Jacobs University Bremen **Spring Semester 2020**
Prof. Preusser

Numerical Methods I

Assignment Sheet 2. Due: February 20, 2017

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Exercise 6 [5 + 10 Points]: Define

$$A = \begin{pmatrix} 1 & 0 & 4 & 1 \\ 2 & 1 & 10 & 1 \\ -1 & 5 & 4 & 3 \\ 5 & 2 & 2 & 2 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

a) Check whether Gaussian elimination with scaled partial pivoting can be applied to solve $Ax = b$.

b) Solve $Ax = b$ for x by Gaussian elimination with scaled partial pivoting. Structure your solution properly and clearly indicate which operations you are performing.

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Exercise 7 [5 + 5 + 5 Points]: Let

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 2 \\ 2 & 2 & 14 \end{pmatrix}$$

a) Show that the matrix A is positive definite.

b) Compute the LU decomposition of the matrix A. Structure your solution properly and clearly indicate which operations you are performing.

c) Compute the Cholesky decomposition of the matrix A. Structure your solution properly and clearly indicate which operations you are performing.

d) **(2 Bonus Points)** Prove that an LU decomposition exists for any positive definite matrix.

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Exercise 8 [not graded, w/o Points]: Given an $n \times n$-matrix A with entries $a_{ij} = (10 \cdot i)^{j-1}$ for $i, j = 1, \ldots, n$. Compute a vector b such that $x = (1, 1, \ldots, 1)$ is the solution to the linear equation system $Ax = b$.

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Exercise 9 [not graded, w/o Points]: Write down an algorithm that does LU decomposition with (scaled) partial pivoting (LUP). When does such LUP factorization exist?

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Exercise 10 [not graded, w/o Points]: Assume normalized floating point representation with base $b = 2$ and precision $k = 23$. If Gaussian elimination is used without pivoting to solve the system

$$\begin{pmatrix} \epsilon & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

what will be the solution vector when $0 < \epsilon < 2^{-22}$?
Hint: This corresponds to IEEE 32bit single precision floating point representations: 1bit is used for the sign, 8 bits are used for the exponent, and the remaining 23bits carry the mantissa.