Lecture 6:

BRDF

Contents

1. Bidirectional Reflectance Distribution Function (BRDF)
2. Reflection models
3. Projection onto spherical basis functions
4. Shading

Reflection equation

$$L(x, \omega_0) = \int_{\Omega_+} f_r(\omega_i, x, \omega_\mathrm{o})\, L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

BRDF

- Ratio of reflected radiance to incident irradiance

$$f_r(\omega_i, x, \omega_\mathrm{o}) = \frac{L_o(x, \omega_o)}{dE_i(x, \omega_i)}$$

BRDF describes surface reflection for light incident from direction (θ_i, φ_i) observed from direction (θ_o, φ_o)

$$f_r(\omega_i, x, \omega_\mathrm{o}) = \frac{L_o(x, \omega_o)}{dE_i(x, \omega_i)} = \frac{L_o(x, \omega_o)}{L_i(x, \omega_i)\cos\theta_i d\omega_i}$$

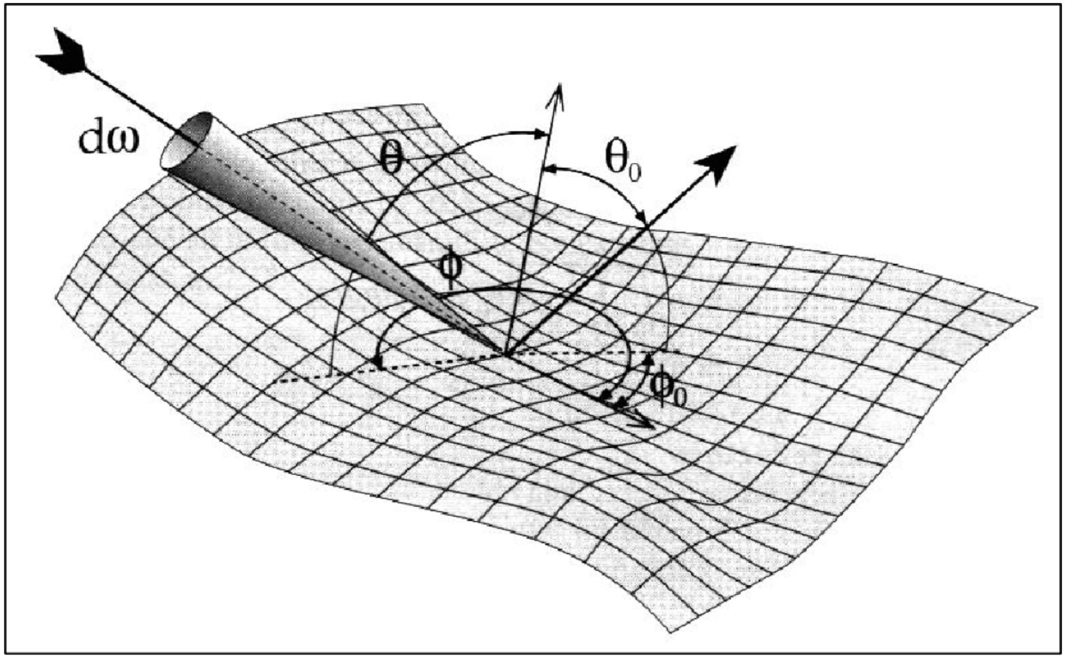
Bidirectional

- Depends on two directions and position (6-D function)

Distribution function

- Can be finite

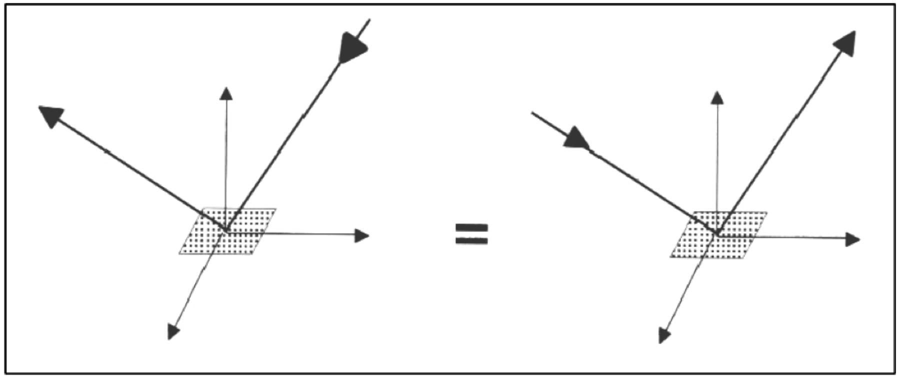
Unit [1 / sr]



Helmholtz reciprocity principle

- BRDF remains unchanged if incident and reflected directions are interchanged

$$f_r(\omega_i, x, \omega_o) = f_r(\omega_o, x, \omega_i)$$

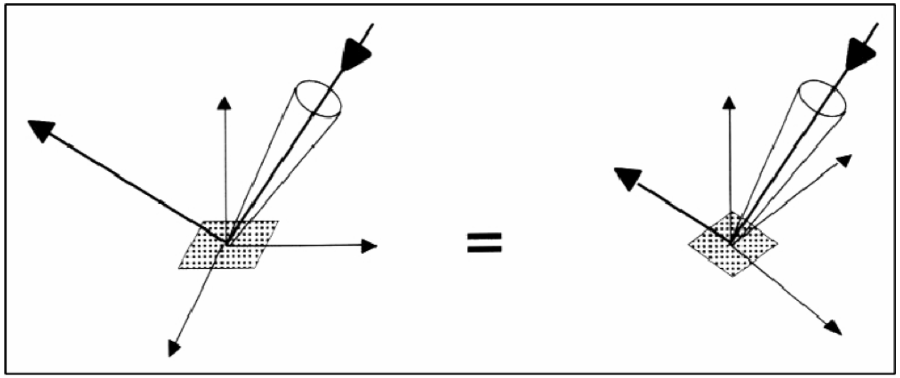


Smooth surface: isotropic BRDF

- Reflectivity independent of rotation around surface normal
- BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(\omega_i, x, \omega_o) \equiv f_r(\theta_i, \varphi_i, x, \theta_o, \varphi_o) \equiv$$

$$\equiv f_r(\theta_i, x, \theta_o, \varphi_i - \varphi_o)$$



Characteristics

- BRDF units [sr^{-1}]
  - Not intuitive
- Range of values:
  - From 0 (absorption) to ∞ (reflection, δ-function)
- Energy conservation law
  - No self-emission
  - Possible absorption

$$\int_{\Omega_+} f_r(\omega_i, x, \omega_o) \cos\theta_o d\omega_o \leq 1 \quad \forall \theta, \varphi$$

Reflection only at the point of entry ($x_i = x_o$)

- No subsurface scattering

Gonio-Reflectometer

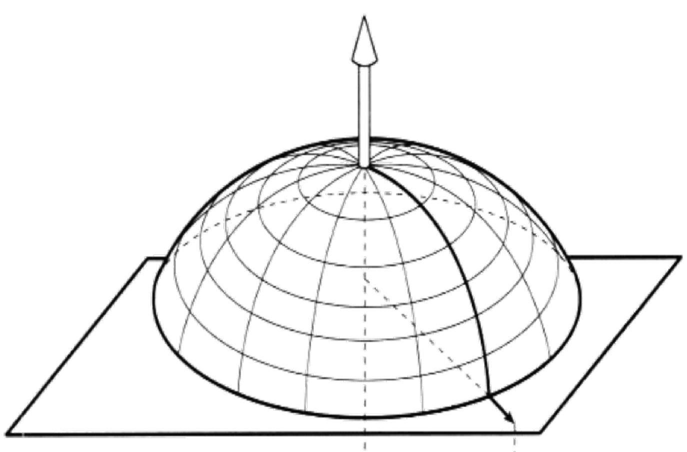
BRDF measurement

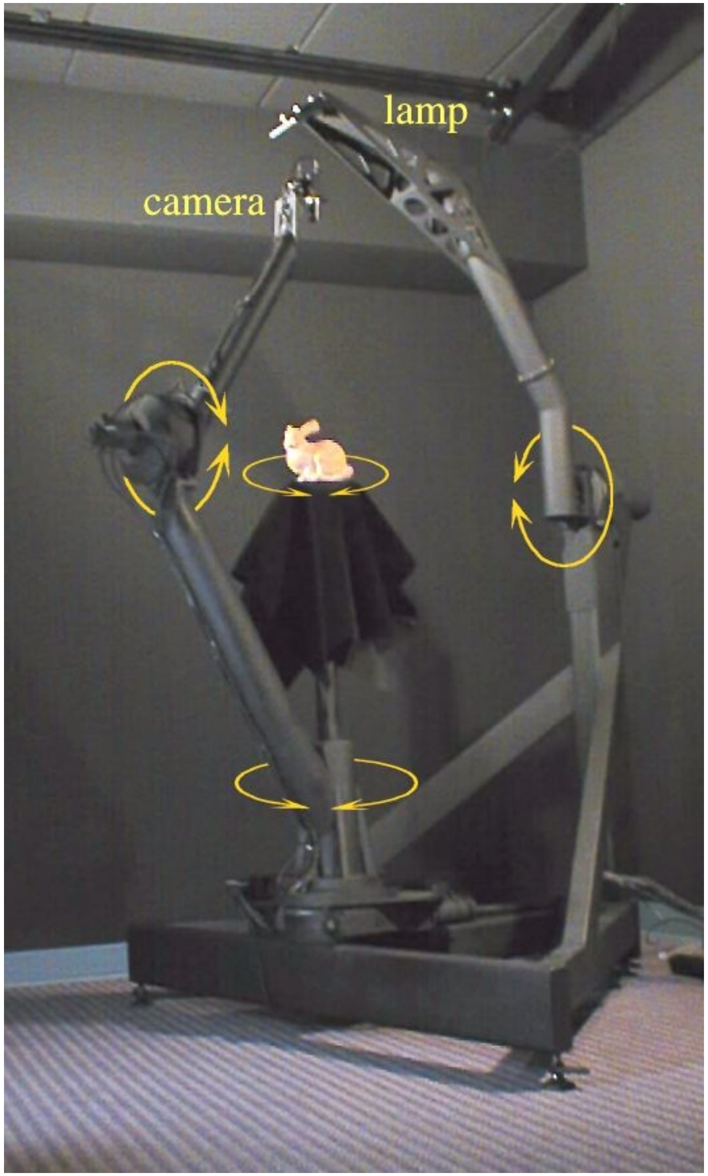
- Point light source position(θ_i, φ_i)
- Light detector position (θ_o, φ_o)

4 directional degrees of freedom

BRDF representation

- *m* incident direction samples (θ_i, φ_i)
- *n* outgoing direction samples (θ_o, φ_o)
- *m***n* reflectance values (large!!!)





Stanford light gantry

Linearity, superposition principle

- Complex illumination: integrating light distribution against BRDF
- Sampled BRDF: superimposed point light sources

Interpolation

- Look-up during rendering
- Sampled BRDF must be filtered

BRDF Modeling

- Fit parameterized BRDF model to measured data
- Continuous function
- No interpolation
- Fast evaluation

Representation in spherical harmonics basis

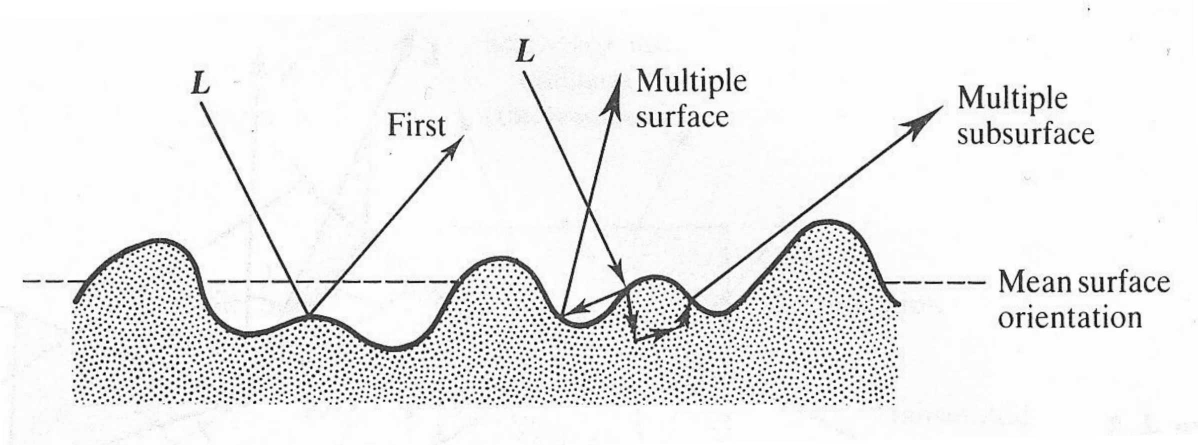
- Mathematically elegant filtering, illumination-BRDF integration
- Soon supported by graphics hardware ?

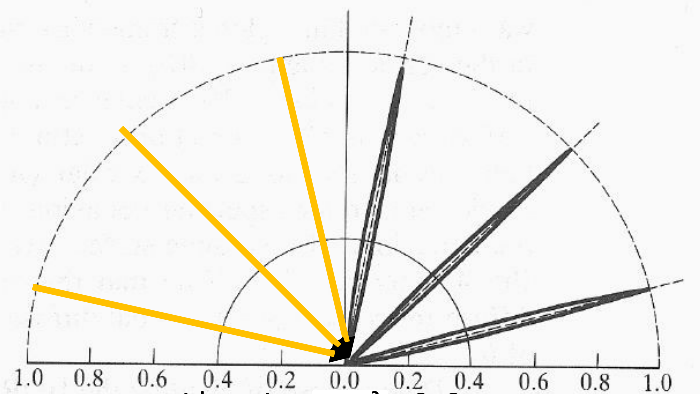
Reflectance may vary with

- Illumination angle
- Viewing angle
- Wavelength
- Polarization
- …

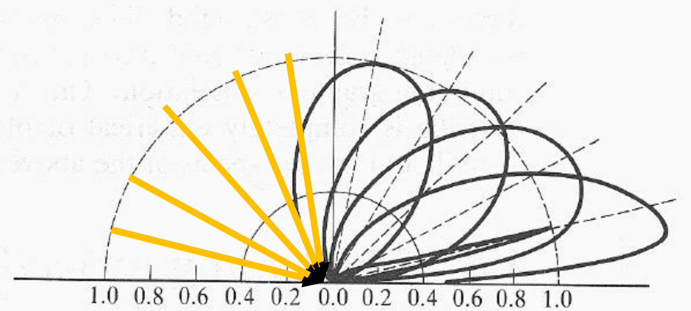
Variations due to

- Absorption
- Surface micro-geometry
- Index of refraction / dielectric constant
- Scattering

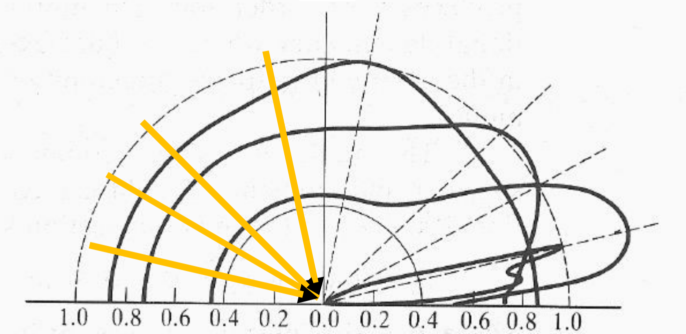




Aluminum; λ=2.0μm



Aluminum; λ=0.5μm



Magnesium; λ=0.5μm

Phenomenological approach

- Description of visual surface appearance

Ideal specular reflection

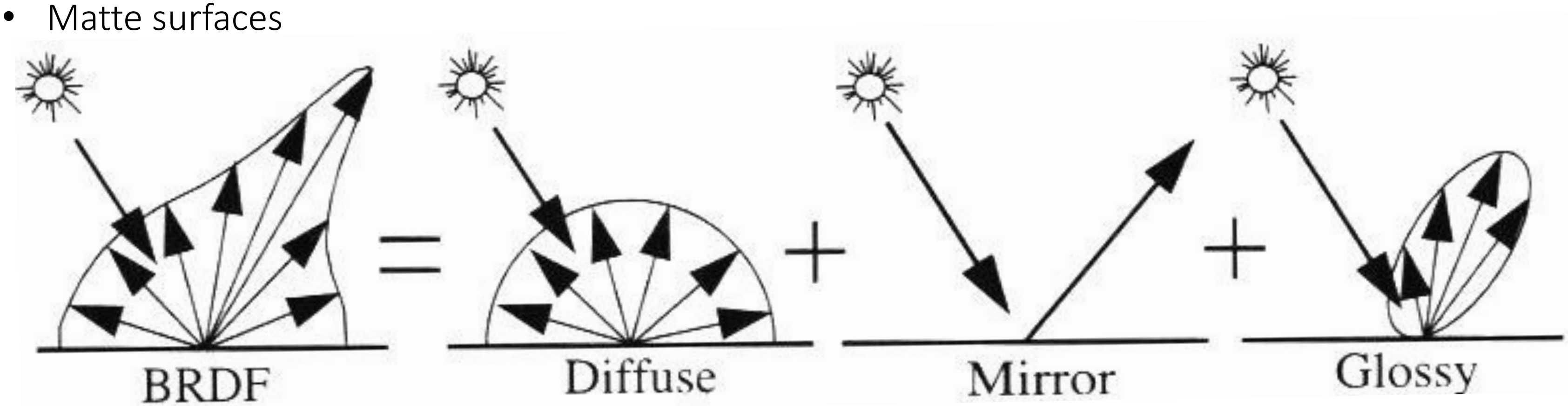
- Reflection law
- Mirror

Glossy reflection

- Directional diffuse
- Shiny surfaces

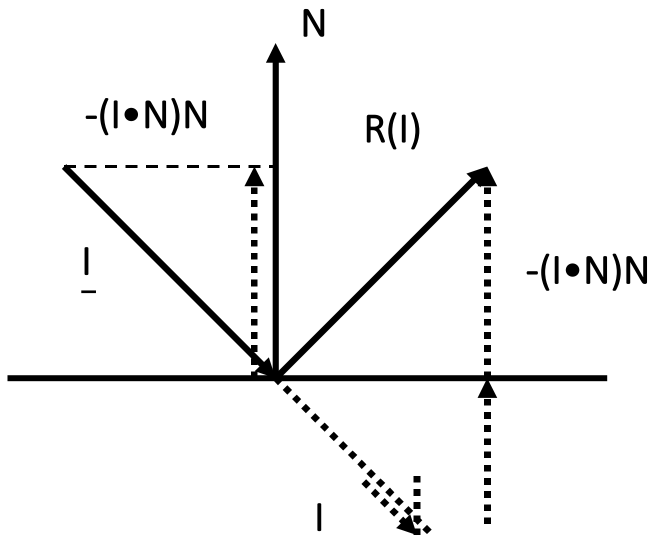
Ideal diffuse reflection

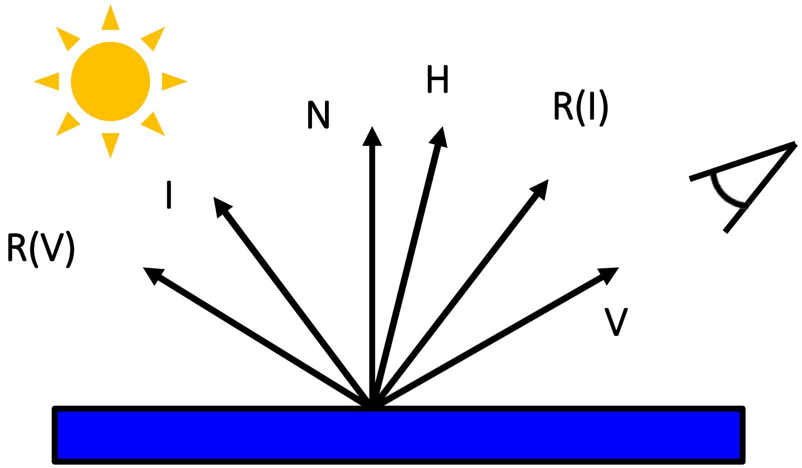
- Lambert's law
- Matte surfaces

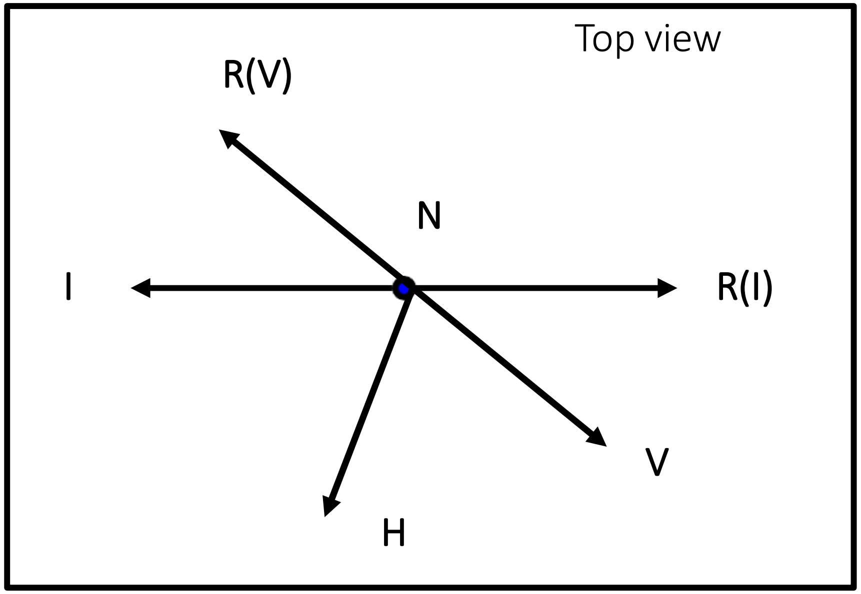


Direction vectors (normalize):

- N: surface normal
- I: vector to the light source
- V: viepoint direction vector
- H: halfway vector: H= (I + V) / |I + V|
- R(I): reflection vector R(I)= I - 2(I•N)N
- Tangential surface: local plane





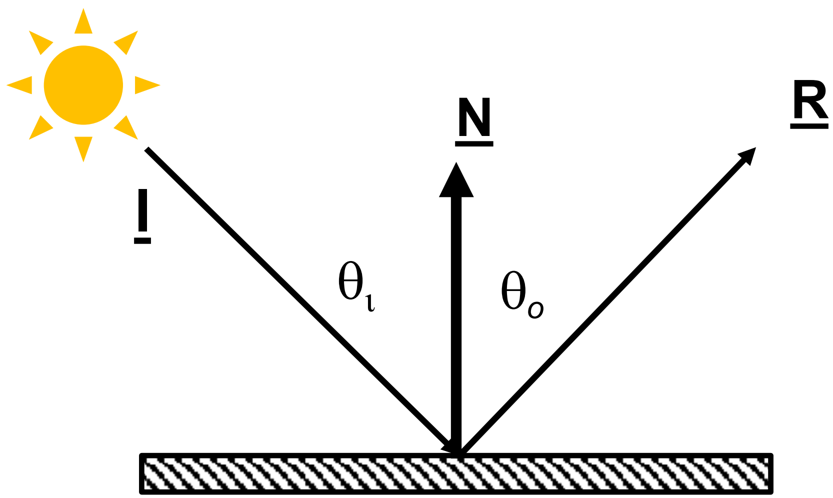


Angle of reflectance equal to angle of incidence

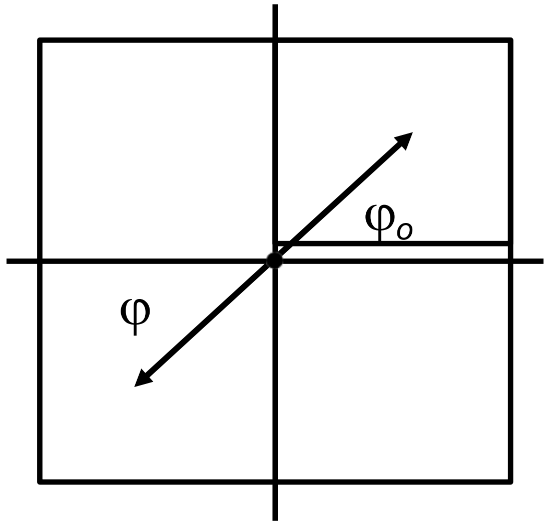
Reflected vector in a plane with incident ray and surface normal vector

$$R+(-I) = 2\cos\theta\, N = -2(I \bullet N)\, N$$

$$R(I) = I - 2(I \bullet N)\, N$$



$\theta_\iota = \theta_o$



$\varphi = \varphi_o + 180°$

Dirac Delta function $\delta(x)$

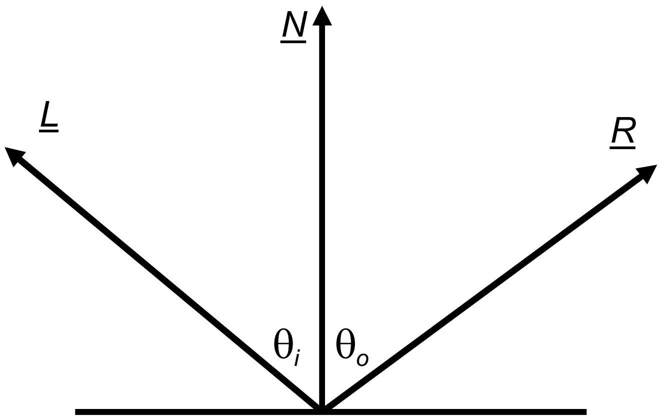
- $\delta(x)$: zero everywhere except at $x = 0$
- Unit integral iff integration domain contains zero (zero otherwise)

$$f_{r,m}(\omega_i, x, \omega_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \cdot \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_0) = \int_{\Omega_+} f_{r,m}(\omega_o, x, \omega_\mathrm{i})\, L_i(\theta_i, \varphi_i) \cos\theta_i \, d\omega_i$$

$$= \int_{\Omega_+} \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \cdot \delta(\varphi_i - \varphi_o \pm \pi)\, L_i(\theta_i, \varphi_i) \cos\theta_i \, d\omega_i$$

$$= L_i(\theta_o, \varphi_o \pm \pi)$$



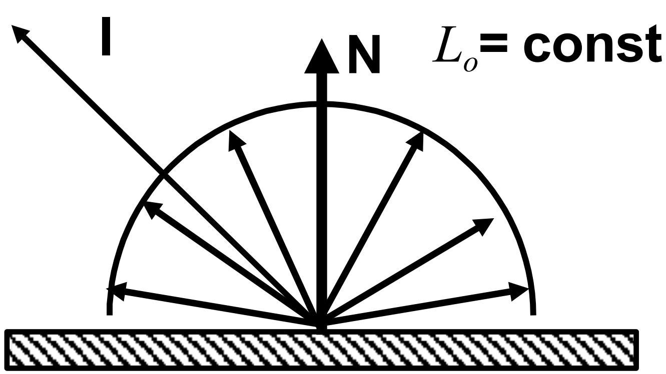
Light equally likely to be reflected in any output direction (independent of input direction)

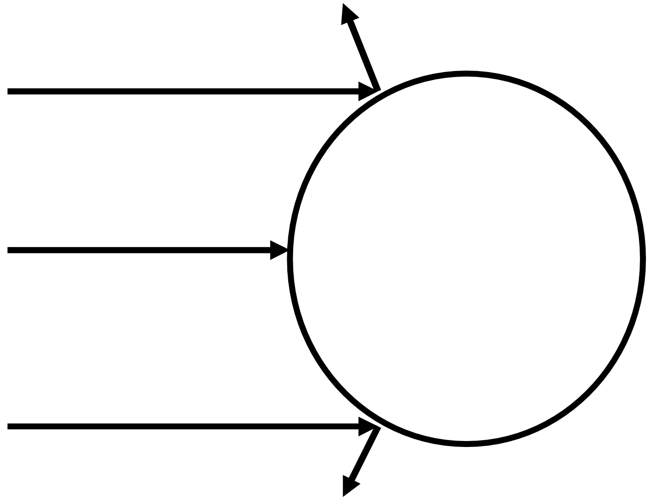
Constant BRDF

$$f_{r,d}(\omega_i, x, \omega_o) = k_d = const$$

$$L_o(x, \omega_0) = \int_{\Omega_+} k_d \, L_i(\mathrm{x}, \omega_i) \cos\theta_i \, d\omega_i = k_d \int_{\Omega_+} L_i(\mathrm{x}, \omega_i) \cos\theta_i \, d\omega_i = k_d E$$

- k_d: diffuse coefficient, material property [1 / sr]





Radiosity

$$B = \int_{\Omega_+} L_o(x,\omega_o)\, cos\,\theta_o\, d\omega_o = L_o \int_{\Omega_+} cos\,\theta_o\, d\omega_o = \pi L_o$$

Diffuse Reflectance

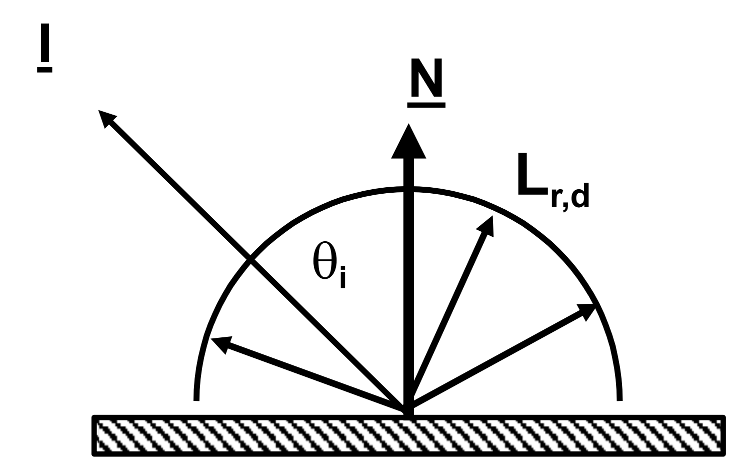
$$\mathrm{B} = \pi k_d E$$

Lambert's Cosine Law

$$\mathrm{B} = \pi k_d E_i \cos\theta_i$$

For each light source:

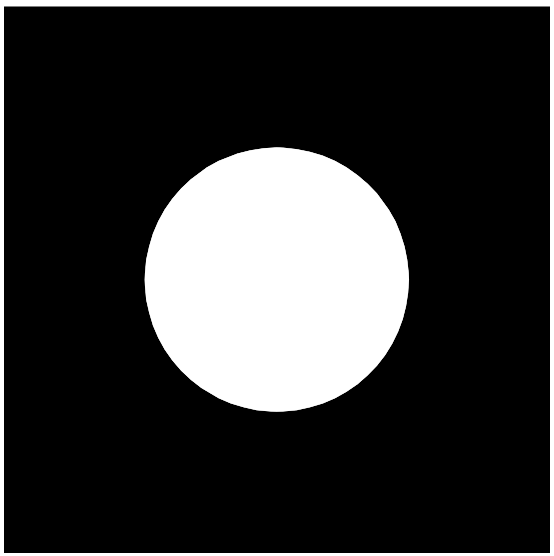
$$L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (I \cdot N)$$



Lambertian Objects

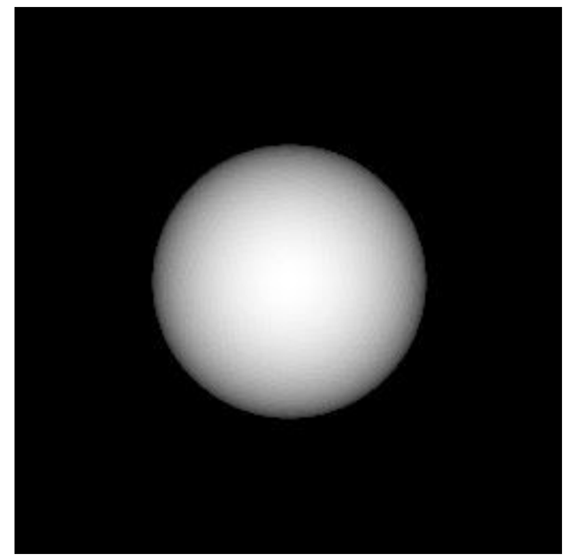
Self-Luminous spherical Lambertian Light Source

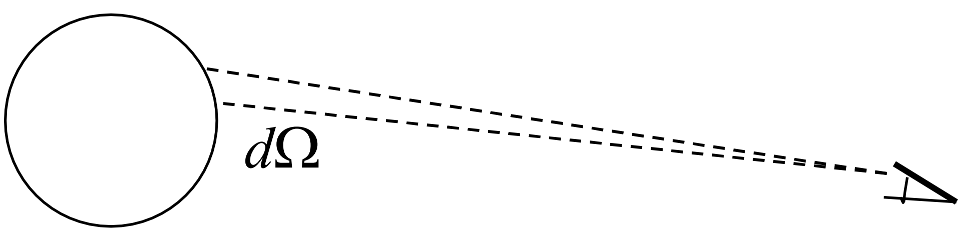
$$\Phi_0 \propto L_0 d\Omega$$

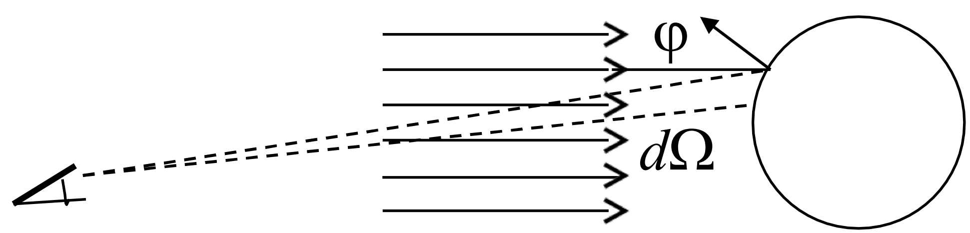


Eyelight illuminated Spherical Lambertian Reflector

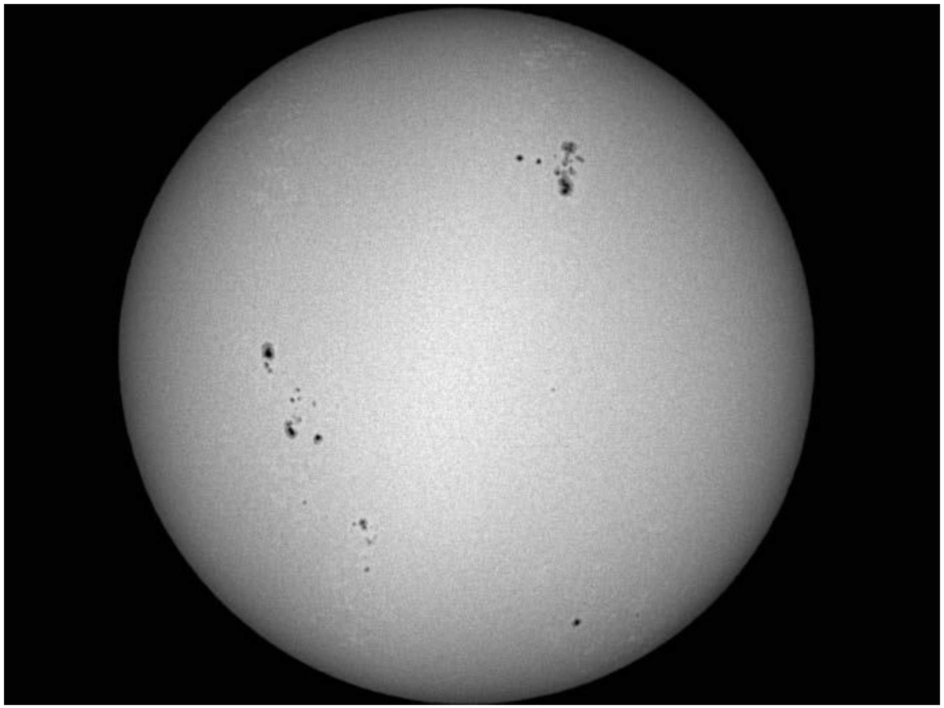
$$\Phi_1 \propto L_0 \cdot \cos\varphi \cdot d\Omega$$







The Sun



- Absorption in photosphere
- Path length through photosphere longer from the Sun’s rim

The Moon

- Surface covered with fine dust
- Dust on TV visible best from slanted viewing angle

Neither the Sun nor the Moon are Lambertian

Theoretical explanation

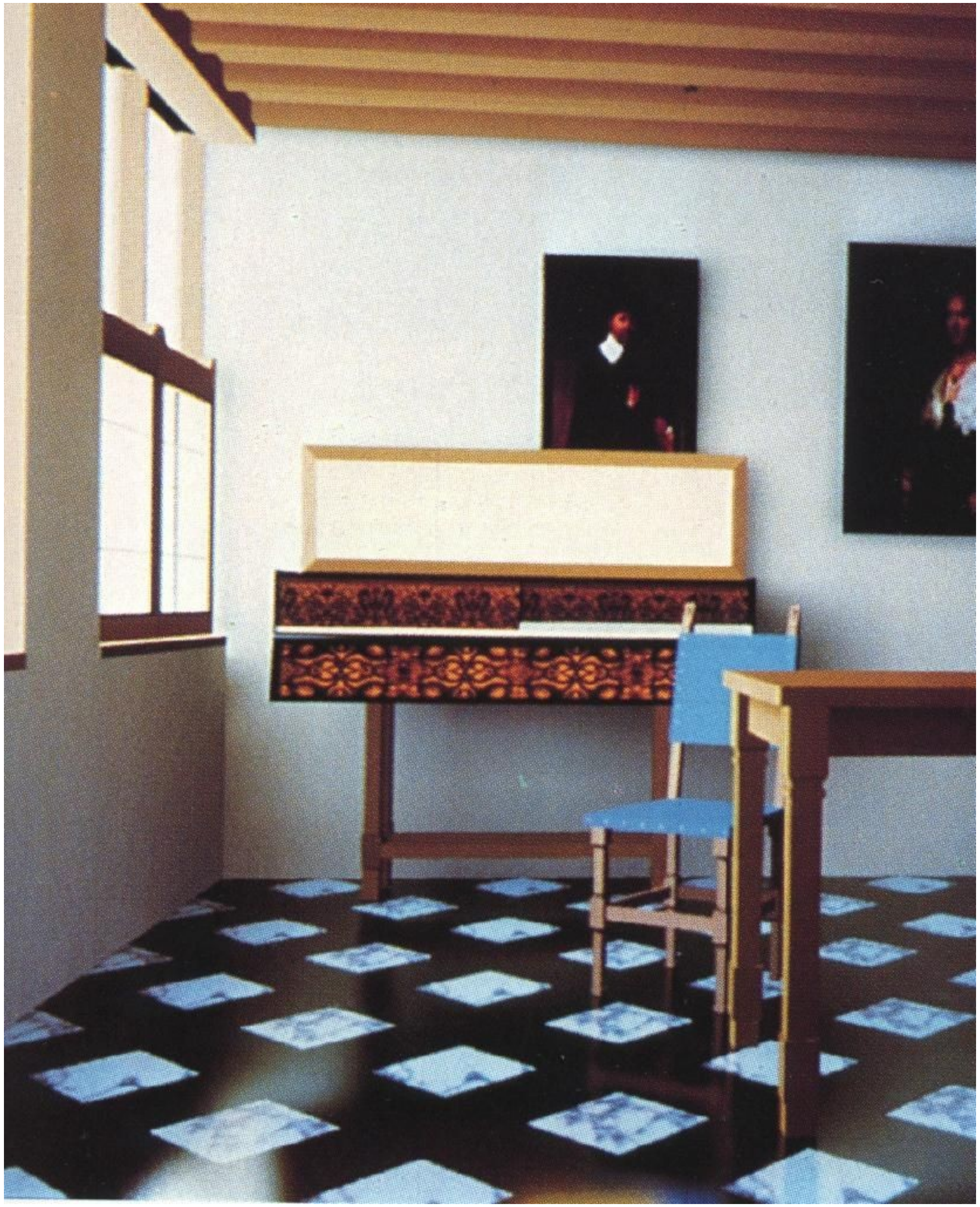
- Multiple scattering

Experimental realization

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufacturers attempt to create ideal diffuse paints

Glossy Reflection



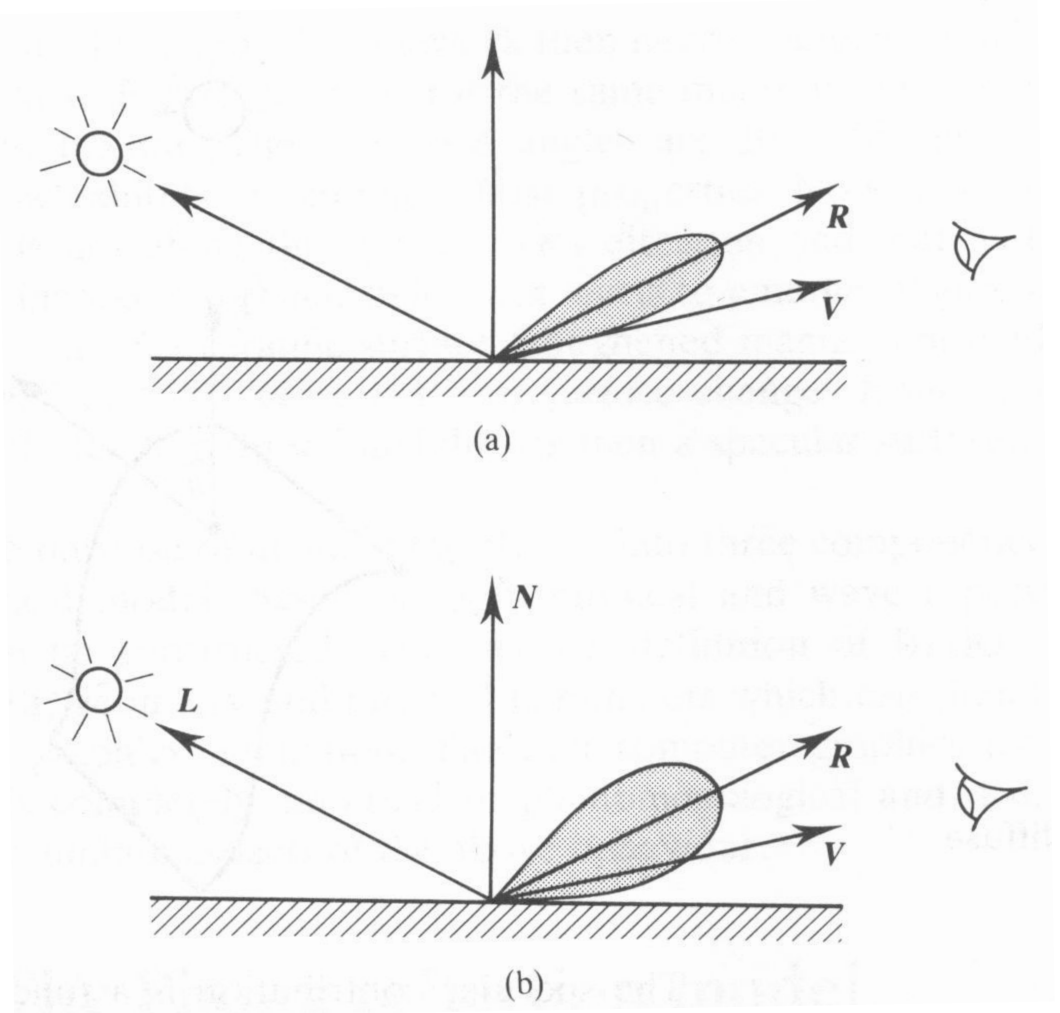
Due to surface roughness

Empirical models

- Phong
- Blinn-Phong

Physical models

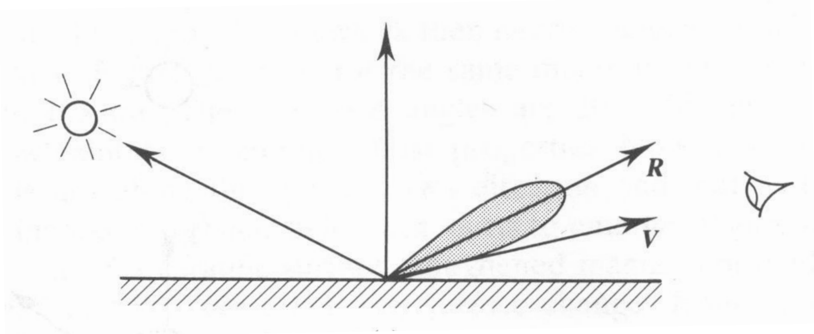
- Blinn
- Cook & Torrance

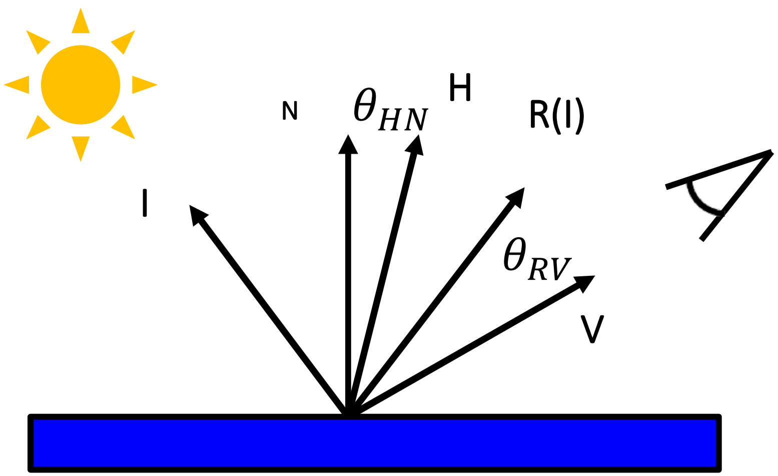


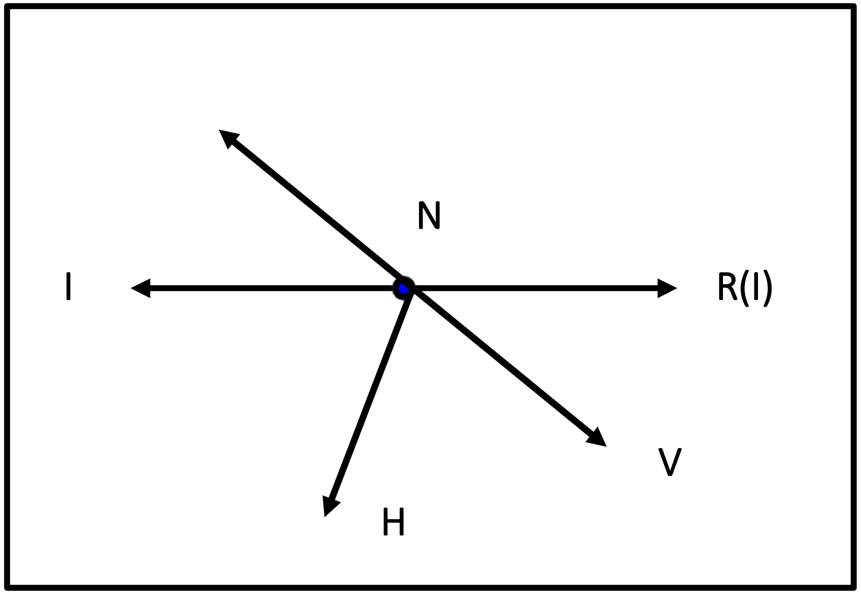
Cosine power lobe

$$f_r(\omega_i, x, \omega_o) = k_s(R(I) \cdot V)^{k_e}$$

- $L_{r,s} = L_i k_s (\cos\theta_{RV})^{k_e}$
- Dot product & power
- Not energy conserving / reciprocal
- Plastic-like appearance



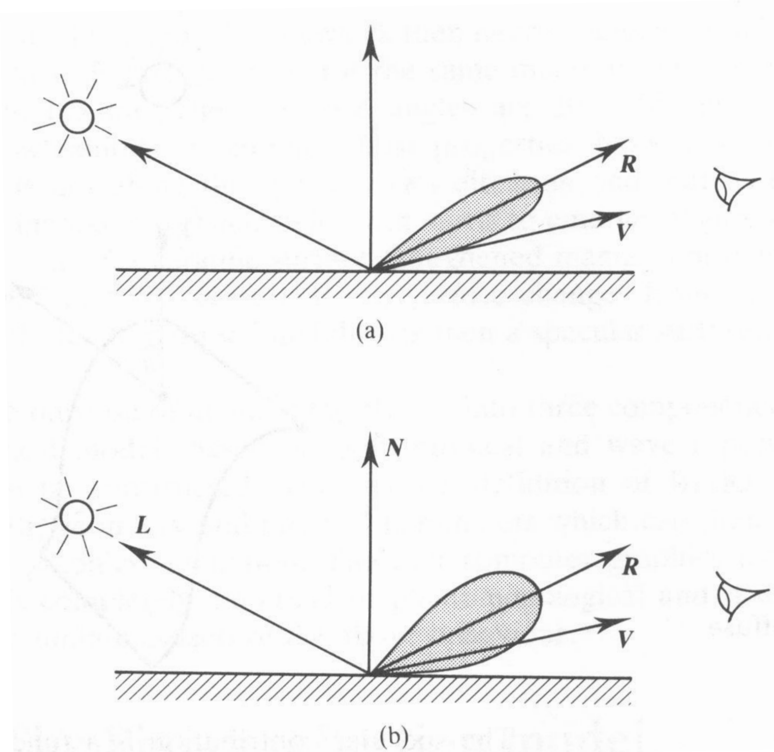


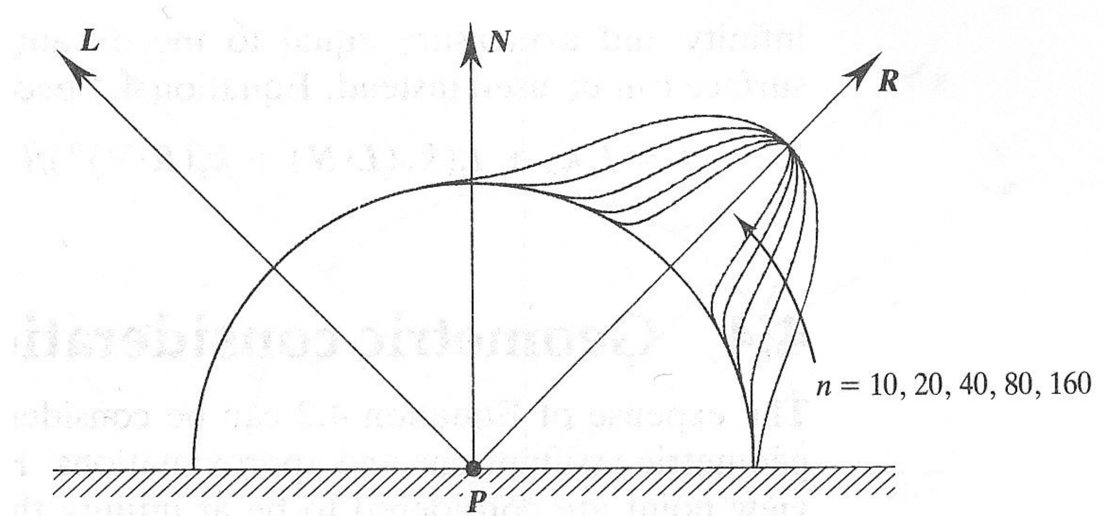


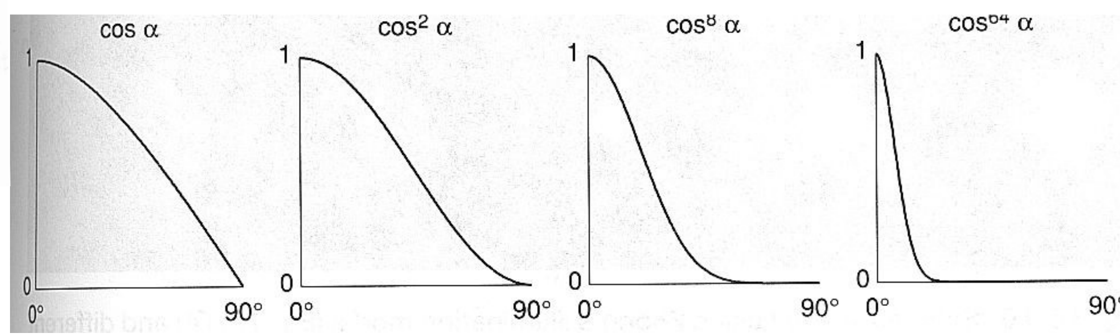
Phong Exponent k_e

$$f_r(\omega_o, x, \omega_i) = k_s(R(I) \cdot V)^{k_e}$$

Determines size of highlight



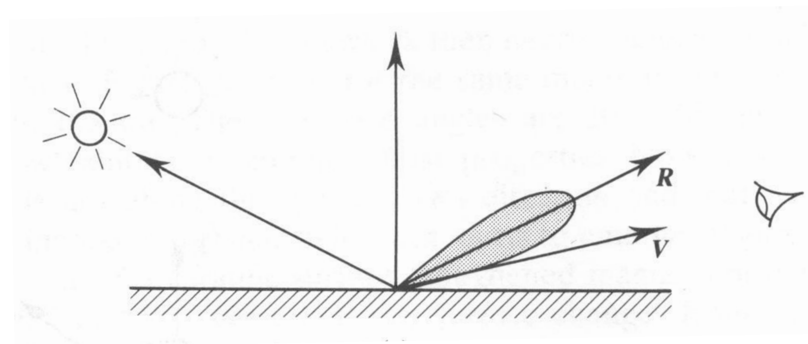


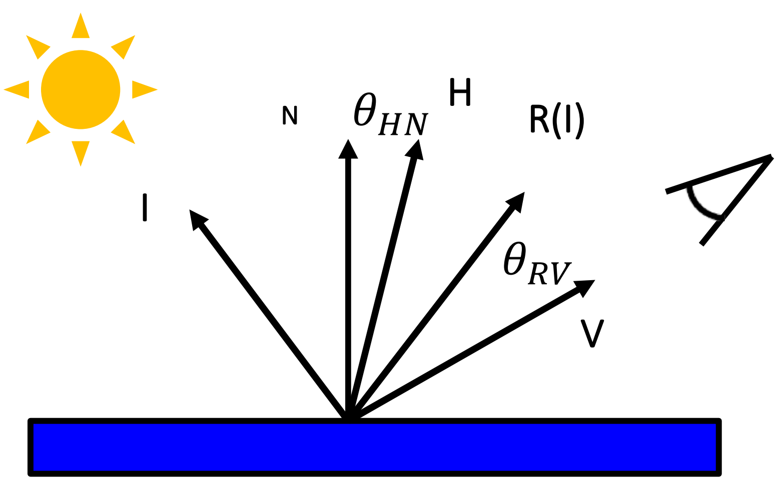


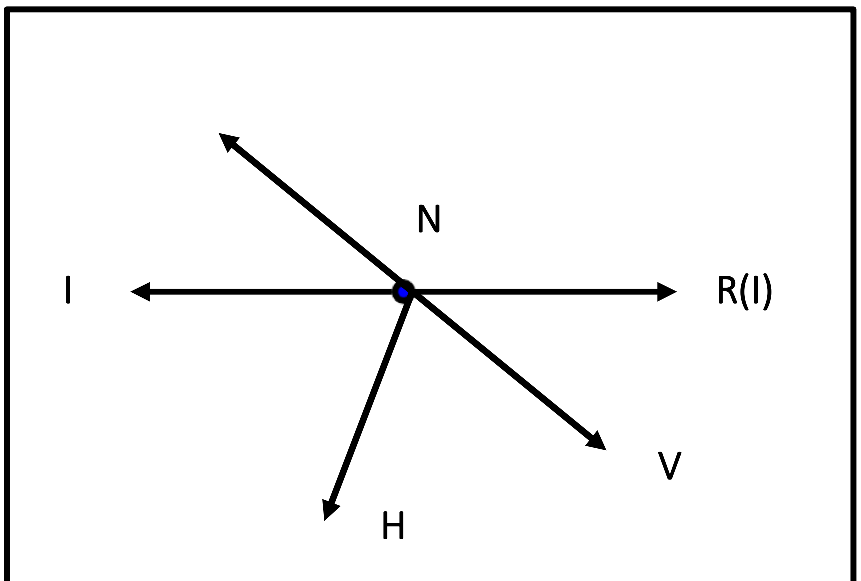
Blinn – Phong Reflection Model

$$f_r(\omega_i, x, \omega_o) = k_s (H \cdot N)^{k_e}$$

- $L_{r,s} = L_i k_s (\cos\theta_{HN})^{k_e}$
- $\theta_{RV} \to \theta_{HN}$
- Light source, viewer far away
- I, R constant: H constant
    - θ_{HN} less expensive to compute







Extended light sources: l point light sources

$$L_r = k_a L_{i,a} + k_d \sum_l L_i (I_l \cdot N) + k_s \sum_l L_i (R(I_l) \cdot V)^{k_e} \qquad \text{(Phong)}$$

$$L_r = k_a L_{i,a} + k_d \sum_l L_i (I_l \cdot N) + k_s \sum_l L_i (H_l \cdot N)^{k_e} \qquad \text{(Blinn)}$$

Color of specular reflection equal to light source

Heuristic model

- Contradicts physics
- Purely local illumination
  - Only direct light from the light sources
  - No further reflection on other surfaces
  - Constant ambient term

Often: light sources & viewer assumed to be far away

Assignment 2 (Theoretical part)

Submission deadline: Friday, 4. October 2019 9:45 (before the lecture)
Written solutions have to be submitted in the lecture room before the lecture. Every assignment sheets counts 100 points (theory and practice)

2.1 Reflection Rays (5 Points)

Given a ray $\vec{r}(t) = \vec{o} + t \cdot \vec{d}$ which hits a reflective surface at $t = t_{hit}$. The surface has the geometry normal $\vec{n}$ at the hit point. Assume that both, the ray direction $\vec{d}$ and the surface normal $\vec{n}$ are normalized. Compute the ray that has been reflected (assuming a perfect mirror reflection) by the surface.

You have to submit your solutions from this exercise in written form.