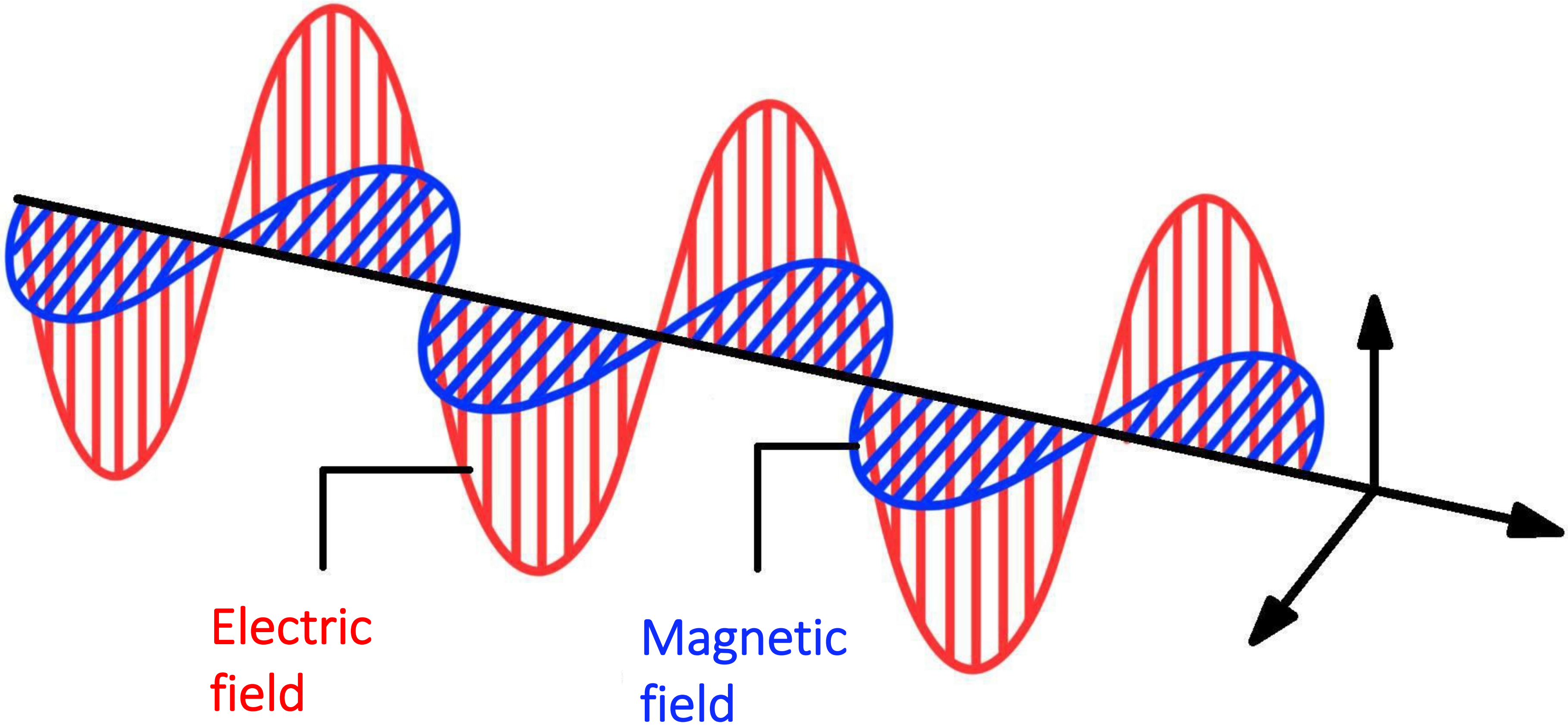
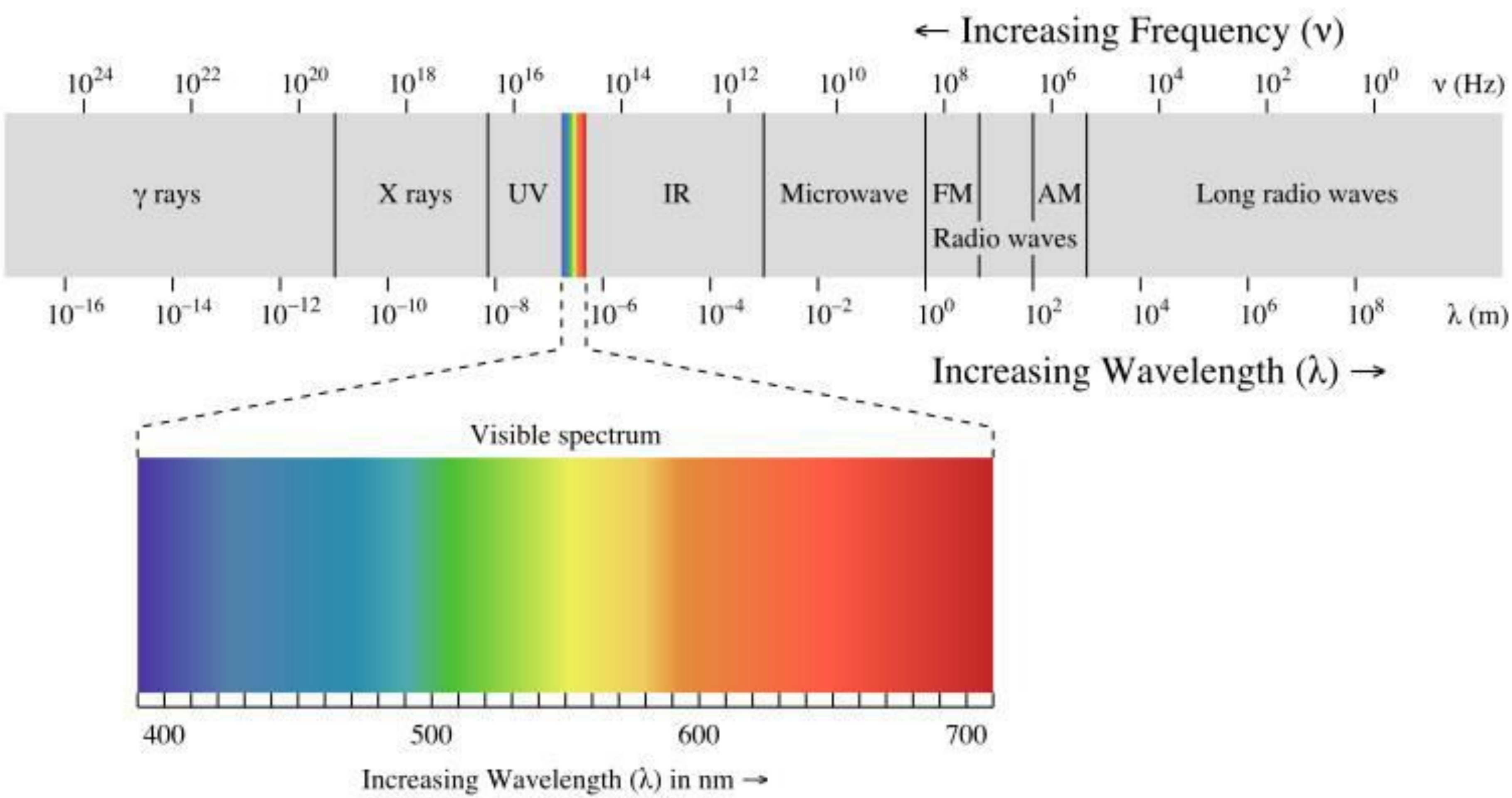
Lecture 5:

Light Transport

Contents

Electro-magnetic wave propagating at speed of light




← Increasing Frequency (ν)
10^{24} 10^{22} 10^{20} 10^{18} 10^{16} 10^{14} 10^{12} 10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Radio waves
Long radio waves
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400 500 600 700
Increasing Wavelength (λ) in nm →

[Wikipedia]

Ray

- Linear propagation
- Geometrical optics

Vector

- Polarization
- **Jones Calculus**: matrix representation

Wave

- Diffraction, interference
- **Maxwell equations**: propagation of light

Particle

- Light comes in discrete energy quanta: photons
- **Quantum theory**: interaction of light with matter

Field

- Electromagnetic force: exchange of virtual photons
- **Quantum Electrodynamics** (QED): interaction between particles

What is Light ?

Ray

- Linear propagation
- Geometrical optics

Vector

- Polarization
- **Jones Calculus**: matrix representation

Wave

- Diffraction, interference
- **Maxwell equations**: propagation of light

Particle

- Light comes in discrete energy quanta: photons
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Field

- Electromagnetic force: exchange of virtual photons
- **Quantum Electrodynamics** (QED): interaction between particles

Based on human visual perception

- Macroscopic geometry (→ Reflection Models)
- Tristimulus color model (→ Human Visual System)
- Psycho-physics: tone mapping, compression, …

Ray optic assumptions

- Macroscopic objects
- Incoherent light
- Light: scalar, real-valued quantity
- Linear propagation
- Superposition principle: light contributions add, do not interact
- No attenuation in free space

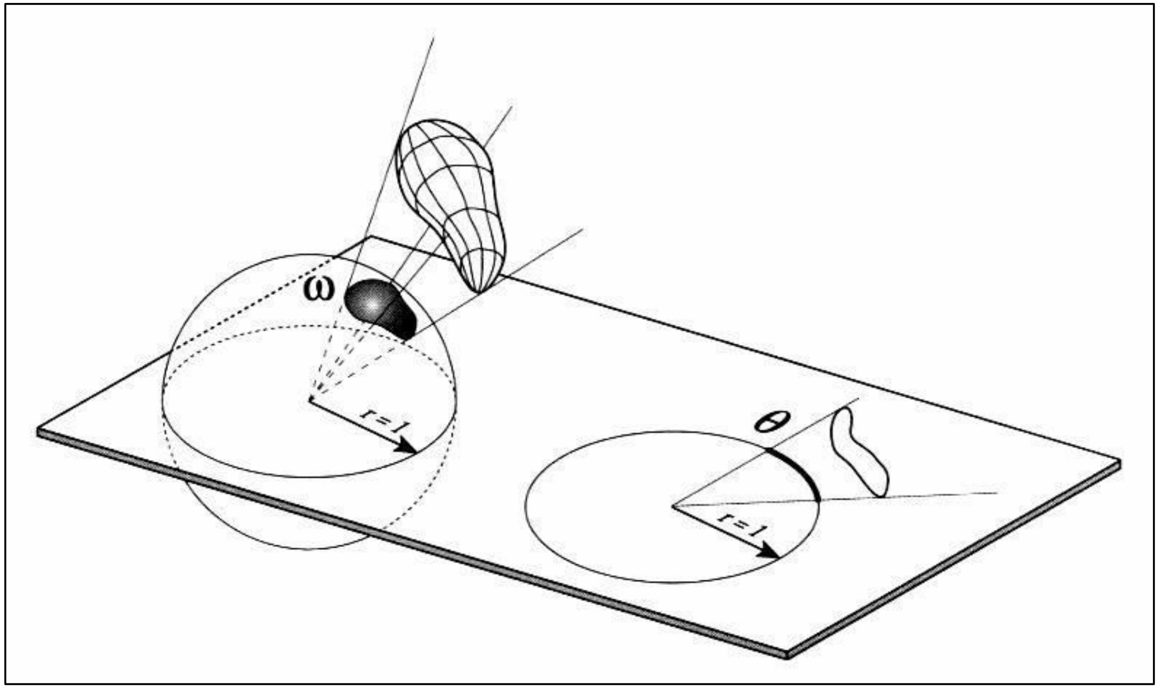
Limitations

- No microscopic structures (≈ λ): diffraction, interference
- No polarization
- No dispersion, …

The angle θ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta; r = 1$.

The solid angle $\Omega, d\omega$ subtended by an object, is the surface area of its projection onto the unit sphere

- Units for measuring solid angle: steradian [sr] (dimensionless)

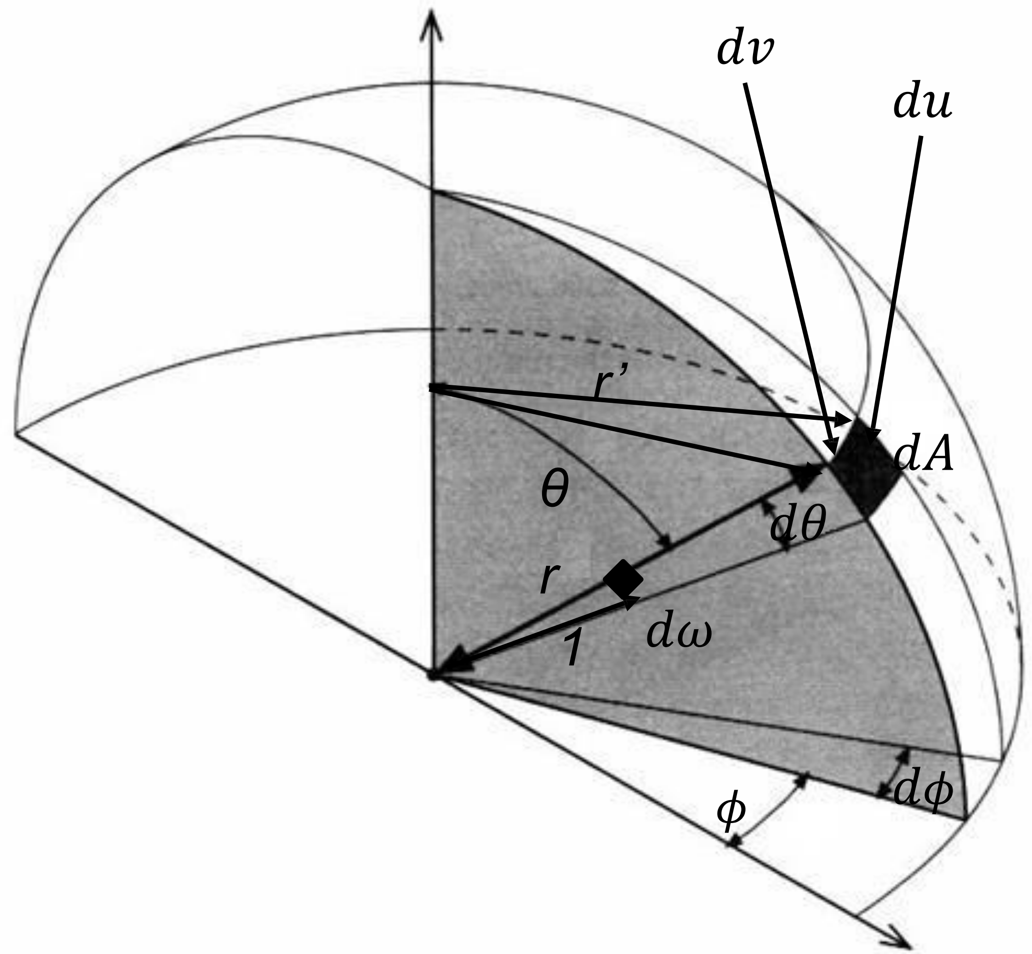


Infinitesimally small solid angle dω

- $du = r\, d\theta$
- $dv = r'\, d\phi = r \sin\theta\, d\phi$
- $dA = du\, dv = r^2 \sin\theta\, d\theta d\phi$
- $d\omega = dA / r^2 = \sin\theta\, d\theta d\phi$

Finite solid angle

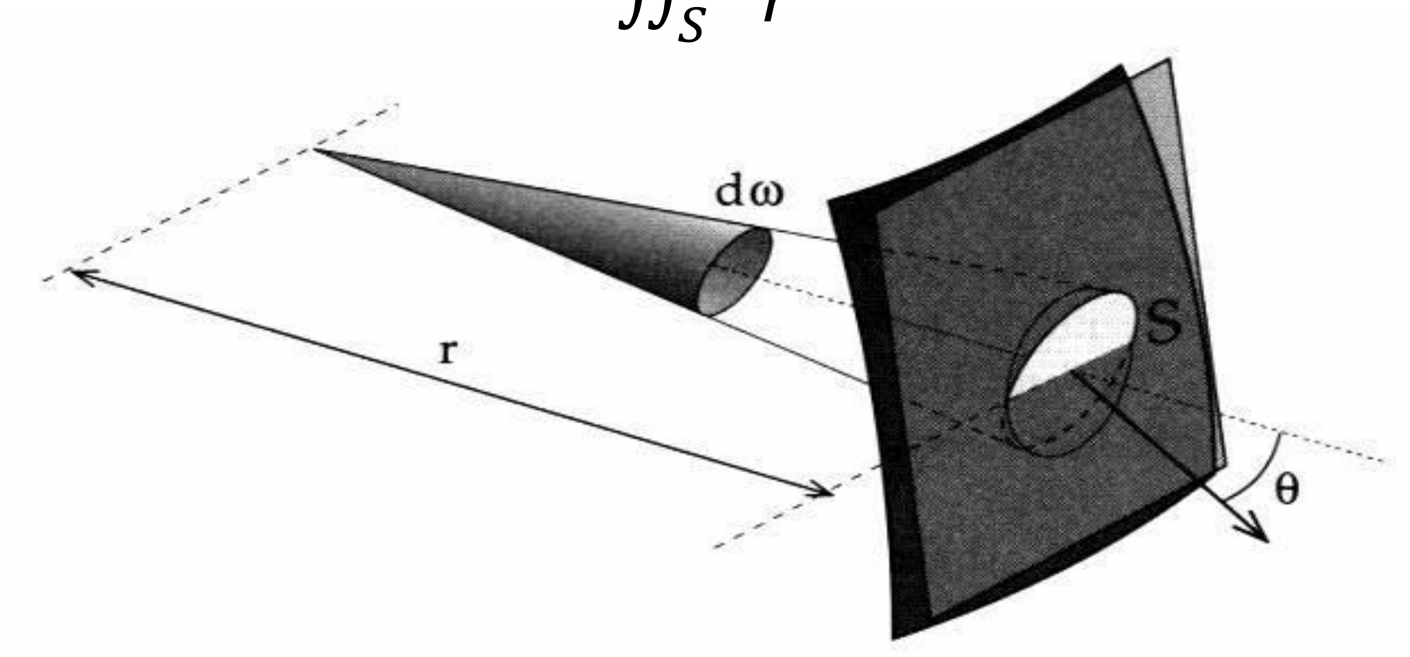
$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\, d\theta$$



The solid angle subtended by a small surface patch S with area dA is obtained

- by projecting it orthogonal to the vector r from the origin: $dA\cos\theta$
- and dividing by the squared distance to the origin: $d\omega = \frac{dA\cos\theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^3}\,dA$$



Definition:

- **Radiometry** is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers

Radiometric Quantities

• Energy	[J]	Q	(#Photons x Energy = $n \cdot hv$)
• Radiant power	[watt = J / s]	Φ	(Total Flux)
• Intensity	[watt / sr]	I	(Flux from a point per solid angle)
• Irradiance	[watt / m^2]	E	(Incoming flux per area)
• Radiosity	[watt / m^2]	B	(Outgoing flux per area)
• Radiance	[watt / (m^2 sr)]	L	(Flux per area & projected solid angle)

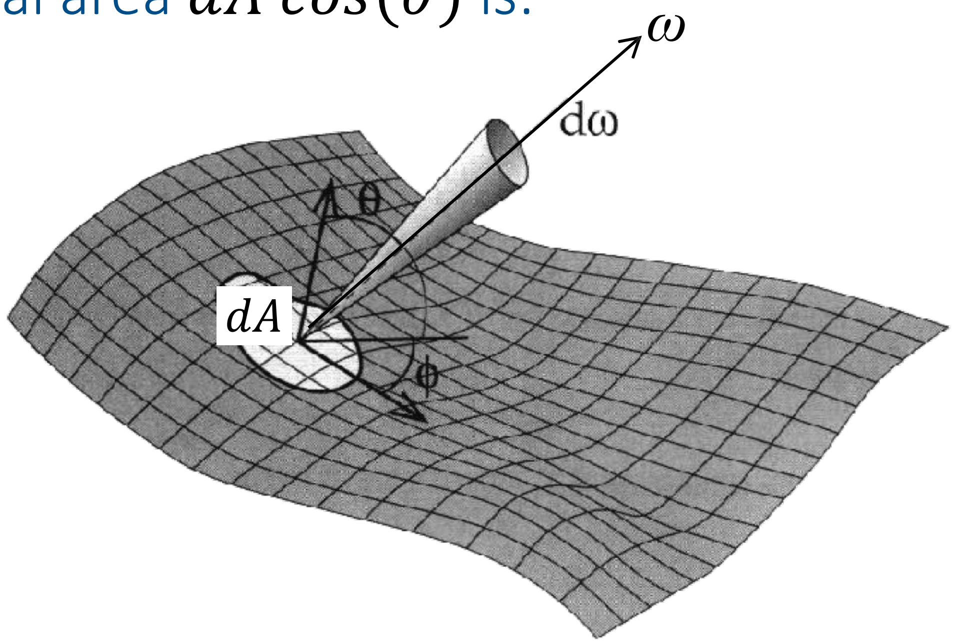
Radiance L is used to describe radiant energy transfer

Radiance L is defined as

- The power (flux) traveling through some point x
- In a specified direction $\omega = (\theta, \varphi)$
- Per unit area perpendicular to the direction of travel
- Per unit solid angle

Thus, the differential power $d^2\Phi$ radiated through the *differential solid angle* $d\omega$, from the projected differential area $dA\, cos(\theta)$ is:

$$d^2\Phi = L(x, \omega) dA \cos(\theta) d\omega$$



Irradiance E is defined as the **total power per unit area** (flux density) **incident onto** a surface. To obtain the total flux incident to dA, the *incoming* radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(x,\omega)\cos(\theta)\,d\omega\right] dA$$

$$E(x) = \int_{\Omega_+} L_i(x,\omega)\cos(\theta)\,d\omega = \int_0^{\pi/2}\int_0^{2\pi} L_i(x,\omega)\cos(\theta)\sin(\theta)\,d\theta d\phi$$

Radiocity B is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to dA, the *outgoing* radiance L_o is integrated over the upper hemisphere Ω_+ above the surface:

$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_o(x,\omega)\cos(\theta)\,d\omega\right] dA$$

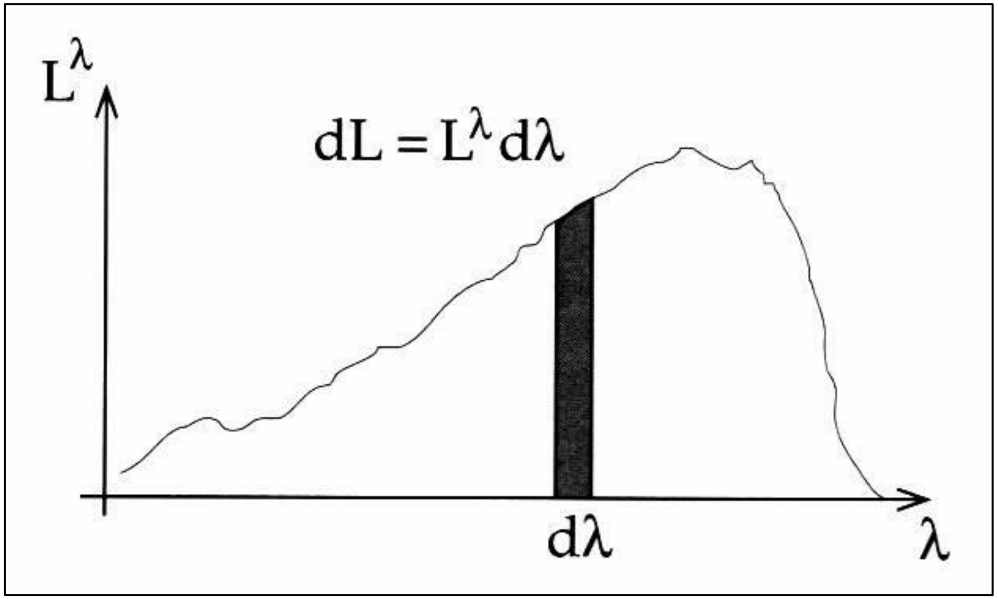
$$B(x) = \int_{\Omega_+} L_o(x,\omega)\cos(\theta)\,d\omega = \int_0^{\pi/2}\int_0^{2\pi} L_o(x,\omega)\cos(\theta)\sin(\theta)\,d\theta d\phi$$

Wavelength

- Light is composed of electromagnetic waves
- These waves have different frequencies and wavelengths
- Most transfer quantities are continuous functions of wavelength: $L(x,\theta,\varphi) = \int_{\lambda_{min}}^{\lambda_{max}} L(x,\theta,\varphi,\lambda)d\lambda$

In graphics

- Each measurement $L(x,\omega)$ is for a discrete band of wavelength only
  - Often R(ed, long), G(reen, medium), B(lue, short) (but see later)



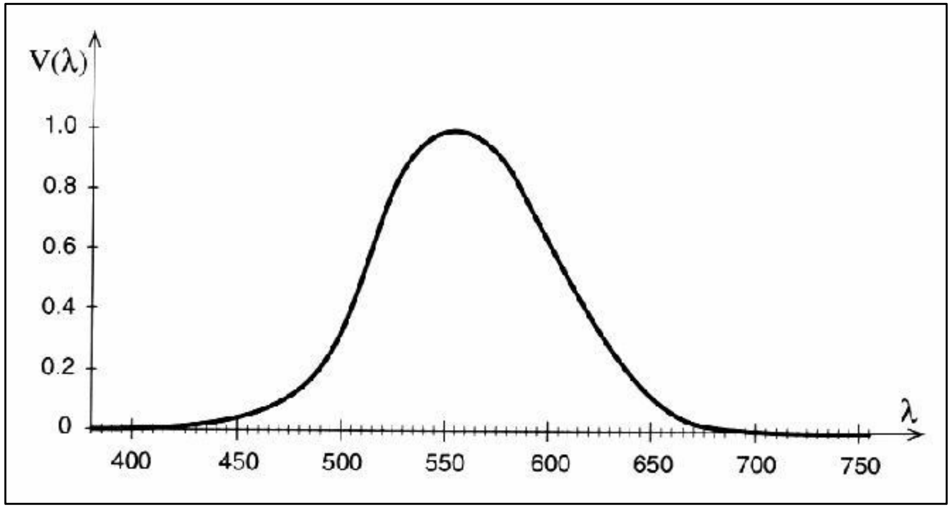
The human eye is sensitive to a limited range of wavelengths

- Roughly from *380 nm* to *780 nm*

Our visual system responds differently to different wavelengths

- Can be characterized by the **Luminous Efficiency Function** $V(\lambda)$
- Represents the average human spectral response
- Separate curves exist for light and dark adaptation of the eye

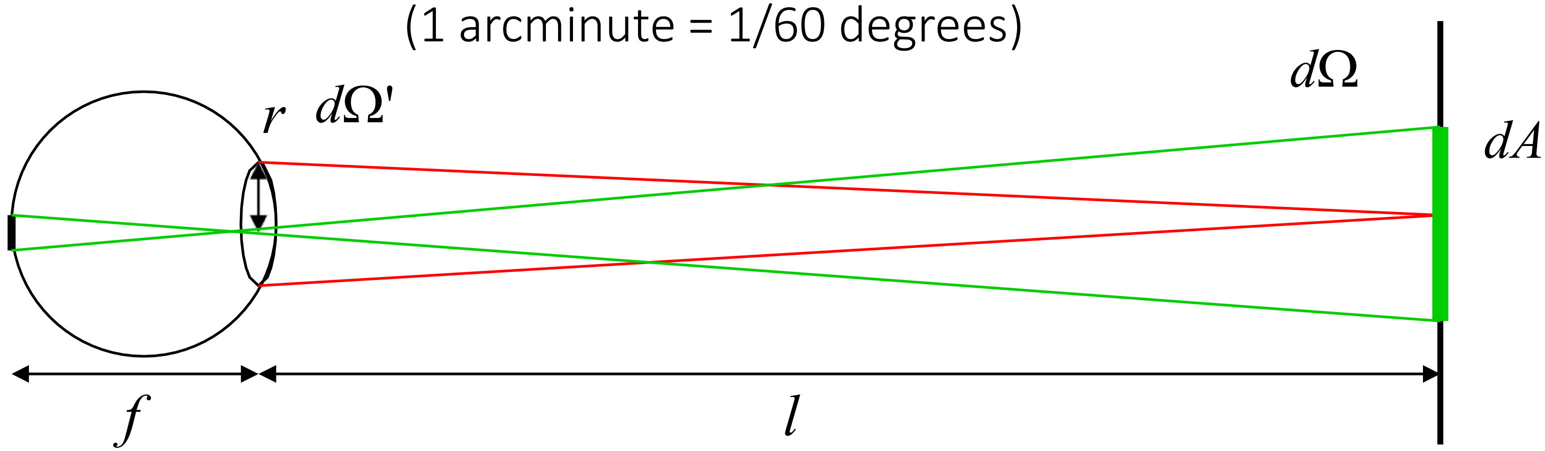
Photometric quantities are derived from radiometric quantities by integrating them against this function



Physics - based quantities / Perception - based quantities

	Radiometry	$\rightarrow$	Photometry	
W	Radiant power	$\rightarrow$	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	$\rightarrow$	Luminosity Illuminance	Lux (lm/m^2)
$W/m^2/sr$	Radiance	$\rightarrow$	Luminance	cd/m^2 ($lm/m^2/sr$)

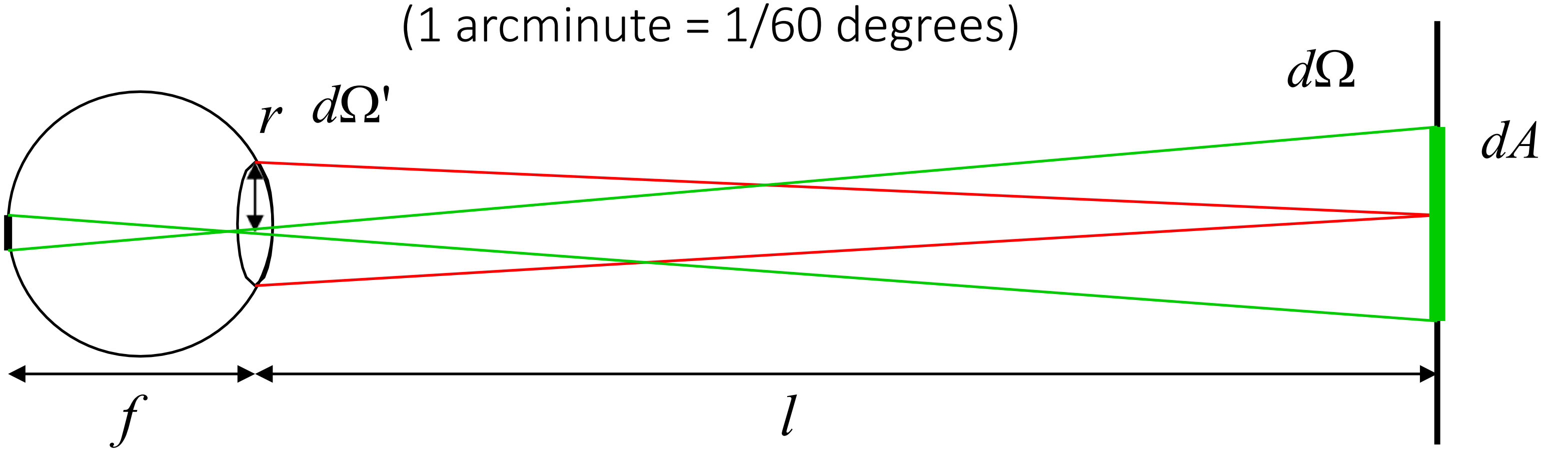
(1 arcminute = 1/60 degrees)



- photons / second = **flux** = **energy / time** = **power** Φ rod sensitive to flux
- angular extent of rod = **resolution** ($\approx$ 1 arcminute^2) $\mathrm{d}\Omega$
- projected rod size = **area** $\mathrm{d}A \approx l^2 \cdot d\Omega$
- angular extent of pupil aperture ($r \leq 4$mm) = **solid angle** $\mathrm{d}\Omega' \approx \pi \cdot r^2 / l^2$
- **flux** proportional to area and solid angle $\Phi \propto d\Omega' \cdot dA$
- **radiance** = flux per unit area per unit solid angle $L = \frac{\Phi}{d\Omega' \cdot dA}$
- As l increases: $\Phi_0 = L \cdot \pi \frac{r^2}{l^2} \cdot l^2 \cdot d\Omega = L \cdot const$

The eye detects radians

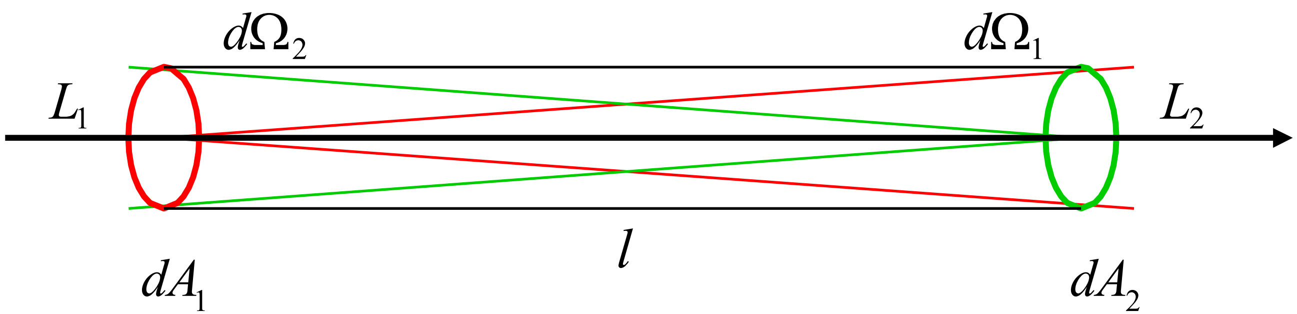
(1 arcminute = 1/60 degrees)



- $dA' > dA$: photon flux per rod stays constant
- $dA' < dA$: photon flux per rod decreases

Where does the Sun turn into a star ?

- Depends on apparent Sun disc size on retina
- Photon flux per rod stays the same on Mercury, Earth or Neptune
- Photon flux per rod decreases when $d\Omega' < 1$ arcminute^2 (beyond Neptune)



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 d\Omega_1 dA_1 = L_2 d\Omega_2 dA_2$$

From geometry follows

$$d\Omega_1 = \frac{dA_2}{l^2} \qquad d\Omega_2 = \frac{dA_1}{l^2}$$

Ray throughput T

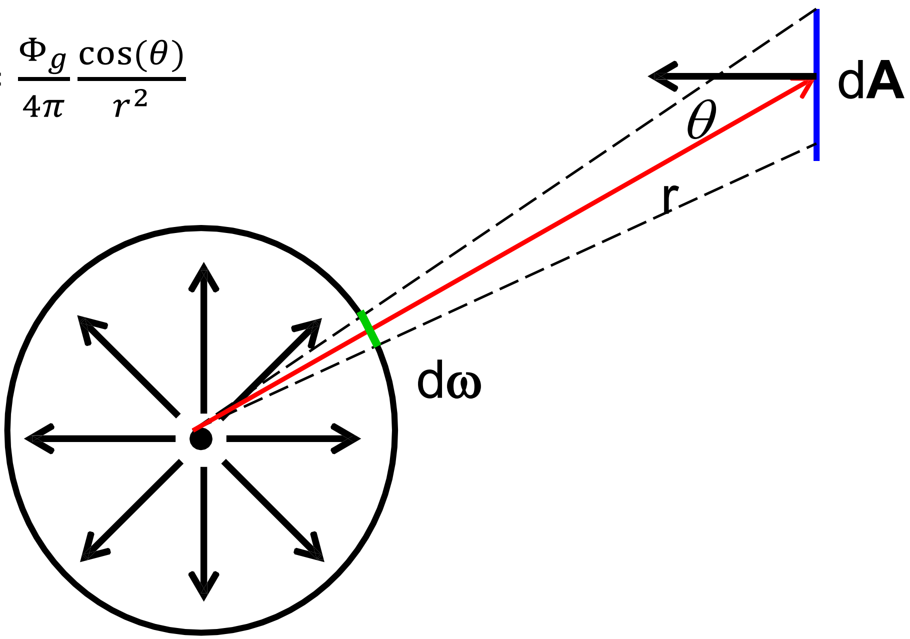
$$T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$$

$$L_1 = L_2$$

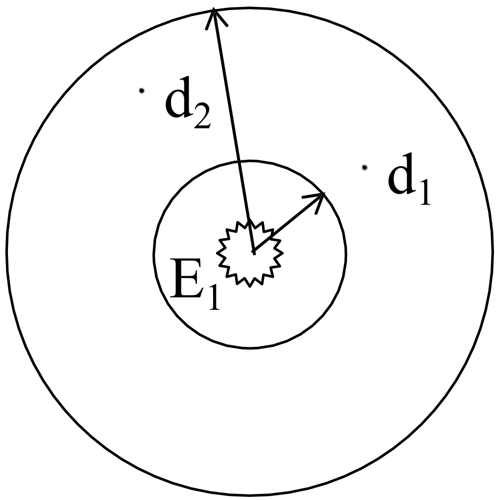
The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

Point light with isotropic radiance

- Power (total flux) of a point light source
  - Φ_g = Power of the light source [watt]
- Intensity of a light source (radiance cannot be defined, no area)
  - $I = \Phi_g/(4\pi)$ [watt / sr]
- Irradiance on a sphere with radius r around light source:
  - $E_r = \Phi_g/(4\pi r^2)$ [watt / m^2]
- Irradiance on some other surface A
  - $E(x) = \frac{d\Phi_g}{dA} = \frac{Id\omega}{dA} = \frac{\Phi_g}{4\pi}\frac{dA\cos(\theta)}{r^2 dA} = \frac{\Phi_g}{4\pi}\frac{\cos(\theta)}{r^2}$



Inverse Square Law



Irradiance E:

$$\frac{E_1}{E_2} = \frac{d_2^{\,2}}{d_1^{\,2}}$$

Irradiance E: power per m^2

- Illuminating quantity

Distance-dependent

- Double distance from emitter: area of sphere is four times bigger

Irradiance falls off with inverse of squared distance

- Only for point light sources (!)

Power (total flux)

- Emitted energy / time

Active emission size

- Point, line, area, volume

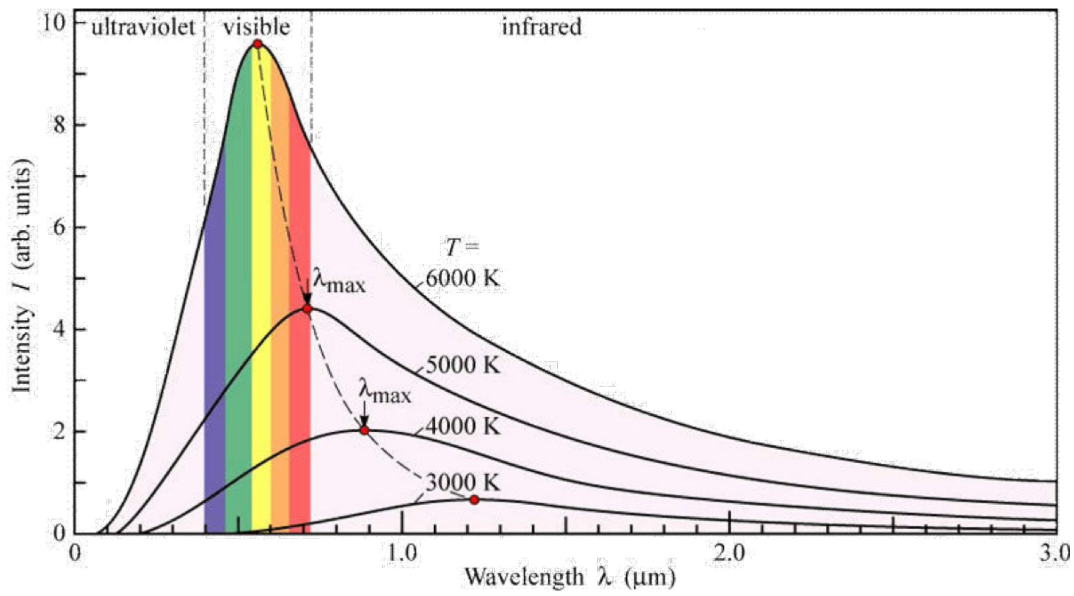
Spectral distribution

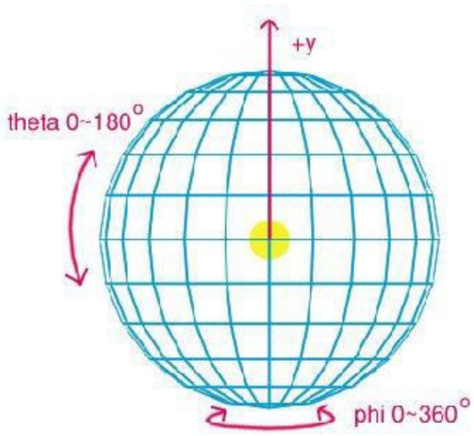
- Thermal, line spectrum

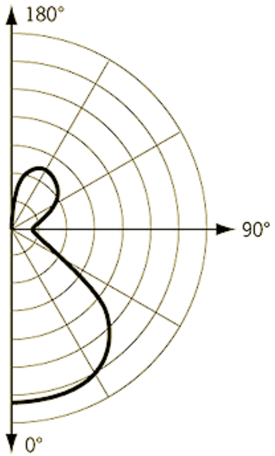
Directional distribution

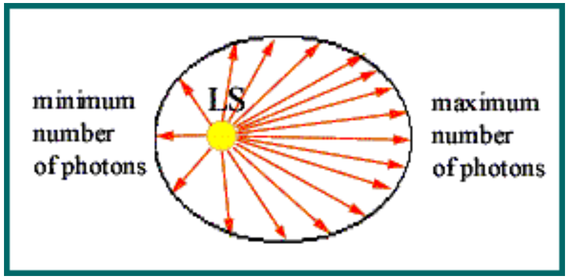
- Goniometric diagram

Black body radiation (see later)









Radiation characteristics:

Directional light

- Spot-lights
- Projectors
- Distant sources

Diffuse emitters

- Torchieres
- Frosted glass lamps

Ambient light

- “Photons everywhere”

Emitting area:

Volume

- Neon advertisements
- Sodium vapor lamps

Area

- CRT, LCD display
- (Overcast) sky

Line

- Clear light bulb, filament

“Point”

- Xenon lamp
- Arc lamp
- Laser diode

Sun

- Point source (approx.)
- White light (by def.)

Sky

- Area source
- Scattering: blue

Horizon

- Brighter
- Haze: whitish

Overcast sky

- Multiple scattering in clouds
- Uniform grey

Several sky models are available

Scene

- Lights (emitters)
- Object surfaces (partially absorbing)

Illuminated object surfaces become emitters, too !

- Radiosity = Irradiance – absorbed photon flux
  - Radiosity: photons per second per m^2 leaving surface
  - Irradiance: photons per second per m^2 incident on surface

Light bounces between all mutually visible surfaces

Invariance of radiance in free space

- No absorption in-between objects

Dynamic Energy Equilibrium

- emitted photons = absorbed photons (+ escaping photons)

→ Global Illumination

$$L(x,\omega_0) = L_e(x,\omega_0) + \int_{\Omega_+} f_r(\omega_i,x,\omega_o)\, L_i(x,\omega_i) cos(\theta_i) d\omega_i$$

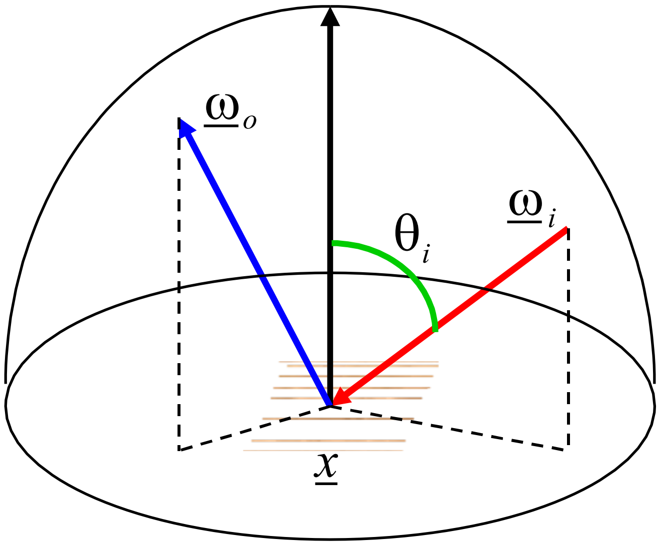
Visible surface radiance $L(x,\omega_o)$

- Surface position x
- Outgoing direction ω_0
- Incoming illumination direction ω_i

Self-emission $L_e(x,\omega_o)$

Reflected light

- Incoming radiance from all directions $L_i(x,\omega_i)$
- Direction-dependent reflectance $f_r(\omega_i,x,\omega_o)$
  (BRDF: bidirectional reflectance distribution function)



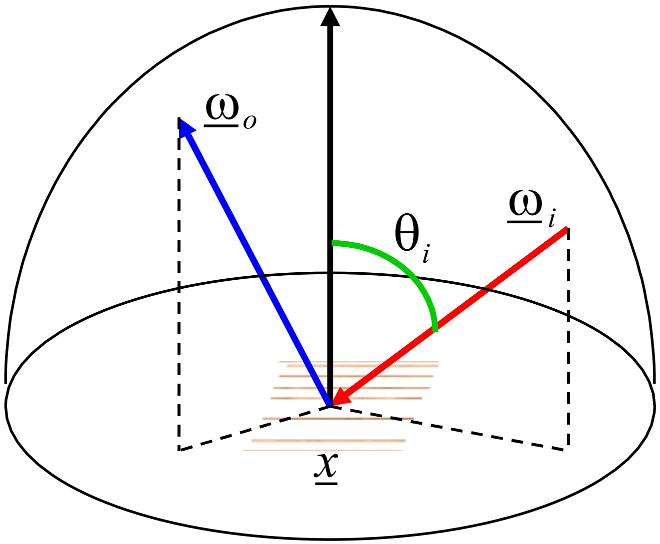
In Physics: *Radiative Transport Equation*

In Graphics: *The most important equation*

- Expresses energy equilibrium in scene:

$$L(x,\omega_0) = L_e(x,\omega_0) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o)\, L_i(x,\omega_i) cos(\theta_i) d\omega_i$$

- total radiance = emitted radiance + reflected radiance



First term: emissivity of the surface

- Non-zero only for light sources

Second term: reflected radiance

- Integral over all possible incoming directions of radiance times angle-dependent surface reflection function

Fredholm integral equation of 2nd kind

- Unknown radiance appears both on the left-hand side and inside the integral
- Numerical methods necessary to compute approximate solution

Approximations based only on empirical foundations

- An example: polygon rendering in OpenGL

Using RGB instead of full spectrum

- Follows roughly the eye's sensitivity

Sampling hemisphere along finite, discrete directions

- Simplifies integration to summation

Reflection function model (BRDF)

- Parameterized function
  - **Ambient**: constant, non-directional, background light
  - **Diffuse**: light reflected uniformly in all directions
  - **Specular**: light from mirror-reflection direction

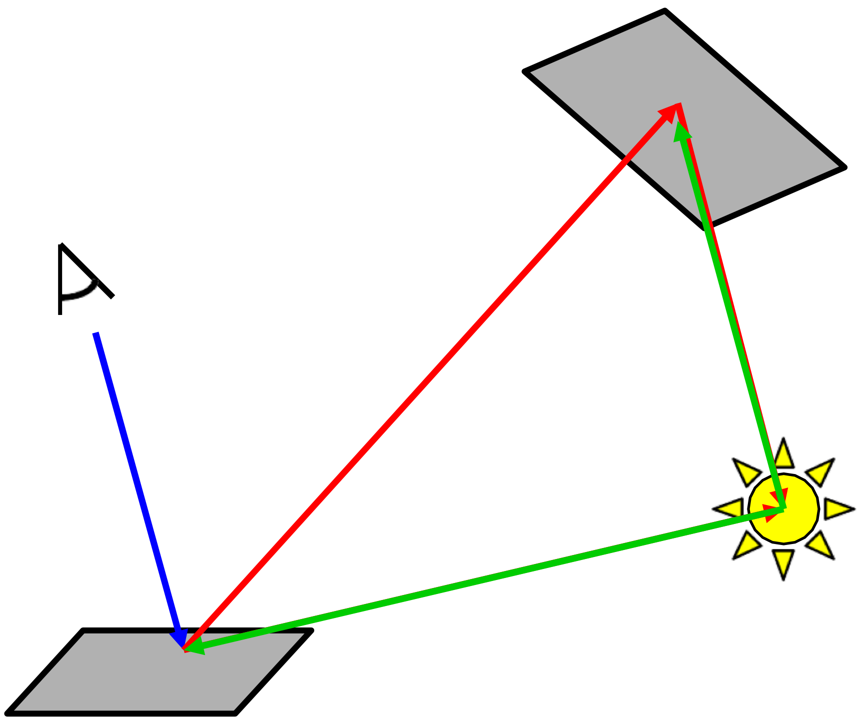
$$L(x,\omega_0) = L_e(x,\omega_0) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o)\, L_i(x,\omega_i) cos(\theta_i) d\omega_i$$

Simple ray tracing

- Illumination from discrete light sources only – **direct illumination only**
  - Integral – sum of contributions from each light
  - **No global illumination**
- Evaluates angle-dependent reflectance function (BRDF) – **shading process**

Advanced ray tracing techniques

- Recursive ray tracing
  - Multiple reflections/refractions (for specular surfaces)
- Forward ray tracing for global illumination
  - Stochastic sampling (Monte Carlo methods)



Outgoing illumination at a point

$$L(x, \omega_0) = L_e(x, \omega_0) + L_r(x, \omega_0)$$

$$L(x, \omega_0) = L_e(x, \omega_0) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o)\, L_i(x, \omega_i) cos(\theta_i) d\omega_i$$

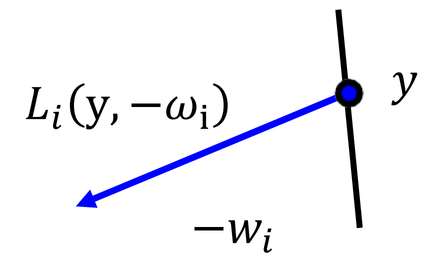
Linking with other surface points

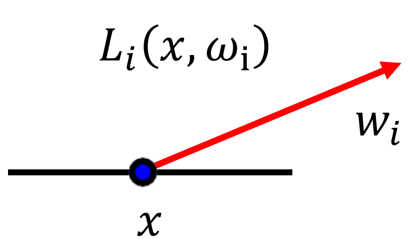
- Incoming radiance at x is outgoing radiance at y

$$L_i(x, \omega_\mathrm{i}) = L(\mathrm{y}, -\omega_\mathrm{i}) = L(RT(x, w_i), -w_i)$$

- Ray-tracing operator

$$y = RT(x, w_i)$$



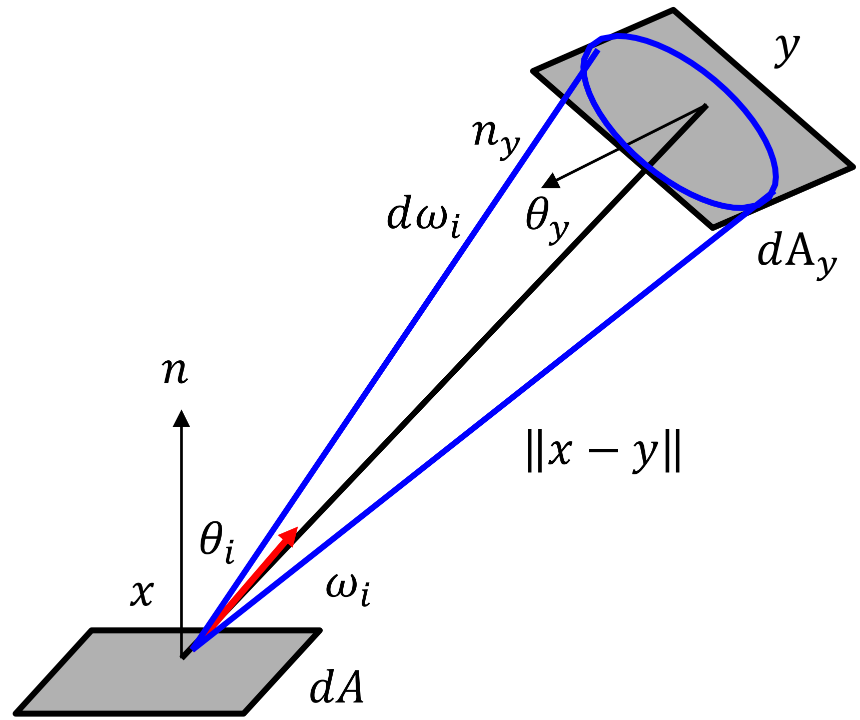


Outgoing illumination at a point

$$L(x,\omega_0) = L_e(x,\omega_0) + \int_{\Omega_+} f_r(\omega_i,x,\omega_o)\, L_i(x,\omega_i) cos(\theta_i) d\omega_i$$

Re-parameterization over surfaces *S*

$$d\omega_i = \frac{\cos(\theta_y)}{\|x-y\|^2} dA_y$$



$$L(x,\omega_0)$$
$$= L_e(x,\omega_0) + \int_{y\in S} f_r(\omega(x,y),x,\omega_o)\, L_i\big(x,\omega(x,y)\big) V(x,y) \frac{cos(\theta_i)cos(\theta_y)}{\|x-y\|^2} dA_y$$

$$L(x, \omega_0)$$
$$= L_e(x, \omega_0) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o)\, L_i\big(x, \omega(x,y)\big) V(x,y) \frac{cos(\theta_i) cos\big(\theta_y\big)}{\|x-y\|^2} d\mathrm{A}_y$$

- Geometry Term: $\mathrm{G}(x,y) = V(x,y) \frac{cos(\theta_i) cos\big(\theta_y\big)}{\|x-y\|^2}$
- Visibility Term: $V(x,y) = \begin{cases} 1, & if\ visible \\ 0, & otherwise \end{cases}$
- Integration over all surfaces: $\int_{y \in S} \cdots d\mathrm{A}_y$

$$L(x, \omega_0) = L_e(x, \omega_0) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o)\, L_i\big(x, \omega(x,y)\big) G(x,y) d\mathrm{A}_y$$

Lambertian surface (only diffuse reflection)

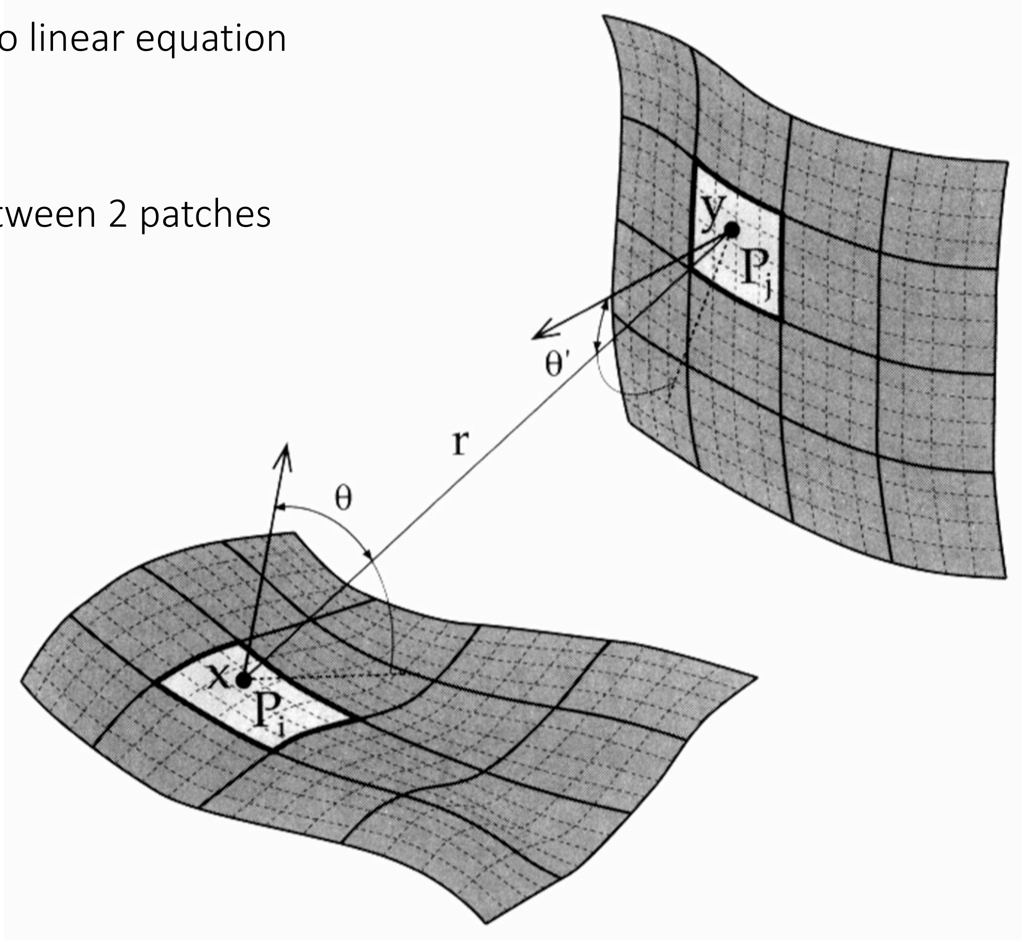
- Radiosity equation: simplified form of the rendering equation

Dividing scene surfaces into small planar patches

- Assumes local constancy: diffuse reflection, radiosity and visibility
  - "Radiosity" algorithms: Discretizes into linear equation

Algorithm

- Form factor: percentage of light flowing between 2 patches
- Form system of linear equations
- Iterative solution



Diffuse reflection ⇒ constant BRDF & emission

$f_r(\omega(x,y),x,\omega_o) = f_r(x) \Longrightarrow$

$$\rho(x\,) = \int_{\Omega_+} f_r(x)\,cos(\theta\,)d\omega \;= f_r(x)\int_0^{\frac{\pi}{2}}\int_0^{2\pi} cos(\theta)sin(\theta)d\theta d\phi = \pi\, f_r(x)$$

- Reflectance factor or albedo: between [0, 1]

Direction-independent out-going radiance

$$L(x,\omega_0) = L_e(x,\omega_0) + f_r(x\,)\int_{y\in S} L_i\big(x,\omega(x,y)\big)G(x,y)d\mathrm{A}_y = L_e(x,\omega_0) + f_r(x\,)E(x)$$

$$= L_o(x)$$

Form factor

$$F(x,y) = \frac{G(x,y)}{\pi}$$

- Defines percentage of light leaving dA_y and arriving at dA

Radiosity

$$B = \int_{\Omega_+} L_o(x, \omega_o) \cos\theta_o \, d\omega_o = L_o f_r(x) \int_{\Omega_+} \cos\theta_o \, d\omega_o = \pi L_o$$

$$B(x) = \pi L_e(x) + \pi f_r(x) E(x) = B_e(x) + \rho(x) E(x)$$

Irradiance

$$E(x) = \int_{y \in S} L_i\big(x, \omega(x,y)\big) G(x,y) \, dA_y = \int_{y \in S} L_o\big(y, -\omega(x,y)\big) G(x,y) \, dA_y$$

$$= \int_{y \in S} \frac{B(y)}{\pi} G(x,y) \, dA_y = \int_{y \in S} B(y) F(x,y) \, dA_y$$

Properties

- Fredholm integral of 2nd kind

$$B(x) = B_e(x) + \rho(x) \int_{y \in S} F(x,y) B(y) dA_y$$

- Global linking
    - Potentially each point with each other
    - Often sparse systems (occlusions)
- No consideration of volume effects!!

$$f(x) = g(x) + K[f(x)]$$

Linear operator

$$K[f(x)] = \int k(x,y) f(y) dy$$

- Acts on functions like matrices act on vectors
- Superposition principle
- Scaling and addition

$$K[af + bg] = aK[f] + bK[g]$$

$$B(x) = B_e(x) + \rho(x) \int_{y \in S} F(x,y) B(y) dA_y$$

Integral equation

$$B(\cdot) = B_e(\cdot) + K[B(\cdot)] \Rightarrow (I - K)[B(\cdot)] = B_e(\cdot)$$

Formal solution

$$B(\cdot) = (I - K)^{-1}[B_e(\cdot)]$$

Neumann series

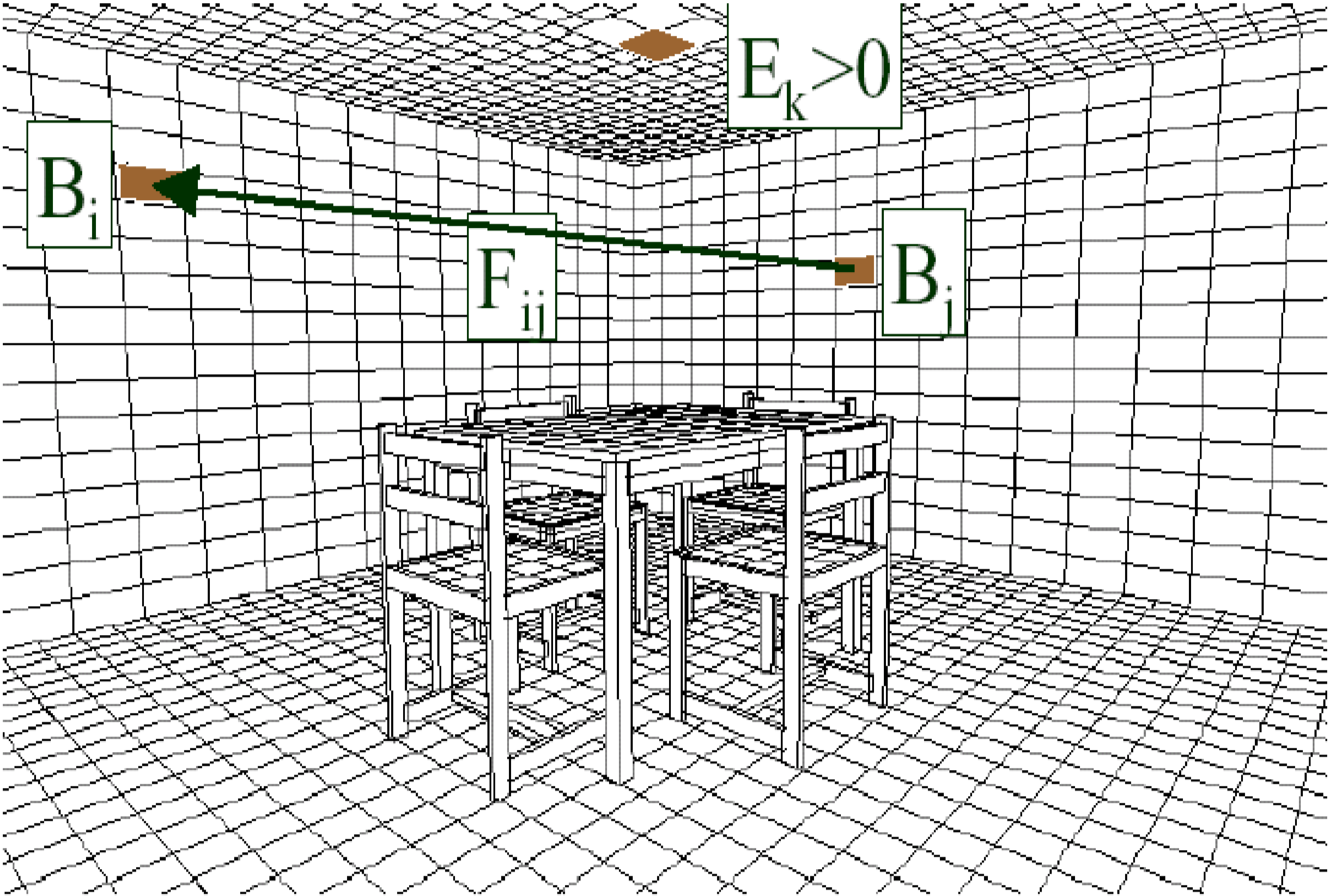
- Converges only if $|K| < 1$ which is true in all physical settings
- $\frac{1}{1-x} = 1 + x + x^2 + \cdots$
- $\frac{1}{I-K} = I + K + K^2 + \cdots$
- $(I - K)\frac{1}{I-K} = (I - K)(I + K + K^2 + \cdots) = I + K + K^2 + \cdots - (K + K^2 + \cdots) = I$

Successive approximation

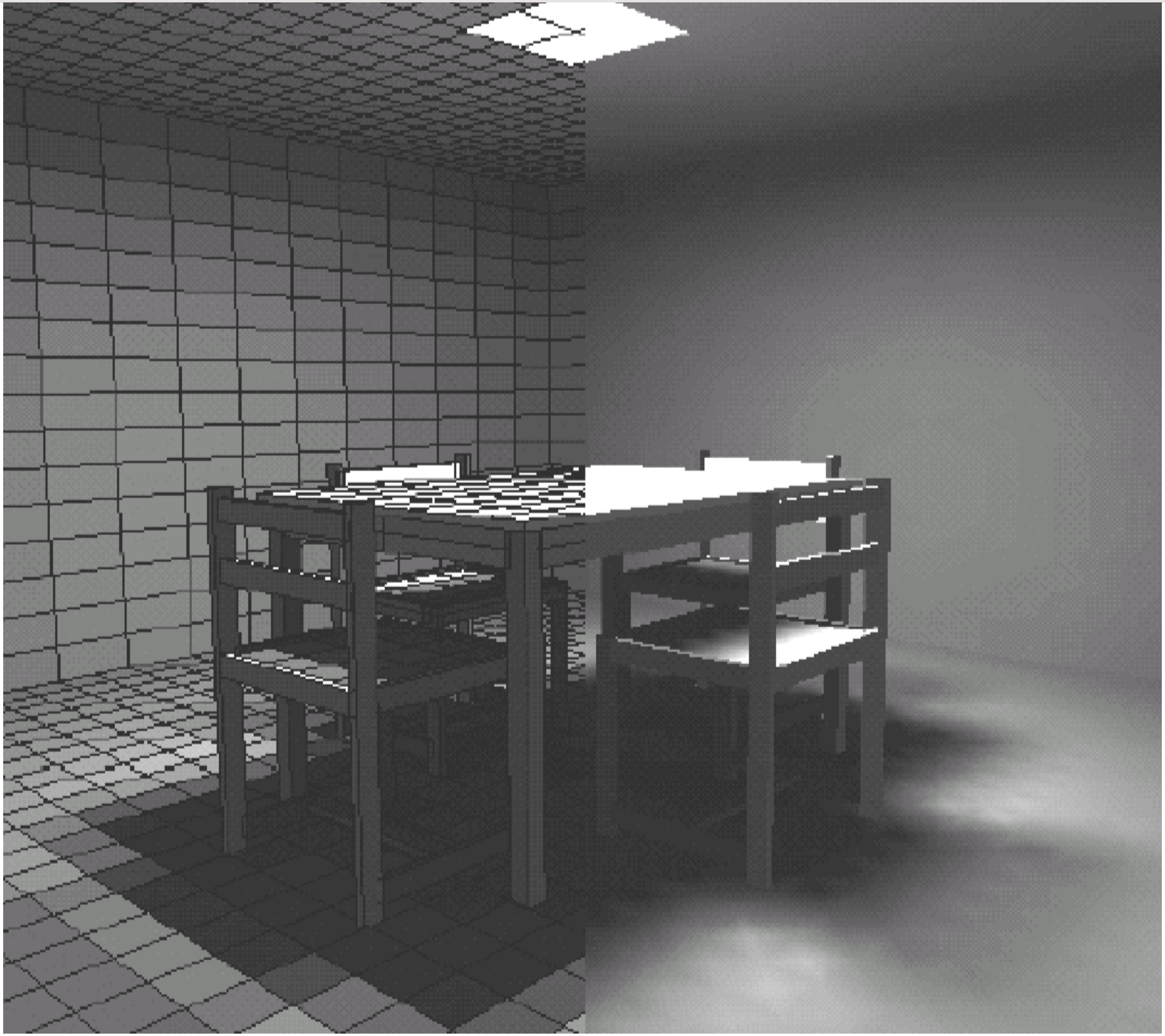
$$\frac{1}{I-K}B_e(\cdot) = B_e(\cdot) + K[B_e(\cdot)] + K^2[B_e(\cdot)] + \cdots$$

$$= B_e(\cdot) + K\big[B_e(\cdot) + K[B_e(\cdot) + \cdots]\big]$$

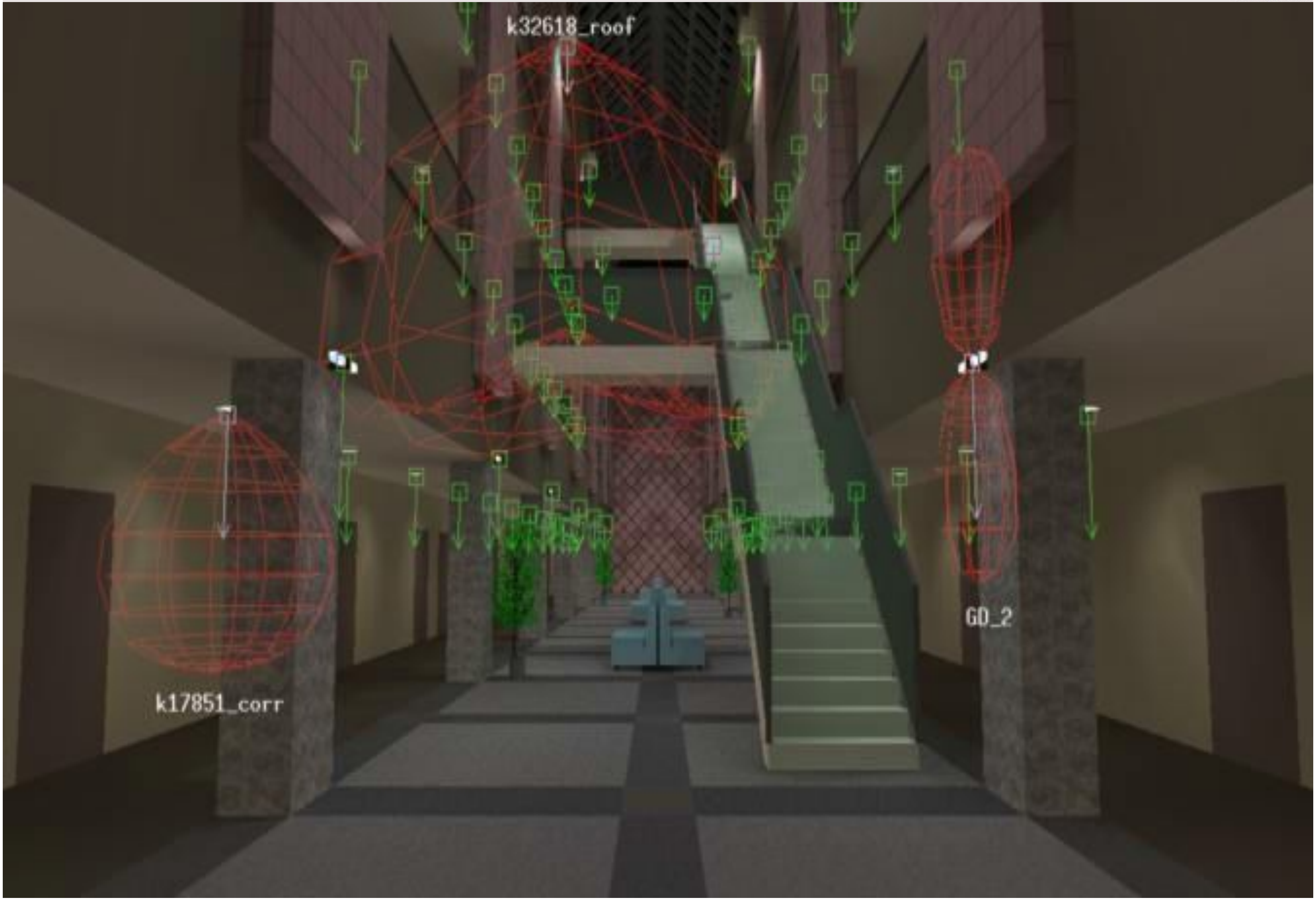
- Direct light from the light source — $B_1(\cdot) = B_e(\cdot)$
- Light which is reflected and transported one time — $B_2(\cdot) = B_e(\cdot) + K[B_e(\cdot)]$
- Light which is reflected and transported *n* times — $B_n(\cdot) = B_e(\cdot) + K[B_{n-1}(\cdot)]$


$E_k>0$
B_i
F_{ij}
B_j

Radiosity Algorithm



Lighting Simulation



Lighting Simulation

Lighting Simulation

Physical Quantities in Rendering

- Radiance
- Radiosity
- Irradiance
- Intensity

Light Perception

Light Source Definition

Rendering Equation

- Key equation in graphics (!)
- Integral equation
- Describes global balance of radiance

Radiosity

- Diffuse reflectance function
- Radiative equilibrium between emission and absorption, escape
- System of linear equations
- Iterative solution