Lecture 3:

Ray Tracing 1

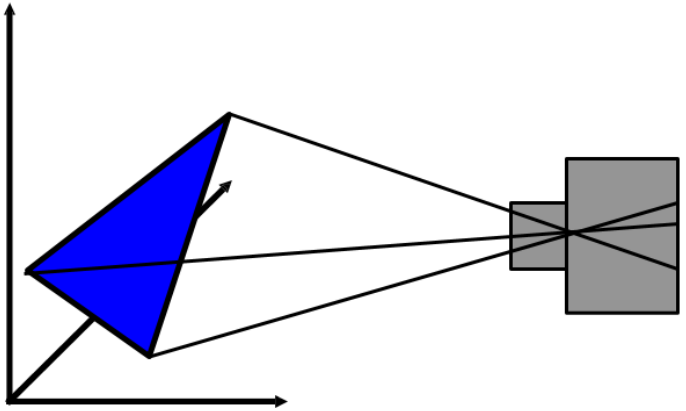
Contents

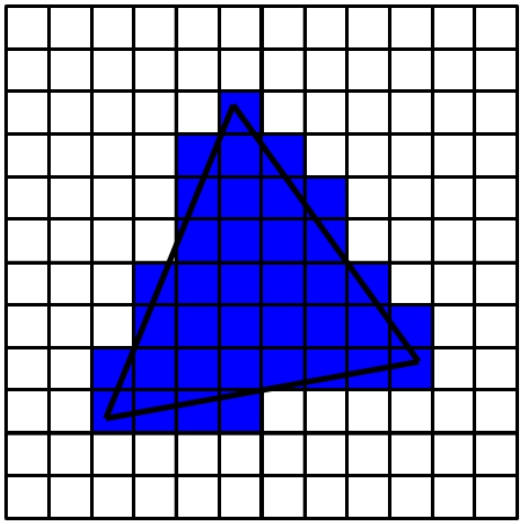
Rendering

- Given a camera and 3D scene as input, generate a 2D image as a view from the camera of the 3D scene

Algorithms: *Rasterization*

- **Primitive operation of all interactive graphics!**
  - Scan convert a single triangle at a time
- **Sequentially processes every triangle individually**
  - Can never access more than one triangle
- But most effects need access to the world: shadows, reflection, global illumination, *etc.*



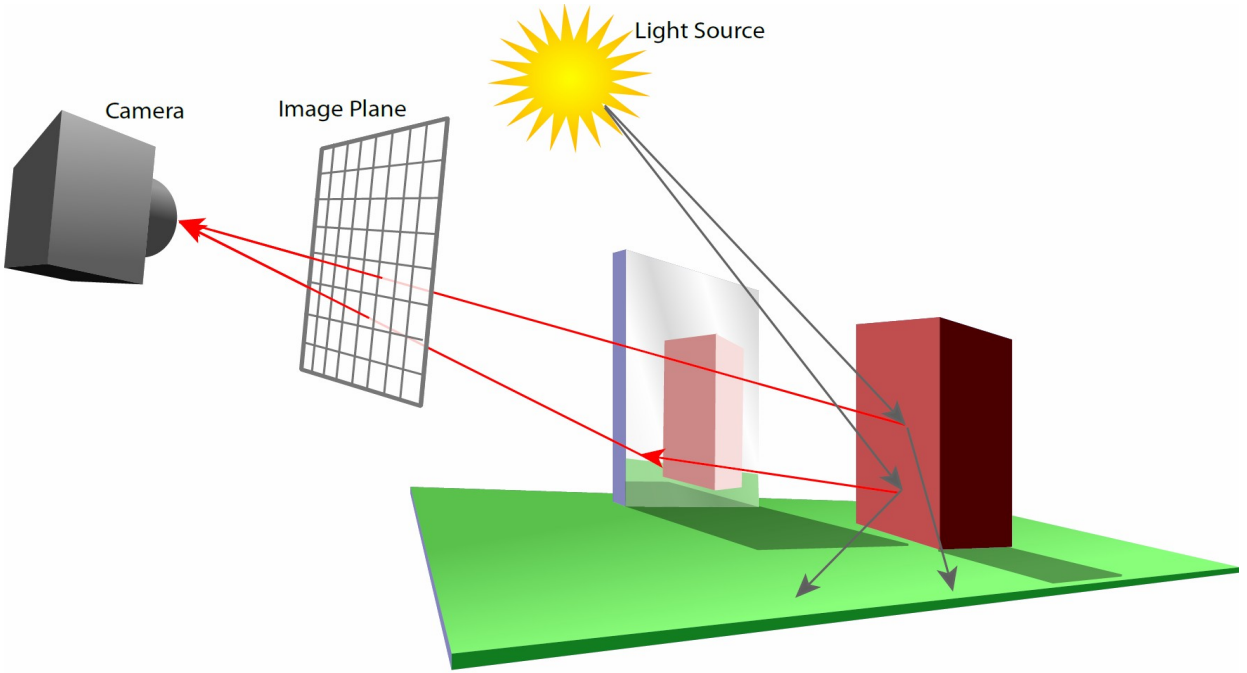


Rendering

- Given a camera and 3D scene as input, generate a 2D image as a view from the camera of the 3D scene

Algorithms: *Ray Tracing*

- Nature:
  - Follow the path of ***many*** photons
  - Record those hitting the film in a camera



Light Distribution in a Scene

- Dynamic equilibrium
- Newly created, scattered, and absorbed photons

Forward Light Transport

- Shoot photons from the light sources into scene
- Reflect at surfaces (according to some reflection model)
- Wait until they are absorbed or hit the camera (very seldom)
- **Nature:** massive parallel processing at the speed of light

Backward Light Transport

- Start at the camera
- Trace only paths that might transport light towards the camera
  - May try to connect to occluded light sources
- Ray Tracing

Surfaces

- 3D geometry of objects in a scene

Surface reflectance characteristics

- Color, texture, absorption, reflection, refraction, subsurface scattering
- Local property may vary over surface
- Mirror, glass, glossy, diffuse, …

Illumination

- Position and emission characteristics of light sources
- Repeatedly reflected light / indirect illumination

Assumption: air/empty space is totally transparent

- Simplification that excludes scattering effects in participating media volumes
- Later also volume objects, *e.g.* smoke, solid object (CT scan), …

Camera

- View point, viewing direction, field of view, resolution, …

One of the two fundamental rendering algorithms

Automatic, simple and intuitive

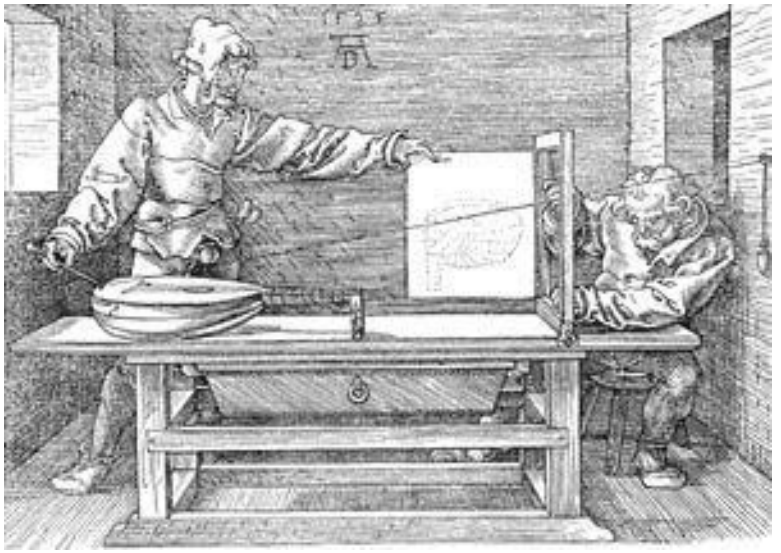
- Easy to understand and implement
- Delivers “correct“ images by default

Powerful and efficient

- Many optical global effects
- Shadows, reflections, refractions, …
- Efficient real-time implementation in SW and HW
- Can work in parallel and distributed environments
- Logarithmic scalability with scene size: *O(log n)* vs. *O(n)*
- Output sensitive and demand driven

Concept of light rays is not new

- Empedocles (492-432 BC), Renaissance (Dürer, 1525), …
- Uses in lens design, geometric optics, …



Perspective Machine, Albrecht Dürer

Produce Highly Realistic Images

- Ray tracing enables *correct* simulation of light transport

Ray generation

- Rays from viewpoint along viewing directions into 3D scene
- (At least) one ray per picture element (pixel)

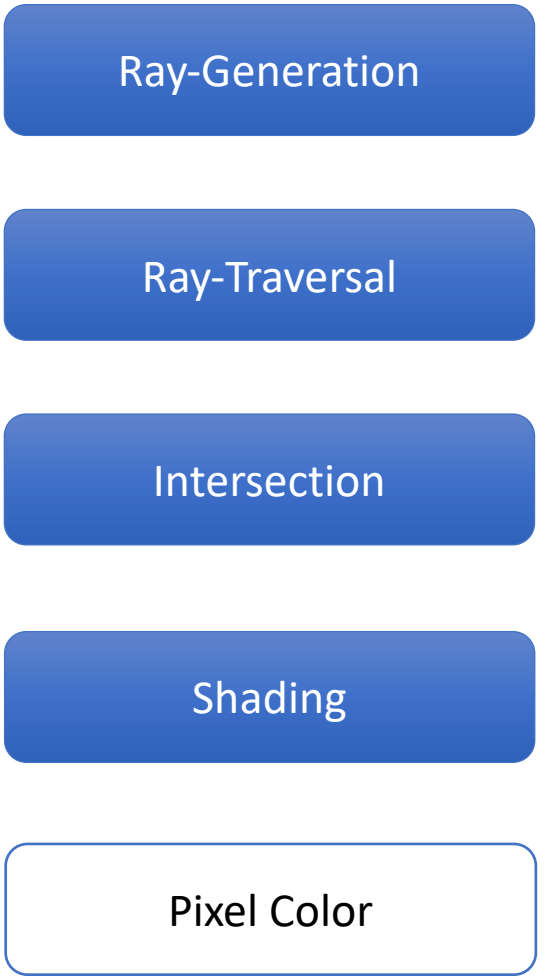
Ray traversal

- Traversal of spatial index structures

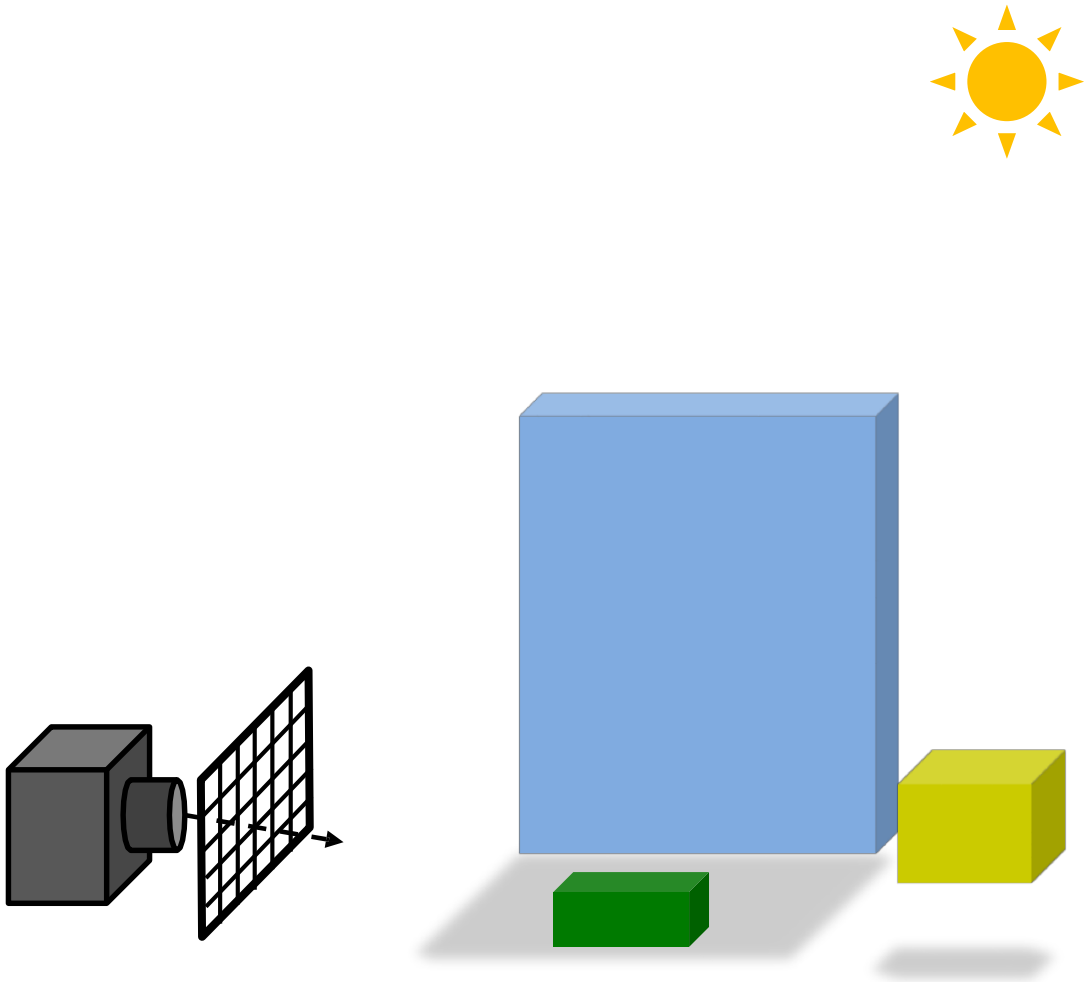
Ray-primitive intersection

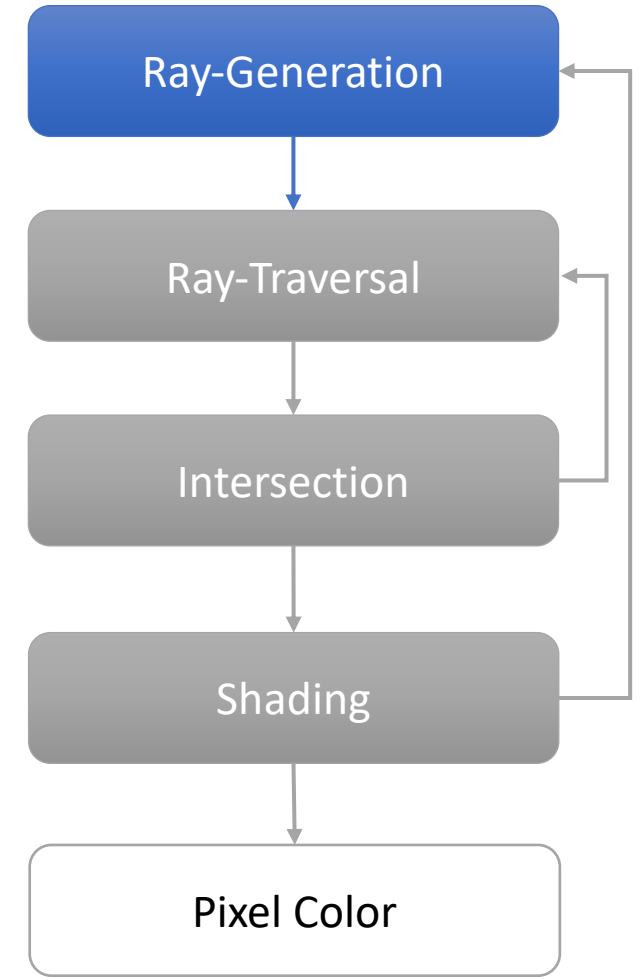
Shading the hit point

- Determine pixel color
  - Energy (color) travelling along primary ray
- Needed
  - Local material color, object texture and reflection properties
  - Local illumination at intersection point
    - Compute through recursive tracing of rays
    - Can be hard to determine correctly



Ray Tracing Pipeline





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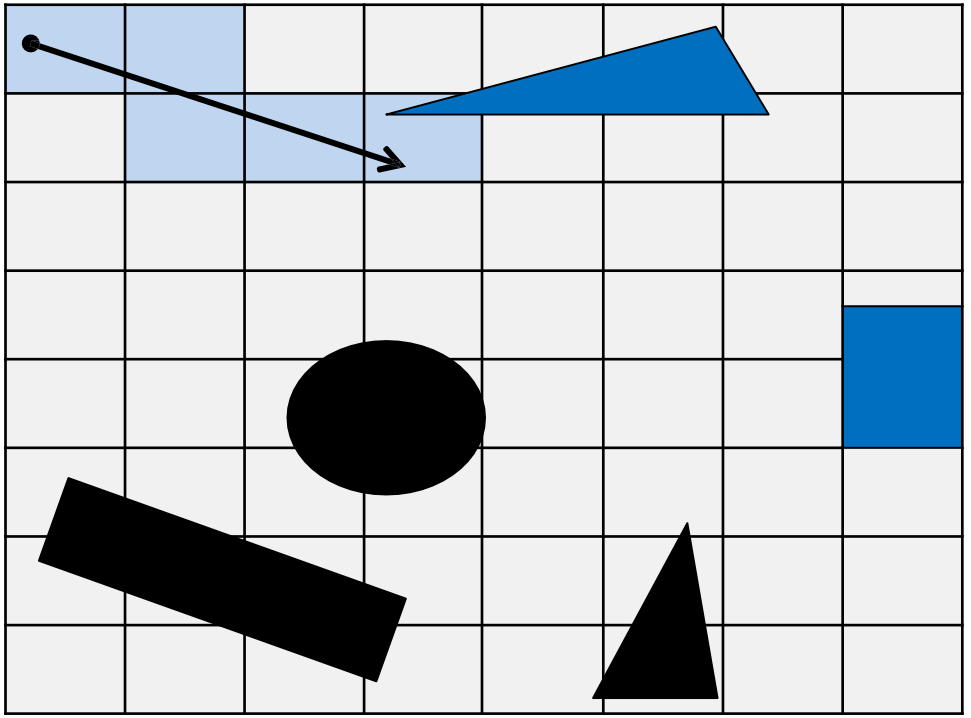

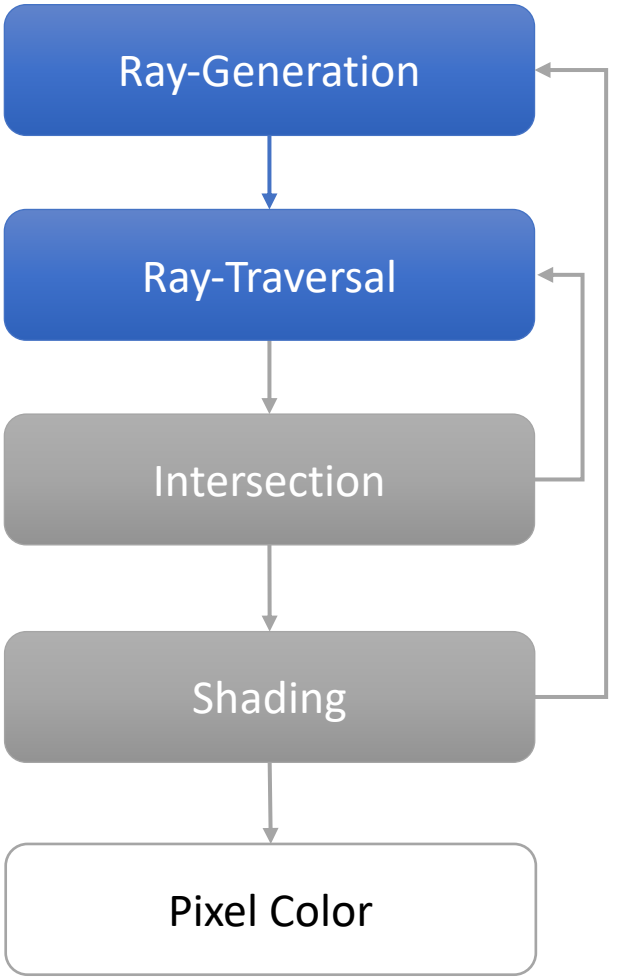

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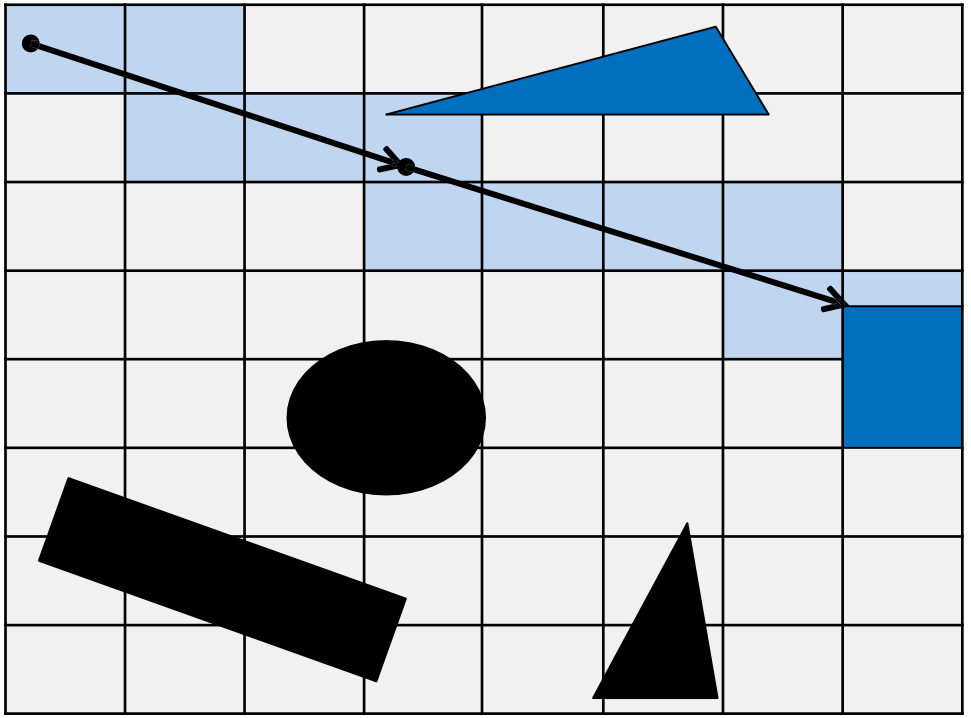

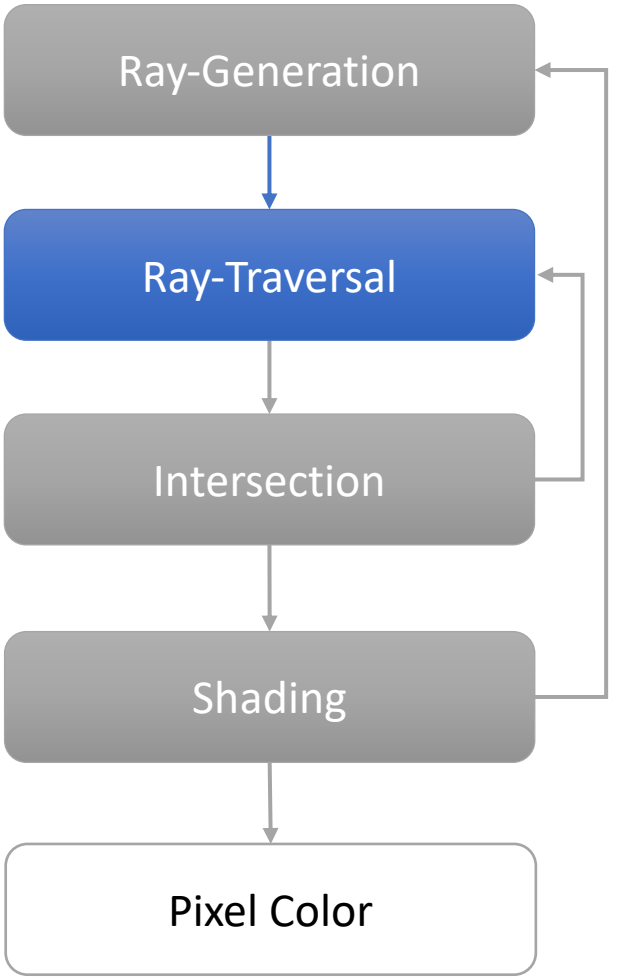

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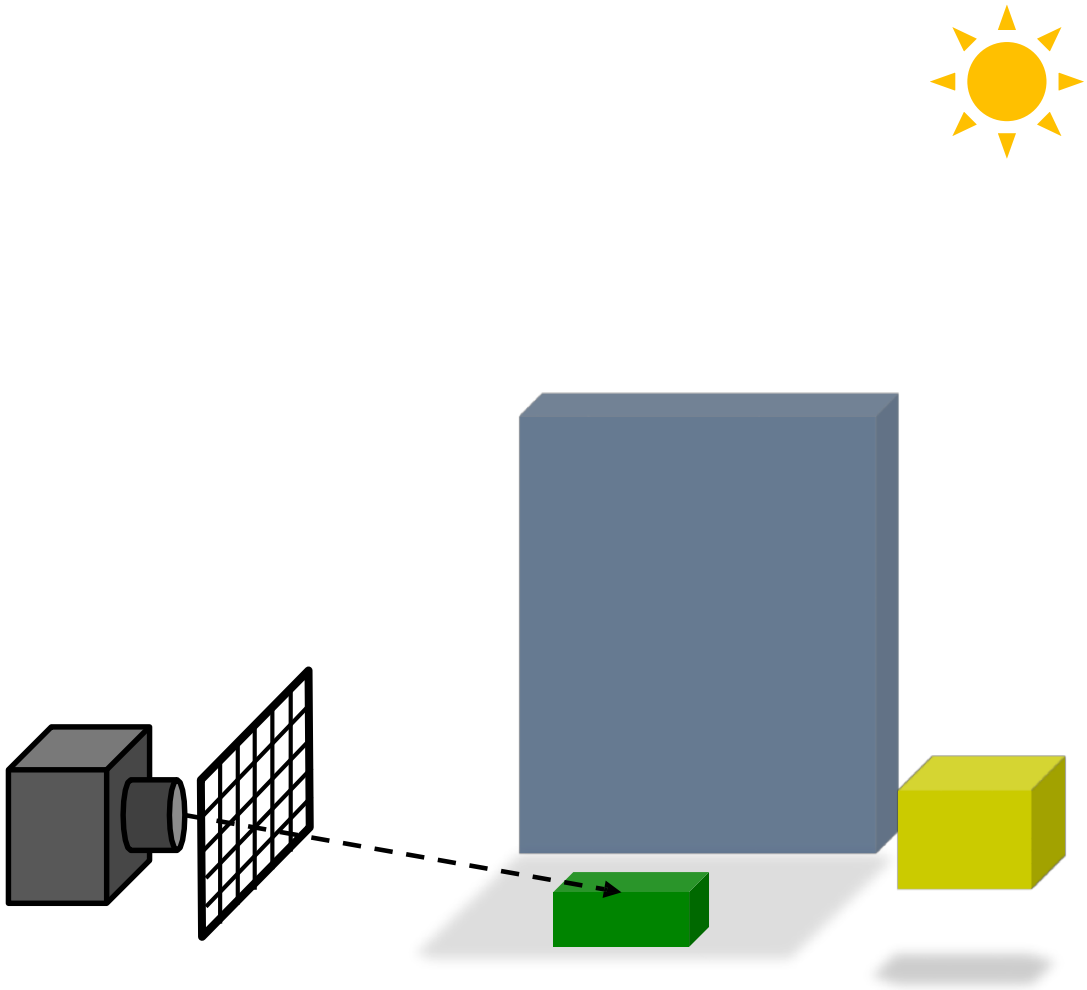

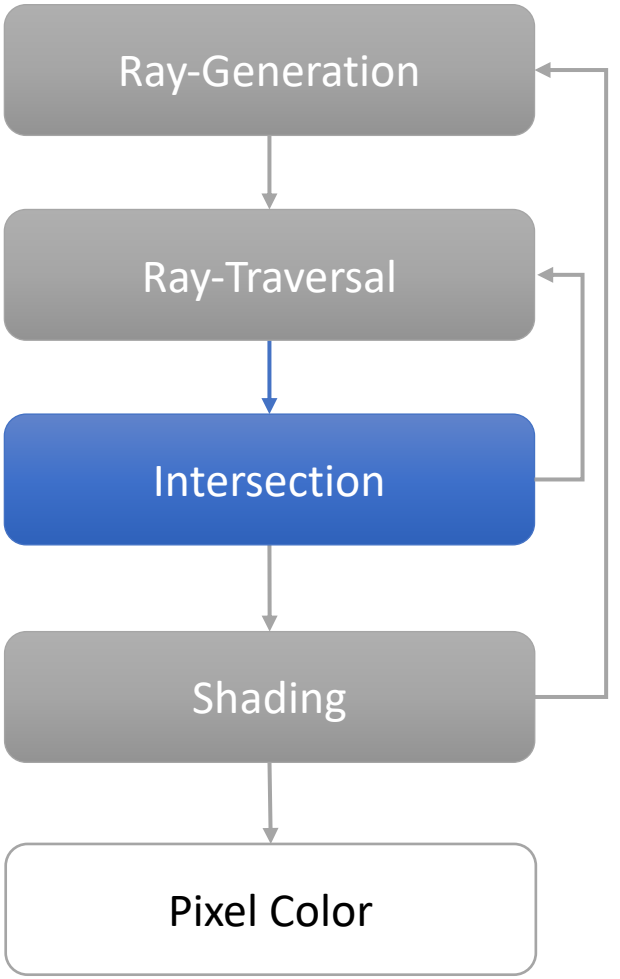

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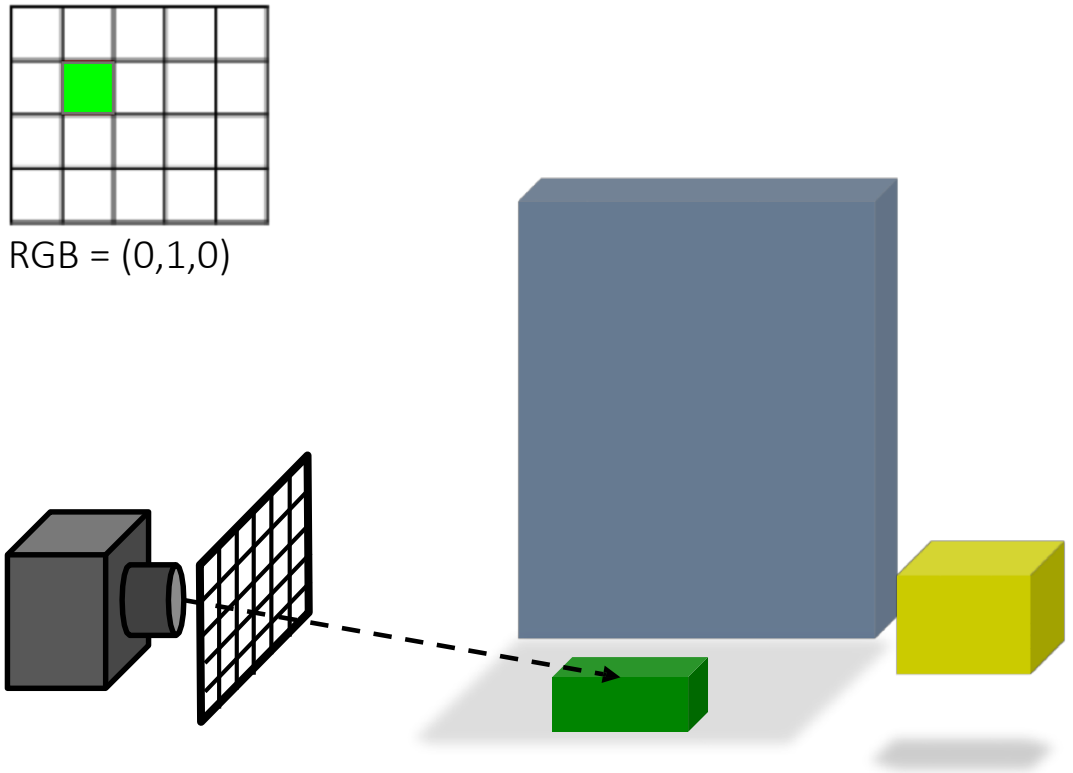

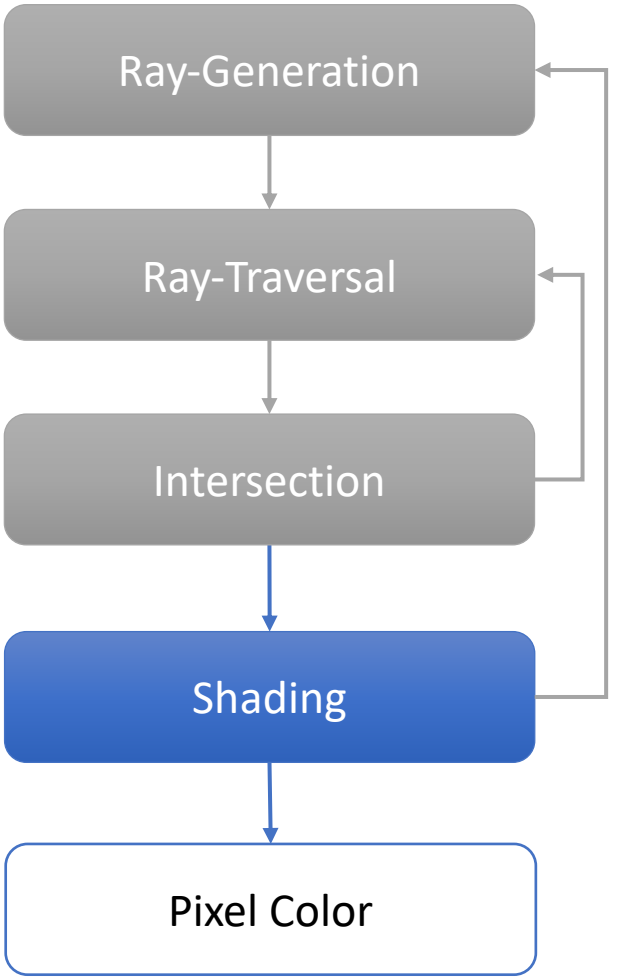

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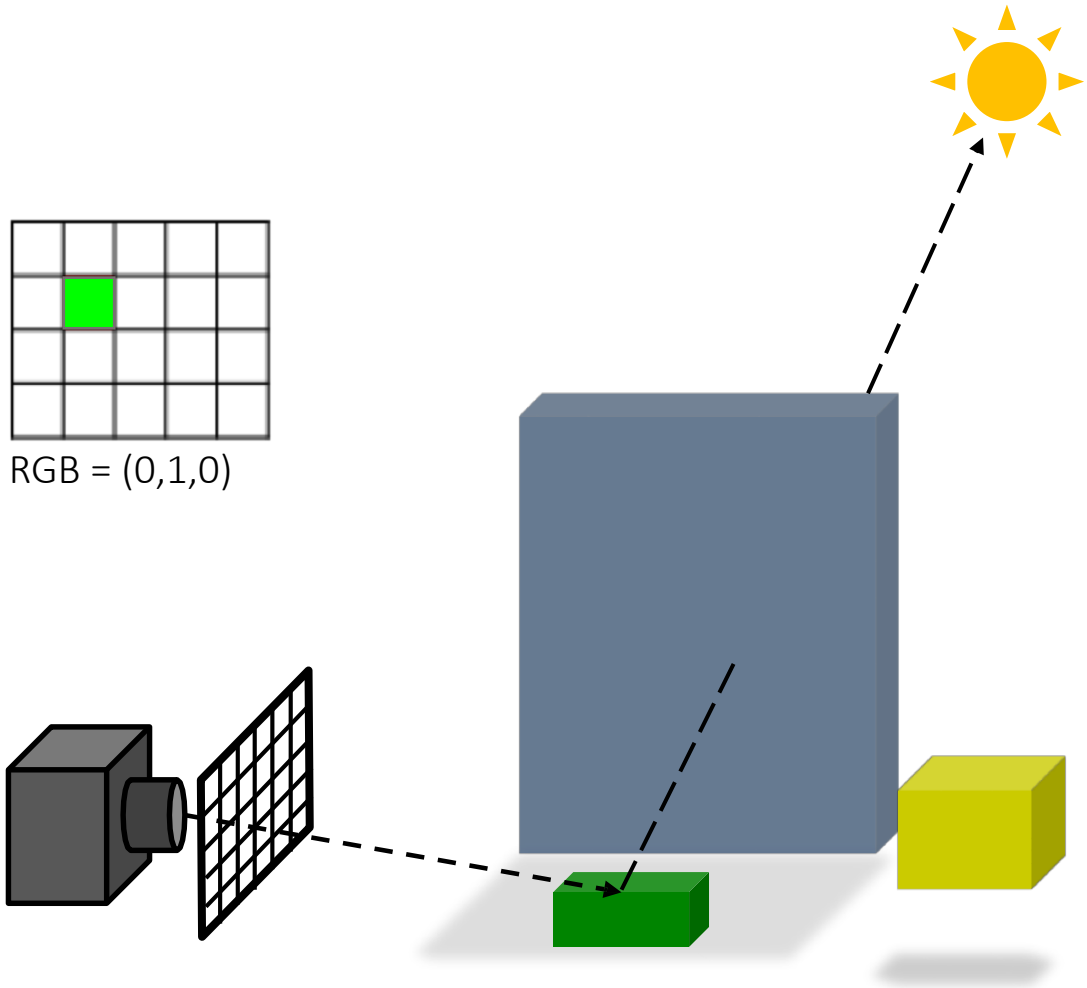

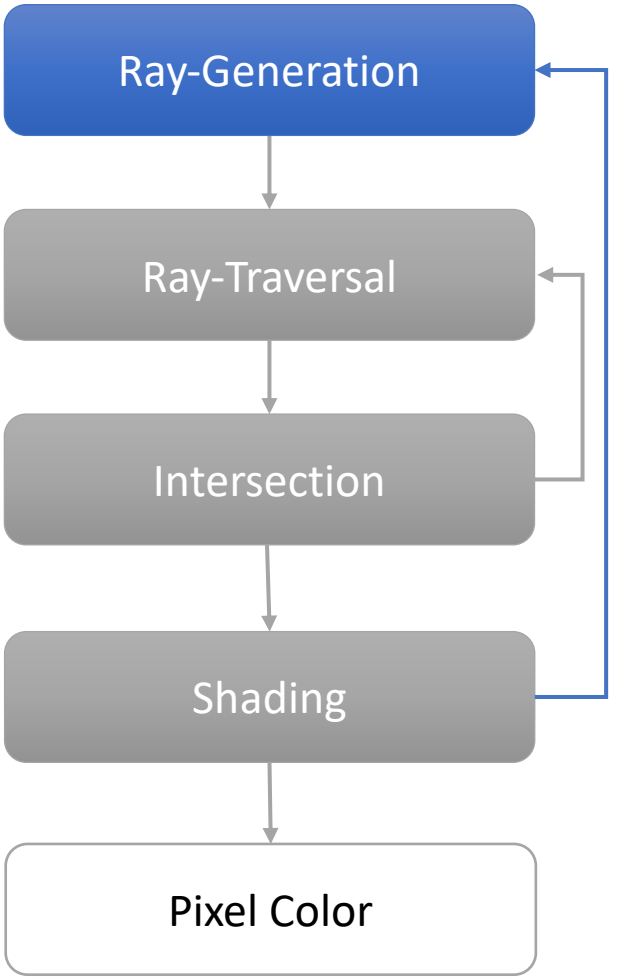

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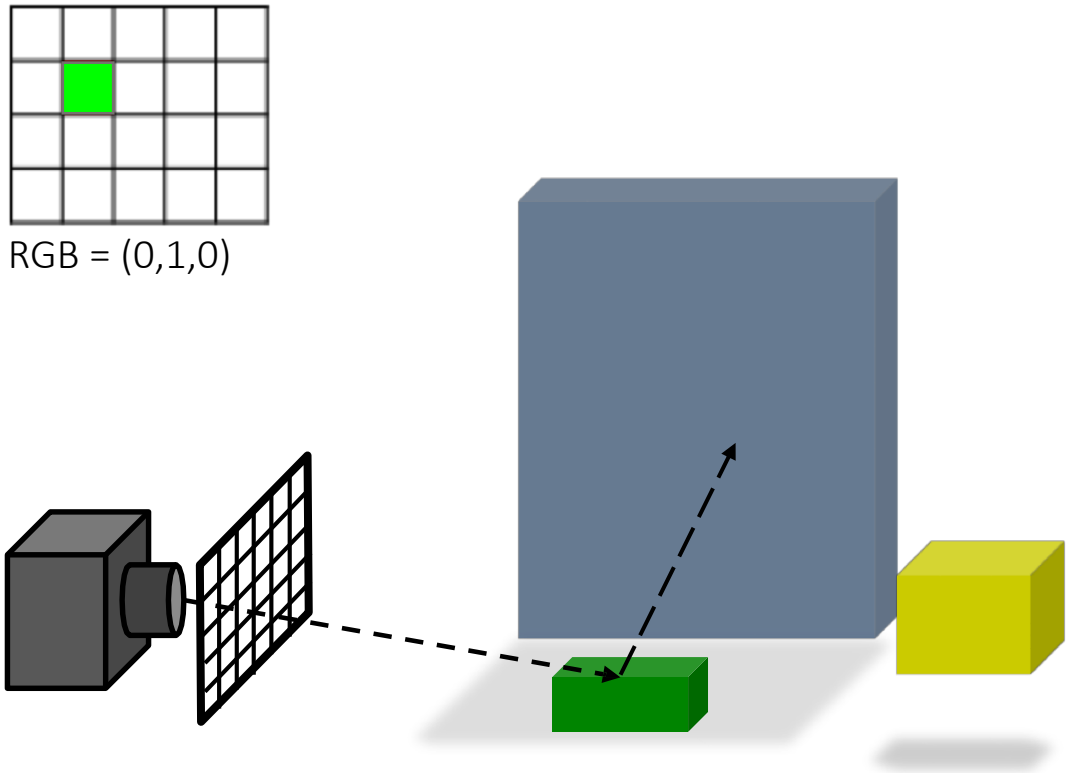

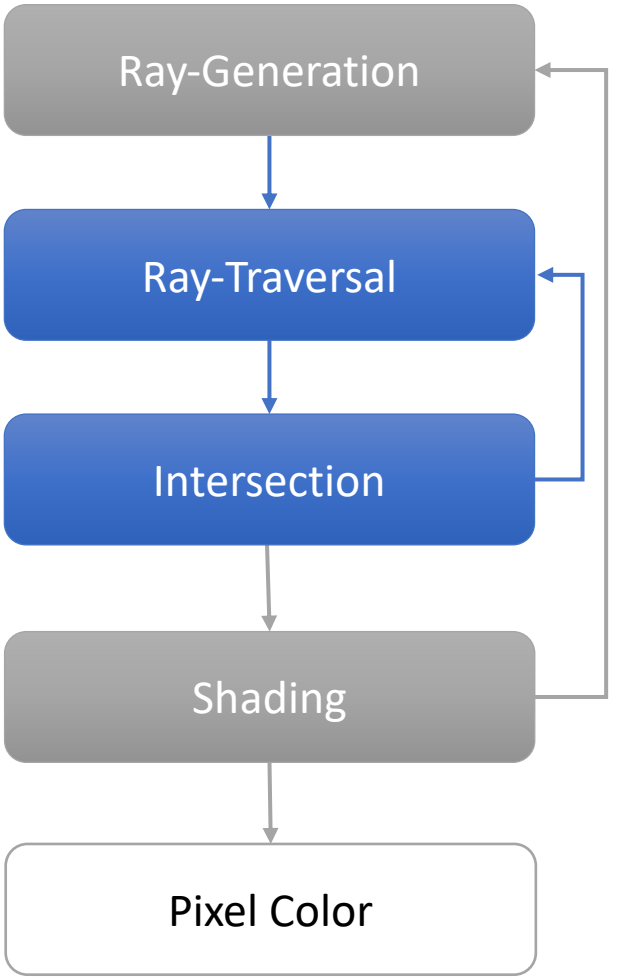

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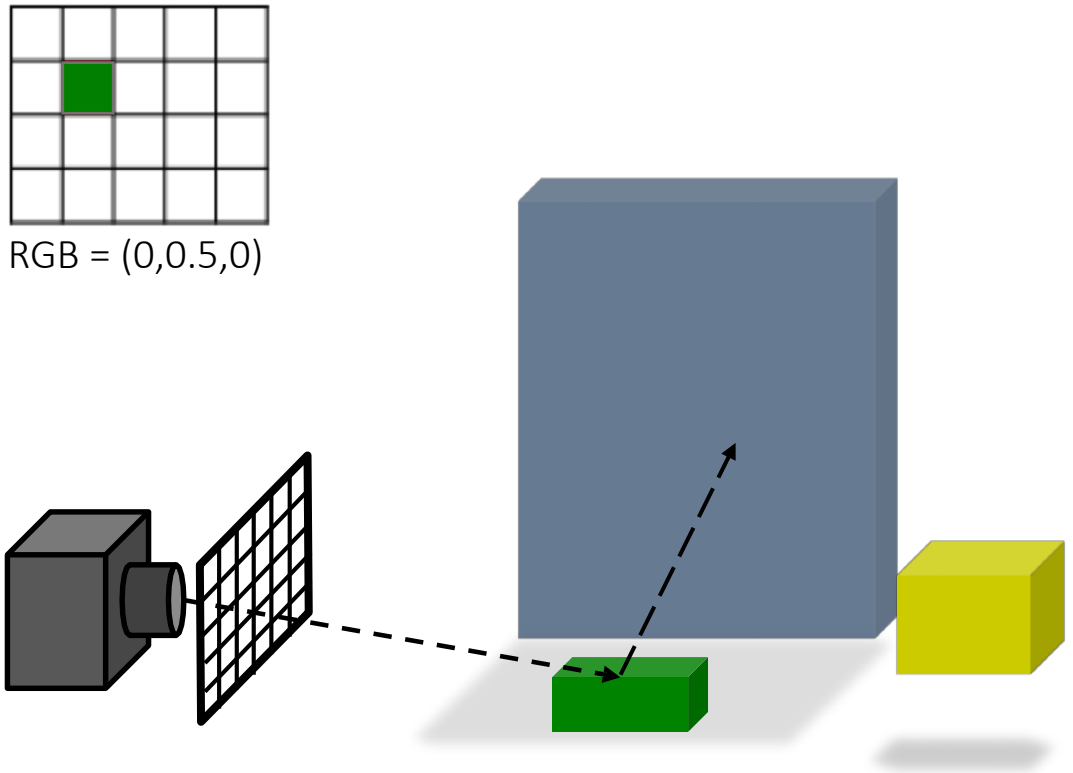

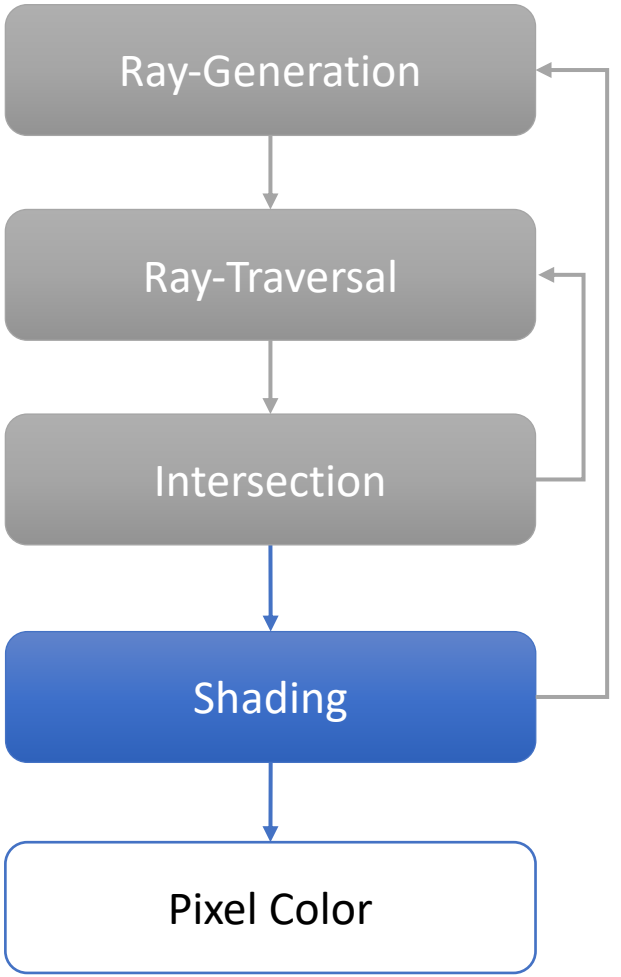

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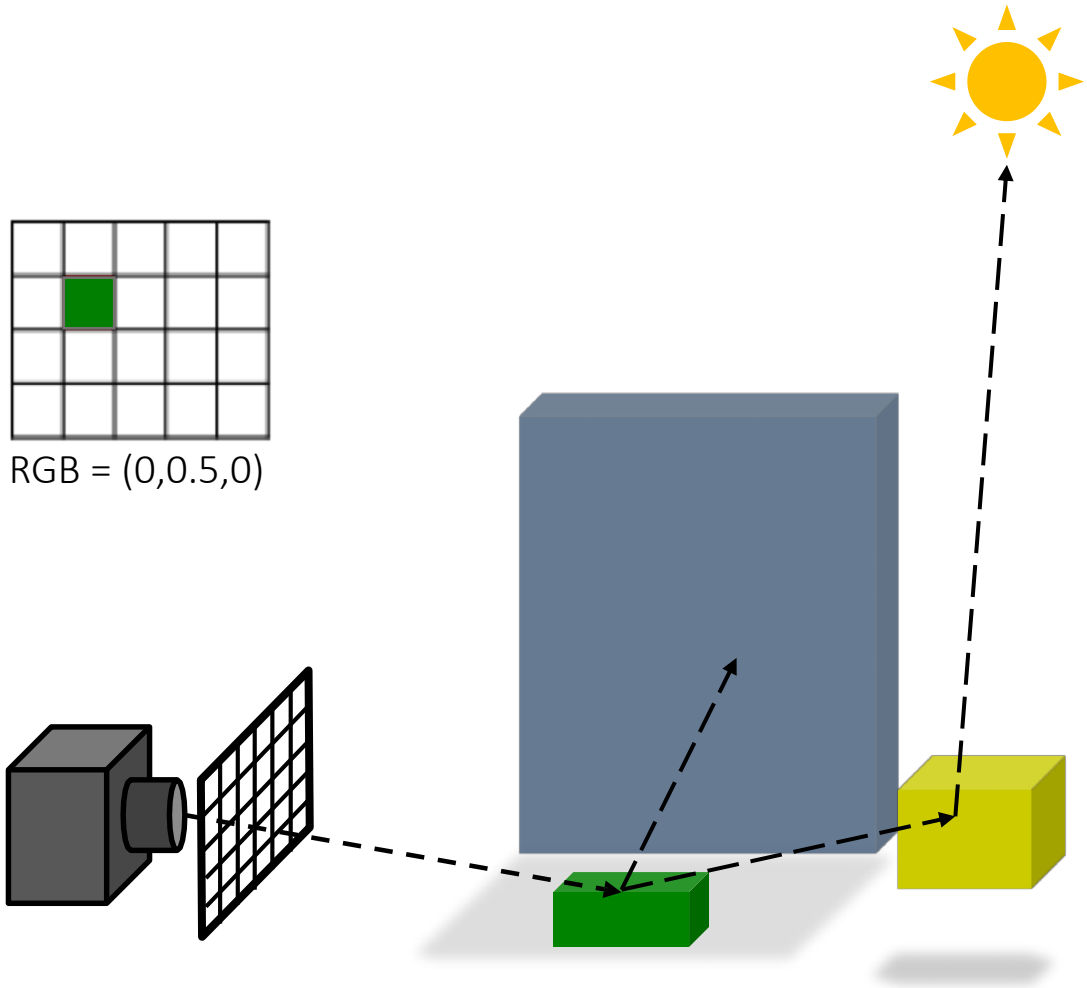

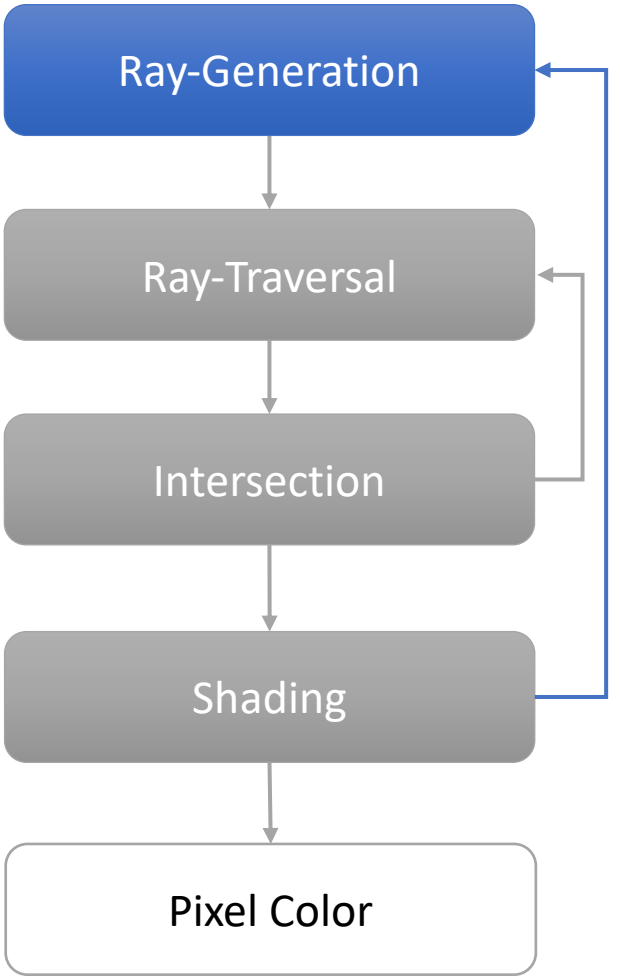

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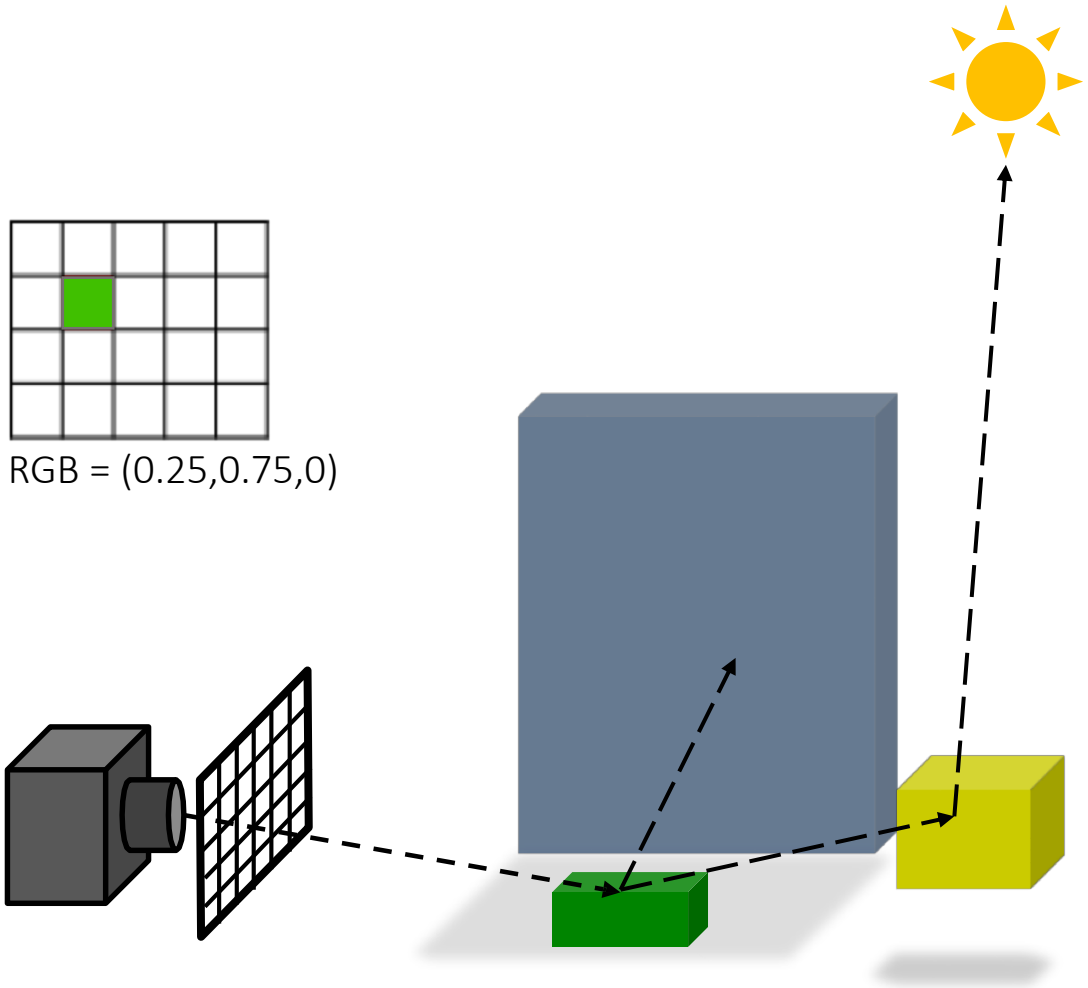

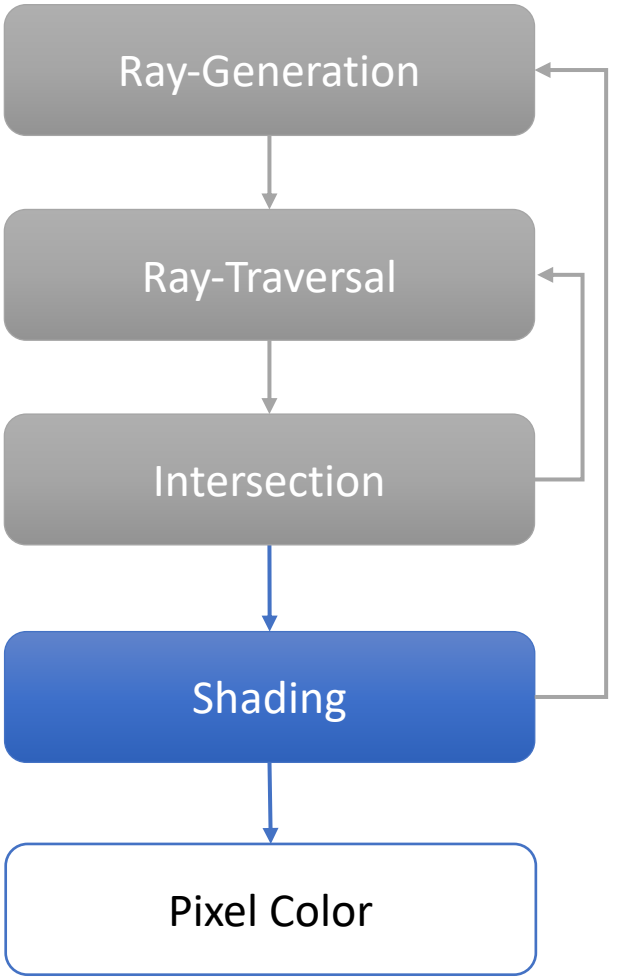

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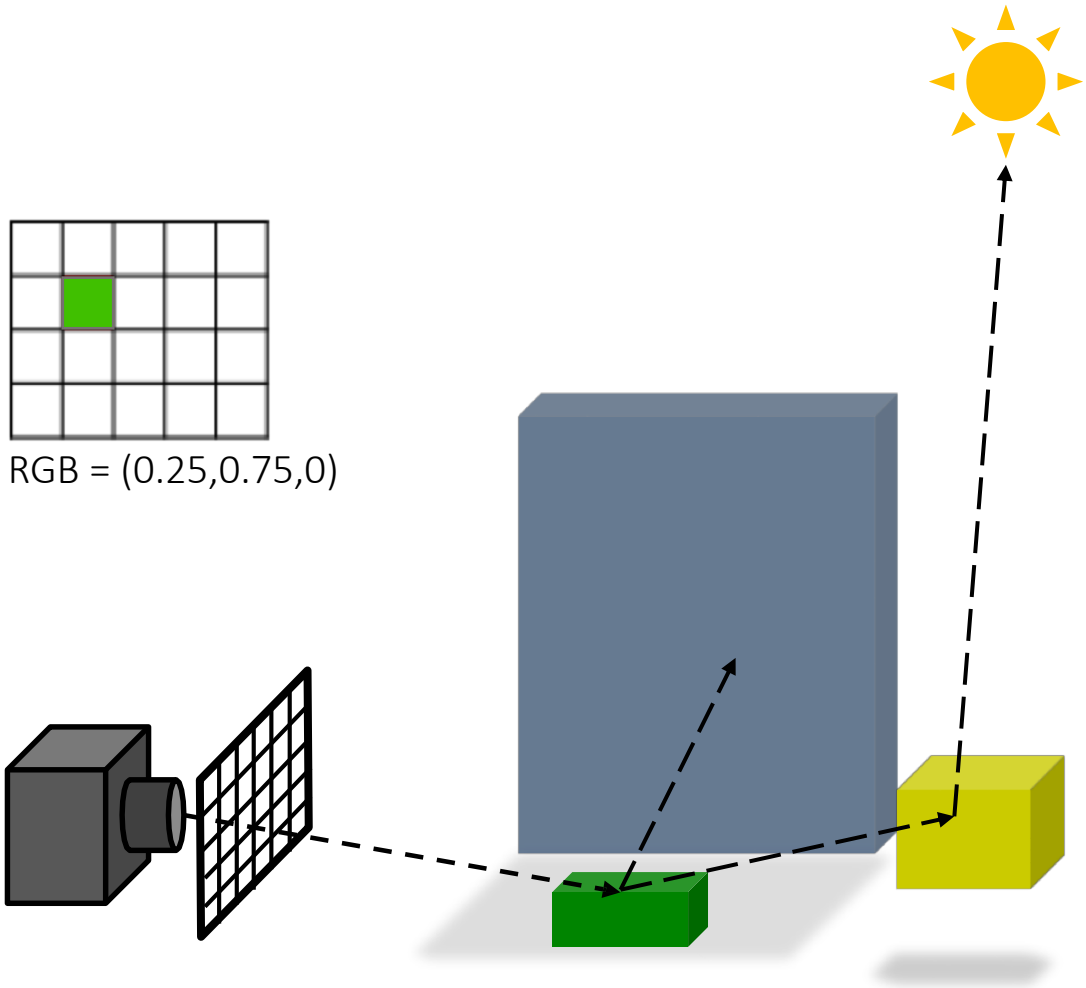
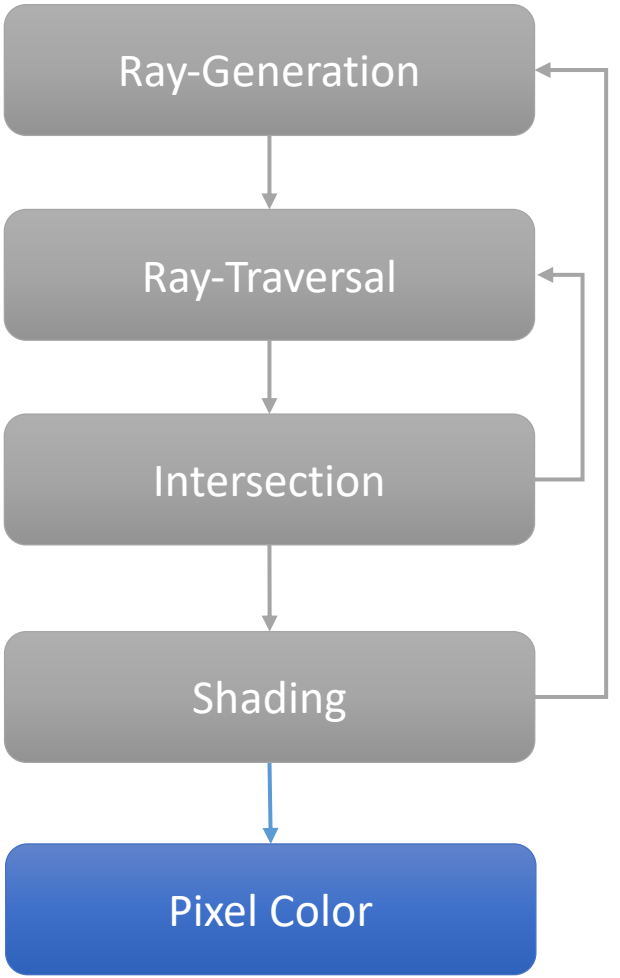
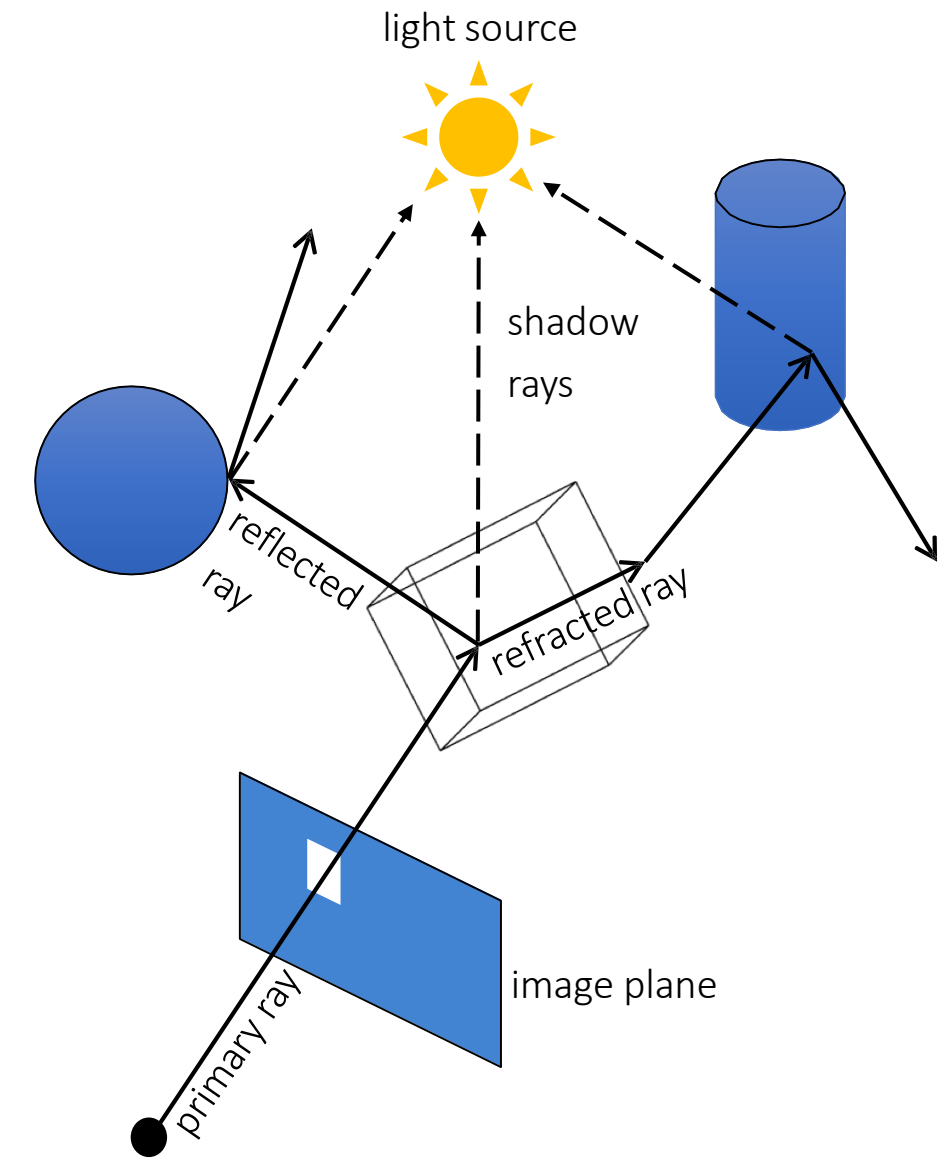
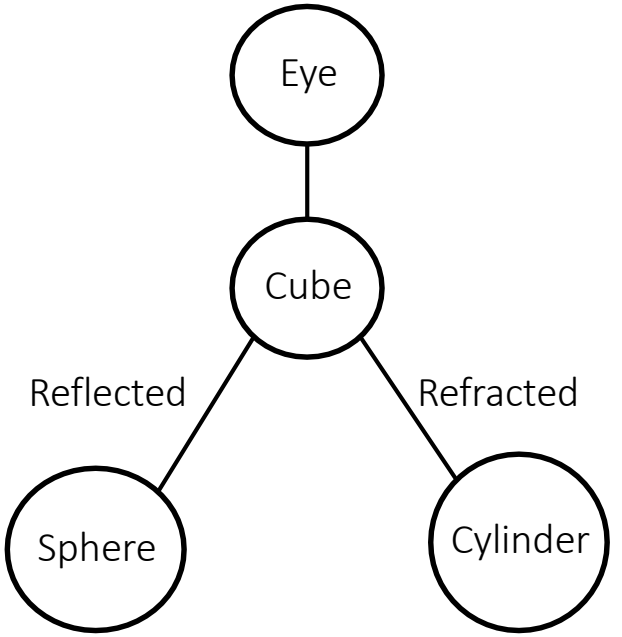






Searching recursively for paths to light sources

- Interaction of light & material at intersections
- Recursively trace new rays in reflection, refraction, and light direction



`trace(ray)`

- Search the next intersection `point(hit, material)`

`return shade(ray, hit, material)`

`shade(ray, hit, material)`

`for_each(light source)`

`if (shadowTrace(ray to light source, distance to light))`

- Calculate reflected radiance (*i.e.* Phong material model)
- Adding to the reflected radiance

`if (mirroring material)`

- Calculate radiance in reflected direction: `trace(R(ray, hit))`
- Adding mirroring part to the reflected radiance

- Same for transmission
- Return reflected radiance

`shadowTrace(ray, dist)`

- `return false`, if intersection with distance < dist has been found
- Can be changed to handle transparent objects as well
  - But not with refraction

Intersection point determines primary ray's "color"

- Diffuse object: color at intersection point
  - No variation with viewing angle: diffuse (Lambertian)
- Perfect reflection / refraction (mirror, glass)
  - Only one outgoing direction → Trace one secondary ray
- Non-Lambertian Reflectance
  - Appearance depends on illumination and viewing direction
  - Local Bi-directional Reflectance Distribution Function (BRDF)

Illumination

- Point / directional light sources
- Area light sources
  - Approximate with multiple samples / shadow rays
- Indirect illumination
  - See Realistic Image Synthesis (RIS) course in next semester

More details later

Usually RGB color model instead of full spectrum

Finite # of point lights instead of full indirect light

Approximate material reflectance properties

- **Ambient**: constant, non-directional background light
- **Diffuse**: reflected uniformly in all directions perfect
- **Specular**: reflection, refraction

Used reflection models are often empirical

- Better physically accurate models are available (*e.g.* Ward model)

Ray Tracing Incorporates into a single framework

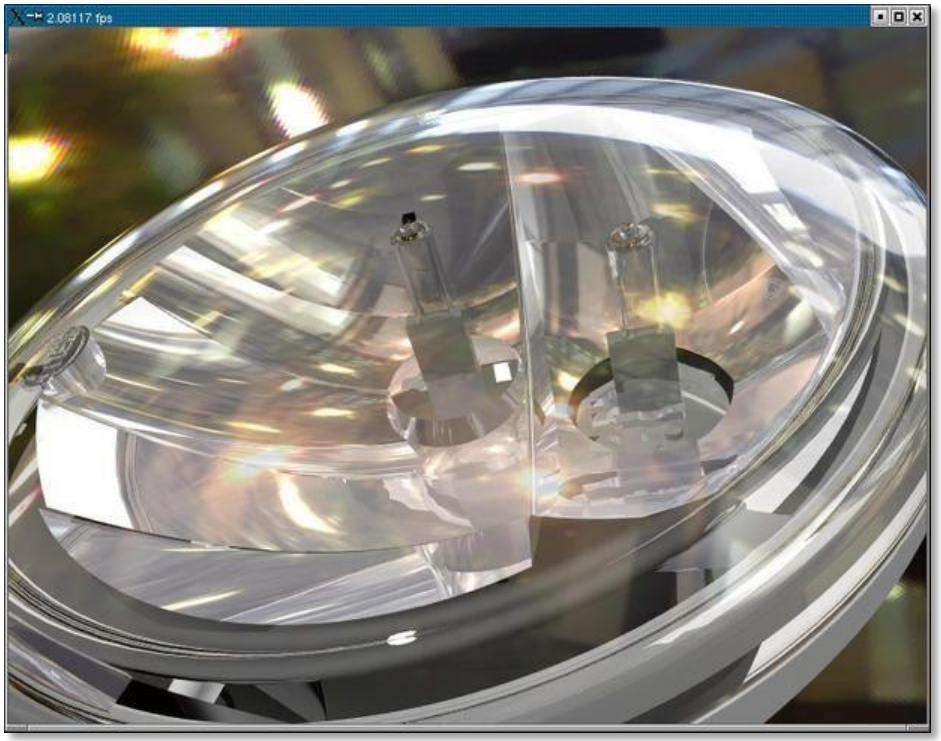
- Hidden surface removal
  - Front to back traversal
  - Early termination once first hit point is found
- Shadow computation
  - Shadow rays / shadow feelers are traced between a point on a surface and a light sources
- Exact simulation of some light paths
  - Reflection (reflected rays at a mirror surface)
  - Refraction (refracted rays at a transparent surface, Snell's law)

Limitations

- Easily gets inefficient for full global illumination computations
- Many reflections (exponential increase in number of rays)
- Indirect illumination requires many rays to sample all incoming directions

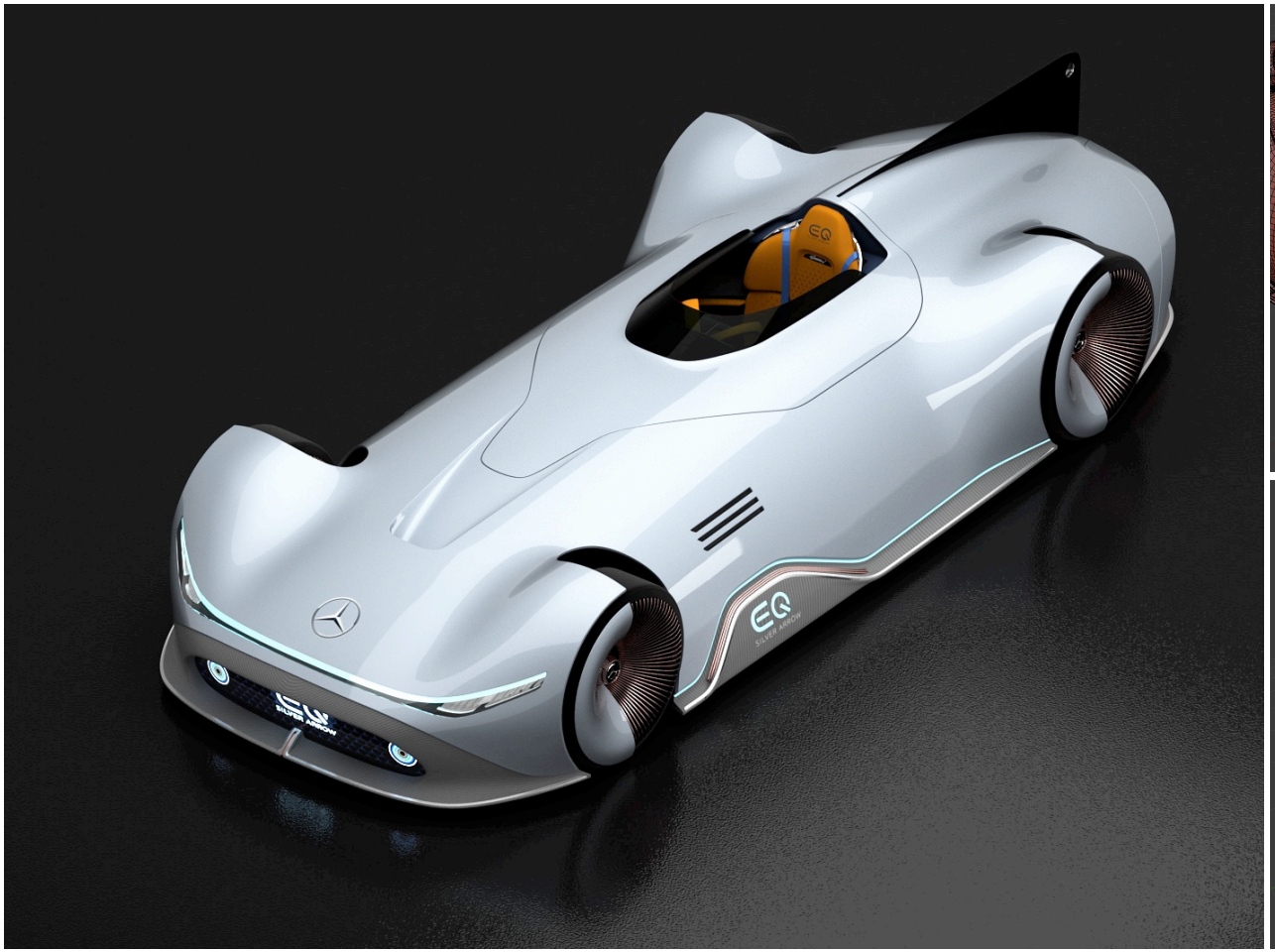
Models Physics of Global Light Transport

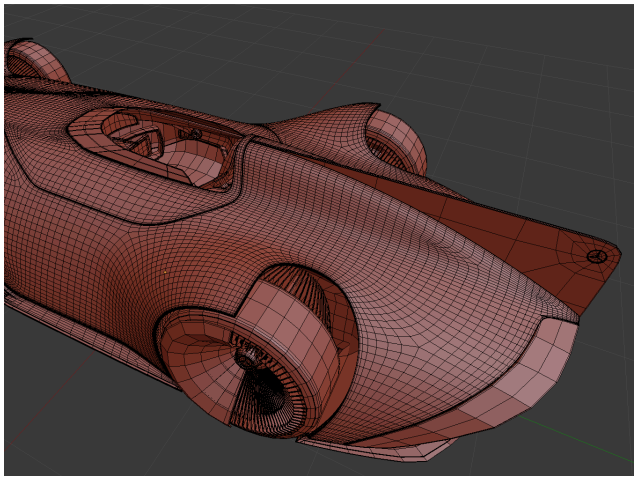
- Dependable, physically-correct visualization

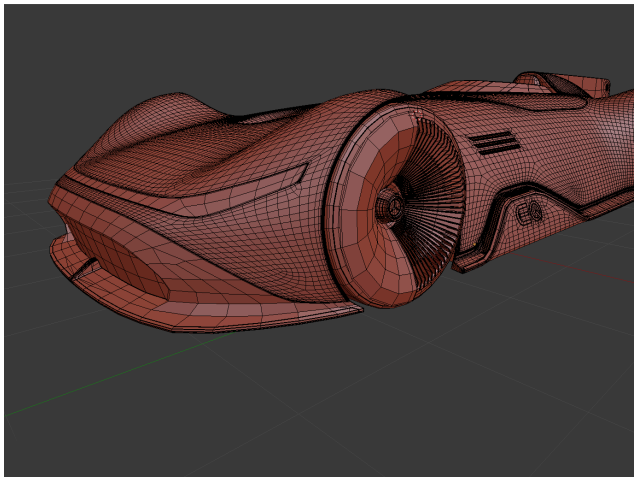


VW Visualization Center

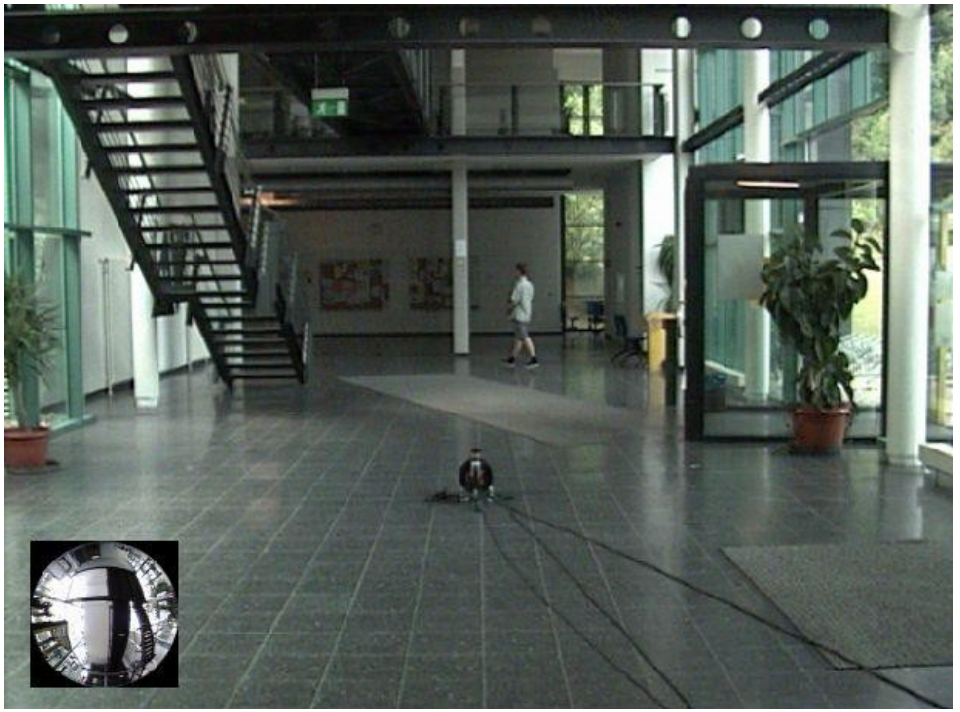
Realistic Visualization: CAD







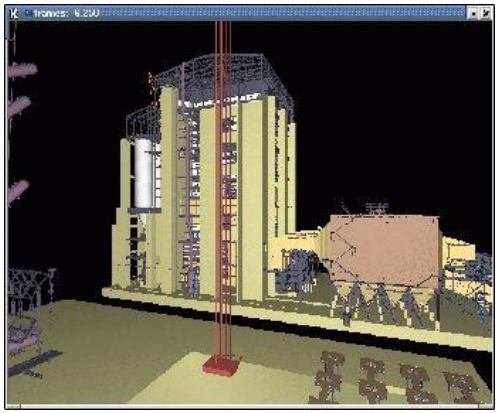
Realistic Visualization: VR / AR

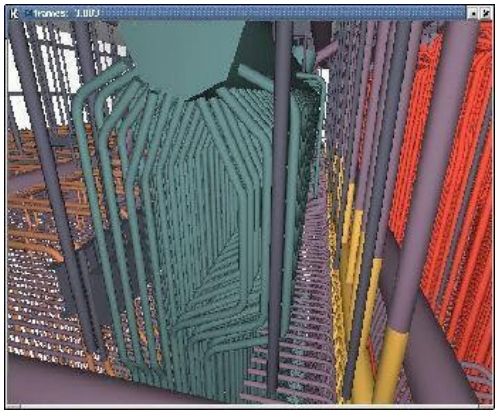


Lighting Simulation

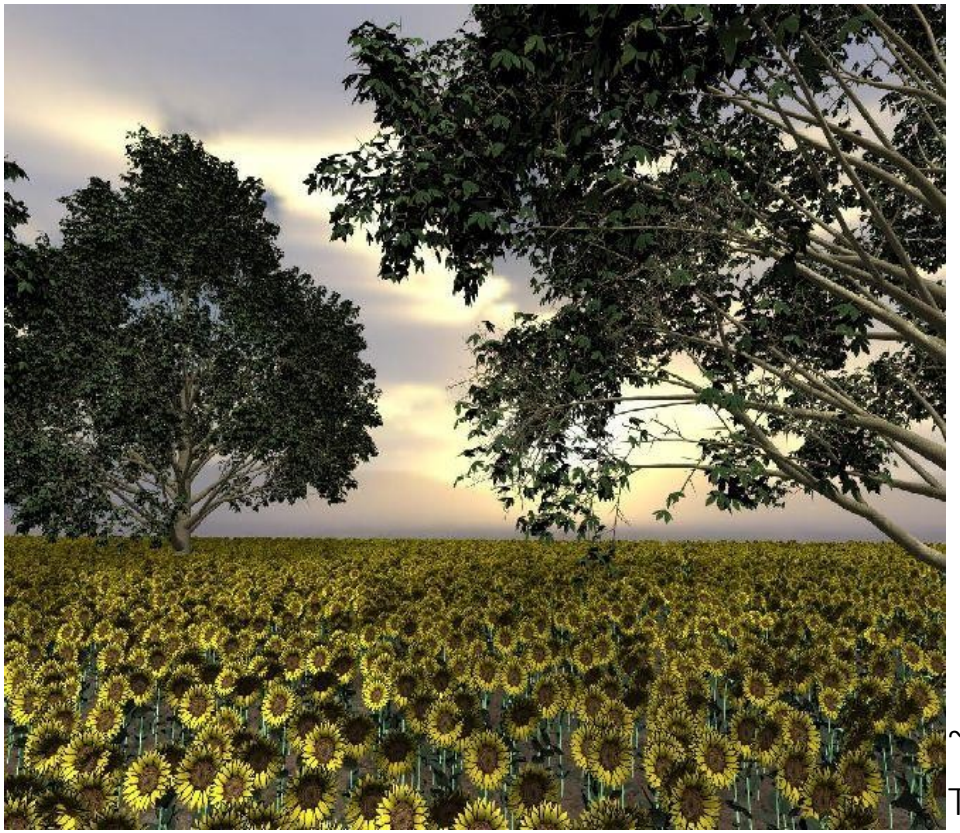
Huge Models

- Logarithmic scaling in scene size





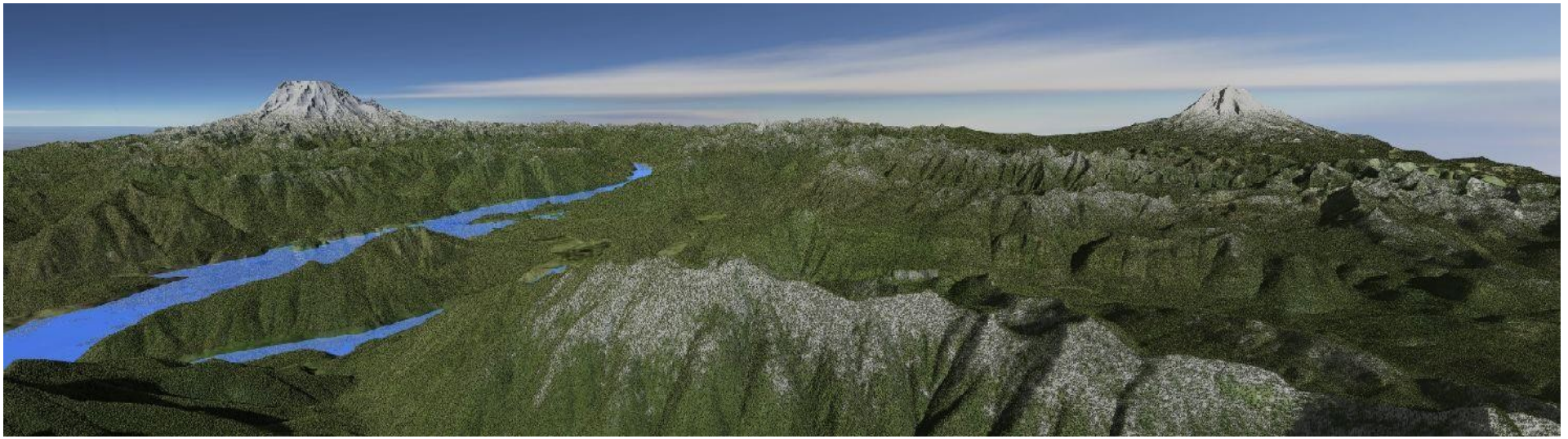
12.5 Million Triangles

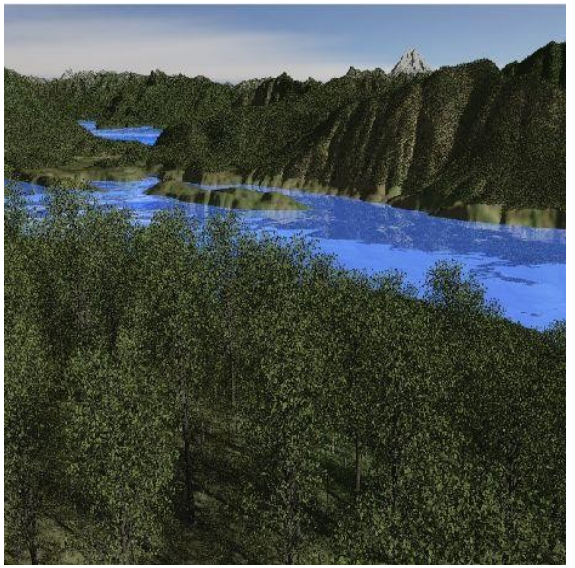


~1 Billion Triangles

Outdoor Environments

- 90 x 1012 (trillion) triangles





Games

In the Past

- Only used as an off-line technique
- Was computationally far too demanding (minutes to hours per frame)
- Believed to not be suitable for a HW implementation

More Recently

- Interactive ray tracing on supercomputers [Parker, U. Utah‘98]
- Interactive ray tracing on PCs [Wald‘01]
- Distributed Real-time ray tracing on PC clusters [Wald’01]
- RPU: First full HW implementation [Siggraph 2005]
- Commercial tools: Embree / OSPRey (Intel / CPU), OptiX (Nvidia / GPU)
- Complete film industry has switched to ray tracing (Monte-Carlo)

Own conference

- Symposium on Interactive RT, now High-Performance Graphics (HPG)

Ray tracing systems

- Research: PBRT (offline, physically-based, OSS),Mitsuba renderer (EPFL), imbatracer (SB), …
- Commercial: V-Ray (Chaos Group), Corona (Render Legion), VRED (Autodesk), MentalRay/iRay (MI), …

Tracing/Casting a ray

- Type of query
  - “Is there a primitive along a ray”
  - “How far is the closest primitive”

Other uses than rendering

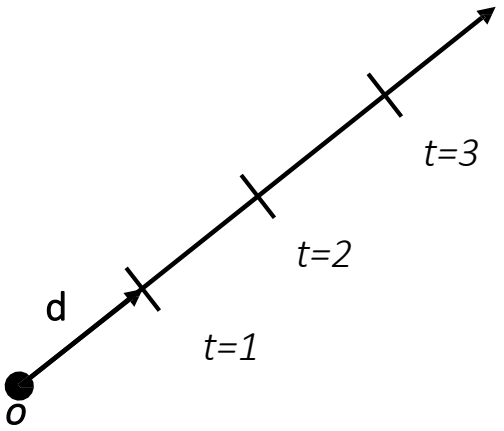
- Volume computation
- Sound waves tracing
- Collision detection
- …

Ray parameterization

- $\vec{r}(t) = \vec{o} + t\vec{d}$
  - Origin: $\vec{o} \in \mathbb{R}^3$
  - Direction: $\vec{d} \in \mathbb{R}^3$

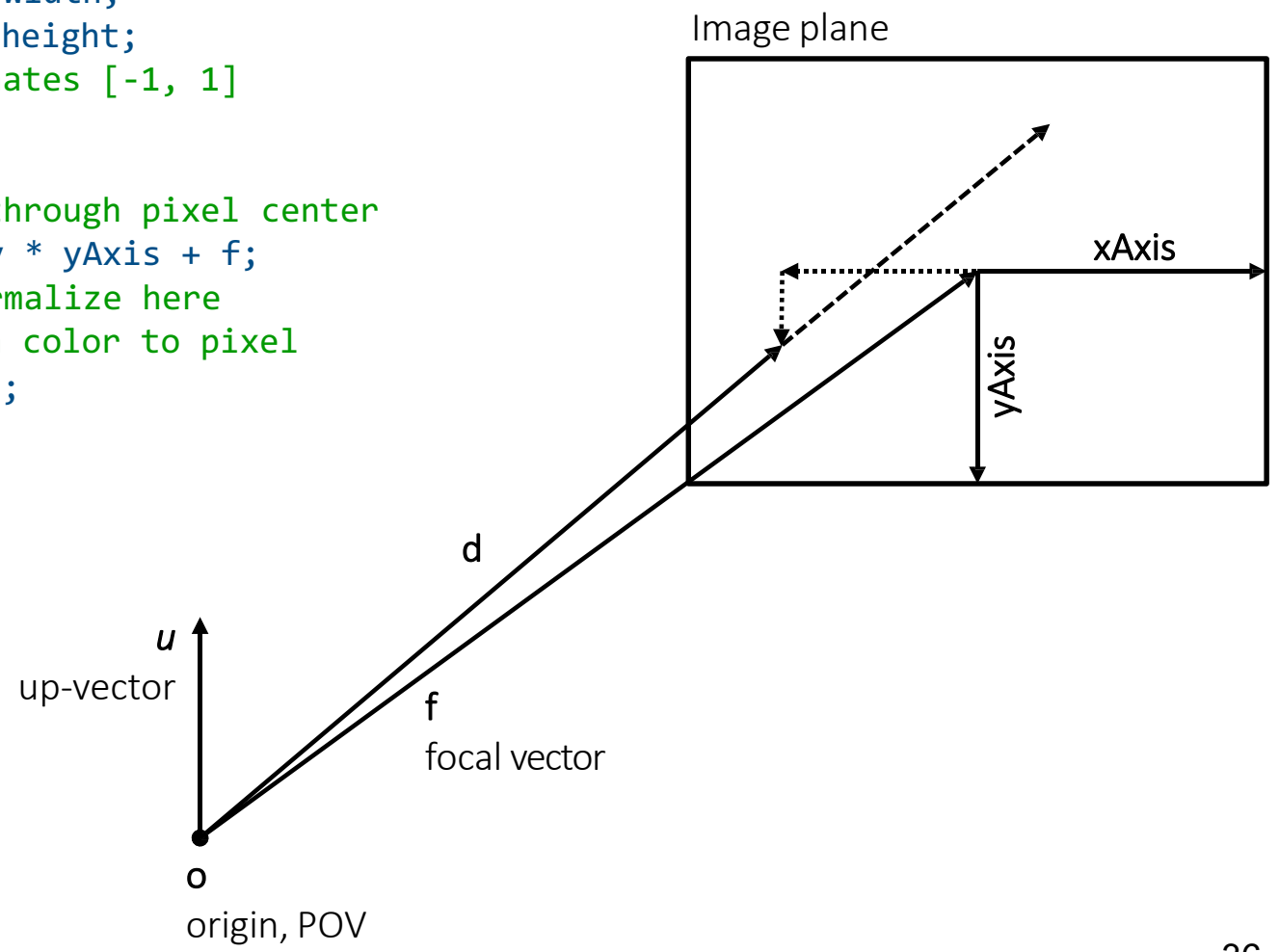
Ray

- All points on the graph of $\vec{r}(t)$, with $t \in \mathbb{R}$;



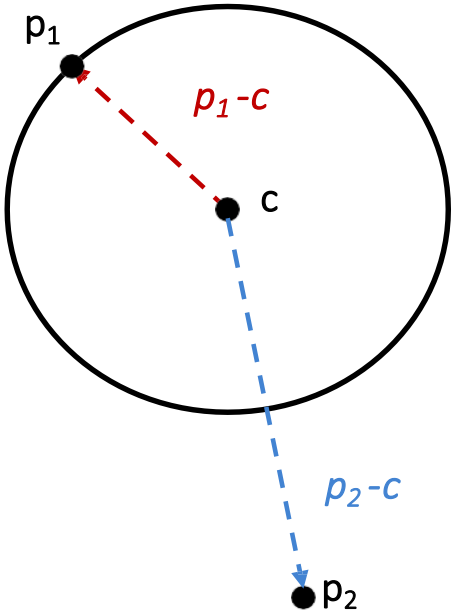
Perspective Camera Model

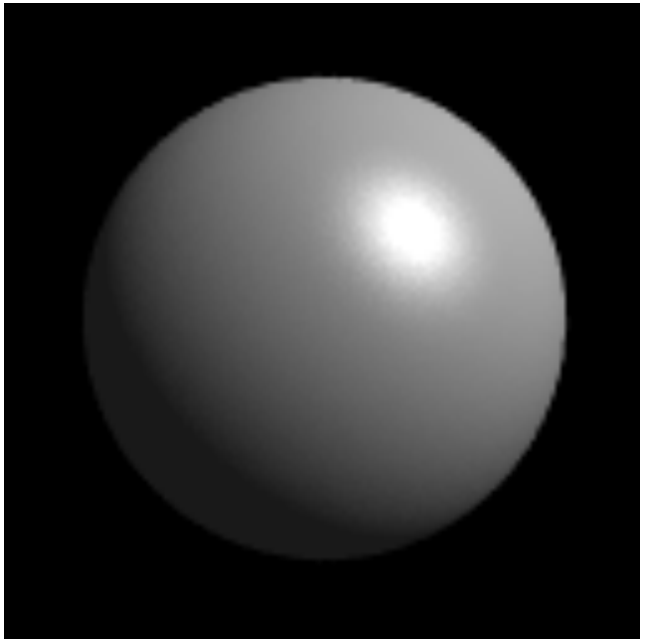
```
// For given image resolution res
// Loop over pixel raster coordinates
for(int y = 0; y < res.height; y++)
   for(int x = 0; x < res.width; x++)
   {
      // Normalized device coordinates [0, 1]
      ndcx = (x + 0.5) / res.width;
      ndcy = (y + 0.5) / res.height;
      // Screen space coordinates [-1, 1]
      sscx = 2 * ndcx - 1;
      sscy = 2 * ndcy - 1;
      // Generate direction through pixel center
      d = sscx * xAxis + sscy * yAxis + f;
      d = d / |d|; // May normalize here
      // Trace ray and assign color to pixel
      color = trace_ray(o, d);
      img(y, x) = color;
   }
```



Sphere S

- $\forall \vec{p} \in \mathbb{R}^3$: $\vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) = r^2$
- Sphere center: $\vec{c} \in \mathbb{R}^3$
- Sphere radius: $r \in \mathbb{R}$





Given

- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Sphere: $\forall \vec{p} \in \mathbb{R}^3: \ \vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) = r^2$

Find closest intersection point

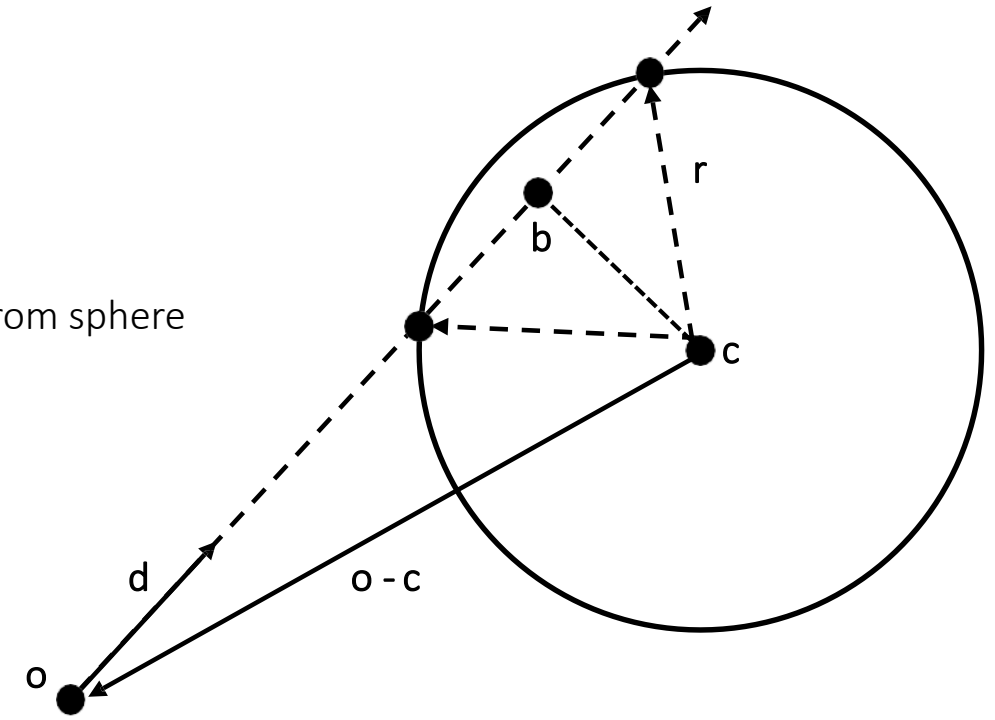
- **Algebraic approach**: substitute ray equation
  - $(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$ with $\vec{p} = \vec{o} + t\vec{d}$
  - $t^2\vec{d} \cdot \vec{d} + 2t\vec{d} \cdot (\vec{o} - \vec{c}) + (\vec{o} - \vec{c}) \cdot (\vec{o} - \vec{c}) - r^2 = 0$
  - Solve for t

Given

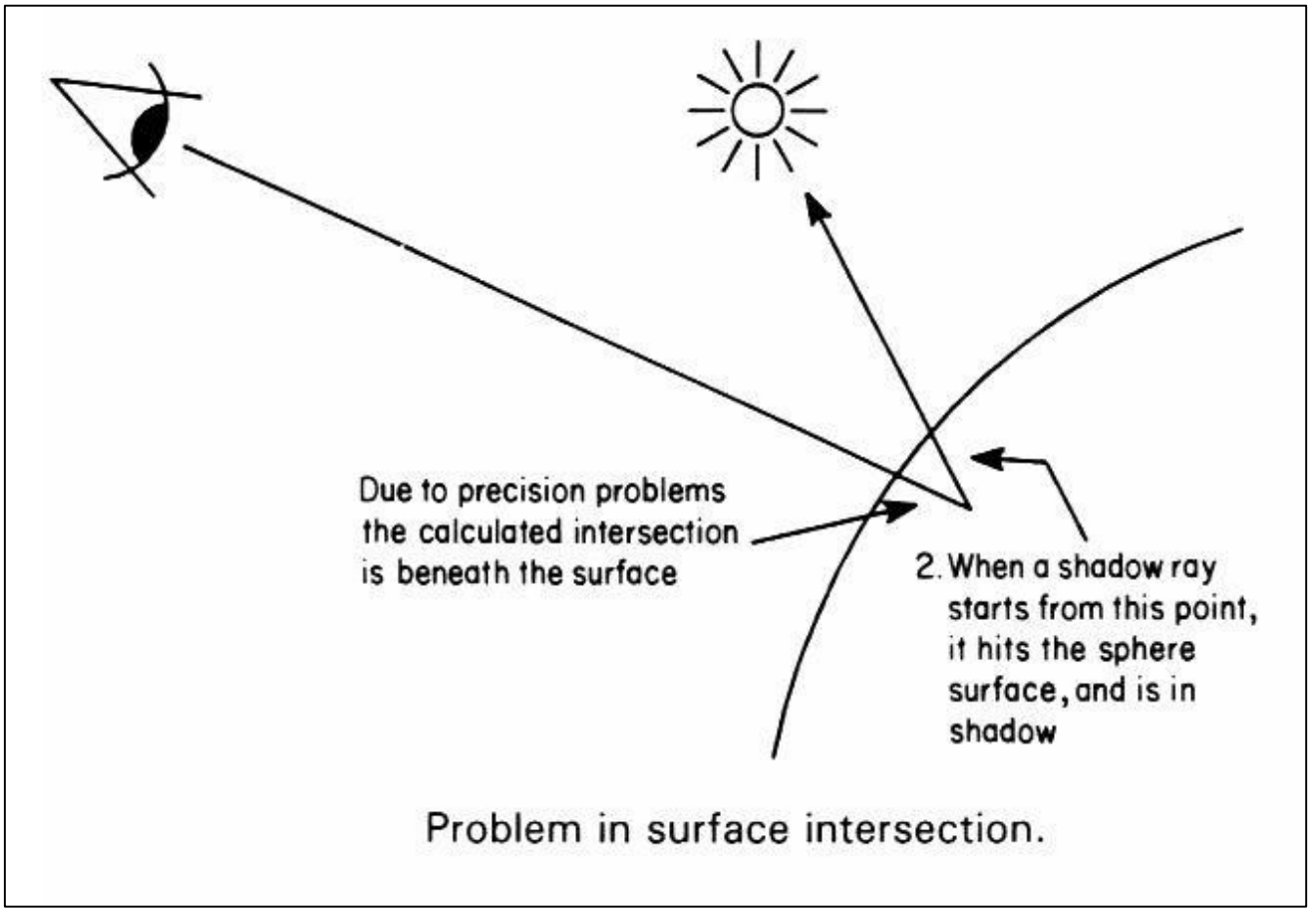
- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Sphere: $\forall \vec{p} \in \mathbb{R}^3: \ \vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) = r^2$

Find closest intersection point

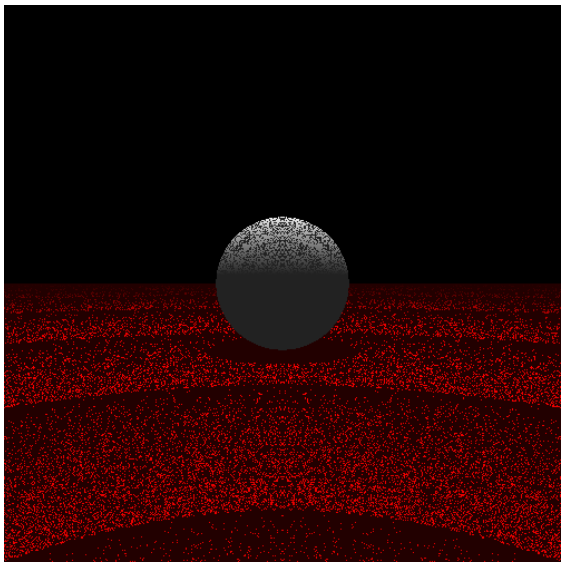
- Geometric approach:
  - Ray and center span a plane
  - Solve in 2D
  - Compute $|\vec{b} - \vec{c}|$, $|\vec{b} - \vec{c}|$
    - $\measuredangle obc = 90°$
  - Intersection(s) if $|\vec{b} - \vec{c}| \leq r$
- Be aware of floating point issue if o is far from sphere



Cause of „surface acne“

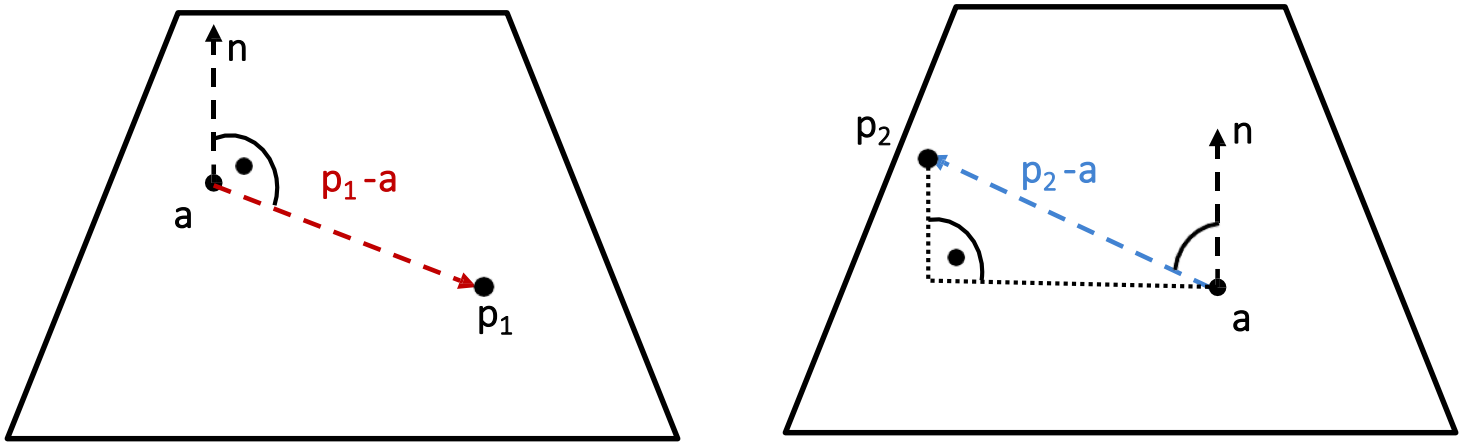


Problem in surface intersection.



Plane P

- $\forall \vec{p} \in \mathbb{R}^3$: $\vec{p} \in P \Leftrightarrow (\vec{p} - \vec{a}) \cdot \vec{n} = 0$
- Plane normal: $\vec{n} \in \mathbb{R}^3$
- Point on a plane: $\vec{a} \in \mathbb{R}^3$
- The difference vector between any two points on the plane is either 0 or orthogonal to the plane's normal



Given

- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Plane: $\forall \vec{p} \in \mathbb{R}^3: \quad \vec{p} \in P \Leftrightarrow (\vec{p} - \vec{a}) \cdot \vec{n} = 0$

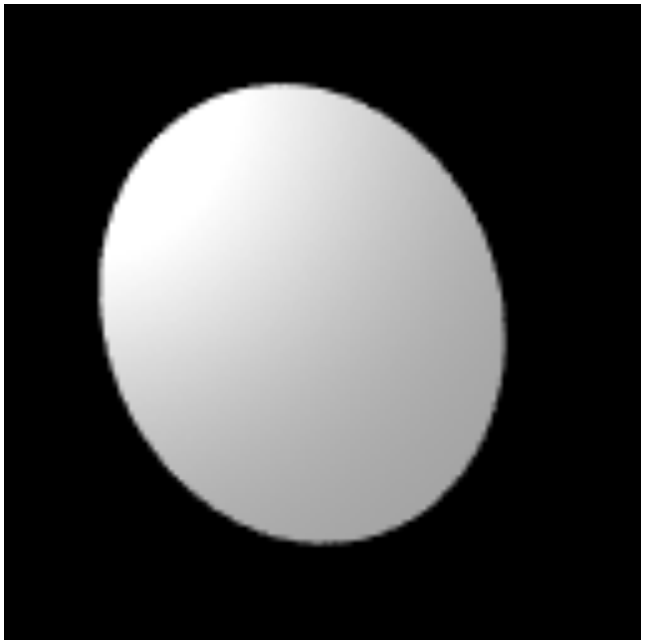
Compute intersection point

- Plane equation: $\vec{p} \in P \Leftrightarrow (\vec{p} - \vec{a}) \cdot \vec{n} = 0$

  $\Leftrightarrow \vec{p} \cdot \vec{n} - D = 0$, with $D = \vec{a} \cdot \vec{n}$
- Substitute ray parameterization: $\left(\vec{o} + t\vec{d}\right) \cdot \vec{n} - D = 0$
- Solve for t
  - 0, 1 or infinitely many solutions

Intersect ray with plane

- Discard intersection if $\|\vec{p} - \vec{a}\| > r$

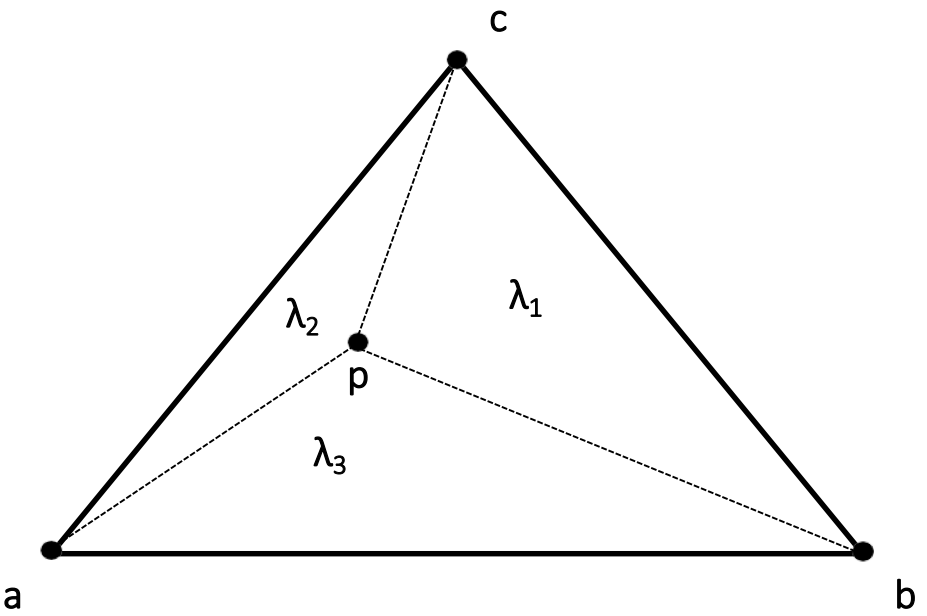


Triangle T

- Vertices: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
- Affine combinations of $\vec{a}, \vec{b}, \vec{c}$ → points in the plane
  - Non-negative coefficients that sum up to 1 → points in the triangle
- $\forall \vec{p} \in \mathbb{R}^3$: $\vec{p} \in T \Leftrightarrow \exists \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}, \lambda_1 + \lambda_2 + \lambda_3 = 1$ and
$$\vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$$

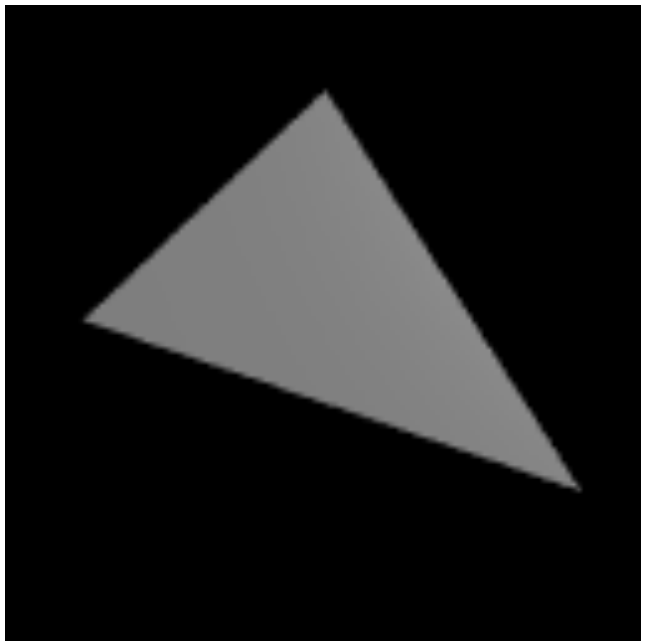
Barycentric coordinates

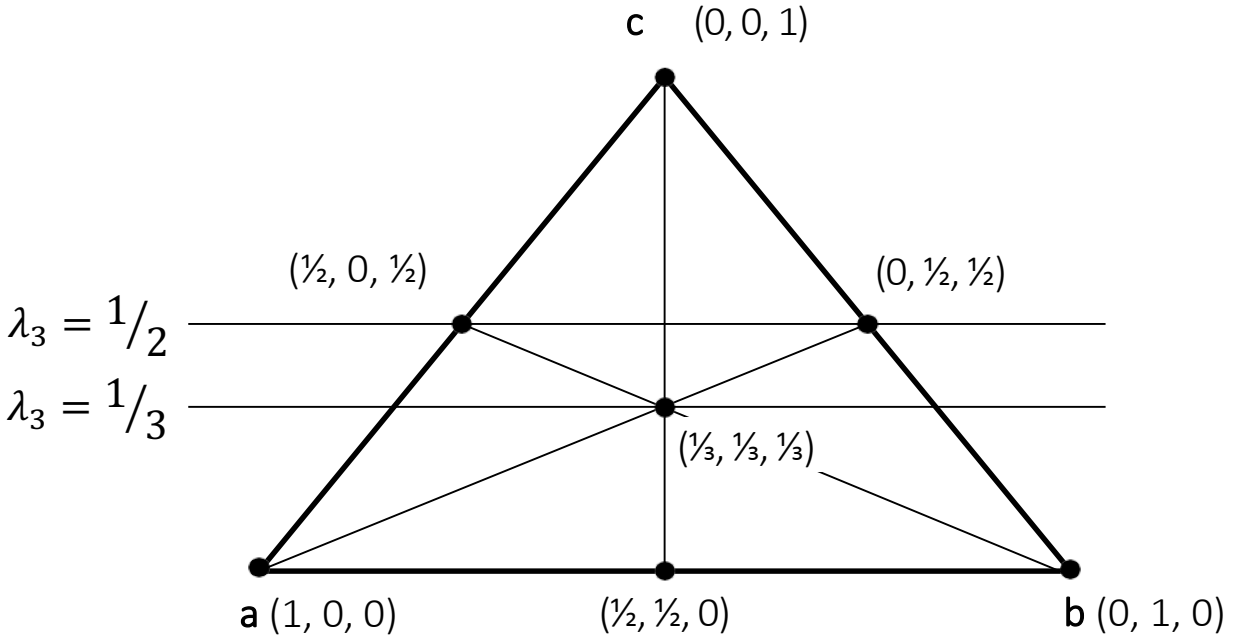
- $\lambda_1, \lambda_2, \lambda_3$
- $\lambda_1 = {}^{S_{pbc}}/_{S_{abc}}$
- S: signed area of triangles



Triangle T

- Vertices: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
- Barycentric coordinates: $\lambda_1, \lambda_2, \lambda_3$
- $\lambda_1 + \lambda_2 + \lambda_3 = 1$
- $\lambda_1 = {}^{S_{pbc}}/_{S_{abc}}$, *etc*



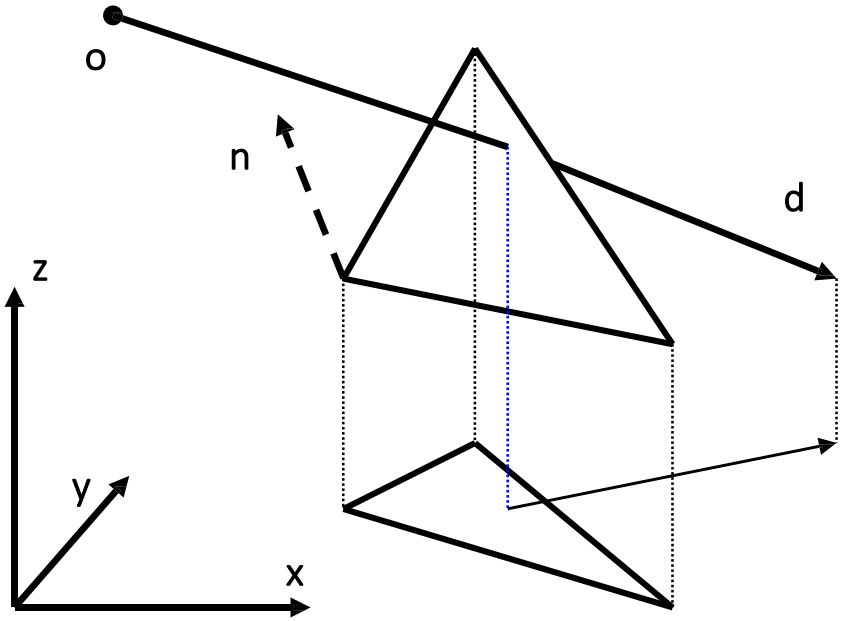


Compute intersection with triangle plane

Compute barycentric coordinates

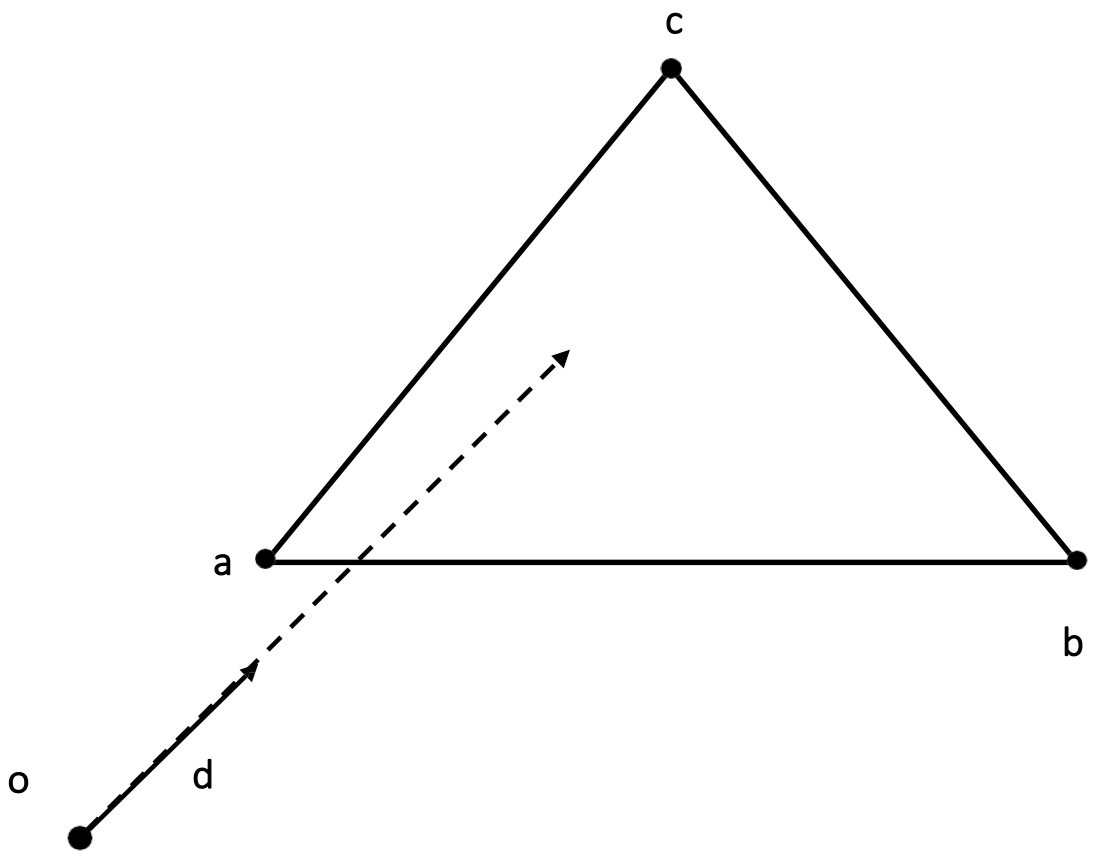
- Signed areas of sub-triangles
- Can be done in 2D, after “projection” onto major plane, depending on largest normal vector component

Test for positive BCs



3D linear function across triangle (3D edge functions)

- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$



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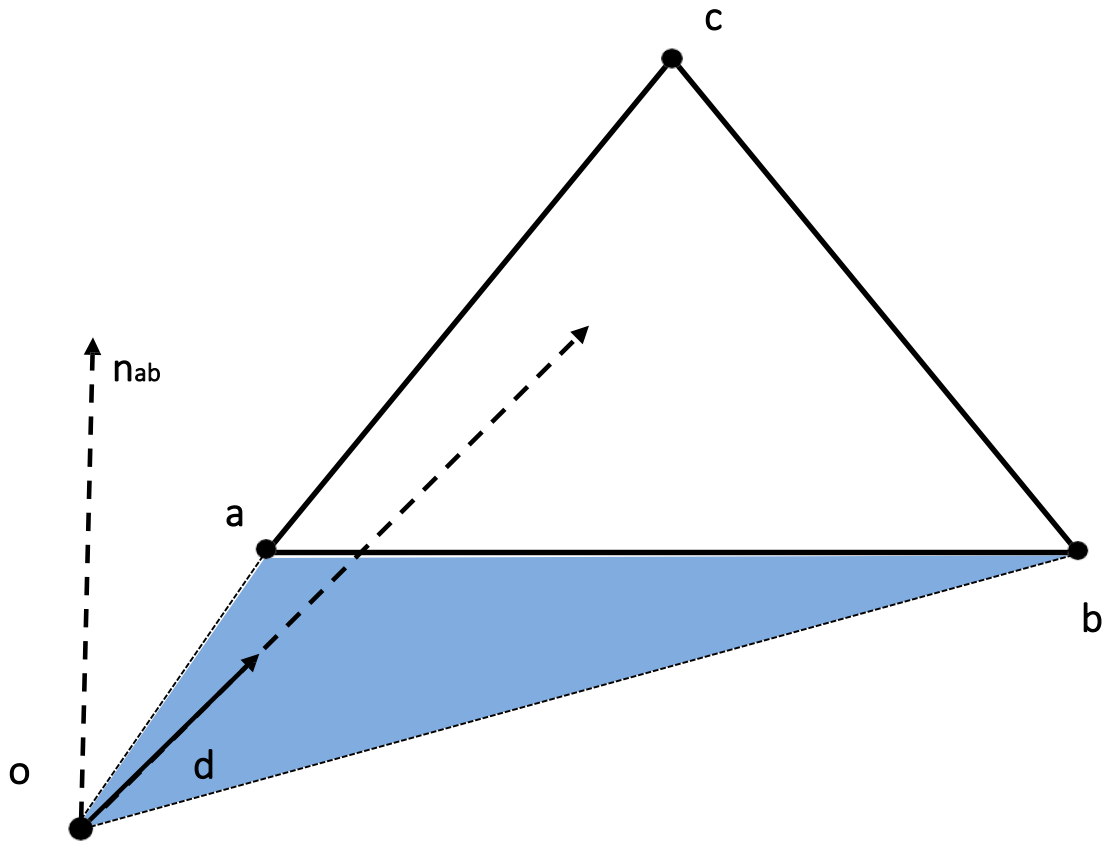
- Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
- $\overrightarrow{n_{ab}} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o})$
- $|\overrightarrow{n_{ab}}|$ is the signed area of OAB (2 times)



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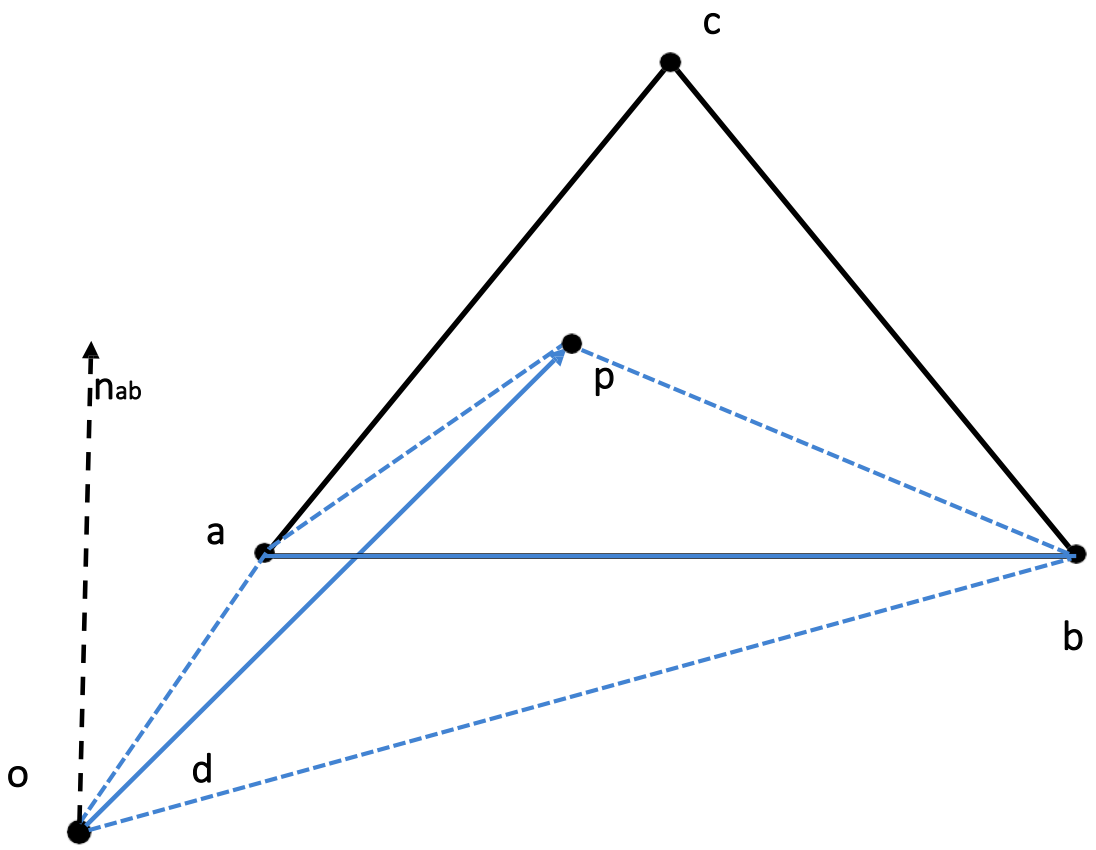
- $\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t\vec{d}$
  - Volume of OABP (6 times)
  - For $t = t_{hit}$



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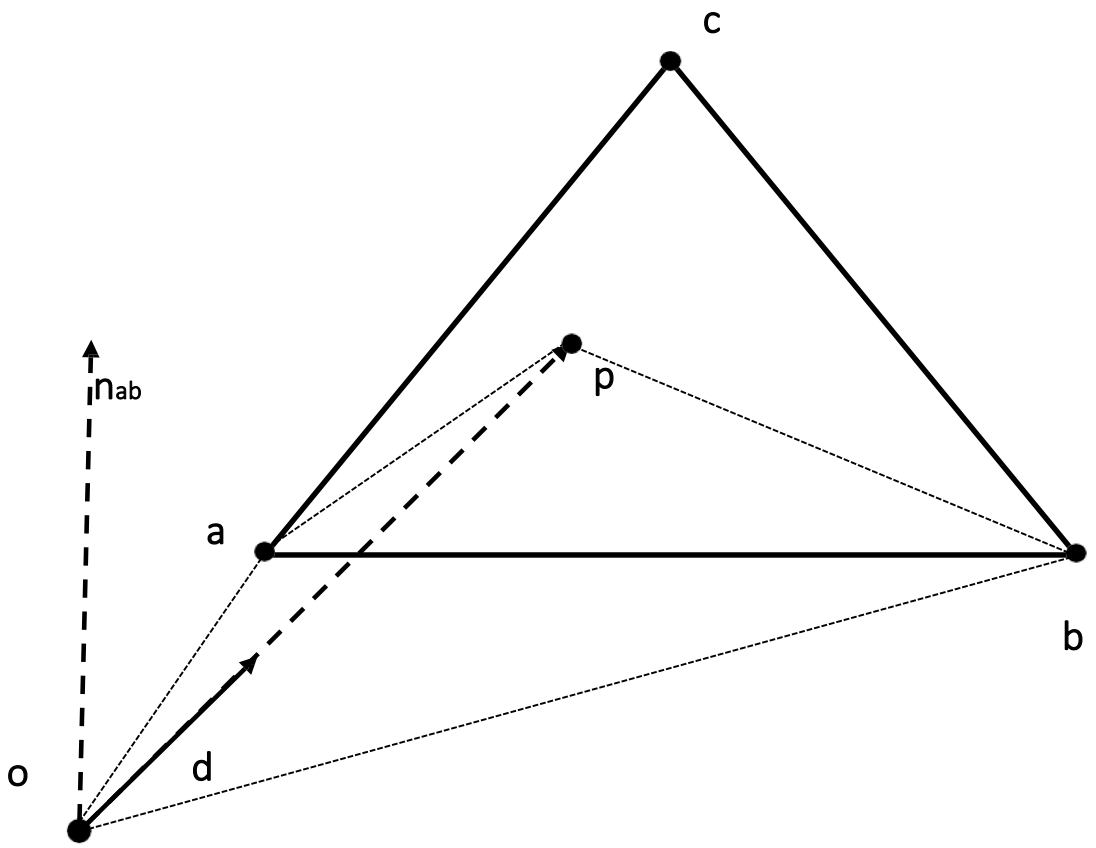
- $\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t\vec{d}$
  - Volume of OABP (6 times)
  - For $t = t_{hit}$
- $\lambda_{1,2}^*(t) = \overrightarrow{n_{bc,ac}} \cdot t\vec{d}$
- Normalize
  - $\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t) + \lambda_2^*(t) + \lambda_3^*(t)}, \quad i = 1, 2, 3$
  - Length of $t\vec{d}$ cancels out



3D linear function across triangle (3D edge functions)

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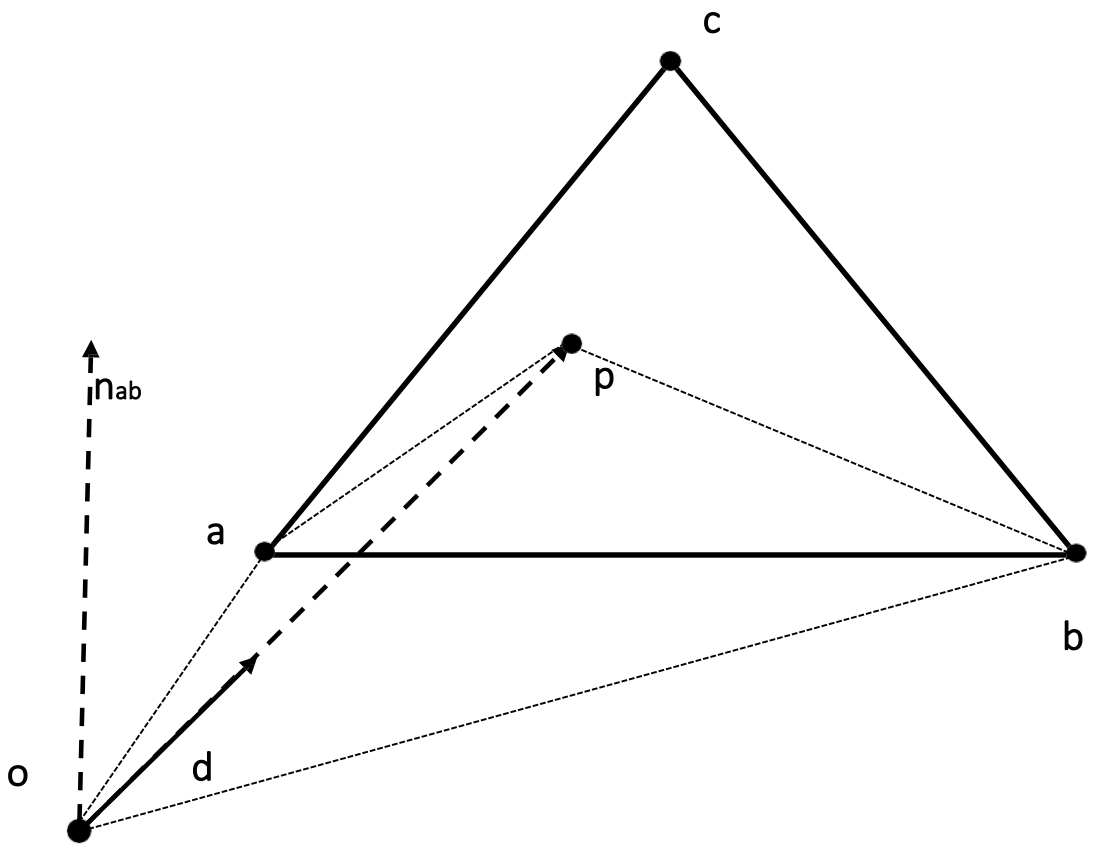
- Normalize
  - $\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t) + \lambda_2^*(t) + \lambda_3^*(t)}, \quad i = 1, 2, 3$

For positive BCs

- Compute $\vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$



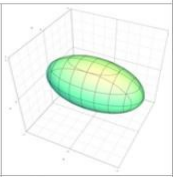
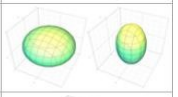
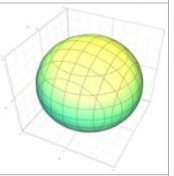
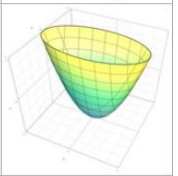
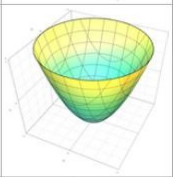
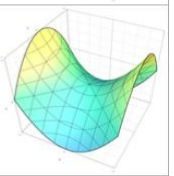
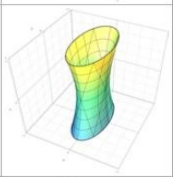
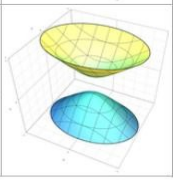
Quadrics

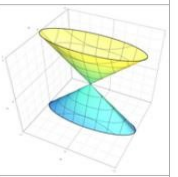
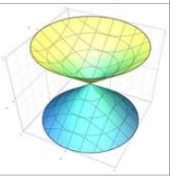
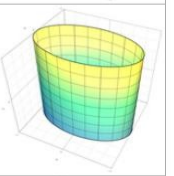
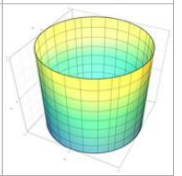
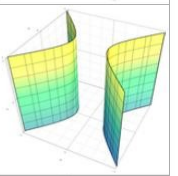
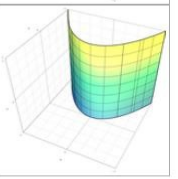
Implicit

- $f(x, y, z) = v$

Ray equation

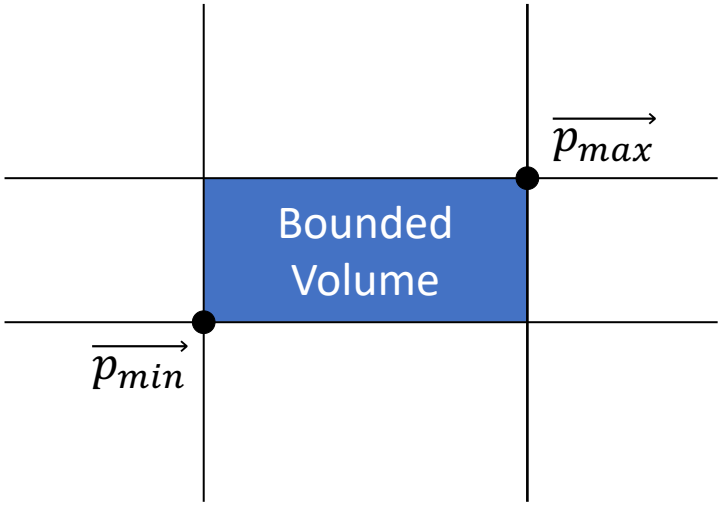
- $x = o_x + td_x$
- $y = o_y + td_y$
- $z = o_z + td_z$
- Solve for t

Non-degenerate real quadric surfaces		
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	
Spheroid (special case of ellipsoid)	$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$	
Sphere (special case of spheroid)	$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2} = 1$	
Elliptic paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - z = 0$	
Circular paraboloid(special case of elliptic paraboloid)	$\frac{x^2}{a^2} + \frac{y^2}{a^2} - z = 0$	
Hyperbolic paraboloid	$\frac{x^2}{a^2} - \frac{y^2}{b^2} - z = 0$	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	
Hyperboloid of two sheets	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$	

Degenerate quadric surfaces		
Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$	
Circular Cone (special case of cone)	$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{b^2} = 0$	
Elliptic cylinder	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	
Circular cylinder (special case of elliptic cylinder)	$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$	
Hyperbolic cylinder	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	
Parabolic cylinder	$x^2 + 2ay = 0$	

Given

- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

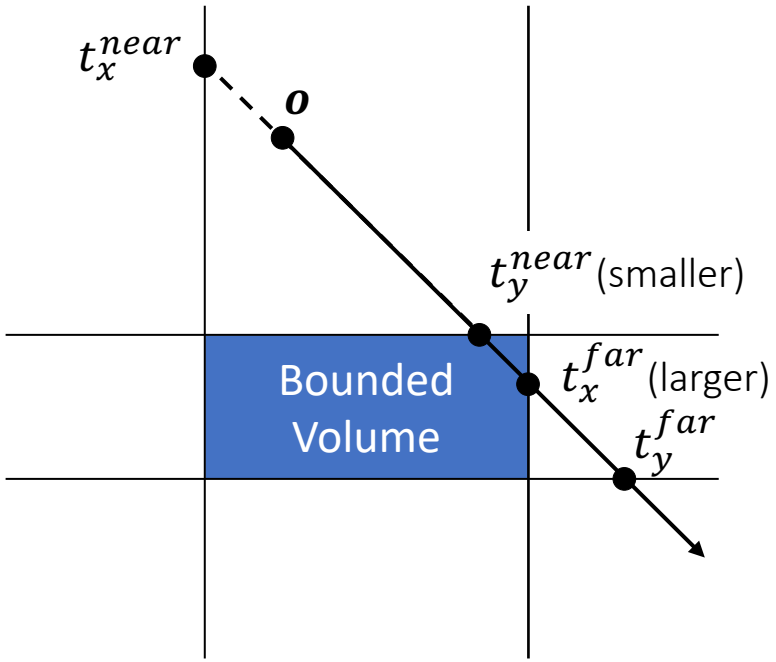


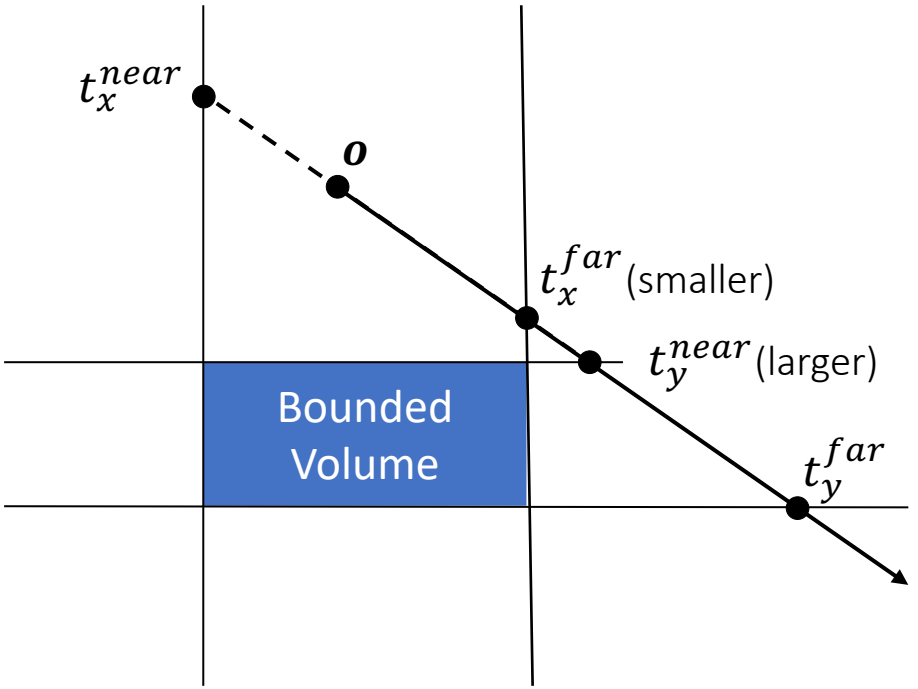
Given

- Ray: $\vec{r}(t) = \vec{o} + t\vec{d}$
- Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

"Slabs test" for ray-box intersection

- Ray enters the box in all dimensions before exiting in any
- $max(\{t_i^{near} | i = x, y, z\}) < min(\{t_i^{far} | i = x, y, z\})$





Ray-geometry intersection algorithms

- Polygons: [Appel ’68]
- Quadrics, CSG: [Goldstein & Nagel ’71]
- Recursive Ray Tracing: [Whitted ’79]
- Tori: [Roth ’82]
- Bicubic patches: [Whitted ’80, Kajiya ’82]
- Algebraic surfaces: [Hanrahan ’82]
- Swept surfaces: [Kajiya ’83, van Wijk ’84]
- Fractals: [Kajiya ’83]
- Deformations: [Barr ’86]
- NURBS: [Stürzlinger ’98]
- Subdivision surfaces: [Kobbelt *et al* ’98]