rasterization

Contents

Visibility can be resolved by ray-tracing or by rasterization



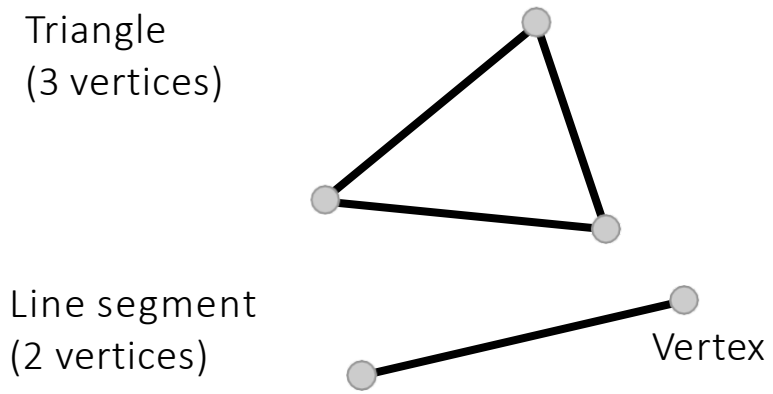
Ray casting computes ray-scene intersections to estimate q from p_1 and p_2.



Rasterizers apply transformations to p_1 and p_2 in order to estimate q.

- If more than one scene point p_i is mapped to the same sensor position q, the scene point closest to the viewer is selected

Computation of pixel positions in an image plane that represent a projected primitive



Primitives represented by vertices

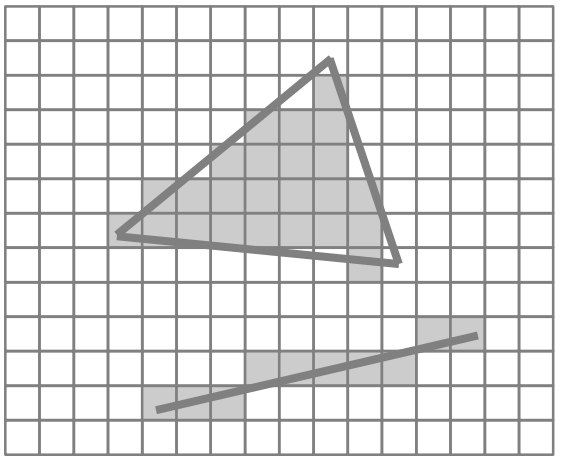
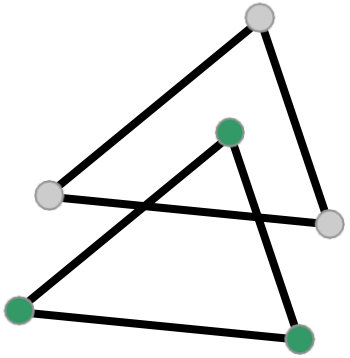
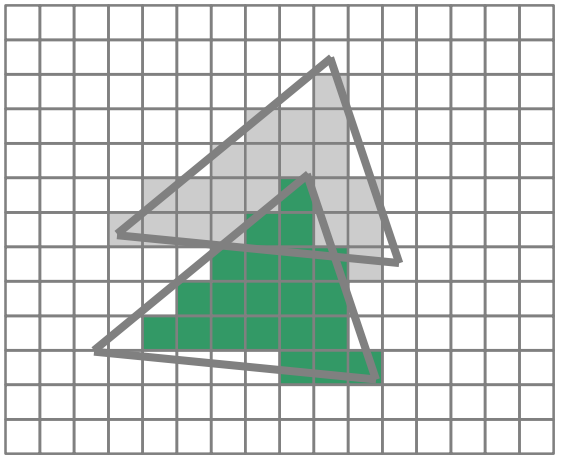


Image plane / 2D array of pixels

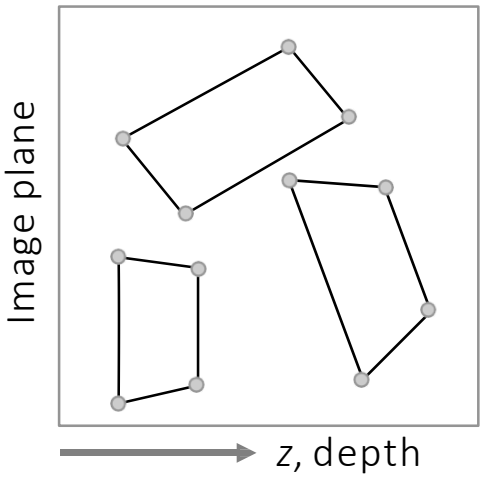
After rasterization, **visibility can be efficiently resolved per pixel position**

- Distances of primitives to the viewer, *i.e.* depth values, can be compared per pixel position

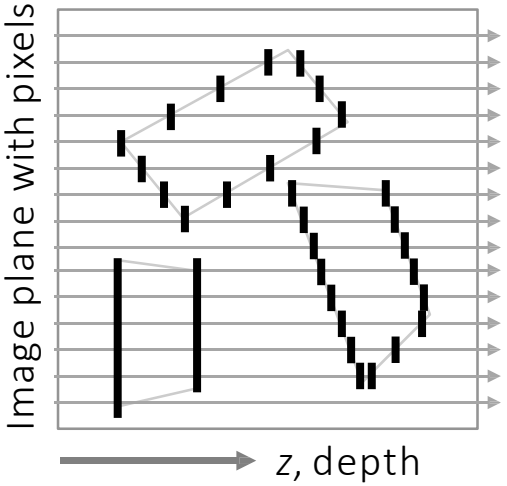




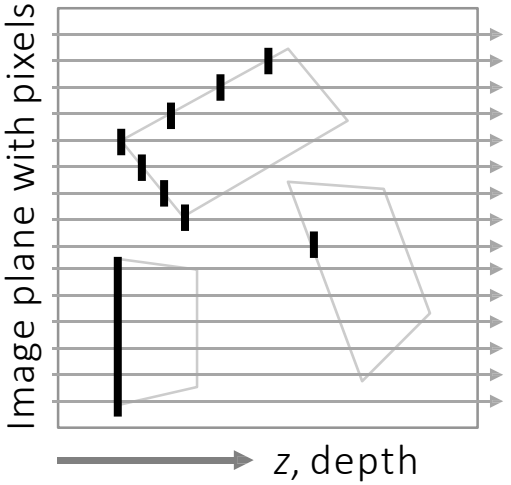
Rasterization is typically implemented for canonical view volumes



Side view of a scene in
a canonical view volume

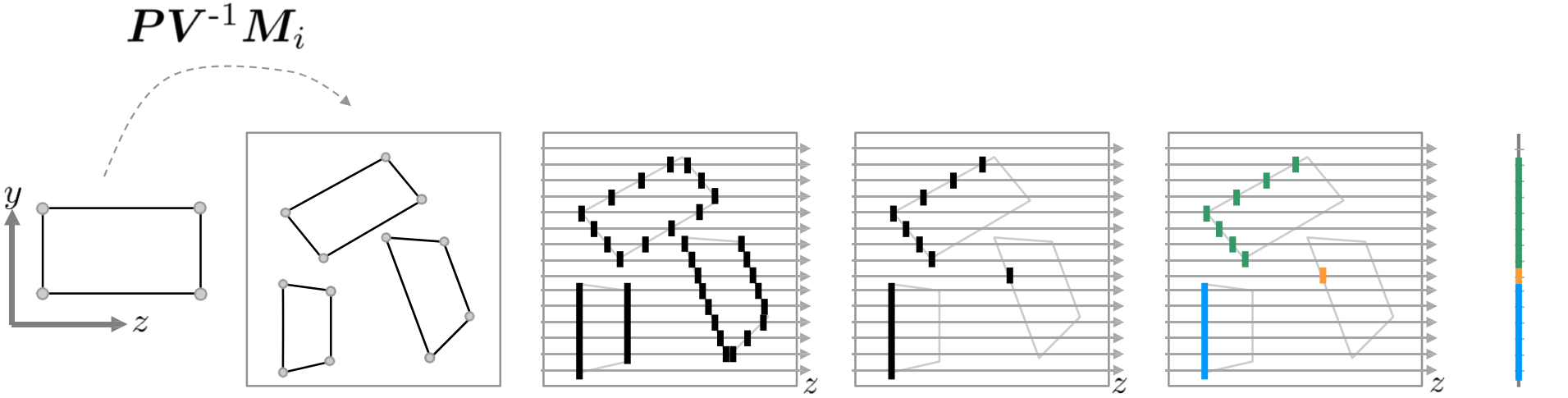


Rasterization result



Resolved visibility

Rasterization is typically implemented for canonical view volumes



Object in local space

Scene in the canonical view volume

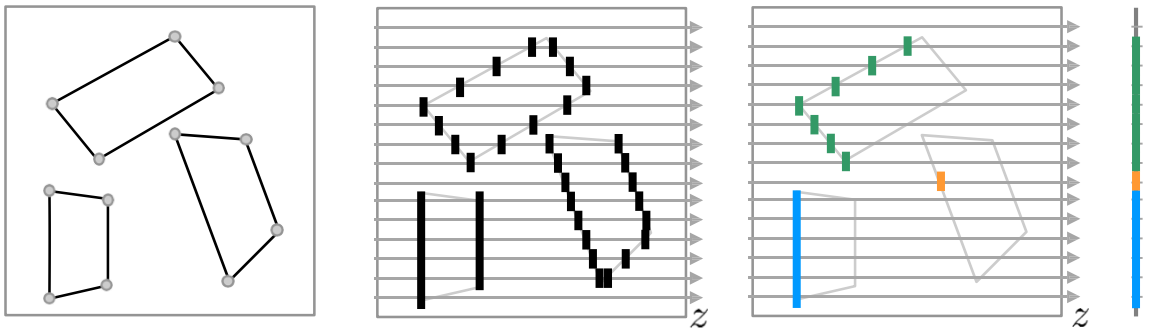
Rasterization

Visibility

Shading

Image

Terms – 2D Illustration



Vertex
Primitive

Object with four vertices and four primitives

Fragment

Pixel

Vertices: have positions and other attributes.

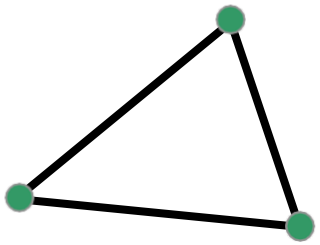
Primitives: are represented by vertices.

Fragments: are pixel candidates with pixel positions and other attributes.

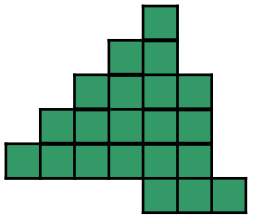
Pixels: have a position and other attributes, in particular color.

Framebuffer: consists of pixels.

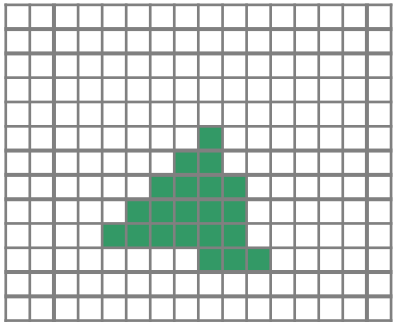
Terms - Illustration



Triangle 1 with three vertices

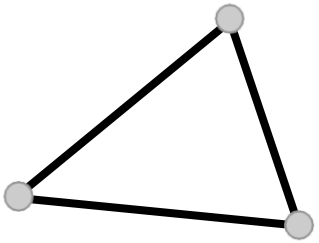


Rasterizer generates fragments.

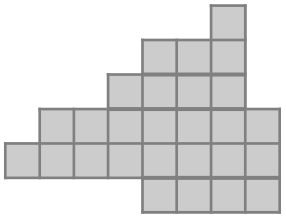


Pixels of the framebuffer

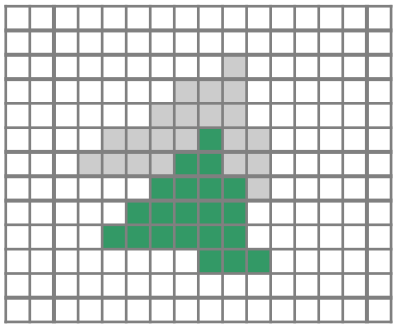
Fragment attributes are used to update pixel attributes in the framebuffer.



Triangle 2 with three vertices



Rasterizer generates fragments.



Pixels of the framebuffer

Framebuffer attributes can be updated. Fragments can be discarded.

Vertex processing

- **Input**: Vertices
- **Output**: Vertices
- Transformations
- Setting, computation, processing of vertex attributes, *e.g.* position, color (Phong), texture coordinates

Rasterization

- **Input**: Vertices and connectivity information
- **Output**: Fragments
- Primitive assembly
- Rasterization of primitives (generates fragments from vertices and connectivity information)
- Sets fragment attributes from vertex attributes, *e.g.* distance to viewer, color, texture coordinates

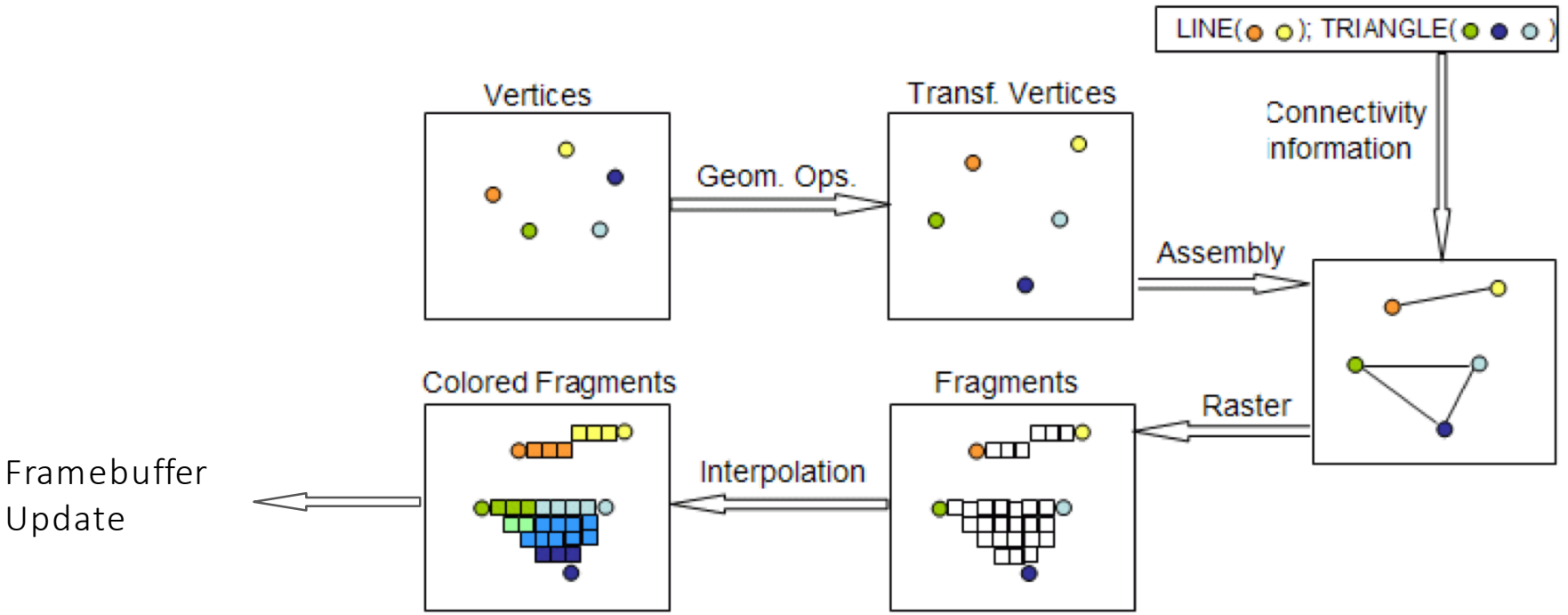
Fragment processing

- **Input**: Fragments
- **Output**: Fragments
- Fragment attributes are processed, *e.g.* color
- Fragments can be discarded

Framebuffer update

- **Input**: Fragments
- **Output**: Framebuffer attributes
- Fragment attributes update framebuffer attributes, *e.g.* color

Main Stages - Overview

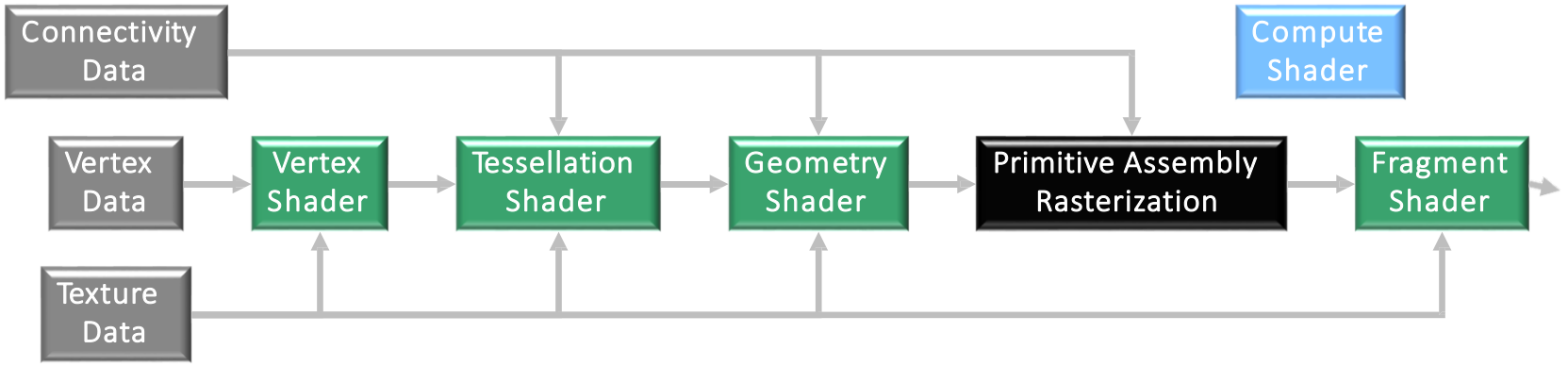


Concept motivated by computational efficiency

- Vertices and fragments are processed independently in the respective stages

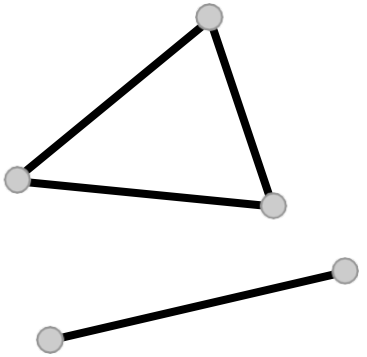
Stages are supported by graphics hardware GPU

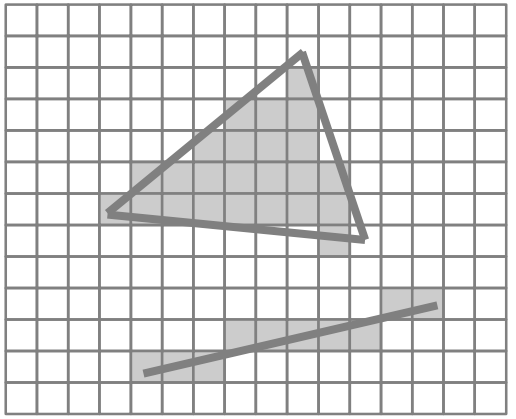
- OpenGL, DirectX, Vulkan are software interfaces to GPUs



Input

- Vertices with connectivity information and attributes:
- **Position**
  - Z-component in NDC space is referred to as depth value. Represents distance to the camera plane.
- **Color**
  - Can optionally be defined or computed with Phong, if surface normal, light and material properties are available
- **Texture coordinates**
  - For lookup and processing of additional data, *i.e.* textures

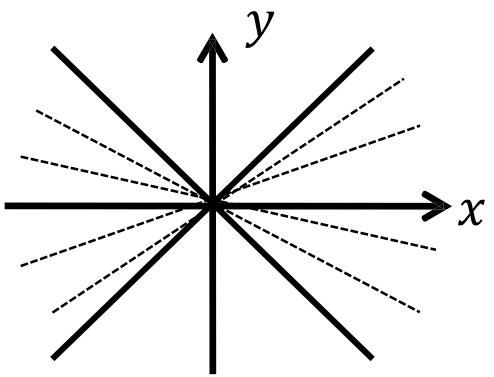




Output

- Fragments with attributes
  - Pixel position
  - Interpolated color, depth, texture coordinates

Assumptions



- Pixels are sample ***points*** on a 2D-integer-grid
  - **OpenGL**: cell bottom-left, integer-coordinate
  - **We**: at the cell center
- Simple raster operations
  - Just setting pixel values or not (binary decision)
    - More complex operations later: compositing / anti-aliasing
- Endpoints at (sub-) pixel coordinates
  - Simple and consistent computations with fixed-point arithmetic
- Limiting to lines with gradient / slope $|m| \leq 1$ (mostly horizontal)
  - Separate handling of horizontal and vertical lines
  - For mostly vertical, swap x and y ($|\frac{1}{m}| \leq 1$), rasterize, swap back
    - Special cases in SW, trivial in HW :-)
- Line width is one pixel
  - $|m| \leq 1$: 1 pixel per column (X-driving axis)
  - $|m| > 1$: 1 pixel per row (Y-driving axis)

Specification

- Initial and end points: $(x_o, y_o), (x_e, y_e)$
- The rational slope $m = {}^{(y_e - y_o)}/_{(x_e - x_o)} = {}^{dy}/_{dx}$
- Functional form: $y = mx + B$, where end points are integer coordinates

Goal

- Find those pixel per column whose distance to the line is smallest

Brute-force algorithm

- Assume that $+x$ is the driving axis → set pixel in every column

```
for (x_i = x_o; x_i < x_e; x_i++) {
    y_i = m * x_i + B
    setPixel(x_i, std::round(y_i))
}
```

Comments

- Variables m and thus y_i need to be calculated in floating-point
- Not well suited for direct HW implementation
  - A floating-point ALU is significantly larger in HW than integer

Digital Differential Analyzer (DDA)

- Origin of incremental solvers for simple differential equations (the Euler method)
- Per time-step: $x' = x + {}^{dx}/_{dt}$, $y' = y + {}^{dy}/_{dt}$

Incremental algorithm

- Choose $dt = dx$, then per pixel

$x_{i+1} = x_i + 1$

$y_{i+1} = m * (x_i + 1) + B = y_i + m$

$setPixel(x_{i+1}, std::round(y_{i+1}))$

Remark

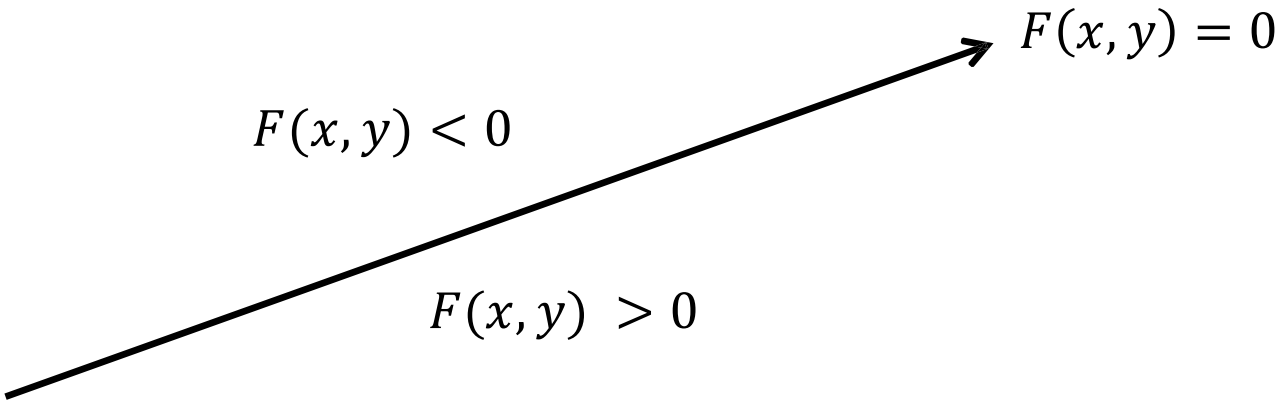
- Utilization of coherence through incremental calculation
  - Avoids the “costly” multiplication
- Accumulates error over length of the line
- Floating point calculations may be moved to fixed point
  - Must control accuracy of fixed point representation

DDA analysis

- Critical point: decision whether rounding up or down

Idea

- Integer-based decision through implicit functions
- Implicit line equation
  - $F(x,y) = ax + by + c = 0$
- Here with $y = mx + B = \frac{dy}{dx}x + B \quad \Rightarrow \quad 0 = xdy - ydx + Bdx$
  - $a = dy, b = -dx, c = Bdx$
- Results in
  - $F(x,y) = xdy - ydx + Bdx = 0$



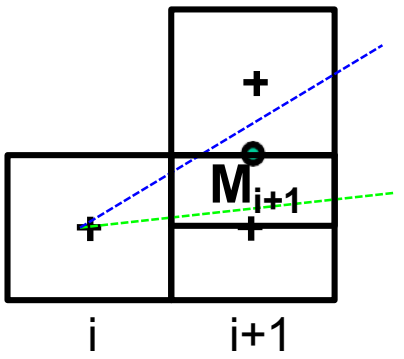
Decision variable d (the midpoint formulation)

- Assume we are at $x = i$, calculating next step at $x = i + 1$
- Measures the vertical distance of midpoint from line:

$$d_{i+1} = F(M_{i+1}) = F\left(x_i + 1, y_i + \frac{1}{2}\right) = a(x_i + 1) + b\left(y_i + \frac{1}{2}\right) + c$$

Preparations for the next pixel

```
if (d_{i+1} <= 0) {                    // Increment in x only
    d_{i+2} = d_{i+1}  + a = d_{i+1} + dy      // Incremental calculation
} else {                               // Increment in x and y
    d_{i+2} = d_{i+1} + a + b = d_{i+1} + dy – dx
    y = y + 1
}
x = x + 1
```



Initialization

- $d_1 = F\left(x_o + 1, y_o + \frac{1}{2}\right) = a(x_o + 1) + b\left(y_o + \frac{1}{2}\right) + c =$
$$= ax_o + by_o + c + a + \frac{b}{2} = F(x_o, y_o) + a + \frac{b}{2} = a + \frac{b}{2}$$
- Because $F(x_o, y_o)$ is zero by definition (line goes through (x_o, y_o))
  - Pixel is always set (but check consistency rules → later)

Elimination of fractions

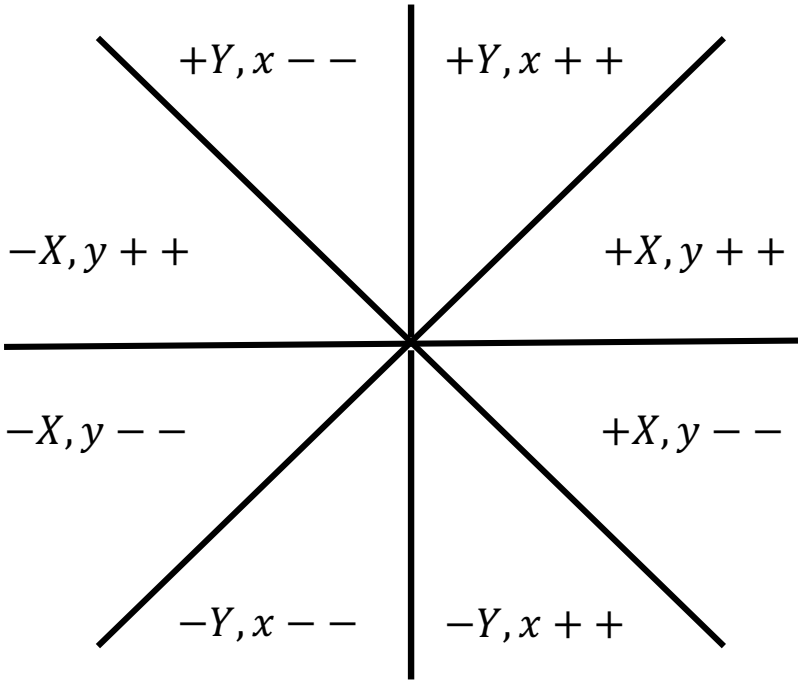
- Any positive scale factor maintains the sign of $F(x, y)$
  - $2F(x_o, y_o) = 2(\mathrm{a}x_o + by_o + c) \rightarrow d_{start} = 2a + b$

Observation:

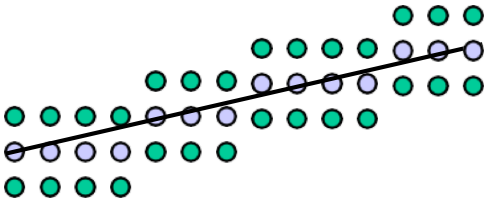
- When the start and end points have integer coordinates then $b = -dx$ and $a = dy$ are also integers
  - Floating point computation can be eliminated
- No accumulated error

8 different cases

- Driving (active) axis: ±X or ±Y
- Increment / decrement of y or x, respectively

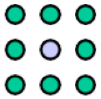


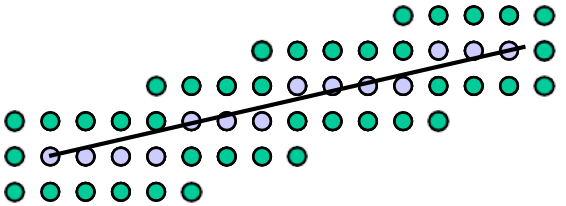
Pixel replication



- Problems with even-numbered widths
- Varying intensity of a line as a function of slope

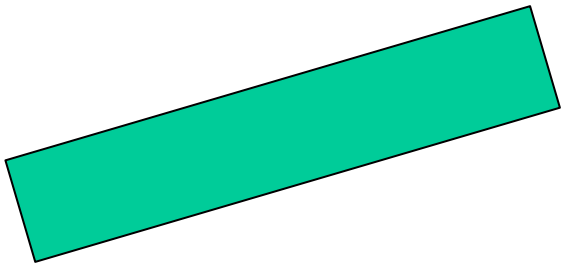
The moving pen





- For some pen footprints the thickness of a line might change as a function of its slope
- Should be as “round” as possible

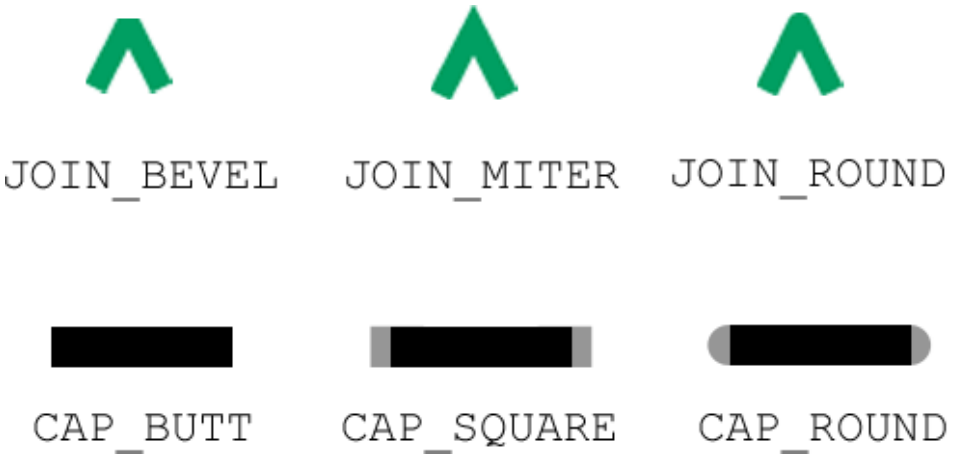
Real Solution: Draw 2D area



- Allows for anti-aliasing and fractional width
- Main approach these days!

End points handling (not available in current OpenGL)

- Joining: handling of joints between lines
  - **Bevel:** connect outer edges by straight line
  - **Miter**: join by extending outer edges to intersection
  - **Round**: join with radius of half the line width

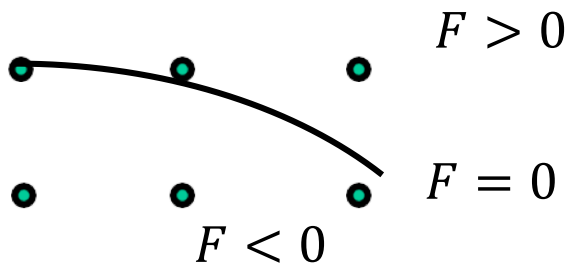


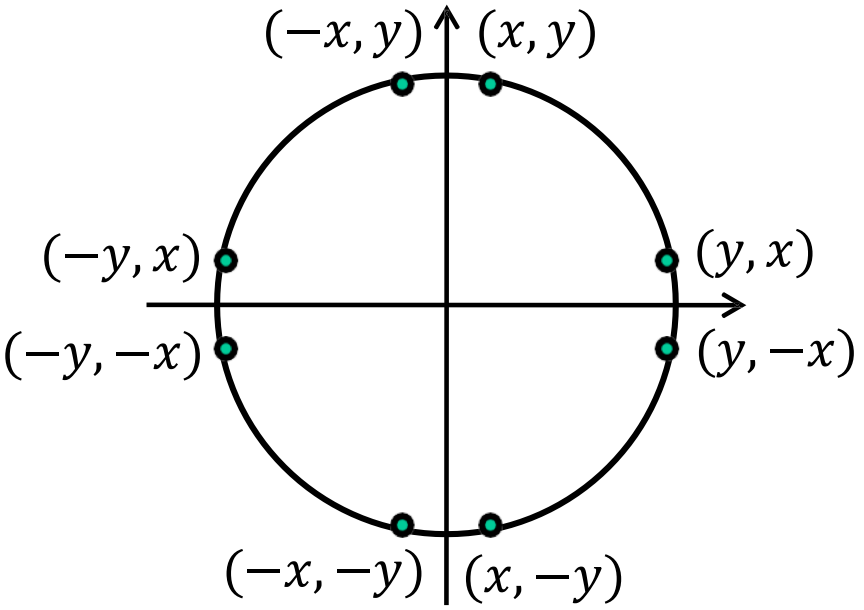
- Capping: handling of end point
  - **Butt**: end line orthogonally at end point
  - **Square**: end line with oriented square
  - **Round**: end line with radius of half the line width

Eight different cases, here $+X, y --$

```
Initialization: x = 0, y = R
F(x,y) = x² + y² - R²
d = F(x+1, y-1/2)
if d < 0
    d = F(x+2, y-1/2)
else if d > 0
    d = F(x+2, y-3/2)
    y = y-1
x = x+1
```

- Works because slope is smaller than 1

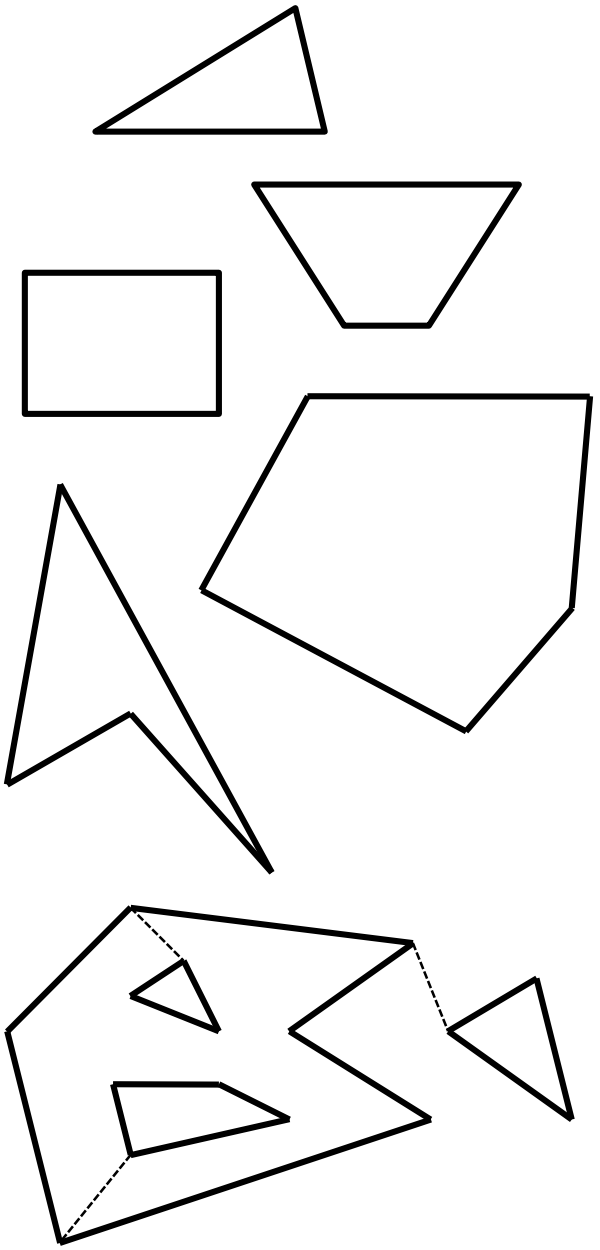




Eight-way symmetry: only one 45° segment is needed to determine all pixels in a full circle

Types

- Triangles
- Trapezoids
- Rectangles
- Convex polygons
- Concave polygons
- Arbitrary polygons
  - Holes
  - Non-coherent

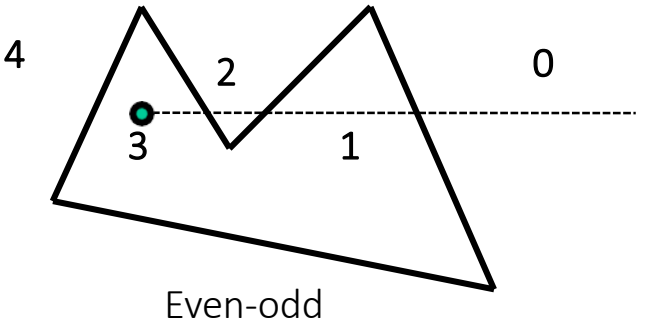


Two approaches

- Polygon tessellation into triangles
  - Only option for OpenGL
  - Needs edge-flags for not drawing internal edges
  - Or separate drawing of the edge
- Direct scan-conversion
  - Mostly in early SW algorithms

What is the interior of a polygon?

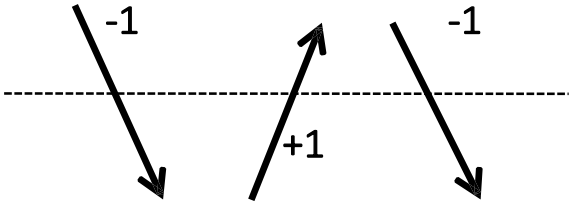
- Jordan Curve Theorem
  - „Any continuous ***simple*** closed curve in the plane, separates the plane into two disjoint regions, the inside and the outside, one of which is bounded.“



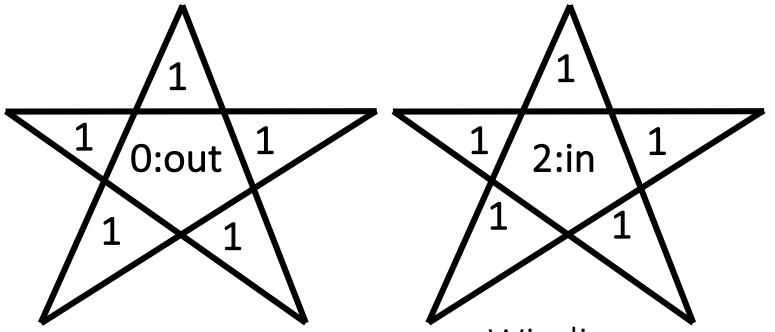
Even-odd

What to do with *non-simple* polygons?

- Even-odd rule (odd parity rule)
  - Counting the number of edge crossings with a ray starting at the queried point **P** till infinity
  - Inside, if the number of crossings is odd
- Non-zero winding number rule
  - Relies on knowing the direction of stroke for each part of the curve
  - Counts # times polygon wraps around **P**
    - Signed intersections with a ray
  - Inside, if the number is not equal to zero
- Differences only in the case of non-simple curves (*e.g.* self-intersection)

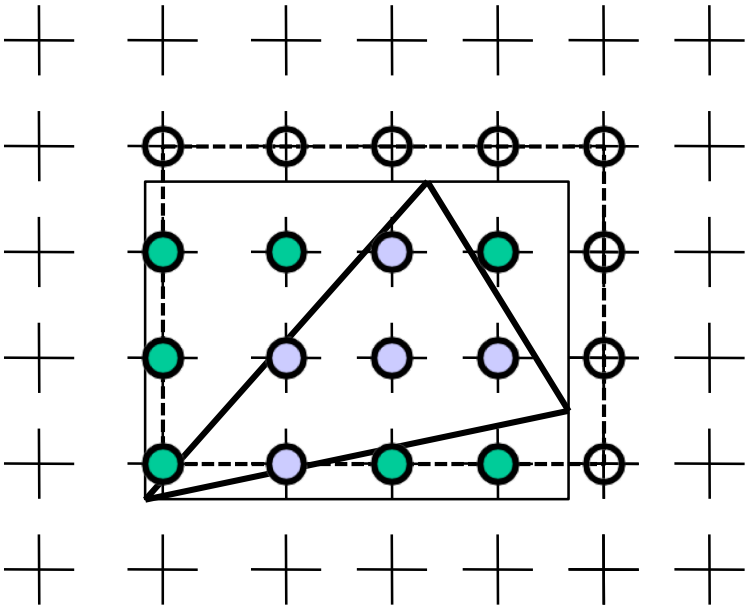


Winding



Even-odd Winding

```
Raster3_box(vertex v[3])
{
    CBoundingBox b;
    bound3(v, &b);
    for (int y = b.ymin; y < b.ymax; y++)
        for (int x = b.xmin; x < b.xmax; x++)
            if (inside(v, x, y)) // upcoming
                fragment(x,y);
}
```



Brute-force algorithm

- Iterate over all pixels within bounding box

Possible approaches for dealing with scissoring

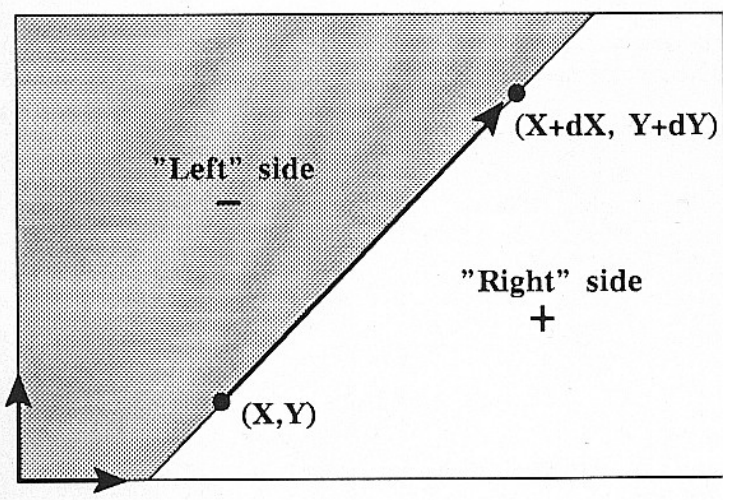
- Scissoring: Only draw on AA-Box of the screen (region of interest)
  - Test triangle for overlap with scissor box, otherwise discard
  - Use intersection of scissor and bounding box, otherwise as above

Approach (Pineda, `88)

- Implicit edge functions for every edge

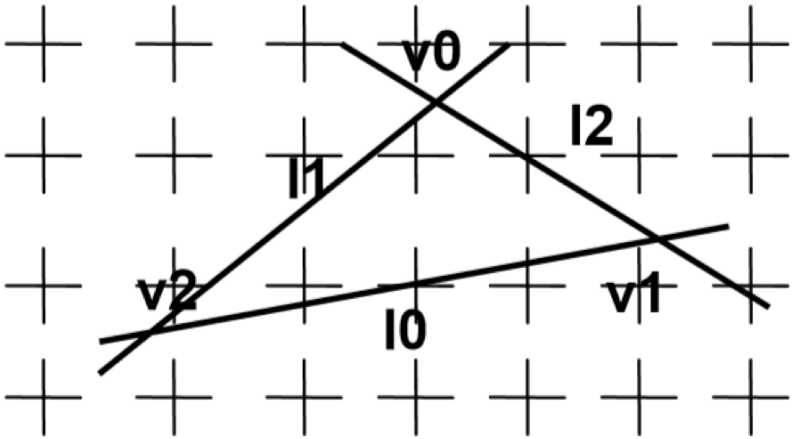
$$F_i(x, y) = ax + by + c$$

- Point is *inside* triangle, if every $F_i(x, y)$ has the same sign
- Perfect for parallel evaluation at many points
  - Particularly with wide SIMD machines (GPUs, SIMD CPU instructions)
- Requires “triangle setup”: Computation of edge function
- Evaluation can also be done in homogeneous coordinates



Hierarchical approach

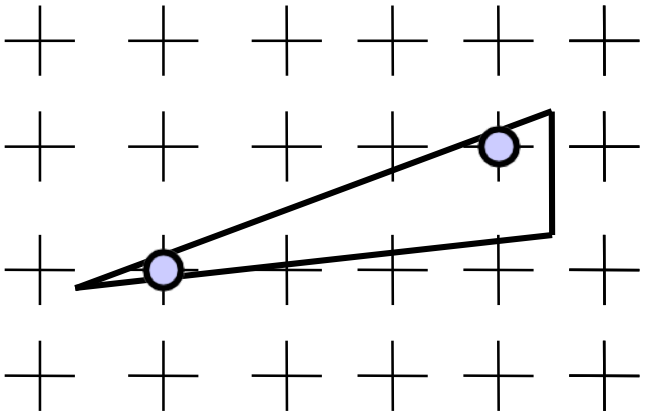
- Can be used to efficiently check large rectangular blocks of pixels
  - Divide screen into tiles / bins (possibly at several levels)
  - Evaluate F at tile corners
  - Recurse only where necessary, possibly until subpixel level



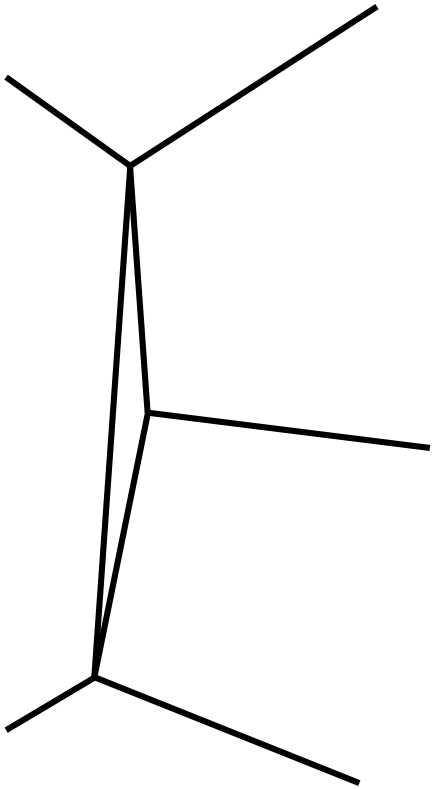

```
Raster3_incr(vertex v[3])
{
    edge l0, l1, l2;
    value d0, d1, d2;
    CBoundingBox b;
    bound3(v, &b);

    mkedge(v[0],v[1],&l2);
    mkedge(v[1],v[2],&l0);
    mkedge(v[2],v[0],&l1);

    d0 = l0.a * b.xmin + l0.b * b.ymin + l0.c;
    d1 = l1.a * b.xmin + l1.b * b.ymin + l1.c;
    d2 = l2.a * b.xmin + l2.b * b.ymin + l2.c;

    for (int y = b.ymin; y < b.ymax, y++) {
        for (int x = b.xmin; x < b.xmax, x++) {
            if (d0 <= 0 && d1 <= 0 && d2 <= 0)
                fragment(x,y);
            d0 += l0.a; d1 += l1.a; d2 += l2.a;
        }
        d0 += l0.a * (b.xmin - b.xmax) + l0.b;
        …
    }
```

Observations

- Pixels set can be non-connected
- May have overlap and gaps at T-edges



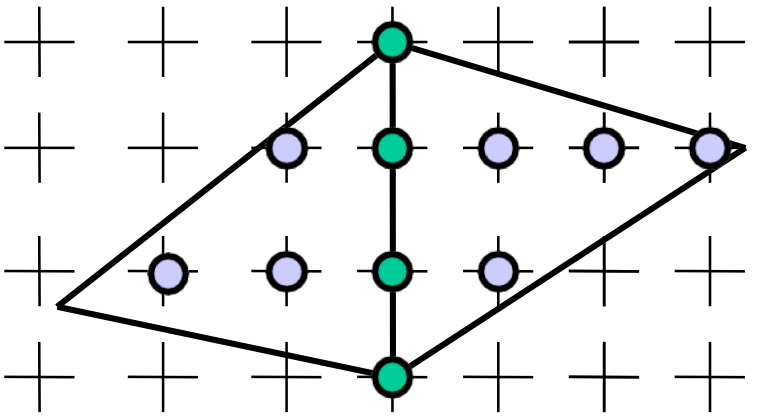
Non-connected pixels: OK



Not OK: Model must be changed

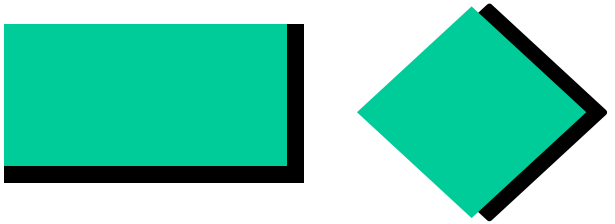
Consistency: edge singularity (shared by 2 triangles)

- What if term $d = ax + by + c = 0$ (pixel centers lies exactly on the line)
- For $d \leq 0$: pixels would get set twice
  - Problem with some algorithms
  - Transparency, XOR, CSG, ...
- Missing pixels for $d < 0$ (set by no triangle)



Solution: "shadow" test

- Pixels are not drawn on the right and bottom edges
- Pixels are drawn on the left and upper edges
  - Evaluated via derivatives a and b
- Test for all edges also solves problem at vertices



```
bool inside(value d, value a, value b)
{       // ax + by + c = 0
    return (d < 0) || (d == 0 && !shadow(a, b));
}
bool shadow(value a, value b)
{
    return (a > 0) || (a == 0 && b > 0);
}
```

In-Triangle test (for common origin)

- Rasterization:
  - Project to 2D, clip
  - Set up 2D edge functions, evaluate for each sample (using 2D point)
- Ray tracing:
  - Set up 3D edge functions, evaluate for each sample (using direction)
- The ray tracing test can also be used for rasterization in 3D
  - Avoids projection & clipping

Enumerating scene primitives

- Rasterization (simple):
  - Linearly test them all in random order
- Rasterization (advanced):
  - Build (coarse) spatial index (typically on application side)
  - Traverse with (large) view frustum
    - Every one separately when using tiled rendering
- Ray Tracing:
  - Build (detailed) spatial index
  - Traverse with (infinitely thin) ray or with some (small) frustum
- Both approaches can benefit greatly from spatial index

Binning

- Test to (hierarchically) find pixels likely to be covered by a primitive
- Rasterization:
  - Great speedup due to very large view frustum (many pixels)
- Ray tracing (frustum tracing)
  - Can speed up, depending on frustum size [Benthin'09]
- Ray Tracing (single / few rays)
  - Not needed

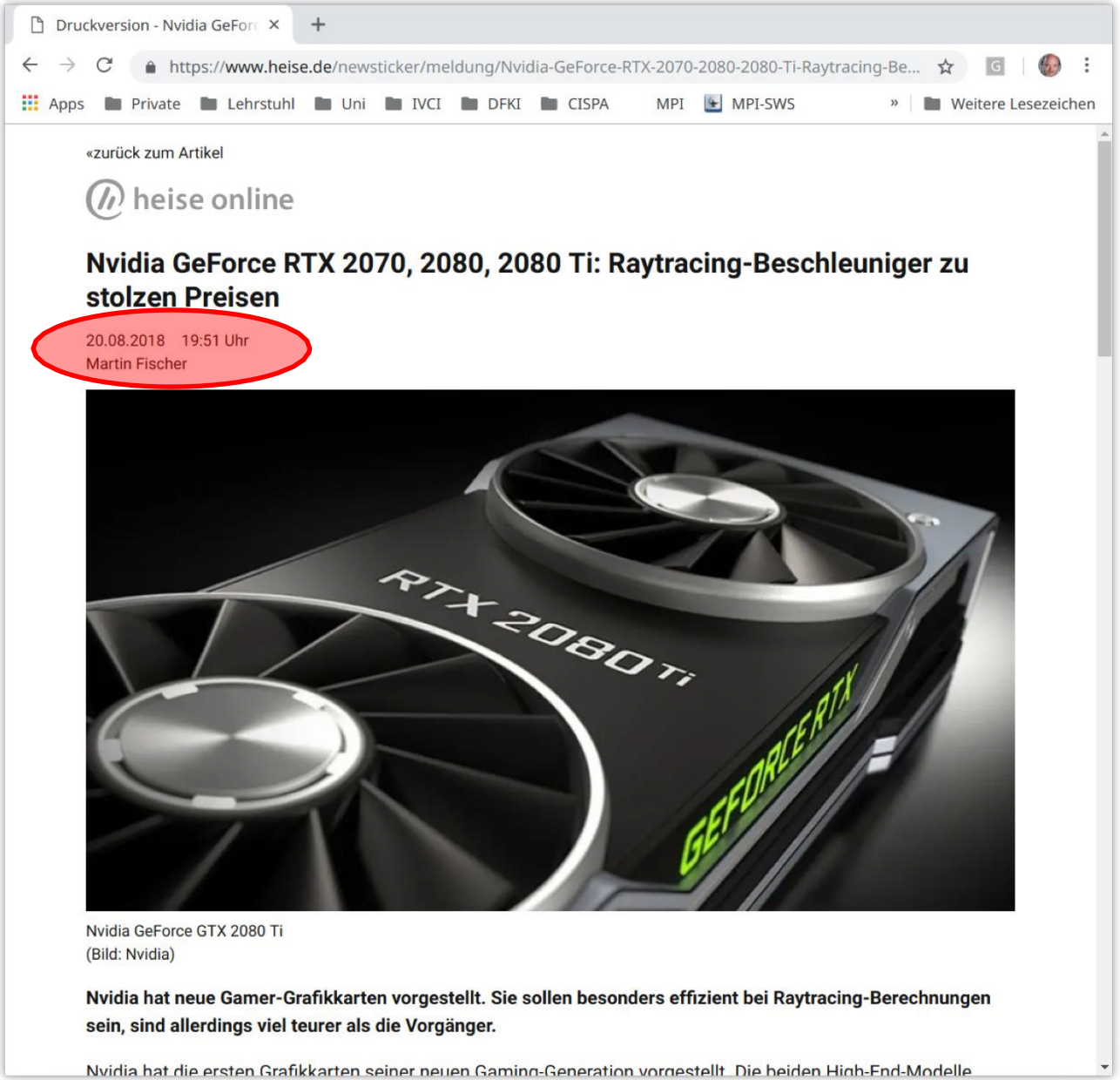
Conclusion

- Both algorithms can use the same in-triangle test
  - In 3D, requires floating point, but boils down to 2D computation
- Both algorithms can benefit from spatial index
  - Benefit depends on relative cost of in-triangle test (HW vs. SW)
- Both algorithms can benefit from 2D binning to find relevant samples
  - Benefit depends on ratio of covered/uncovered samples per frustum

Both approaches are essentially the same

- Different organization (size of frustum, binning)
- There is no reason RT needs to be slower for primary rays (exc. FP)

HW-Supported Ray Tracing



ACMSIGGRAPH | Blog

Ray Tracing Will be Everywhere in 2020

8
OCT 2019

by Jon Peddie | posted in: Augmented Reality, Gaming, Graphics, Hardware, Production, Real-Time | 1

Not just PCs, but consoles, and mobile devices will show the rays

Photo by Davide Cantelli on Unsplash

Ray tracing, the long-held goal for computer graphics, has been in the movies, CAD design, and visualization for decades. The physically accurate, beautiful images have historically made realistic scenes and impressions; however, those images came at a price,

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