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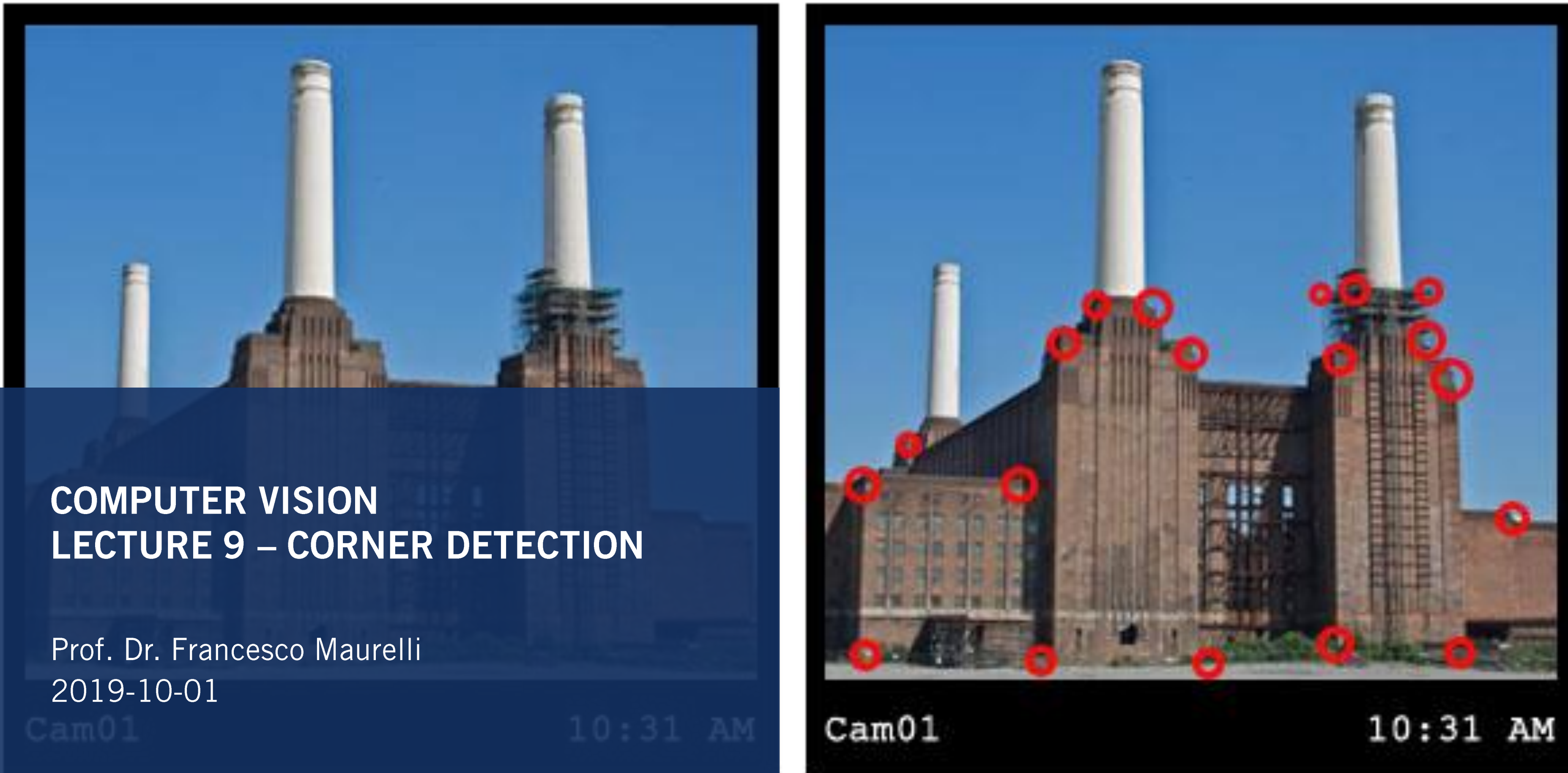

A: Original image

B: Detected image

COMPUTER VISION
LECTURE 9 – CORNER DETECTION

Prof. Dr. Francesco Maurelli
2019-10-01

WHAT WE WILL LEARN TODAY

- Intro to Features
- Keypoint localization
  - Harris corner detector

Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

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Problem 1:

- Detect the same point *independently* in both images

No chance to match!

We need a repeatable detector!

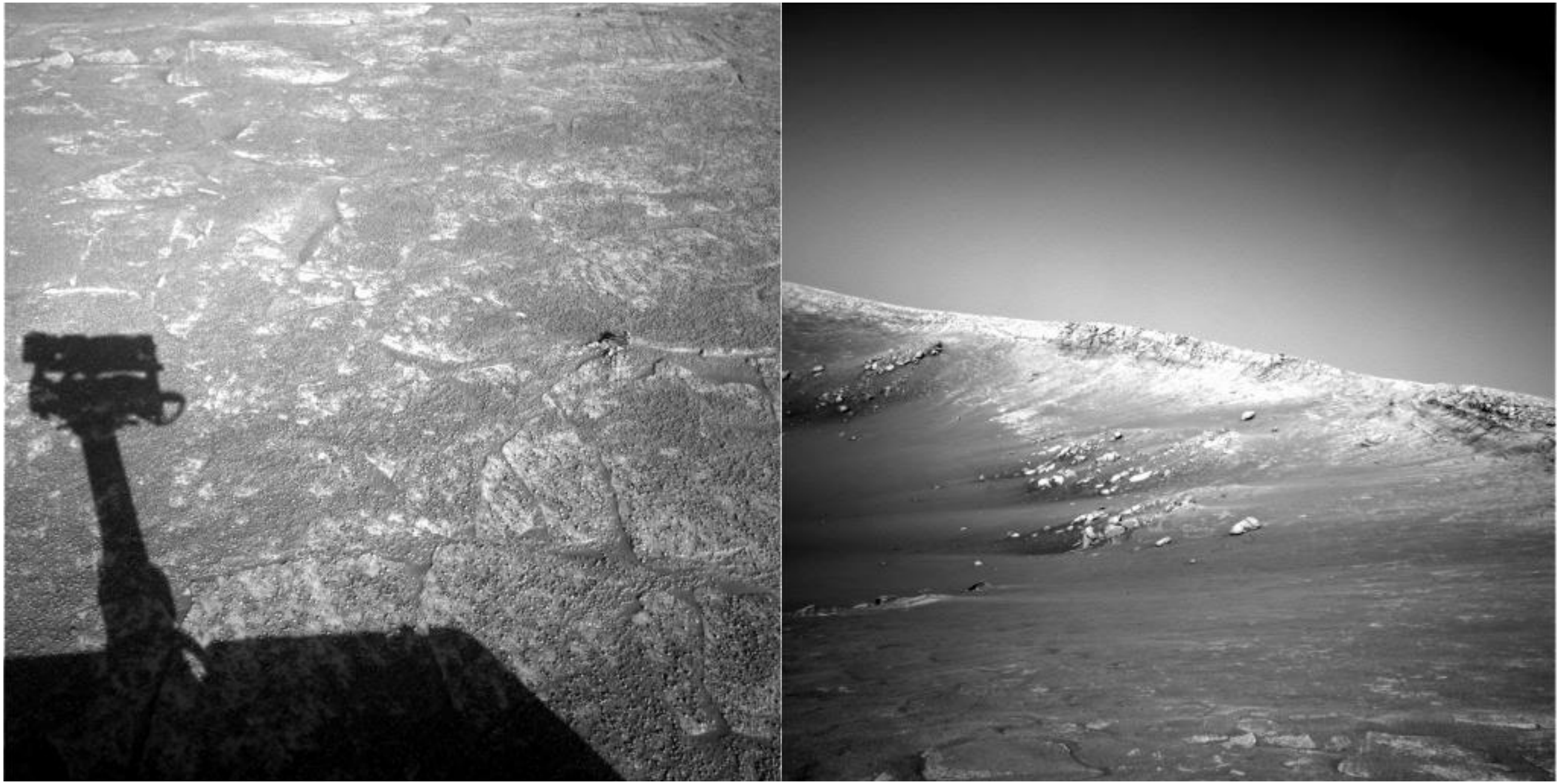
IMAGE MATCHING: A CHALLENGING PROBLEM

by Diva Sian

by swashford

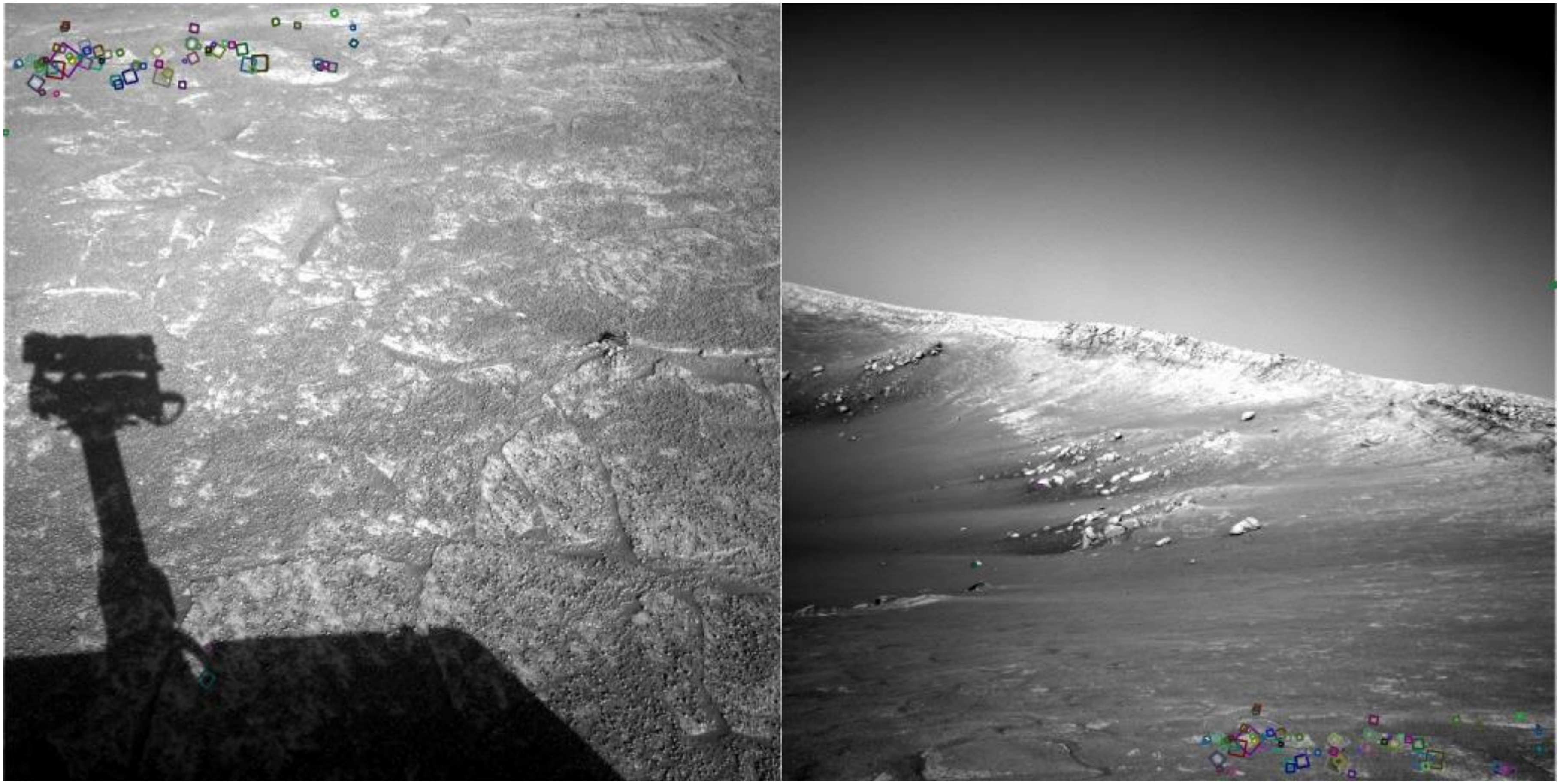
HARDER CASE

EVEN HARDER ...



NASA Mars Rover images

EVEN HARDER ...



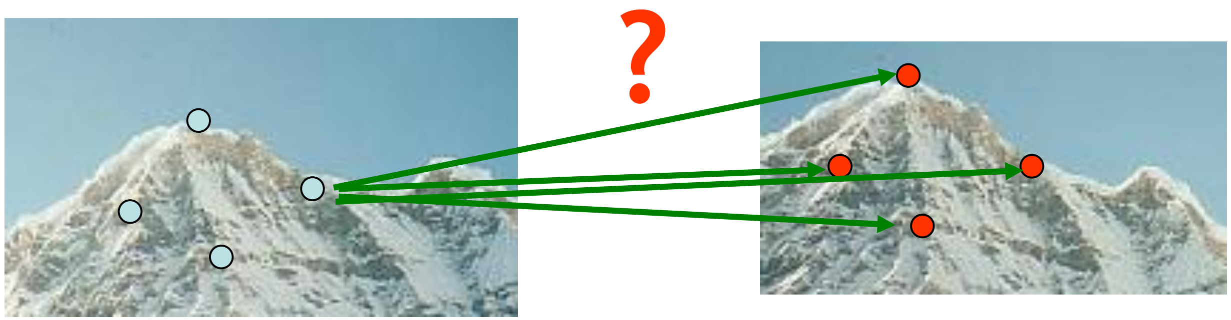
NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Problem 1:

- Detect the same point *independently* in both images

Problem 2:

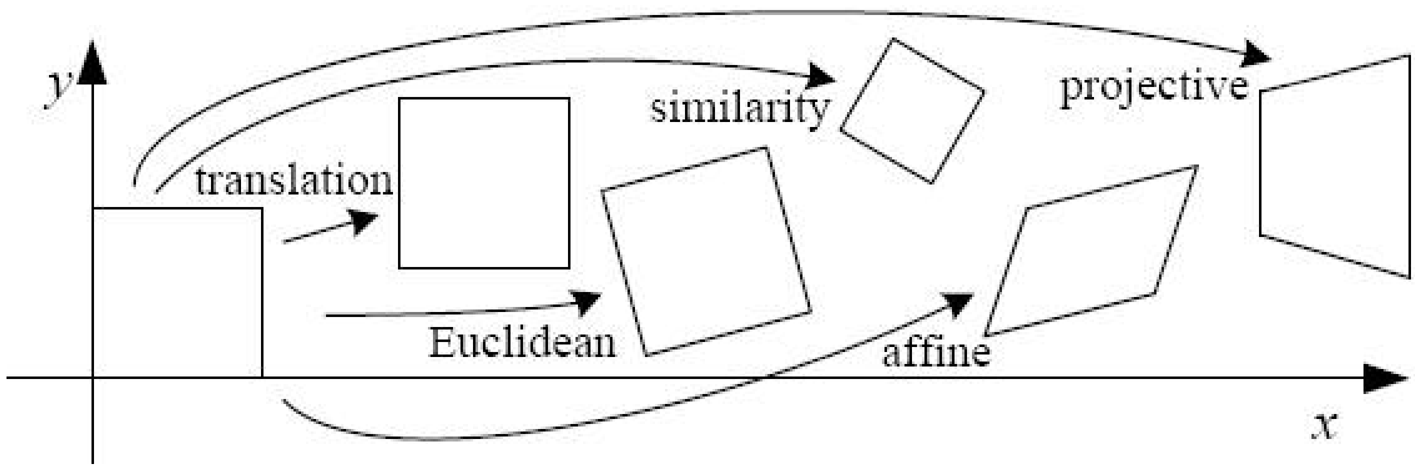
- For each point correctly recognize the corresponding one



We need a reliable and distinctive descriptor!

INVARIANCE: GEOMETRIC TRANSFORMATIONS

LEVELS OF GEOMETRIC INVARIANCE



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1. Region extraction needs to be repeatable and accurate
   - Invariant to translation, rotation, scale changes
   - Robust or covariant to out-of-plane (≈affine) transformations
   - Robust to lighting variations, noise, blur, quantization

2. Locality: Features are local, therefore robust to occlusion and clutter.

3. Quantity: We need a sufficient number of regions to cover the object.

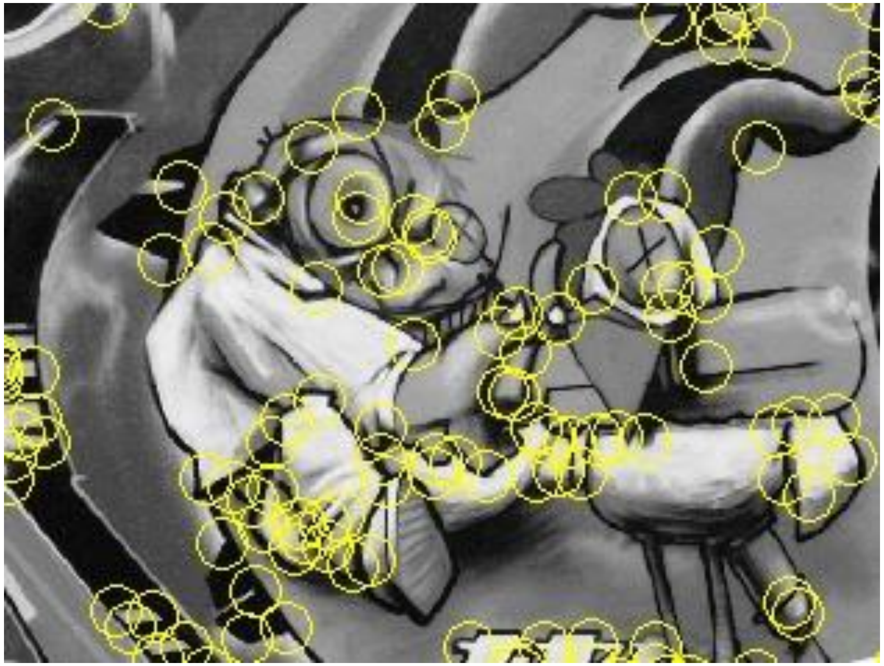
4. Distinctivenes: The regions should contain “interesting” structure.

5. Efficiency: Close to real-time performance.

MANY DETECTORS AVAILABLE

- Hessian & Harris — [Beaudet '78], [Harris '88]
- Laplacian, DoG — [Lindeberg '98], [Lowe '99]
- Harris-/Hessian-Laplace — [Mikolajczyk & Schmid '01]
- Harris-/Hessian-Affine — [Mikolajczyk & Schmid '04]
- EBR and IBR — [Tuytelaars & Van Gool '04]
- MSER — [Matas '02]
- Salient Regions — [Kadir & Brady '01]
- Others…

Those detectors have become a basic building block for many recent applications in Computer Vision.



Goals:

- Repeatable detection
- Precise localization
- Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*

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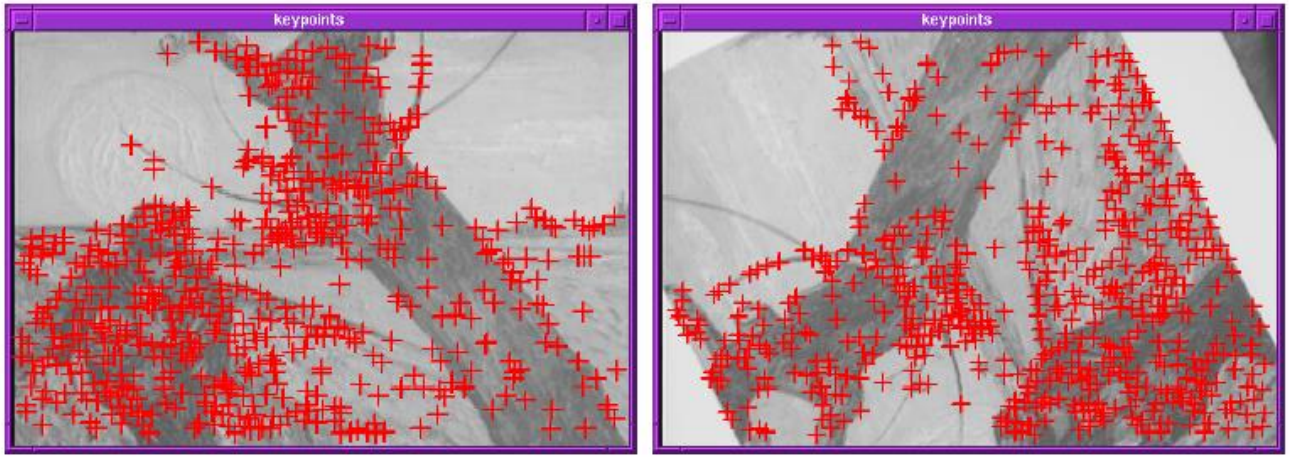

Key property:

- In the region around a corner, image gradient has two or more dominant directions

Corners are *repeatable* and *distinctive*

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*, 1988.

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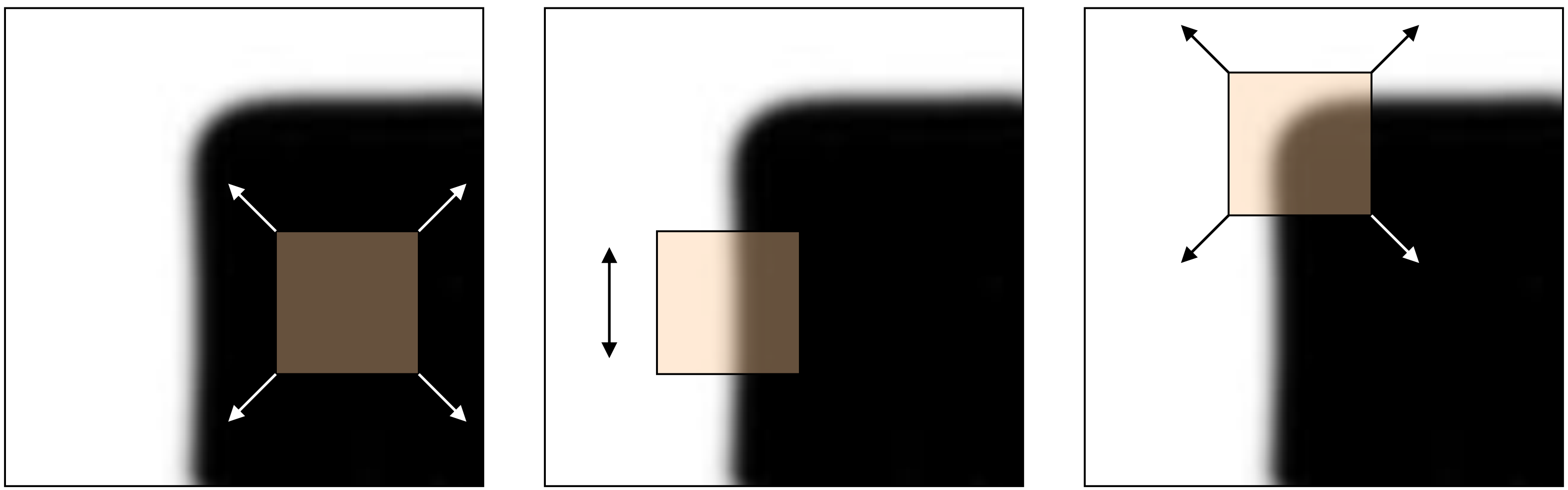
Design criteria

- We should easily recognize the point by looking through a small window (*locality*)
- Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)

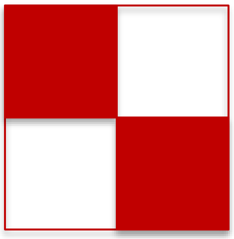


"flat" region:
no change in all directions

"edge":
no change along the edge direction

"corner":
significant change in all directions

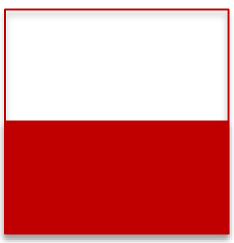
CORNERS VERSUS EDGES



$\sum I_x^2 \longrightarrow$ Large

$\sum I_y^2 \longrightarrow$ Large

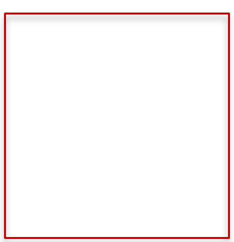
Corner



$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Large

Edge

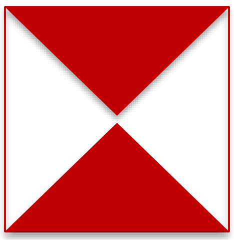


$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Small

Nothing

CORNERS VERSUS EDGES



$$\sum I_x^2 \longrightarrow ??$$

$$\sum I_y^2 \longrightarrow ??$$

Corner

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Change of intensity for the shift [u,v]:

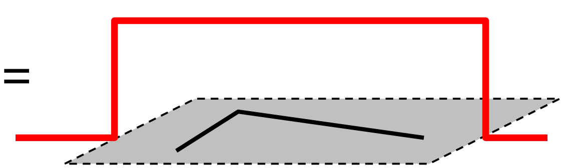
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

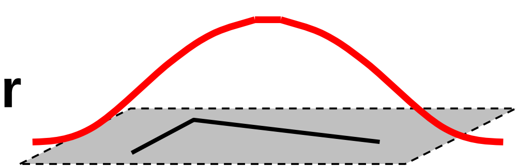
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

Slide credit: Rick Szeliski

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This measure of change can be approximated by:

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \;\; I_y]$$

HARRIS DETECTOR FORMULATION

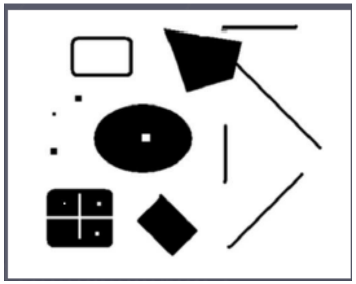
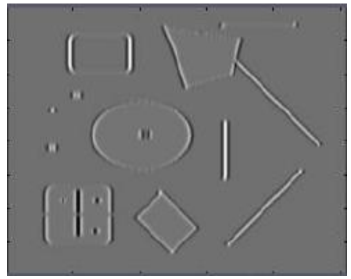
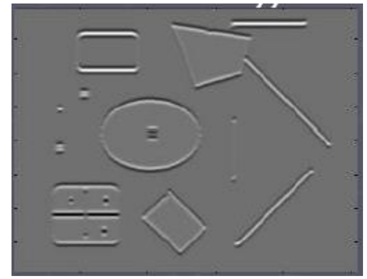


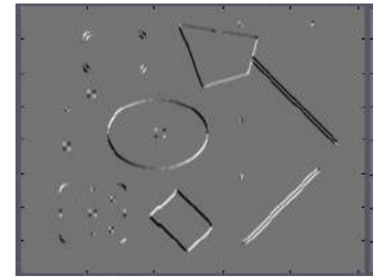
Image I



I_x



I_y



$I_x I_y$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x**, times gradient with respect to** y

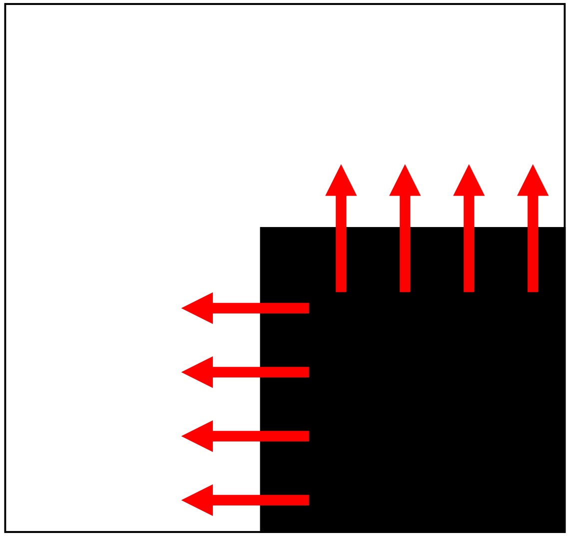
Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

WHAT DOES THIS MATRIX REVEAL?

First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



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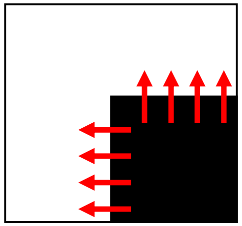


This means:

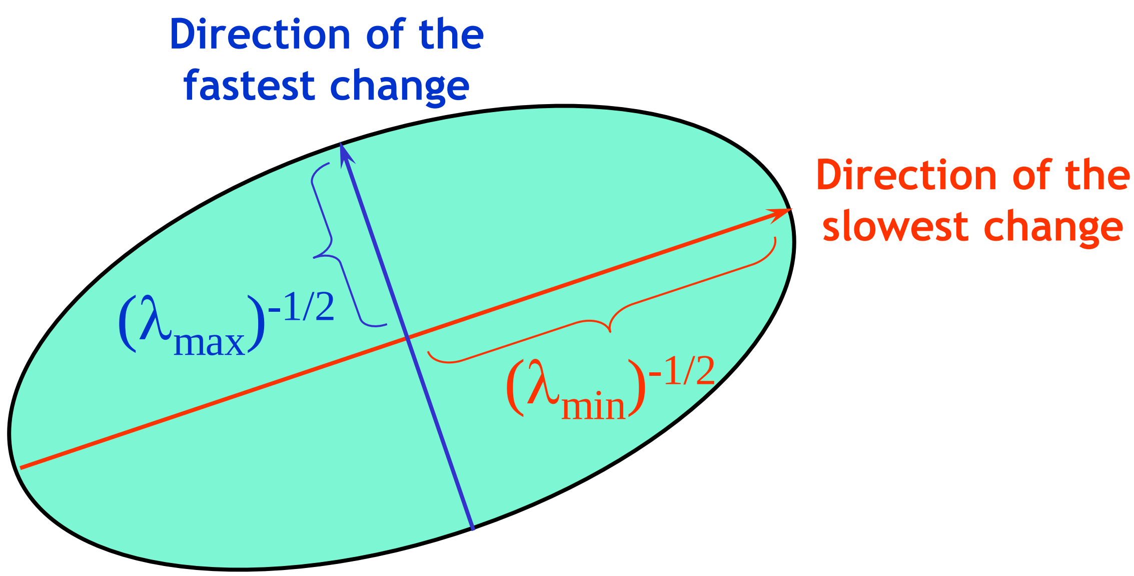
- Dominant gradient directions align with x or y axis
- If either λ is close to 0, then this is not a corner, so look for locations where both are large.

What if we have a corner that is not aligned with the image axes?

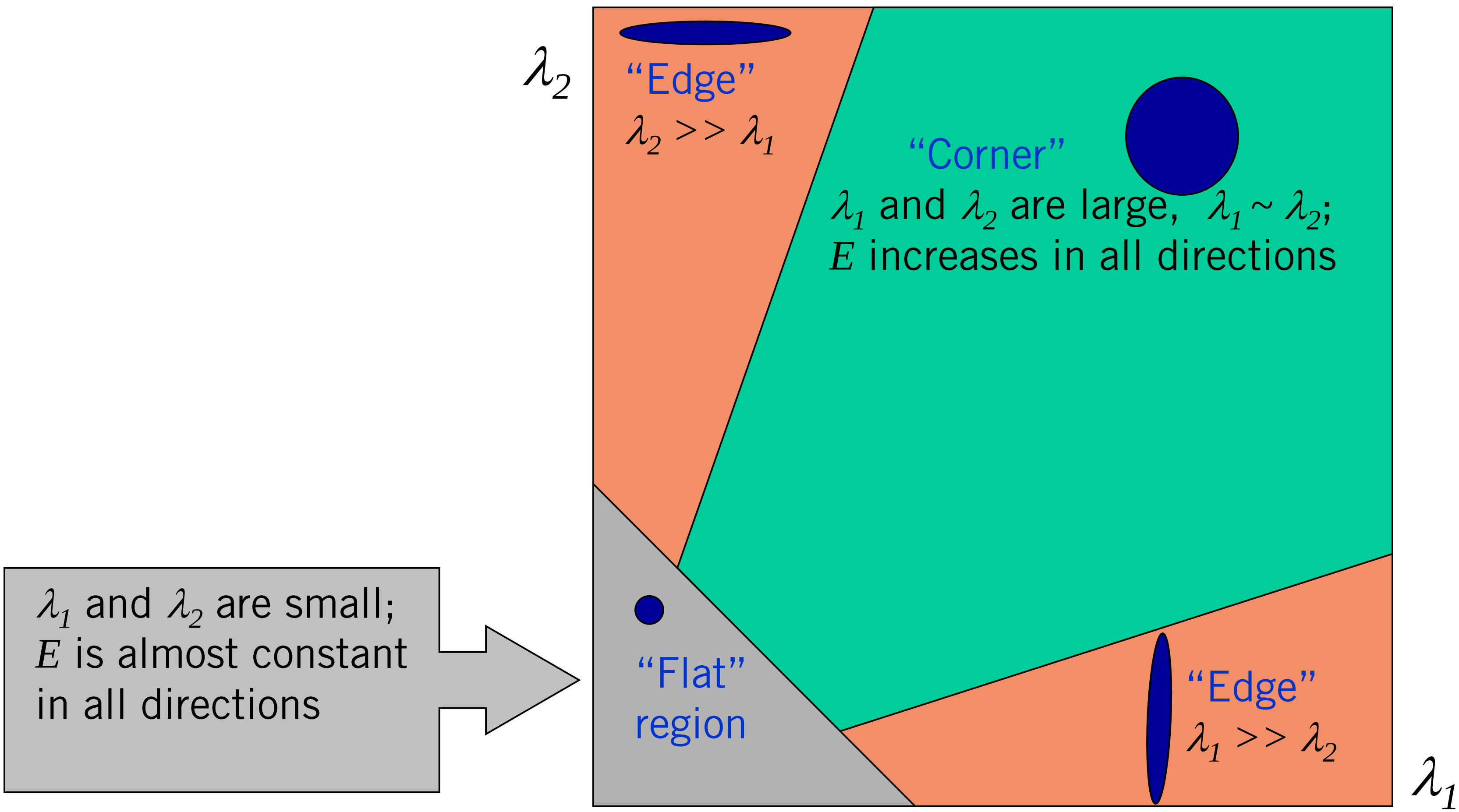
- Since M is symmetric, we have $M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

(Eigenvalue decomposition)

- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

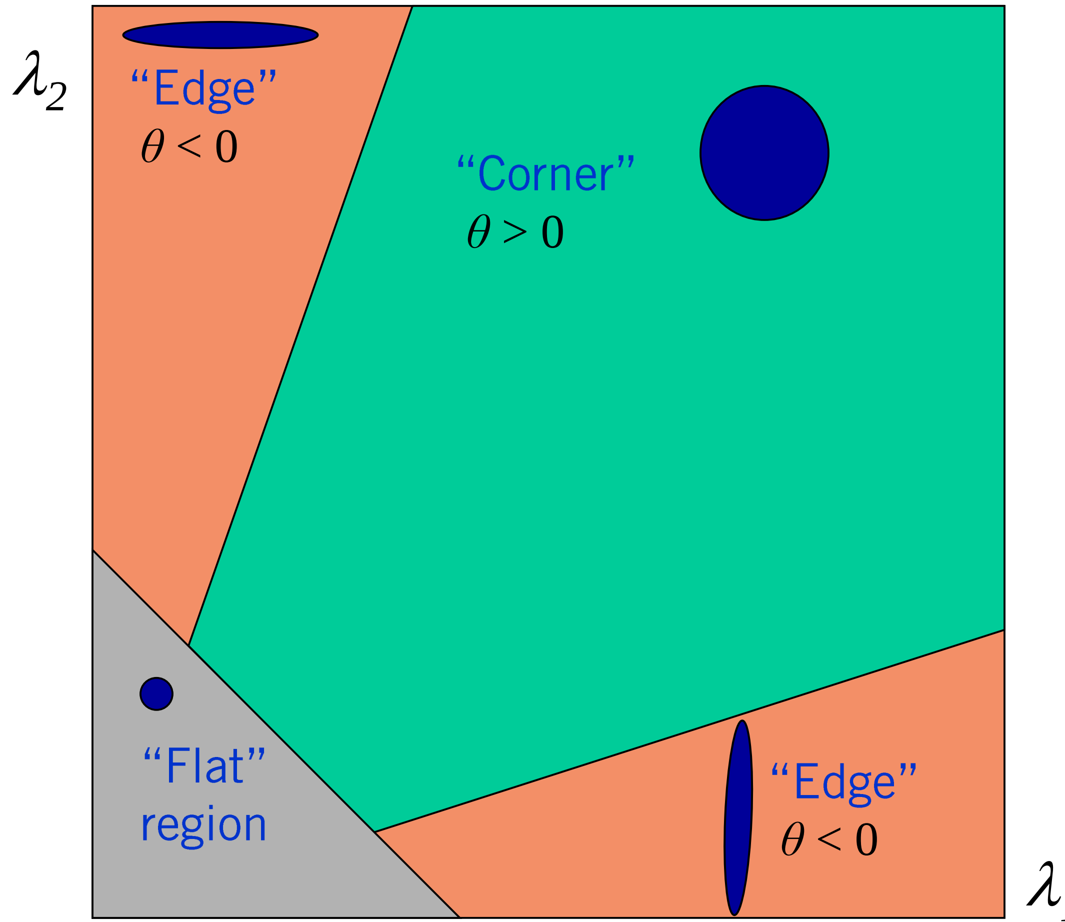


Classification of image points using eigenvalues of M:



CORNER RESPONSE FUNCTION

$$q = \det(M) - a\,\mathrm{trace}(M)^2 = /_1 /_2 - a(/_1 + /_2)^2$$



Fast approximation

- Avoid computing the eigenvalues
- α: constant (0.04 to 0.06)

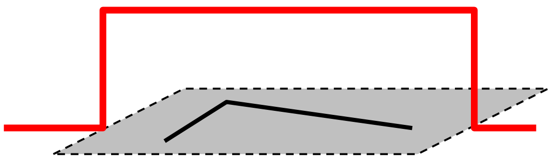
WINDOW FUNCTION *W(X,Y)*

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

- Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



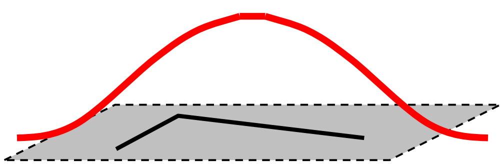
1 in window, 0 outside

- Problem: not rotation invariant

Option 2: Smooth with Gaussian

- Gaussian already performs weighted sum

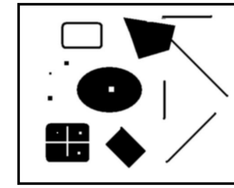
$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Gaussian

- Result is rotation invariant

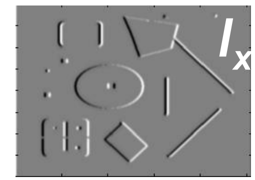
SUMMARY: HARRIS DETECTOR

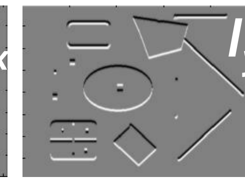


Compute second moment matrix (autocorrelation matrix)

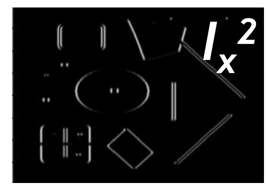
$$M(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

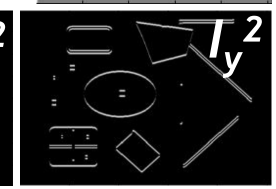
1. Image derivatives

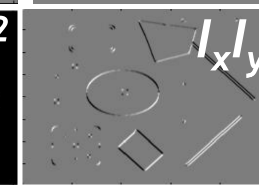




2. Square of derivatives







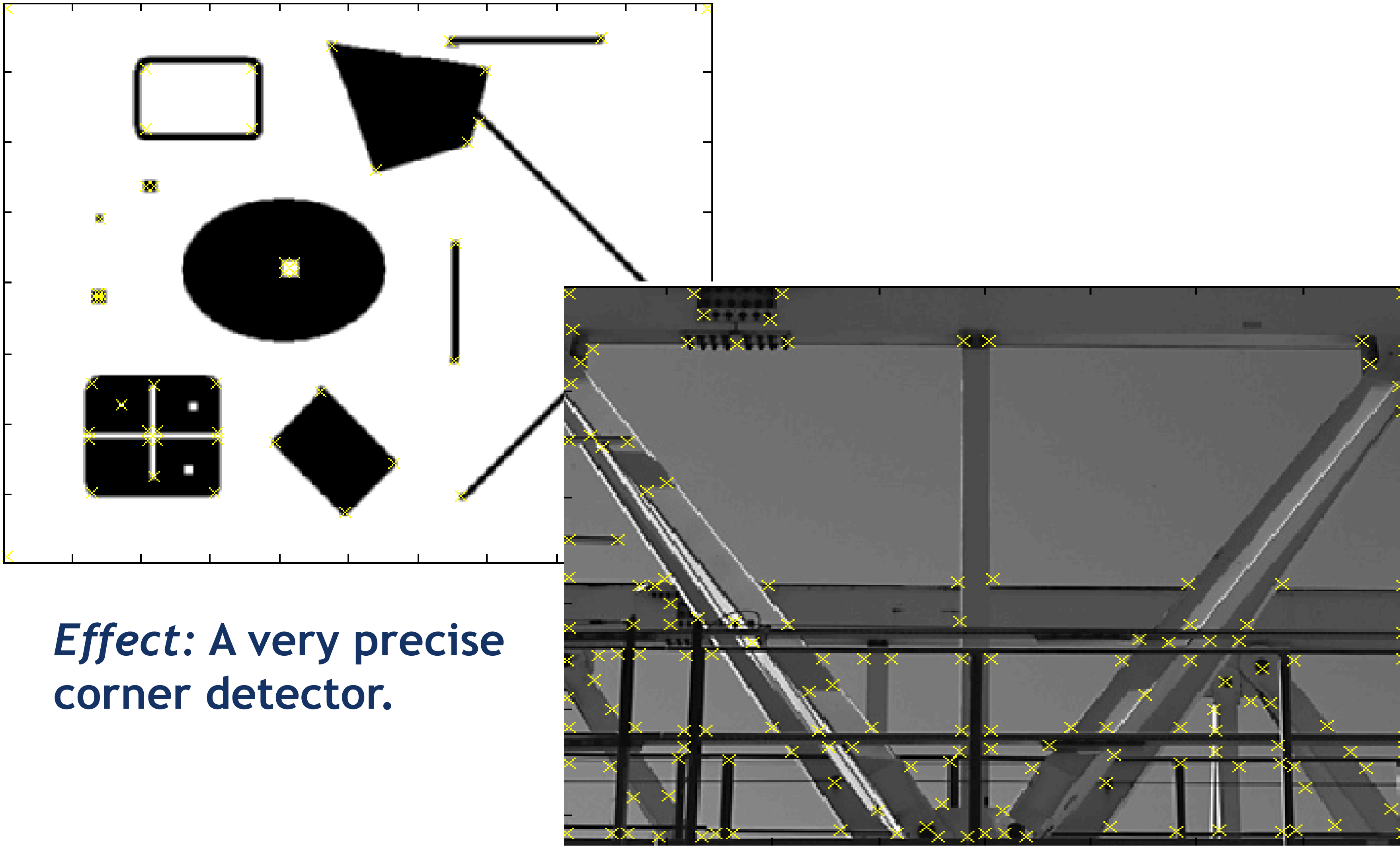
3. Gaussian filter $g(\sigma_I)$

4. Cornerness function - two strong eigenvalues

$$\theta = \det[M(\sigma_I,\sigma_D)] - \alpha[\mathrm{trace}(M(\sigma_I,\sigma_D))]^2$$
$$= g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Perform non-maximum suppression

HARRIS DETECTOR - RESPONSES



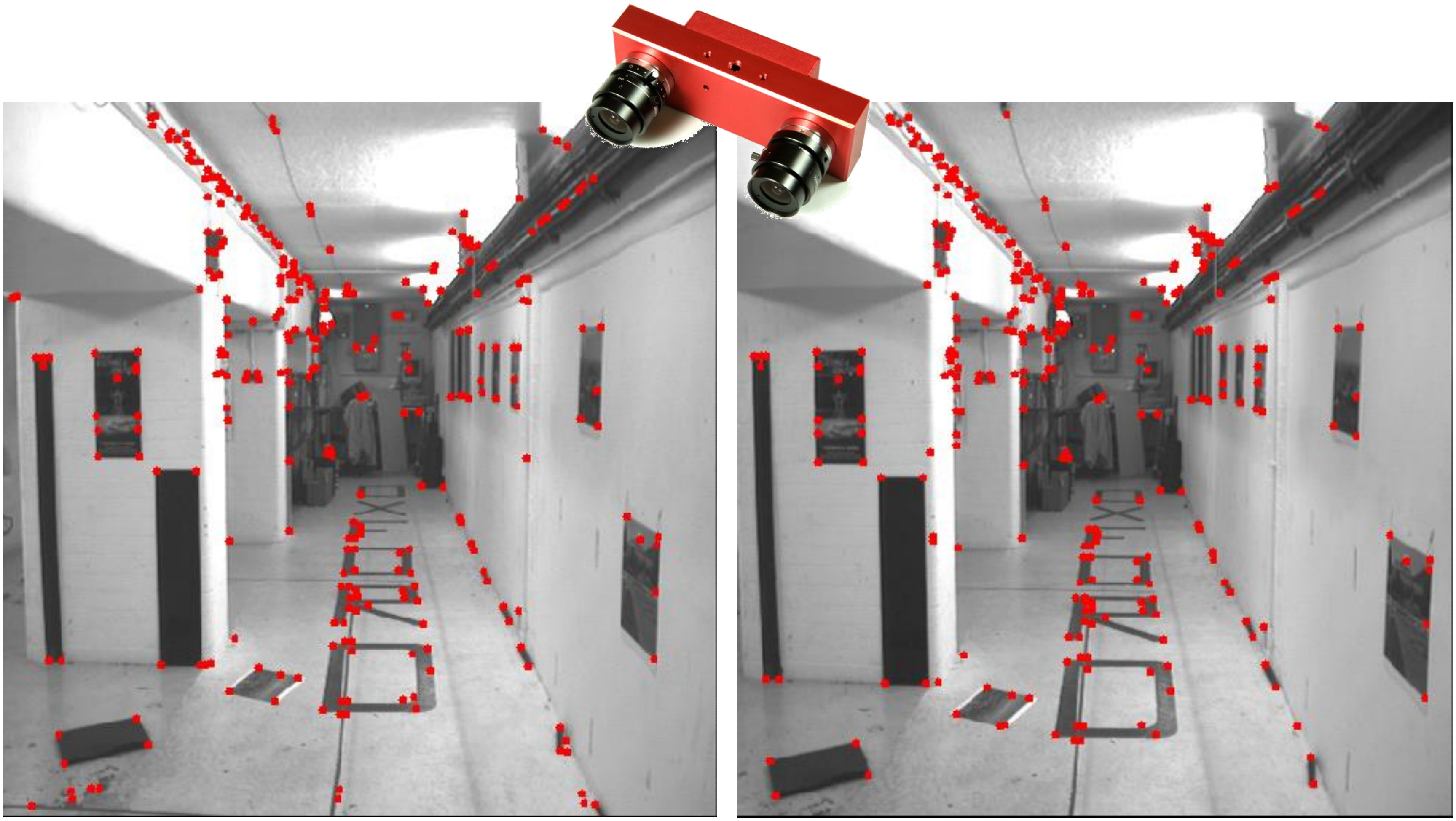
Effect: **A very precise corner detector.**

Slide credit: Krystian Mikolajczyk

HARRIS DETECTOR - RESPONSES

Slide credit: Krystian Mikolajczyk

HARRIS DETECTOR - RESPONSES



Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

HARRIS DETECTOR - PROPERTIES

1. Translation invariance?

Slide credit: Kristen Grauman

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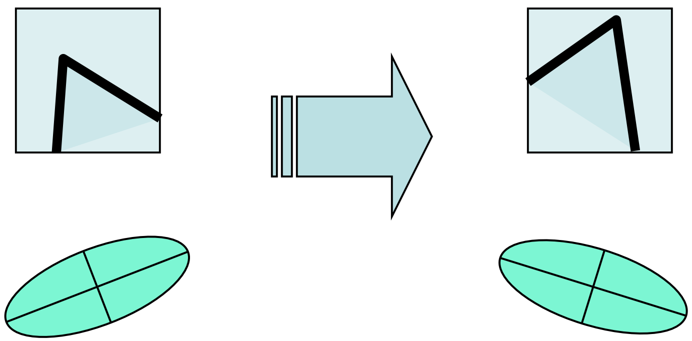
1. Translation invariance
2. Rotation invariance?



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

***Corner response θ* is invariant to image rotation**

Slide credit: Kristen Grauman

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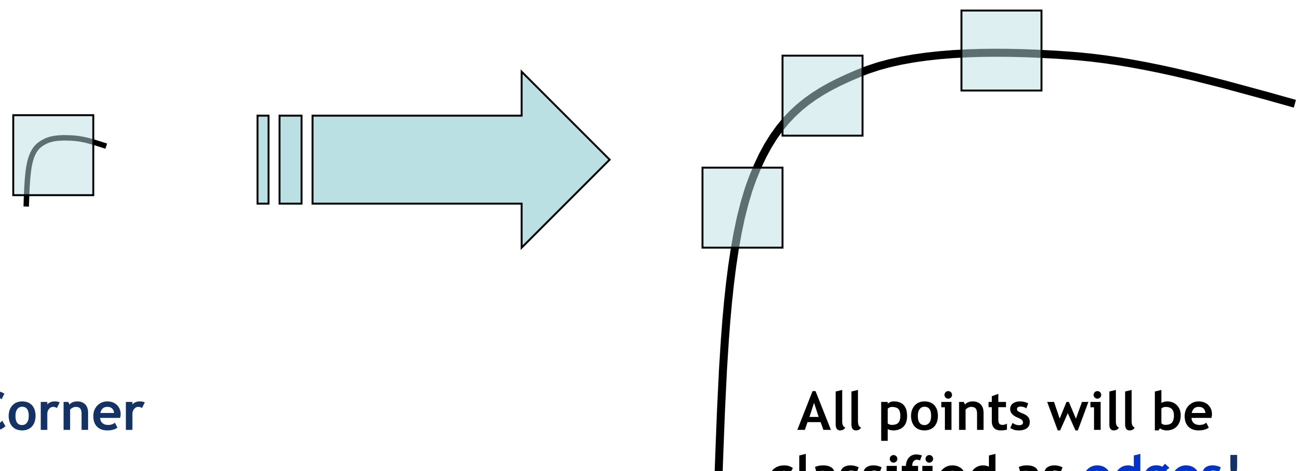
1. Translation invariance
2. Rotation invariance
3. Scale invariance?



Corner

All points will be classified as **edges!**

Not invariant to image scale!

Slide credit: Kristen Grauman

DOES SCALE MATTER?

When detecting corners, the `scale` of the window you use can change the corners you detect.

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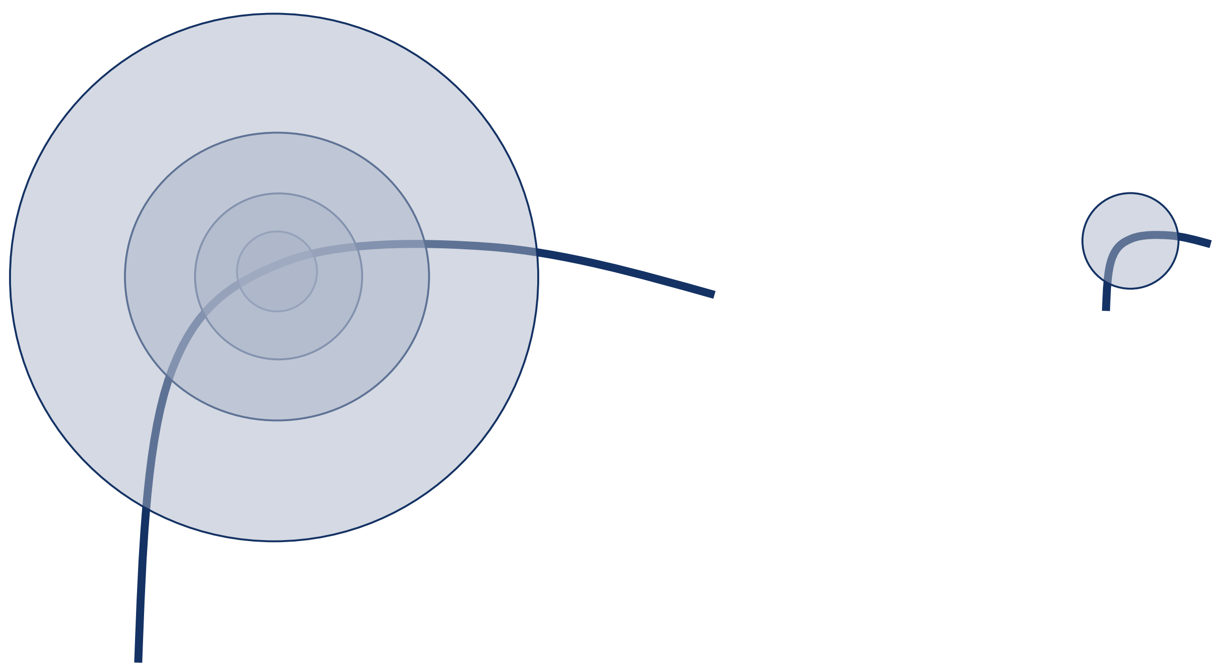
1. Consider regions (e.g. circles) of different sizes around a point
2. Find regions of corresponding sizes that will look the same in both images?



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The problem: how do we choose corresponding circles ***independently*** in each image?