Computer Architecture and Programming Languages
Jacobs University Bremen
Romelda Blaceri

Course: CO20-320241
September 30, 2019

Homework 3

Problem 3.1

Solution:

a)

$$
\begin{aligned}
x &= (M+N)(\overline{M}+P)(\overline{N}+\overline{P}) && \\
&= (M\overline{M}+N\overline{M}+MP+NP)(\overline{N}+\overline{P}) && \text{R1} \\
&= (0+N\overline{M}+MP+NP)(\overline{N}+\overline{P}) && \text{R4} \\
&= \overline{N}N\overline{M}+\overline{N}MP+\overline{N}NP+\overline{P}N\overline{M}+\overline{P}MP+\overline{P}NP && \text{R1, R2, R8} \\
&= 0\cdot\overline{M}+\overline{N}MP+0\cdot P+\overline{P}N\overline{M}+0\cdot M+0\cdot N && \text{R2,R4} \\
&= M\overline{N}P+\overline{M}N\overline{P} && \text{R2, R9}
\end{aligned}
$$

b)

$$
\begin{aligned}
z &= \overline{A}B\overline{C}+AB\overline{C}+B\overline{C}D && \\
&= B\overline{C}(\overline{A}+A+D) && \text{R1,R2} \\
&= B\overline{C}(1+D) && \text{R4} \\
&= B\overline{C}\cdot 1 && \text{R9} \\
&= B\overline{C} && \text{R8}
\end{aligned} \tag{1}
$$

c)

$$
\begin{aligned}
x &= \overline{(M+N+P)Q} && \\
&= \overline{(M+N+P)}+\overline{Q} && \text{R6} \\
&= \overline{M}\cdot\overline{N}\cdot\overline{P}+\overline{Q} && \text{R6}
\end{aligned}
$$

d)

$$
\begin{aligned}
z &= \overline{ABC+DEF} && \\
&= \overline{ABC}\cdot\overline{DEF} && \text{R6} \\
&= (\overline{A}+\overline{B}+\overline{C})(\overline{D}+\overline{E}+\overline{F}) && \text{R6} \\
&= \overline{A}(\overline{D}+\overline{E}+\overline{F})+\overline{B}(\overline{D}+\overline{E}+\overline{F})+\overline{C}(\overline{D}+\overline{E}+\overline{F}) && \text{R1} \\
&= \bar{A}\bar{D}+\bar{A}\bar{E}+\bar{A}\bar{F}+\bar{B}\bar{D}+\bar{B}\bar{E}+\bar{B}\bar{F}+\bar{C}\bar{D}+\bar{C}\bar{E}+\bar{C}\bar{F} && \text{R1}
\end{aligned}
$$

e)

$$
\begin{aligned}
z &= \overline{A\overline{B}+C\overline{D}+EF} && \\
&= \overline{A\overline{B}}\cdot\overline{C\overline{D}}\cdot\overline{EF} && \text{R6} \\
&= (\overline{A}+\overline{\overline{B}})(\overline{C}+\overline{\overline{D}})(\overline{E}+\overline{F}) && \text{R6} \\
&= (\overline{A}+B)(\overline{C}+D)(\overline{E}+\overline{F}) && \text{R7} \\
&= (\overline{A}\cdot\overline{C}+B\overline{C}+\overline{A}D+BD)(\overline{E}+\overline{F}) && \text{R1} \\
&= \overline{A}\cdot\overline{C}\cdot\overline{E}+\overline{C}B\overline{E}+\overline{A}D\overline{E}+BD\overline{E}+\overline{A}\cdot\overline{C}\cdot\overline{F}+\overline{C}B\overline{F}+\overline{A}D\overline{F}+BD\overline{F} && \text{R1, R2}
\end{aligned}
$$

f)

$$
\begin{aligned}
z &= \overline{\overline{A+B\overline{C}} + D(\overline{E+\overline{F}})} \\
&= \overline{\overline{A+B\overline{C}}} \cdot \overline{D(\overline{E+\overline{F}})} && \text{R6} \\
&= (A+B\overline{C})(\overline{D} + (\overline{E+\overline{F}})) && \text{R6, R7} \\
&= (A+B\overline{C})(\overline{D} + \overline{E}\cdot\overline{\overline{F}}) && \text{R6} \\
&= (A+B\overline{C})(\overline{D} + \overline{E}F) && \text{R7} \\
&= A\overline{D} + B\overline{C}\cdot\overline{D} + A\overline{E}F + B\overline{C}\cdot\overline{E}F && \text{R1, R2}
\end{aligned}
$$

Where:

R1 - Distributivity: $XY + YZ = Y(X+Z)$
R2 - Commutativity: $X+Y = Y+X$ or $XY = YX$
R3 - Associativity: $(XY)Z = X(YZ)$ or $(X+Y)+Z = X+(Y+Z)$
R4 - Complement: $\overline{X} + X = 1$ or $X\overline{X} = 0$
R5 - Indermpotent: $X = X + X$ or $XX = X$
R6 - De Morgan: $\overline{X}\,\overline{Y} = \overline{X+Y}$
R7 - Involution: $\overline{\overline{X}} = X$
R8 - Identity: $X + 0 = X$ or $X \cdot 1 = X$
R9 - Annihilator for OR and And: $X + 1 = 1$ and $X \cdot 0 = 0$

Problem 3.2

Solution:

Truth Table

A	B	C	D	x
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Therefore the K-map is:

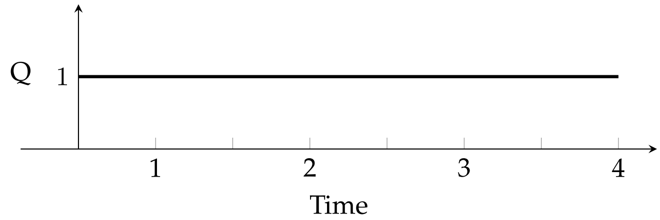
	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	1	1	0
$\overline{A}B$	0	0	0	0
AB	0	0	0	0
$A\overline{B}$	1	1	0	0

Using K-map to simplify:
The loops that are form are: the pair at cells 2, 3 and the quad at cells 1, 2, 13, 14. Therefore, we get this result:
$x = \overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}D$

Problem 3.3

Solution:

During the first positive transition of CLK, S is 1, R is 0, so according to the S-R flip-flop that responds only to the positive-going edge of a clock pulse table, Q stays at 1. During the second positive transition we have the same thing as before, so Q doesn't change. During the third one, S is 0 and R is 0, so Q is 1 again (no change state). During the fourth one, we have same conditions as in the first two, so Q will stay at 1. Therefore we have the following waveform and the corresponding state table:

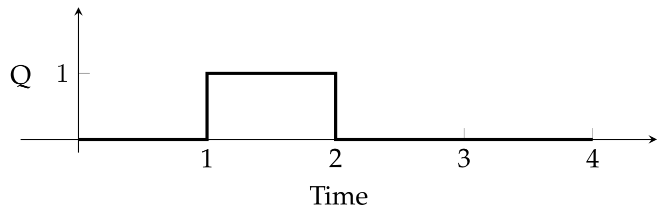


Input		Output	CLK pos.	Action
S	R			
1	0	1	↑	SET
1	0	1	↑	SET
0	0	1	↑	Q_0 (No change)
1	0	1	↑	SET

Problem 3.4

Solution:

During the first negative transition of CLK, S is 1, R is 0, so Q will SET (it will go from initial 0 state to 1). After that, during the second transition, S is 0, R is 1, so Q will RESET. During the 2 last negative transitions, both R and S will be 0, so Q won't change value, it will stay at 0. The waveform is as follows:

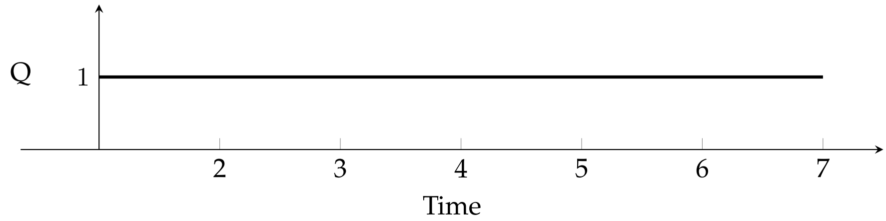


Input		Output	CLK pos.	Action
S	R			
1	0	1	↓	SET
0	1	0	↓	RESET
0	0	0	↓	Q_0 (No change)
0	0	0	↓	Q_0 (No change)

Problem 3.5

Solution:

Since in the first positive transition of CLK, up to the fourth, J and K are both 0, so according to the J-K Flip-Flop table, Q will not change, so since Q_0 is 1, Q will be 1 all along. In the fifth transition, J is 1, K is 0, so Q will be 1 again (SET). The sixth transition gives the same result as the first 4, so Q will be 1 along all the waveform, which is as follows:



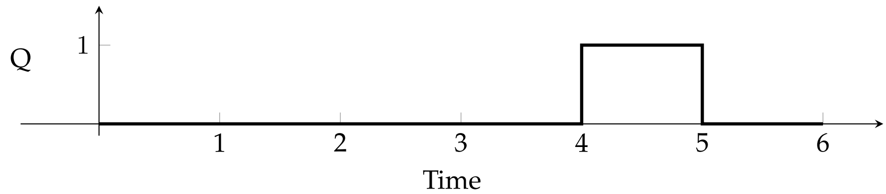
State table:

Input		Output	CLK pos.	Action
J	K			
0	0	1	↑	Q_0 (No change)
0	0	1	↑	Q_0 (No change)
0	0	1	↑	Q_0 (No change)
0	0	1	↑	Q_0 (No change)
1	0	1	↑	SET
0	0	1	↑	Q_0 (No change)

Problem 3.6

Solution:

During the first three negative transitions of CLK, both J and K are 0, so Q will be the same as the initial value, which is 0. During the fourth negative transition, J and K are 1, so value of Q will have to TOGGLE (it becomes 1). During the fifth transition, we have a TOGGLE case again, so Q turns to 0, and it remains 0 even during the last transition since the values of J and K are 0. The waveform will be:



State table:

Input		Output	CLK pos.	Action
J	K			
0	0	0	↑	Q_0 (No change)
0	0	0	↑	Q_0 (No change)
0	0	0	↑	Q_0 (No change)
1	1	1	↑	TOGGLE
1	1	0	↑	TOGGLE
0	0	0	↑	Q_0 (No change)

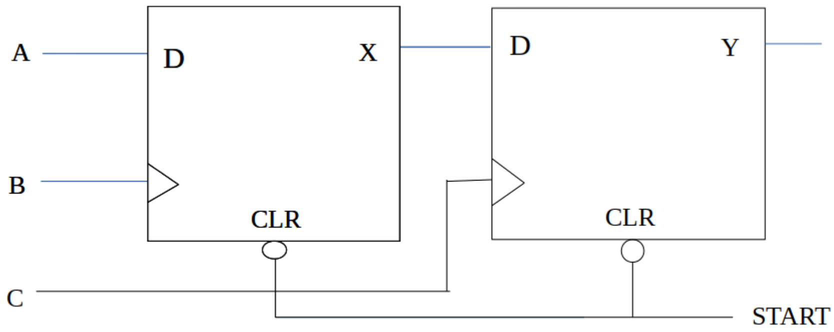
Problem 3.7

Solution:

a) Considering the J-K Flip Flop table, for Y to be 1, since K_Y is 0, J_Y must be 1. For J_Y to be 1, X should also be 1. For X to be 1, since K_X is 0, we need J_X to be 1, therefore, the first thing is that A should be 1. The two CLKs should also be in transition state, so they need to be 1. So, first B need to be 1 and then C.

b) The START pulse is needed to make sure that X and Y are LOW.

c) The required D flip flop circuit is:


A
B
C
D
X
CLR
D
Y
CLR
START