Jacobs University Bremen **Spring Semester 2020**
Prof. Preusser

Numerical Methods I

Assignment Sheet 1. Due: February 12, 2020, 11:15 sharp

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Exercise 1 [3 x 5 Points]: Let $f(x) = \ln\left(\frac{x}{2}\right)$.

a) Compute the Taylor series for f developed at $c = 2$.

b) Use the Taylor series of f when truncated after the n-th term to compute $f\left(\frac{5}{2}\right)$ for $n = 1, \ldots, 4$.

c) Compare the values computed in b) with the actual value of $f\left(\frac{5}{2}\right)$ and plot the errors over n.

d) **(4 Bonus Points)** Show that the Taylor series for $f(x) = \ln\left(\frac{x}{2}\right)$ developed at $c = 2$ represents the function f for $x \in [2, 3]$.

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Exercise 2 [5 + 5 + 5 Points]: The solutions x_1 and x_2 of a quadratic equation $\alpha x^2 + \beta x + \gamma = 0$ can be found by the equation

$$x_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}.$$

a) Compute the solutions to the quadratic equation with $\alpha = 1$, $\beta = 200$, and $\gamma = 0.000015$ when assuming normalized floating-point representations with a mantissa of $k = 10$ digits precision and base $b = 10$.

b) Use the theorem from class to predict the number of lost significant bits when executing the problematic subtraction.

c) The absolute error is calculated by the magnitude of the difference between the computed solution and the actual solution. Compute the absolute error for the solutions in a).

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Exercise 3 [not graded, w/o Points]: Let $f(x) = \log_2(2x)$.

a) Derive the truncated Taylor series expansion and the respective error term when truncating the Taylor series for $f(x + h)$ developed at $x = 1$ after the n-th term.

b) For which values of h does the Taylor series in a) represent the function? Prove your claim.

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Exercise 4 [not graded, w/o Points]: Determine the best integer value of $k \in \mathbb{N}$ in the equation

$$\arctan(x) = x + \mathcal{O}(x^k) \text{ as } x \to 0.$$

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Exercise 5 [not graded, w/o Points]: How many digits of precision are lost in the subtraction $1 - \cos(x)$ for $x = \frac{1}{4}$?