COMPUTER VISION
LECTURE 11-12 – CLUSTERING AND SEGMENTATION

Prof. Dr. Francesco Maurelli

2019-10-08

2019-10-09

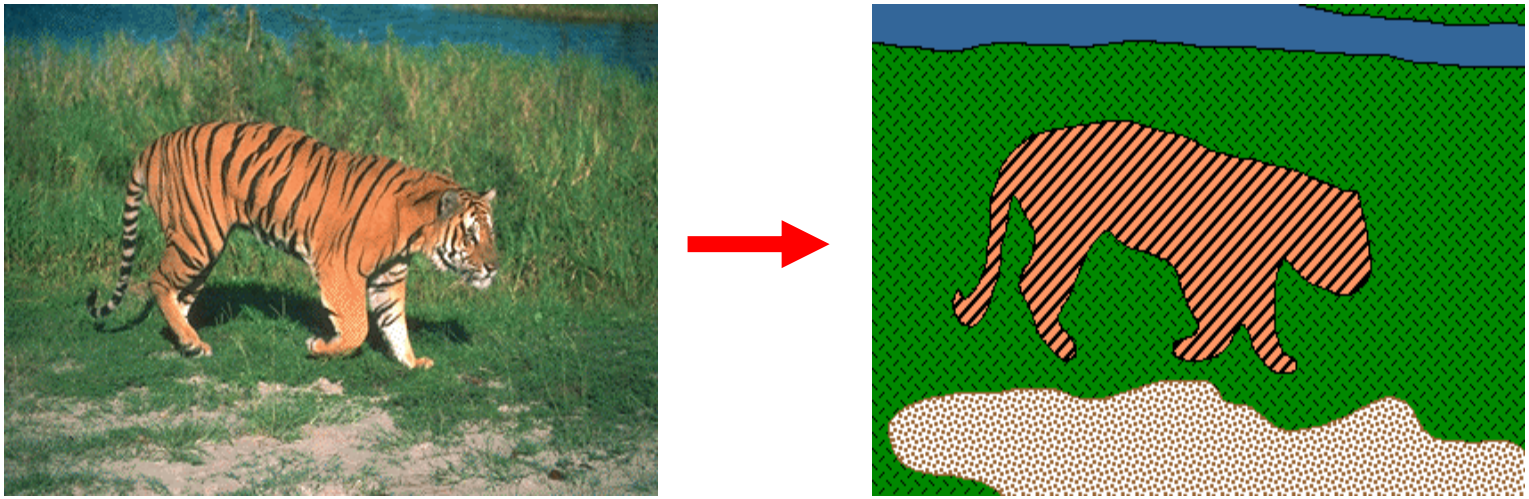
What we will learn today

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Agglomerative clustering
- Oversegmentation

Reading: [Forsyth & Ponce] Chapters: 14.2, 14.4

Image Segmentation

- Goal: identify groups of pixels that go together

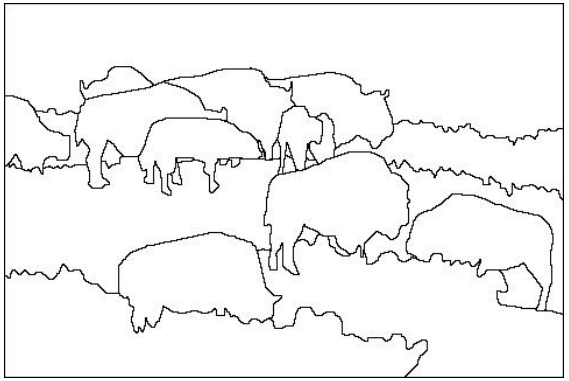


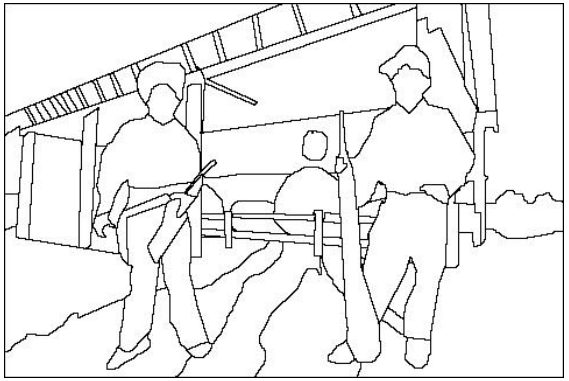
Slide credit: Steve Seitz, Kristen Grauman

The Goals of Segmentation

- Separate image into coherent “objects”

Image | Human segmentation

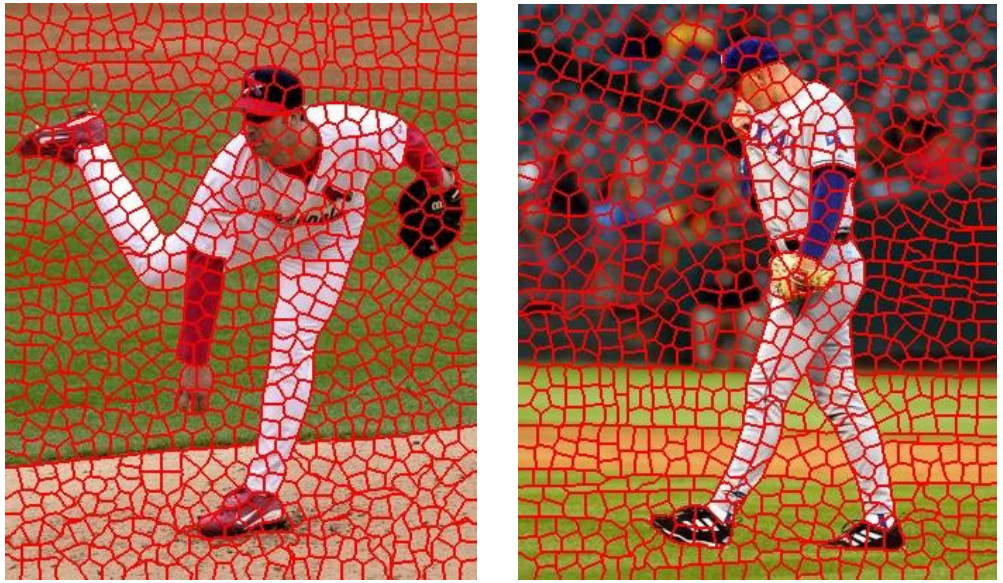



Slide credit: Svetlana Lazebnik

The Goals of Segmentation

- Separate image into coherent “objects”
- Group together similar-looking pixels for efficiency of further processing

“superpixels”



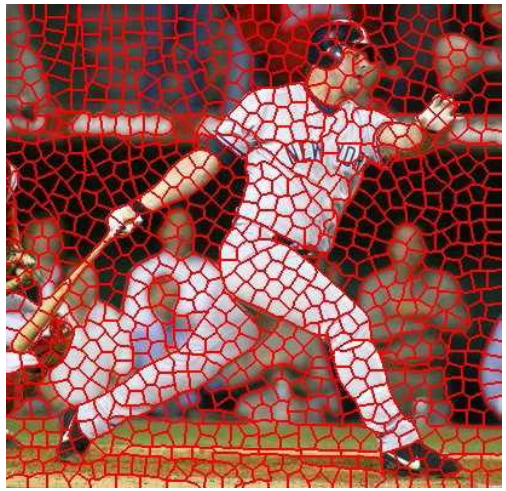
X. Ren and J. Malik. Learning a classification model for segmentation. ICCV 2003.

Slide credit: Svetlana Lazebnik

Segmentation for efficiency

[Felzenszwalb and Huttenlocher 2004]

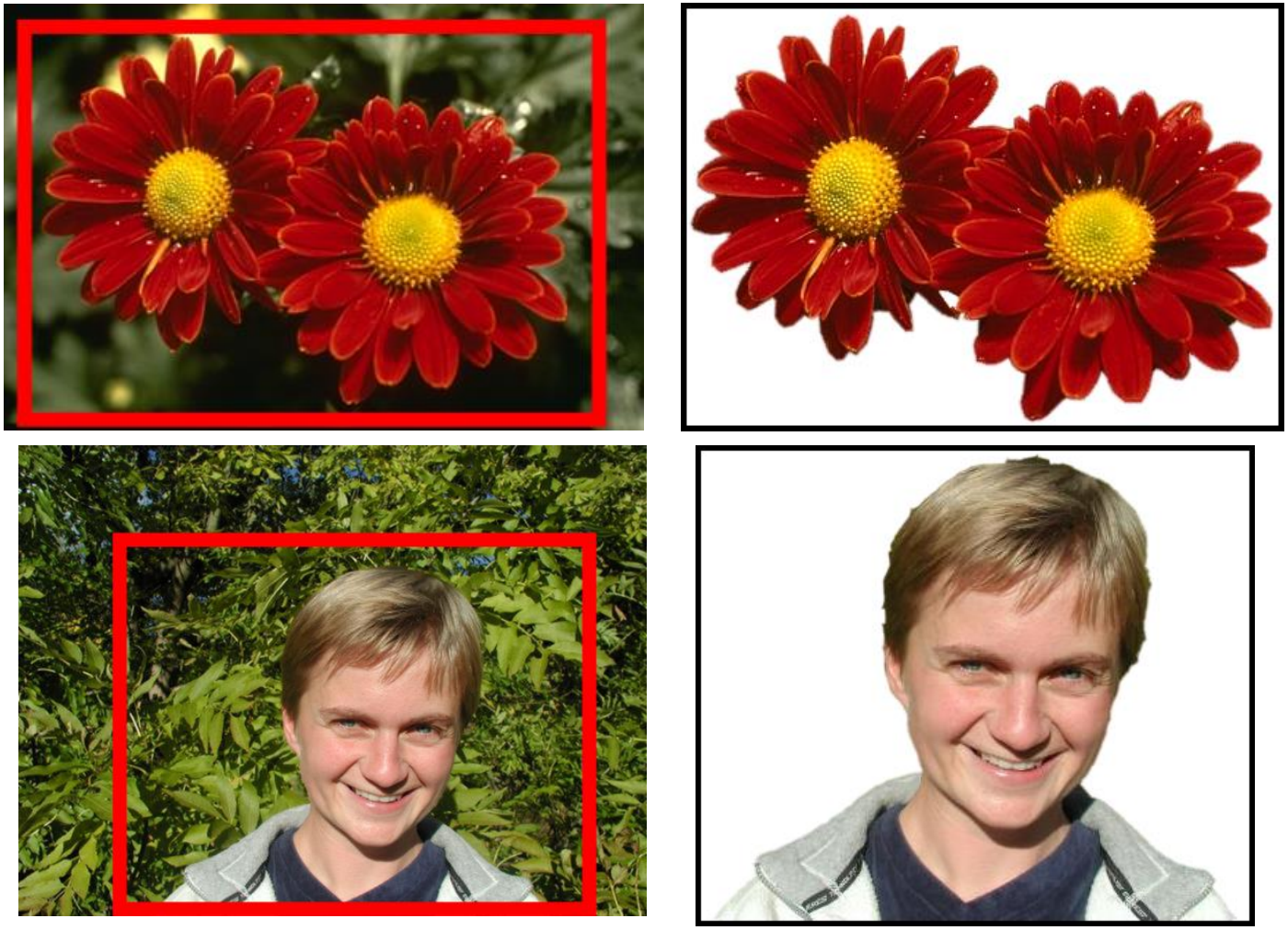
[Hoiem et al. 2005, Mori 2005]



[Shi and Malik 2001]

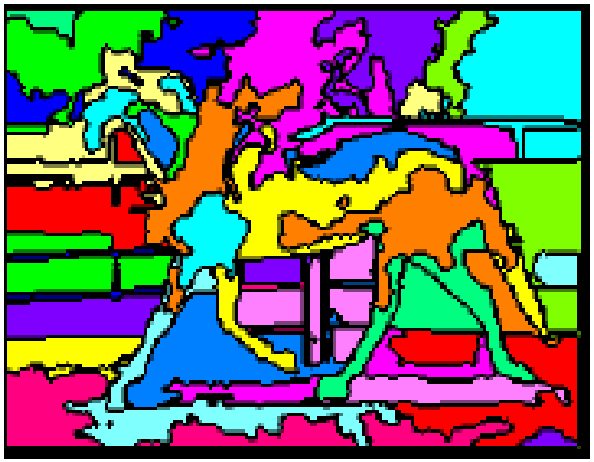
Slide: Derek Hoiem

Segmentation as a result

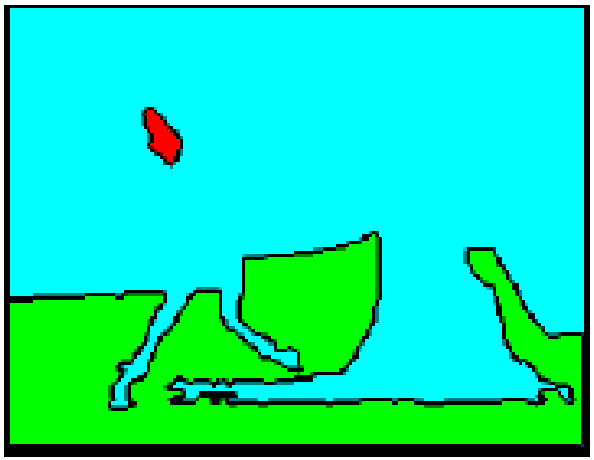


Rother et al. 2004

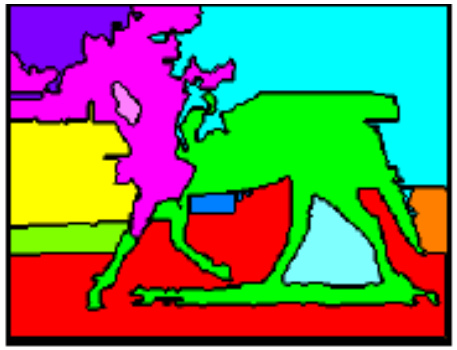
Types of segmentations

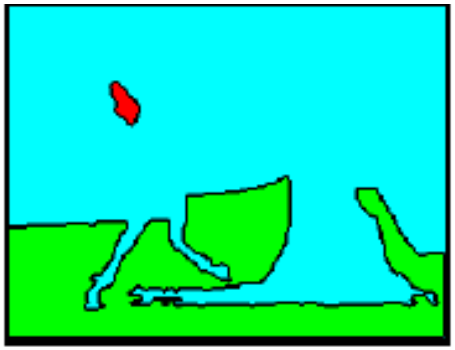


Oversegmentation



Undersegmentation





Multiple Segmentations

One way to think about "segmentation" is Clustering

Clustering: group together similar data points and represent them with a single token

Key Challenges:

1) What makes two points/images/patches similar?

2) How do we compute an overall grouping from pairwise similarities?

Slide: Derek Hoiem

Why do we cluster?

- **Summarizing data**
  - Look at large amounts of data
  - Patch-based compression or denoising
  - Represent a large continuous vector with the cluster number
- **Counting**
  - Histograms of texture, color, SIFT vectors
- **Segmentation**
  - Separate the image into different regions
- **Prediction**
  - Pixels in the same cluster may have the same labels

Slide: Derek Hoiem

How do we cluster?

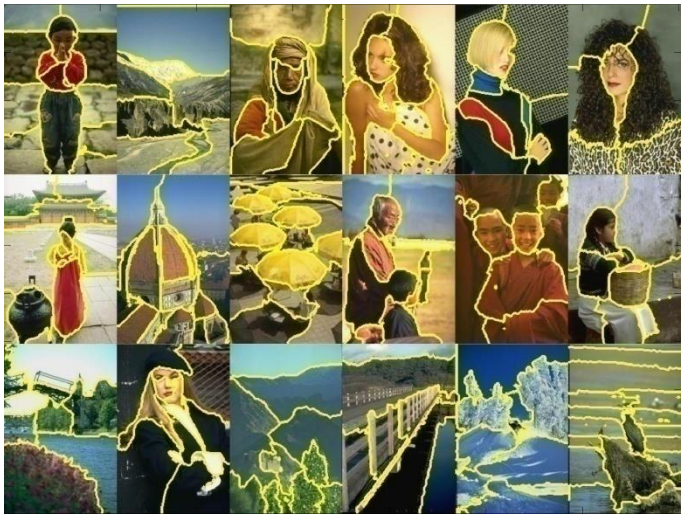
- Agglomerative clustering
  - Start with each point as its own cluster and iteratively merge the closest clusters
- K-means (next lecture)
  - Iteratively re-assign points to the nearest cluster center
- Mean-shift clustering (next lecture)
  - Estimate modes of pdf

General ideas

- Tokens
  - whatever we need to group (pixels, points, surface elements, etc., etc.)
- Bottom up clustering
  - tokens belong together because they are locally coherent
- Top down clustering
  - tokens belong together because they lie on the same visual entity (object, scene…)

> These two are not mutually exclusive

Examples of Grouping in Vision



Determining image regions

Grouping video frames into shots

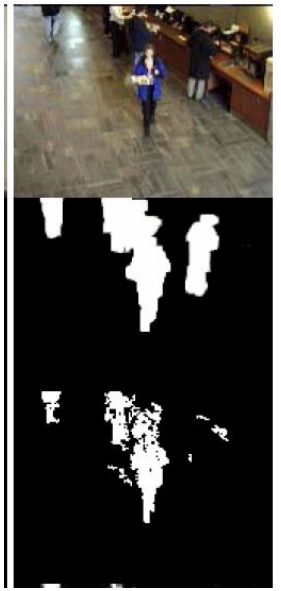
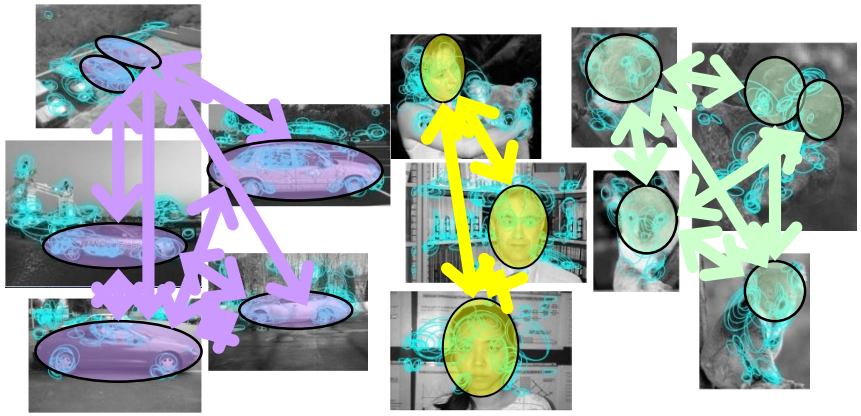


Figure-ground

What things should be grouped?

What cues indicate groups?



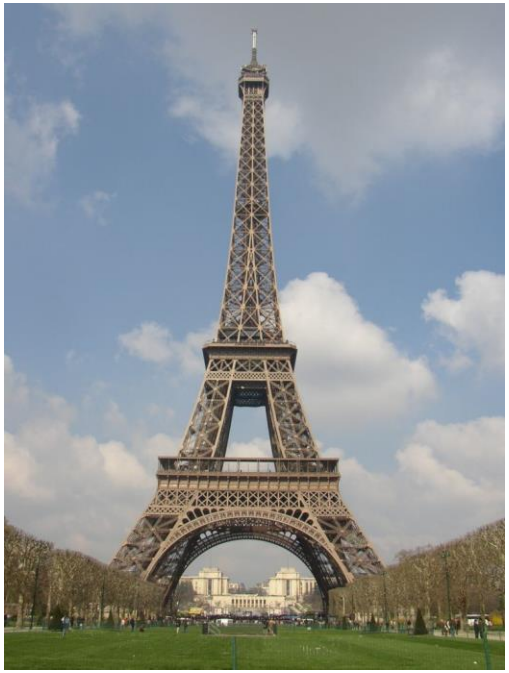
Object-level grouping

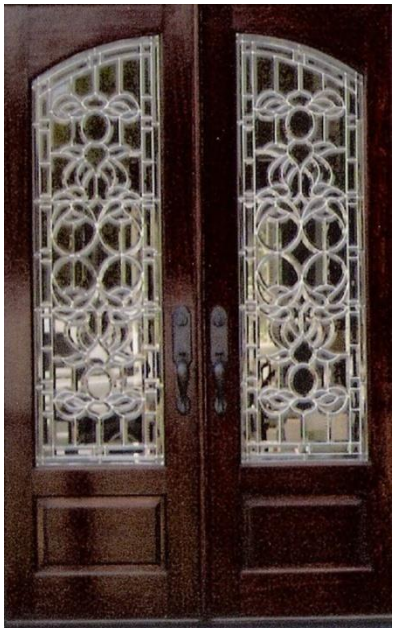
Slide credit: Kristen Grauman

Similarity

Slide credit: Kristen Grauman

Symmetry





Slide credit: Kristen Grauman

Common Fate

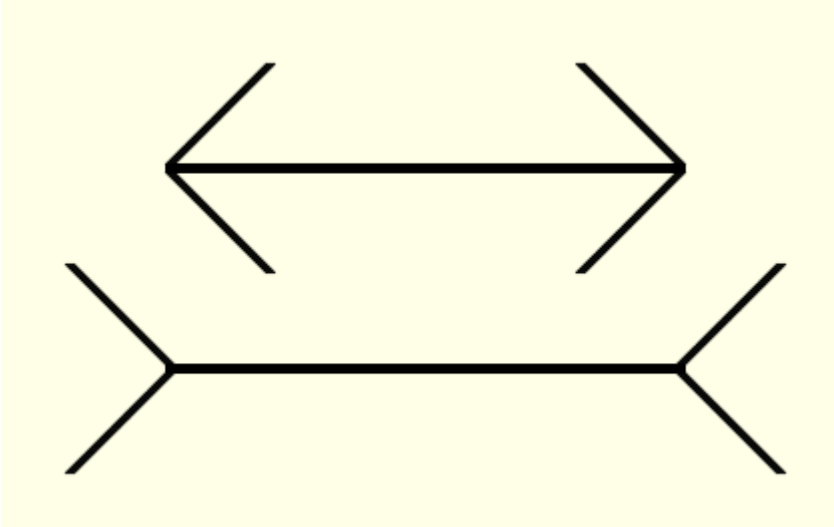
Image credit: Arthus-Bertrand (via F. Durand)

Slide credit: Kristen Grauman

Proximity

Slide credit: Kristen Grauman

Muller-Lyer Illusion



- What makes the bottom line look longer than the top line?

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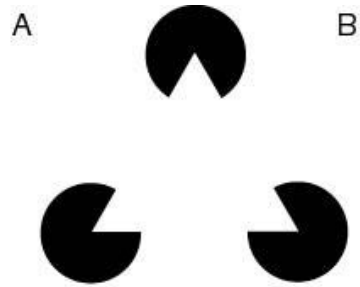
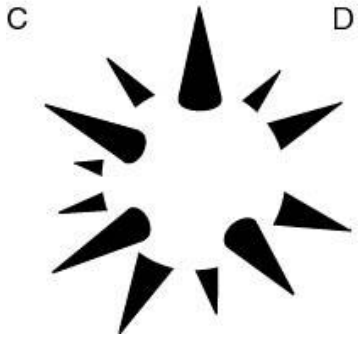
The Gestalt School

- Grouping is key to visual perception
- Elements in a collection can have properties that result from **relationships**
  - "The whole is greater than the sum of its parts"

Illusory/subjective contours



Occlusion



Familiar configuration

http://en.wikipedia.org/wiki/Gestalt_psychology

Slide credit: Svetlana Lazebnik

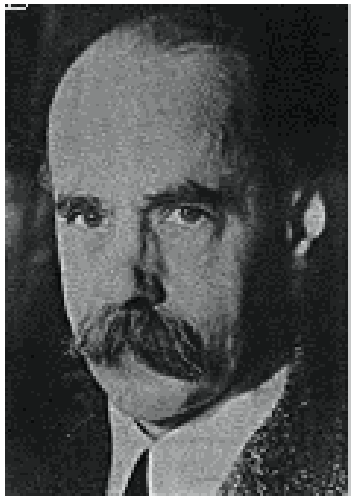
Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features

- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

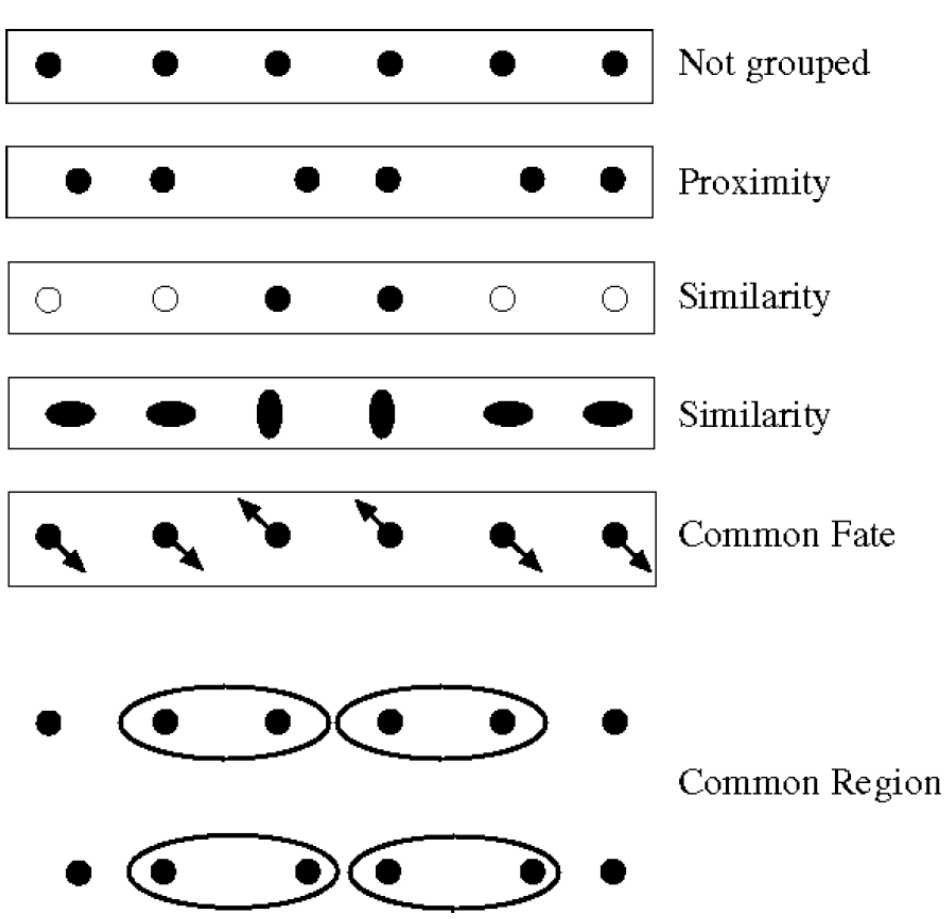
"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

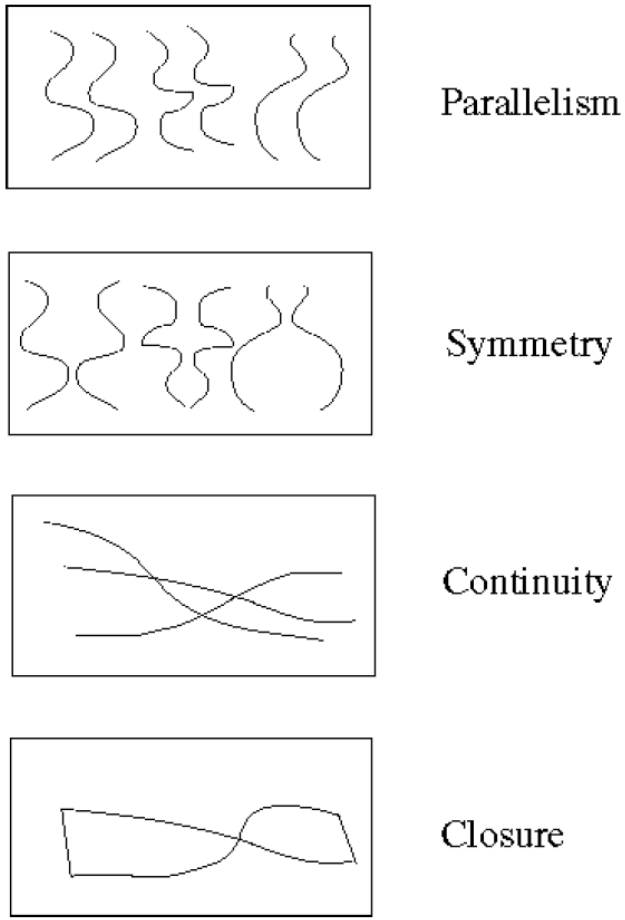
Max Wertheimer
(1880-1943)



Untersuchungen zur Lehre von der Gestalt,
***Psychologische Forschung*, Vol. 4, pp. 301-350, 1923**
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

Gestalt Factors

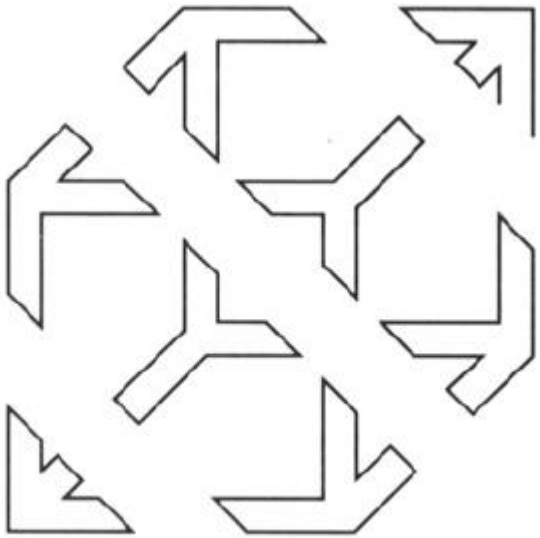




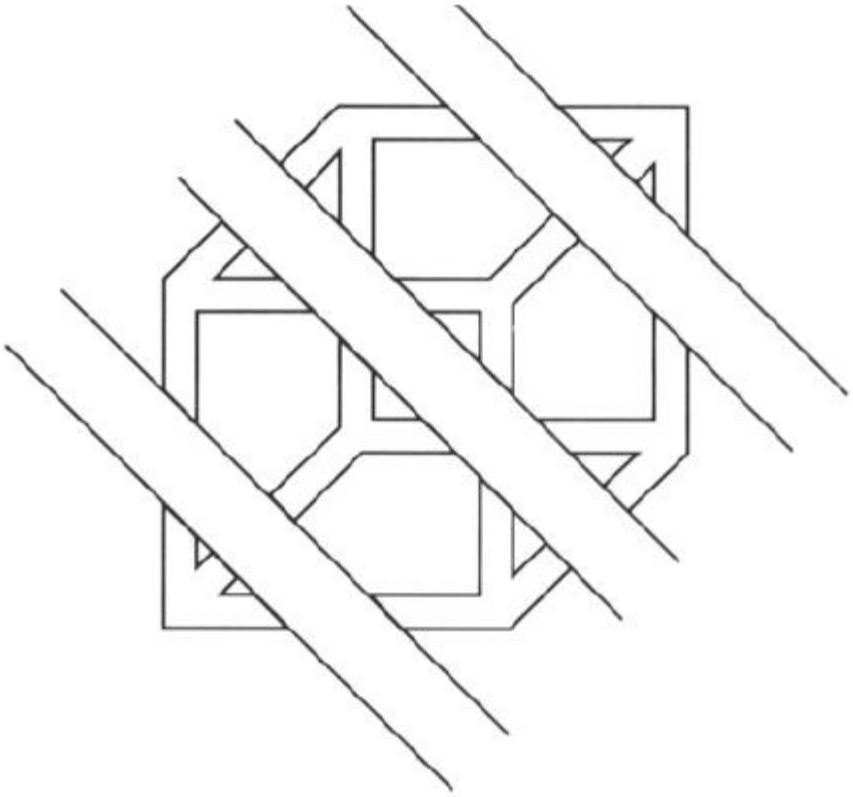
- These factors make intuitive sense, but are very difficult to translate into algorithms.

Image source: Forsyth & Ponce

Continuity through Occlusion Cues

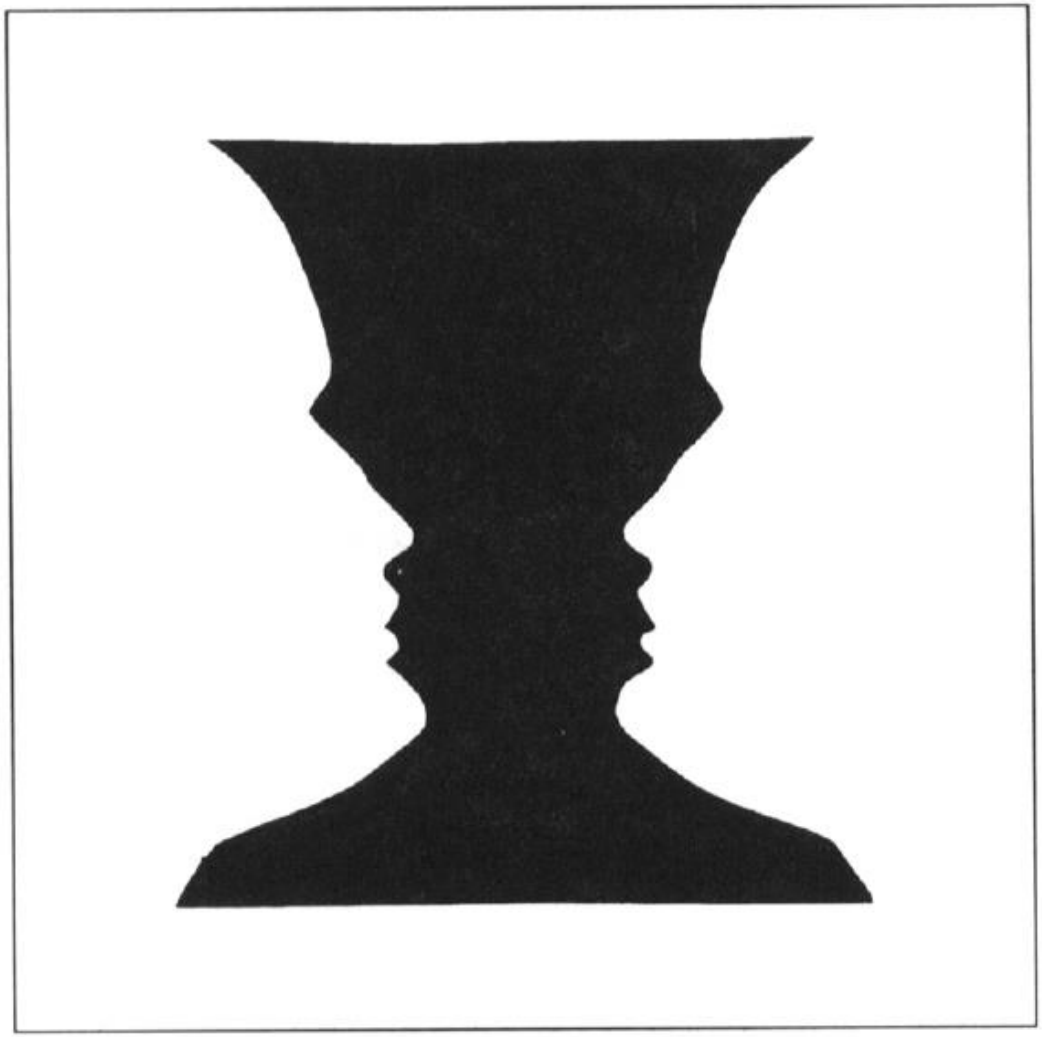


Continuity through Occlusion Cues



Continuity, explanation by occlusion

Figure-Ground Discrimination



The Ultimate Gestalt?

What we will learn today

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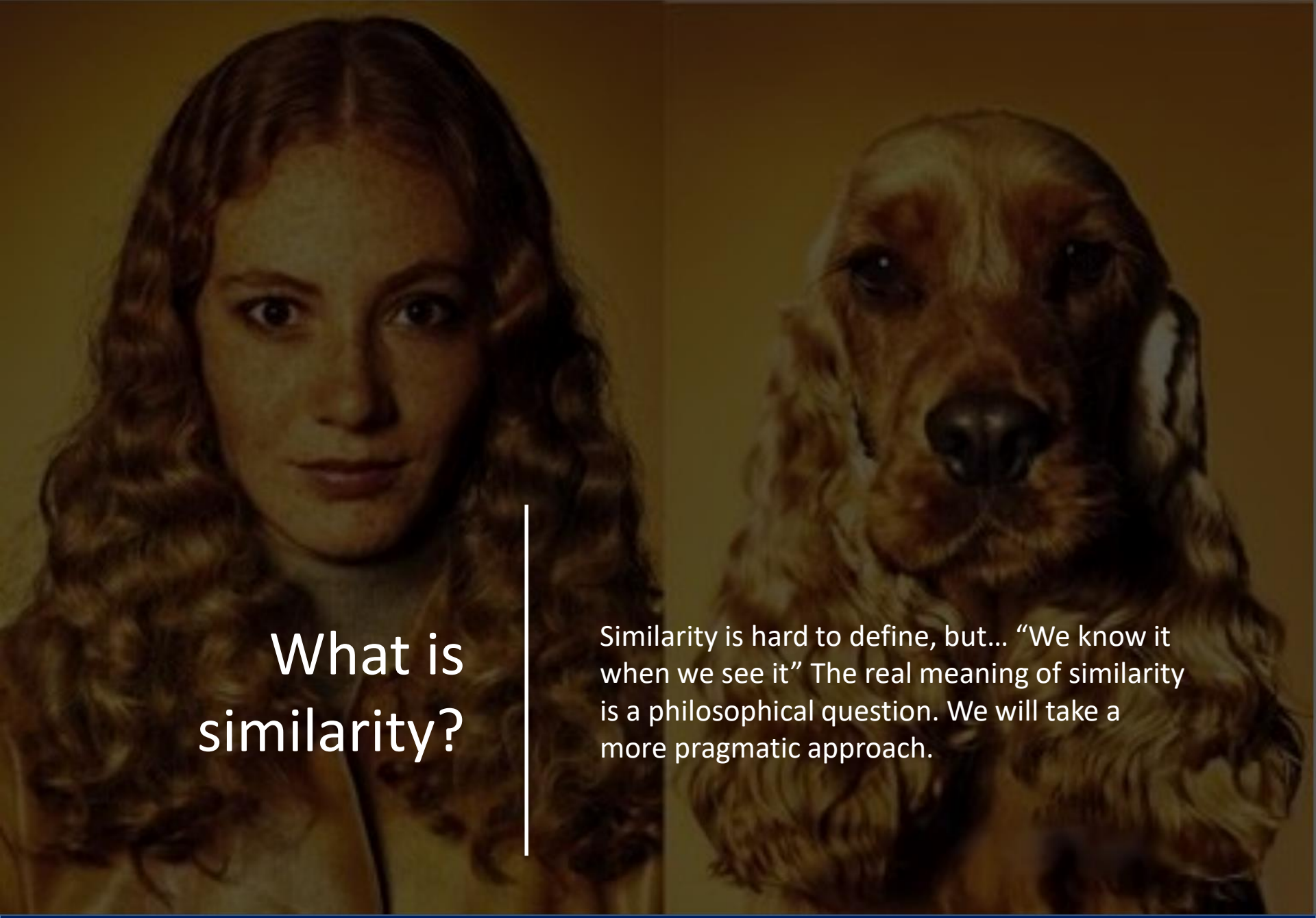


What is similarity?

Similarity is hard to define, but… “We know it when we see it” The real meaning of similarity is a philosophical question. We will take a more pragmatic approach.

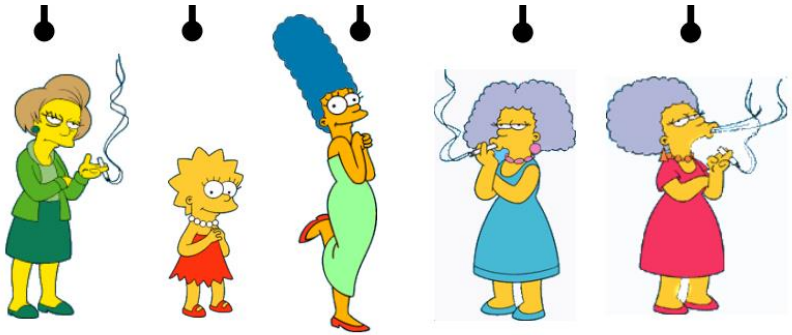
Clustering: distance measure

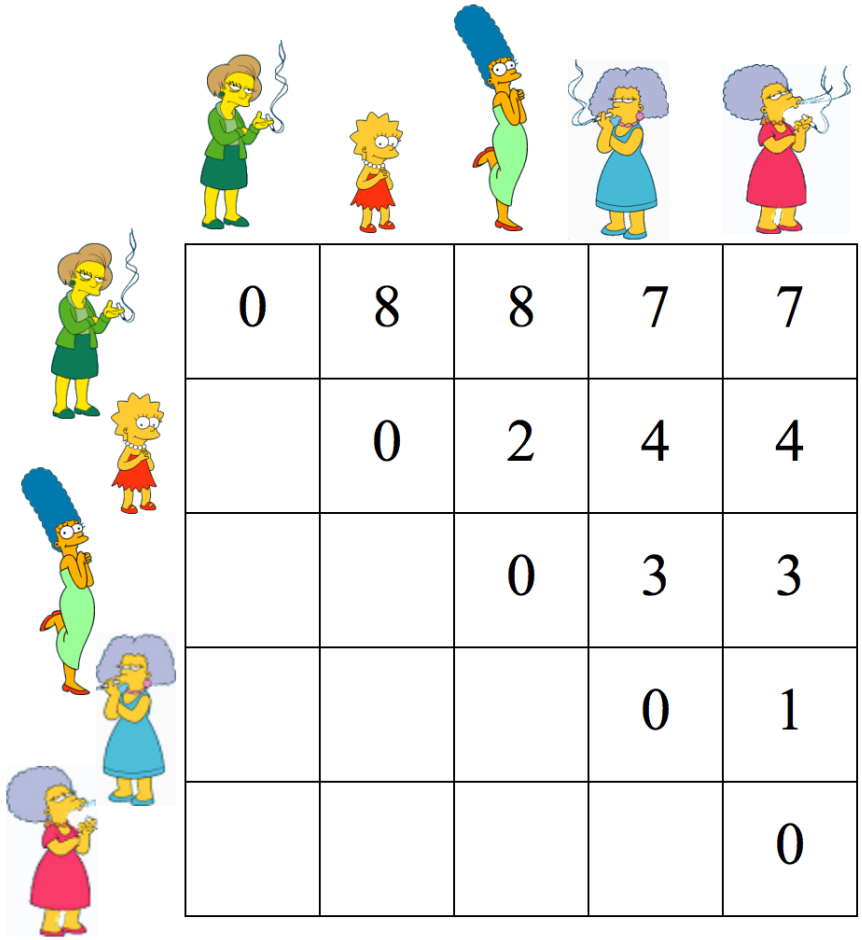
Clustering is an unsupervised learning method. Given items $x_1, \ldots, x_n \in \mathbb{R}^D$, the goal is to group them into clusters. We need a pairwise distance/similarity function between items, and sometimes the desired number of clusters.

Desirable Properties of a Clustering Algorithms

- Scalability (in terms of both time and space)
- Ability to deal with different data types
- Minimal requirements for domain knowledge to determine input parameters
- Interpretability and usability Optional
  - Incorporation of user-specified constraints

Animated example

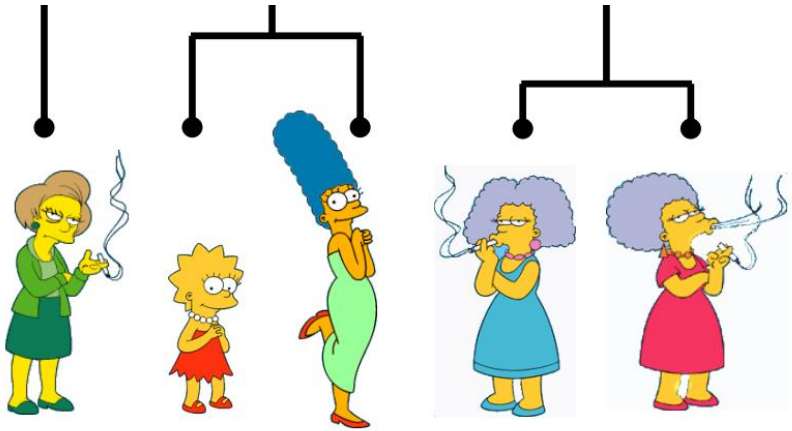


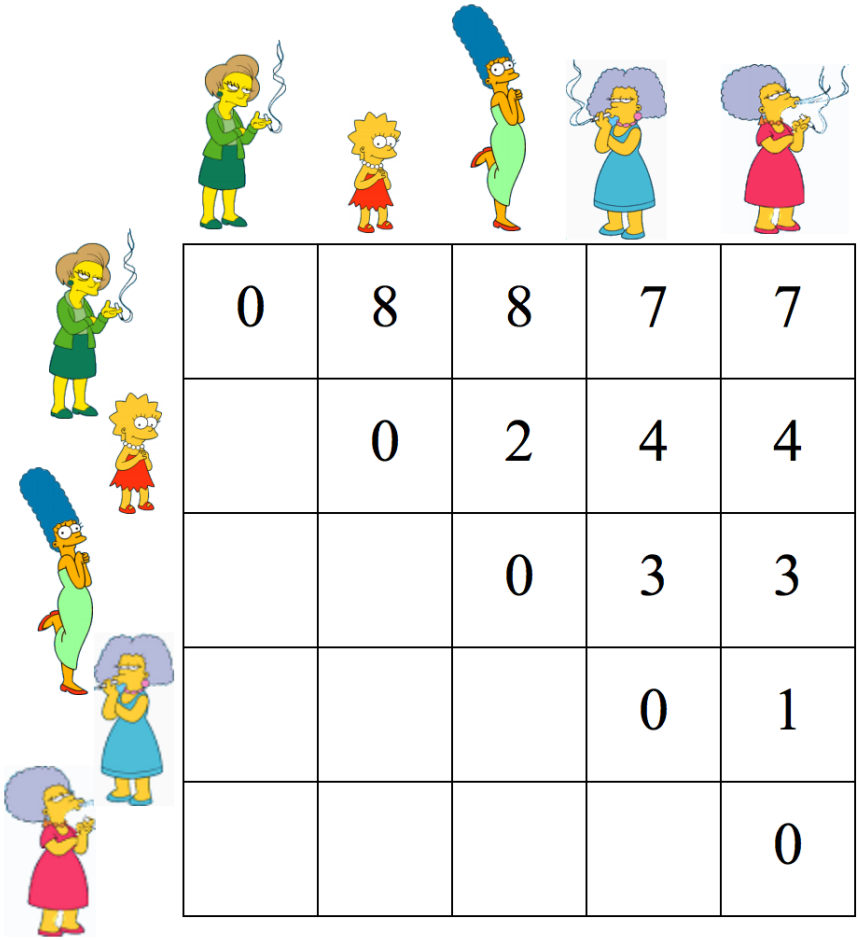


0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
				0

source

Animated example





0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
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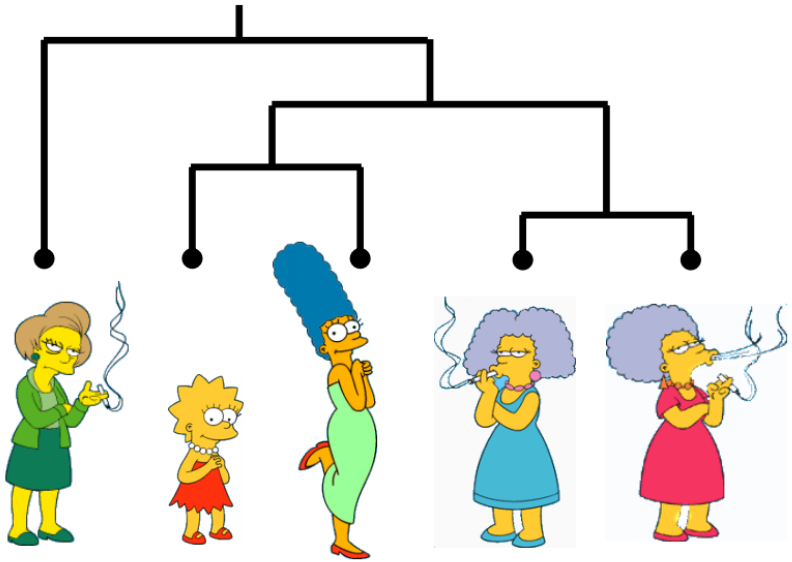
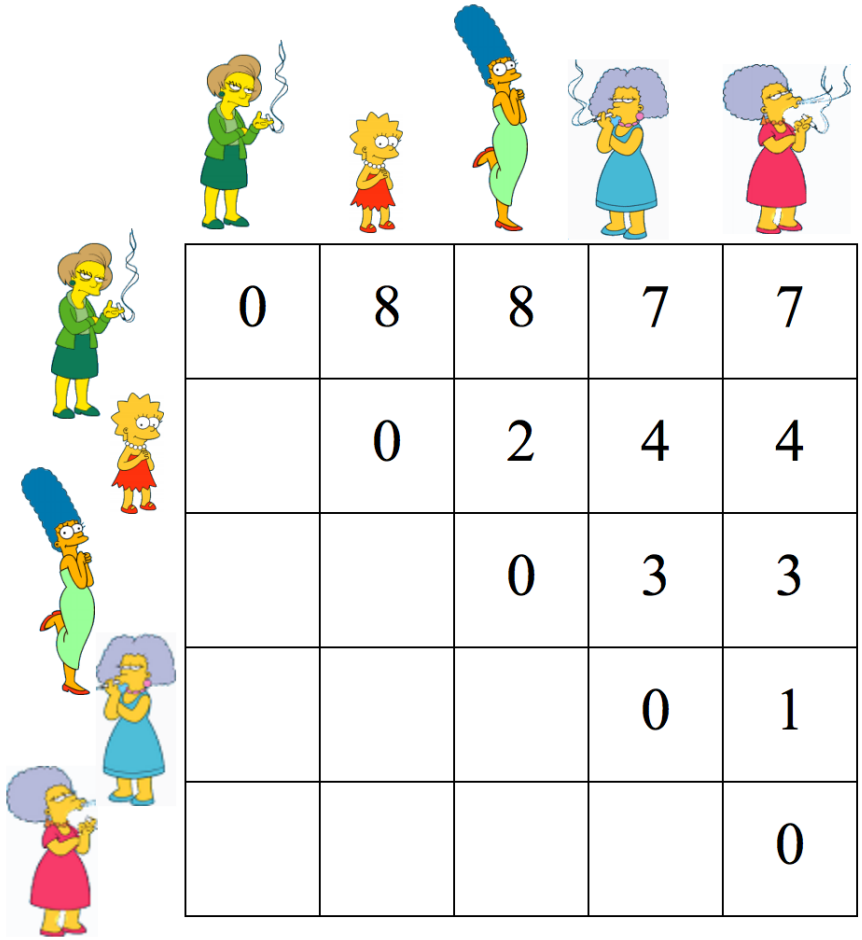
source

Animated example

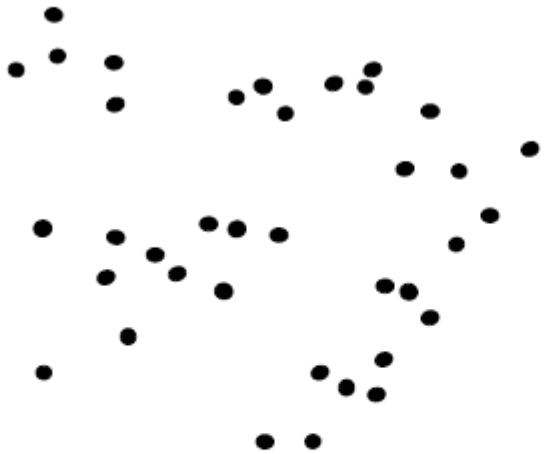




0	8	8	7	7
	0	2	4	4
		0	3	3
			0	1
				0

source

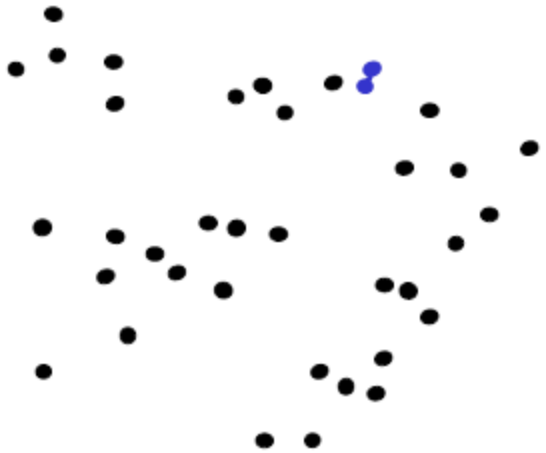
Agglomerative clustering



1. Say "Every point is its own cluster"

Slide credit: Andrew Moore

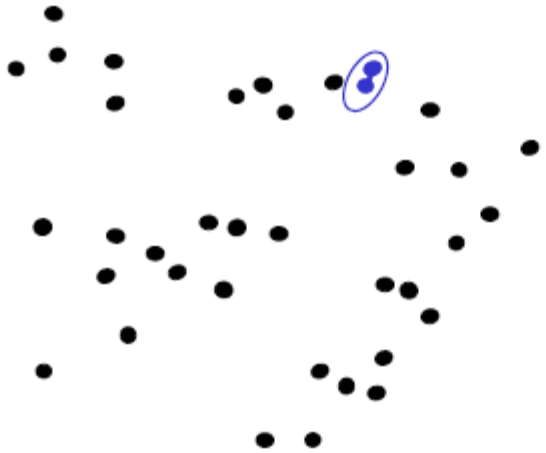
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters

Slide credit: Andrew Moore

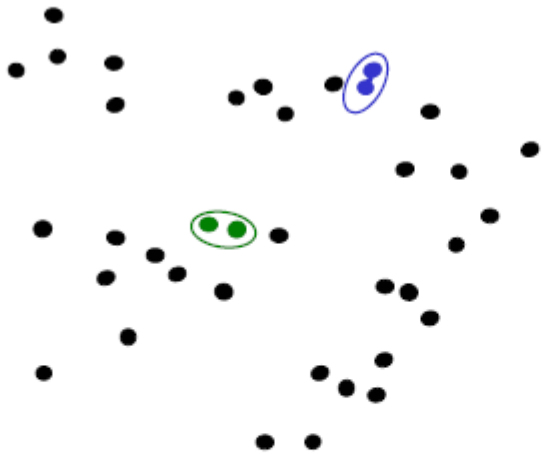
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster

Slide credit: Andrew Moore

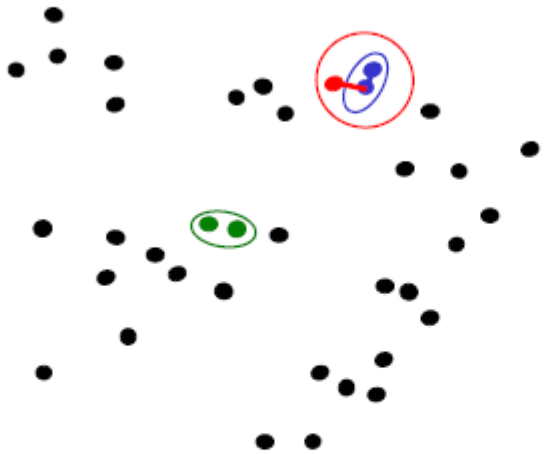
Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster
4. Repeat

Slide credit: Andrew Moore

Agglomerative clustering



1. Say "Every point is its own cluster"
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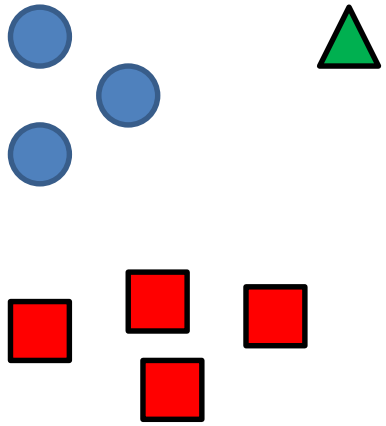
Slide credit: Andrew Moore

Agglomerative clustering

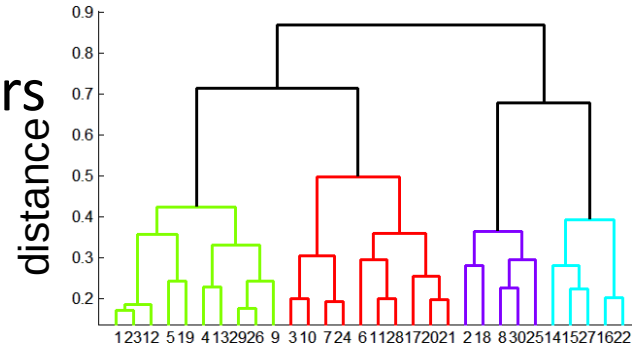
How to define cluster similarity?

- Average distance between points,
- maximum distance
- minimum distance
- Distance between means or medoids



How many clusters?

- Clustering creates a dendrogram (a tree)
- Threshold based on max number of clusters or based on distance between merges



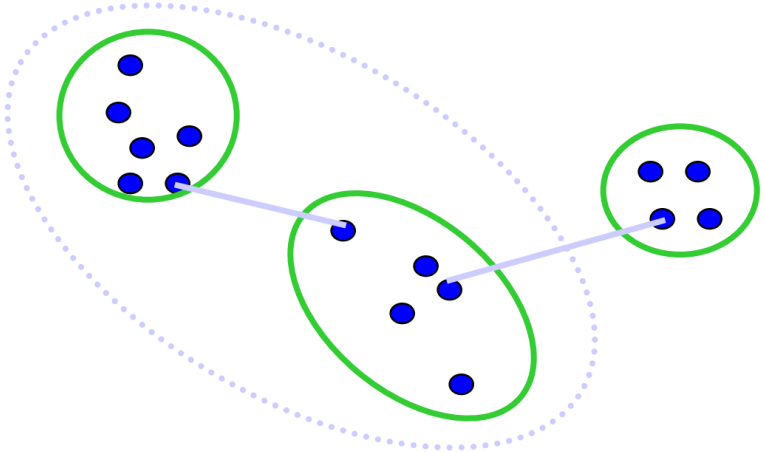
Agglomerative Hierarchical Clustering - Algorithm

1. Initially each item $x_1, \ldots, x_n$ is in its own cluster $C_1, \ldots, C_n$.
2. Repeat until there is only one cluster left:
3. Merge the nearest clusters, say C_i and C_j.

Different measures of nearest clusters

Single Link

- $d(C_i, C_j) = \min_{x \in C_i, x' \in C_j} d(x, x')$. This is known as *single-linkage*. It is equivalent to the minimum spanning tree algorithm. One can set a threshold and stop clustering once the distance between clusters is above the threshold. Single-linkage tends to produce long and skinny clusters.

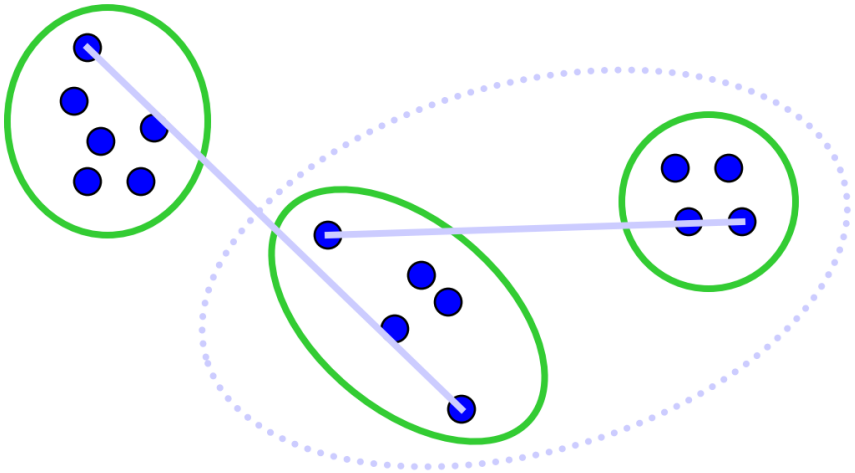


Long, skinny clusters

Different measures of nearest clusters

Complete Link

- $d(C_i, C_j) = \max_{x \in C_i, x' \in C_j} d(x, x')$. This is known as *complete-linkage*. Clusters tend to be compact and roughly equal in diameter.

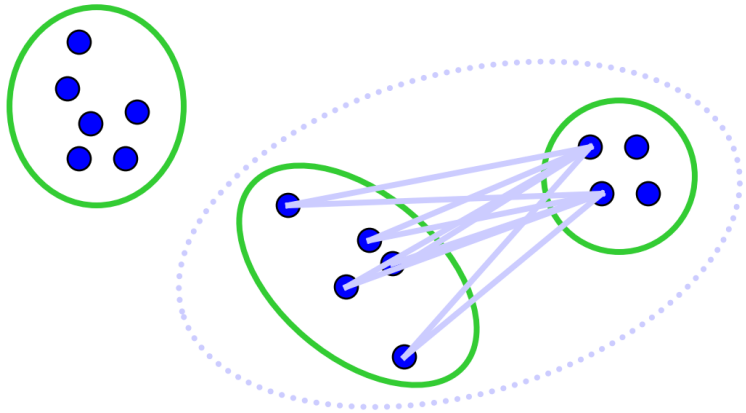


Tight clusters

Different measures of nearest clusters

Average Link

- $d(C_i, C_j) = \frac{\sum x \in C_i, x' \in C_j d(x,x')}{|C_i| \cdot |C_j|}$. This is the average distance between items. Somewhere between single-linkage and complete-linkage.



Robust against noise.

Conclusions: Agglomerative Clustering

Good

- Simple to implement, widespread application.
- Clusters have adaptive shapes.
- Provides a hierarchy of clusters.
- No need to specify number of clusters in advance.

Bad

- May have imbalanced clusters.
- Still have to choose number of clusters or threshold.
- Does not scale well. Runtime of $O(n^3)$.
- Can get stuck at a local optima.

What we will learn today?

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- Agglomerative clustering
- Oversegmentation

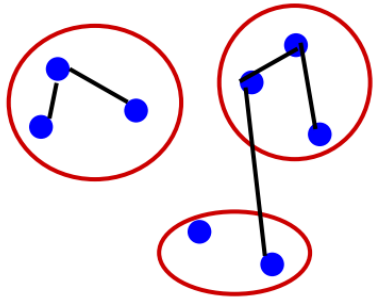
How do we segment using Clustering?

Oversegmentation algorithm

Introduced by *Felzenszwalb and Huttenlocher* in the paper titled:

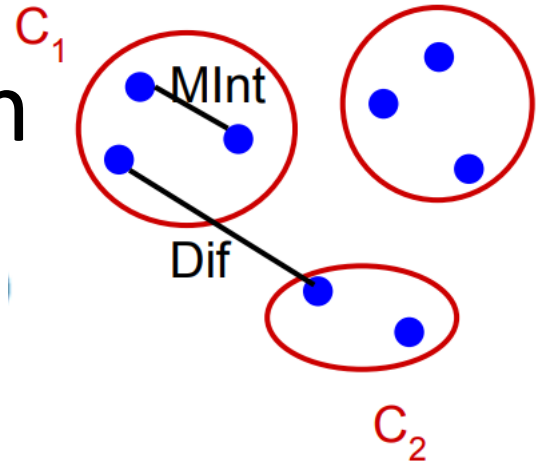
Efficient Graph-Based Image Segmentation.

Problem Formulation



- Graph G = (V, E)
- V is set of nodes (i.e. pixels)
- E is a set of undirected edges between pairs of pixels
- w(vi , vj) is the weight of the edge between nodes vi and vj .
- S is a segmentation of a graph G such that G’ = (V, E’) where E’ $\subset$ E.
- S divides G into G’ such that it contains distinct clusters C.

Predicate for Segmentation



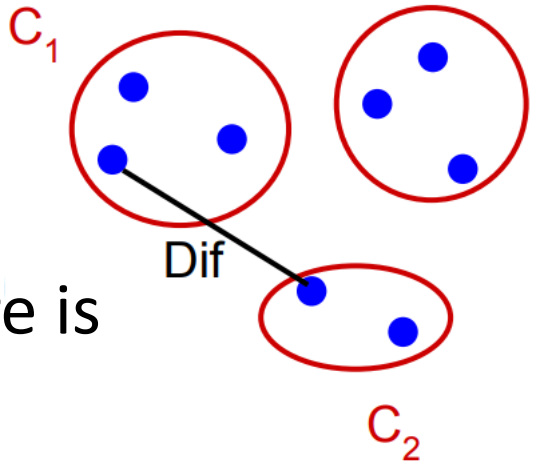
- Predicate D determines whether there is a boundary for segmentation.

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

Where

- dif(C1 , C2) is the difference between two clusters.
- in(C1 , C2) is the internal different in the clusters C1 and C2

Predicate for Segmentation



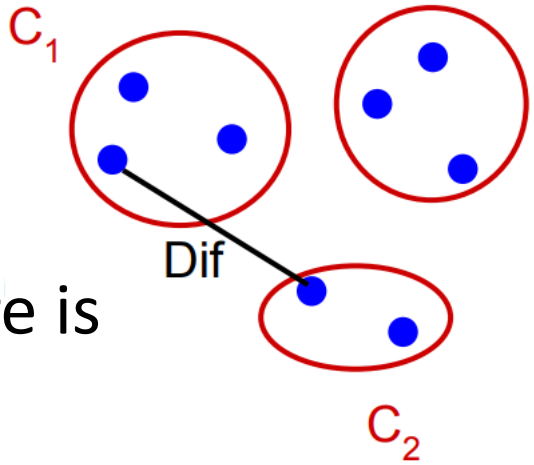
- Predicate D determines whether there is a boundary for segmentation.

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

$$dif(C_1, C_2) = \min_{v_i \in C_1, v_j \in C_2, (C_1, C_2) \in E} w(v_i, v_j)$$

The different between two components is the minimum weight edge that connects a node v_i in clusters C1 to node v_j in C2

Predicate for Segmentation



- Predicate D determines whether there is a boundary for segmentation.

$$Merge(C_1, C_2) = \begin{cases} True & if\ dif(C_1, C_2) < in(C_1, C_2) \\ False & otherwise \end{cases}$$

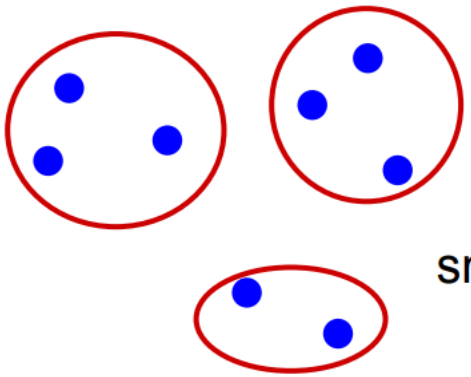
$$dif(C_1, C_2) = \min_{v_i \in C_1, v_j \in C_2, (C_1, C_2) \in E} w(v_i, v_j)$$

$$in(C_1, C_2) = \min_{C \in \{C_1, C_2\}} \left[\max_{v_i, v_j \in C} \left[w(v_i, v_j) + \frac{k}{|C|} \right] \right]$$

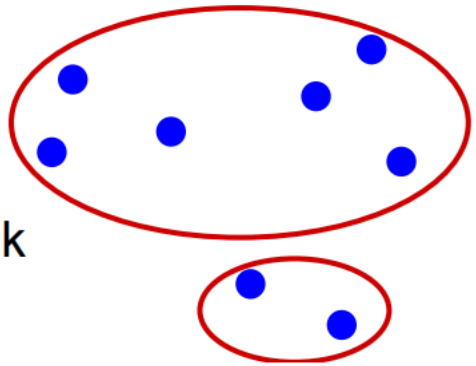
In(C1, C2) is to the maximum weight edge that connects two nodes in the same component.

Predicate for Segmentation

- k/|C| sets the threshold by which the components need to be different from the internal nodes in a component.
- Properties of constant k:
  - If k is large, it causes a preference of larger objects.
  - k does not set a minimum size for components.



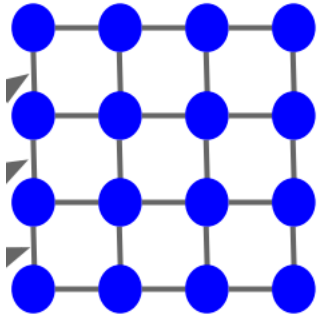
small k



large k

Features and weights

- Project every pixel into feature space defined by (x, y, r, g, b).
- Every pixel is connected to its 8 neighboring pixels and the weights are determined by the difference in intensities.
- Weights between pixels are determined using L2 (Euclidian) distance in feature space.



- Edges are chosen for only top ten nearest neighbors in feature space to ensure run time of O(n log n) where n is number of pixels.

Results

What will we learn today?

- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

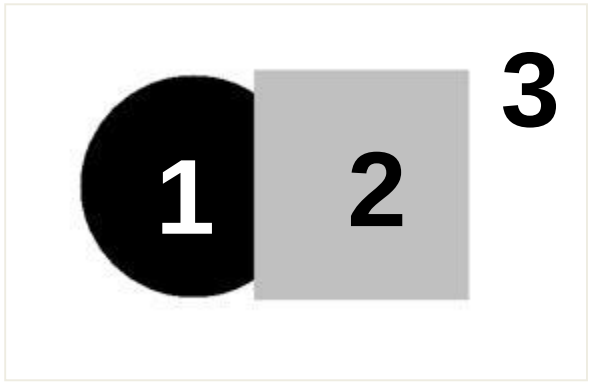
What will we learn today?

- K-means clustering
- Mean-shift clustering

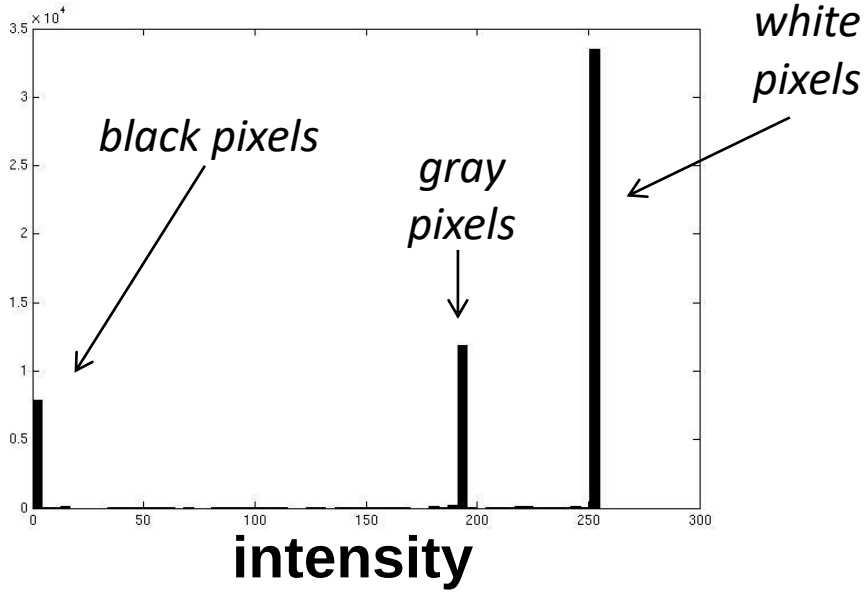
Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Image Segmentation: Toy Example

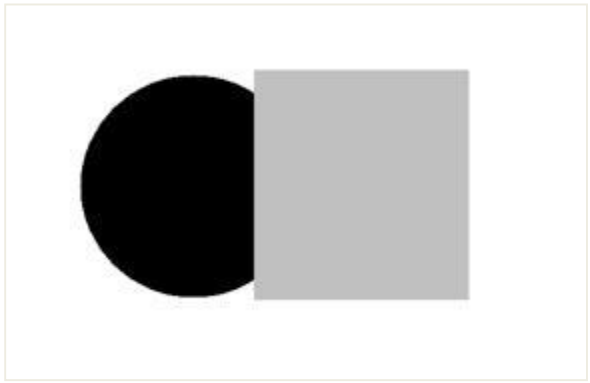


input image

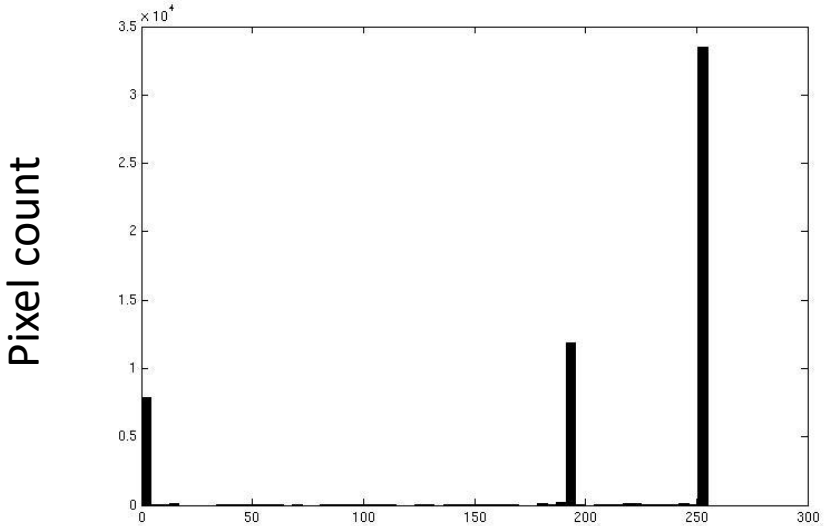


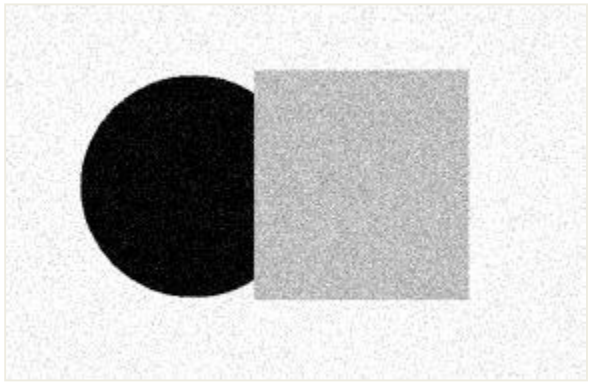
- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

Slide credit: Kristen Grauman

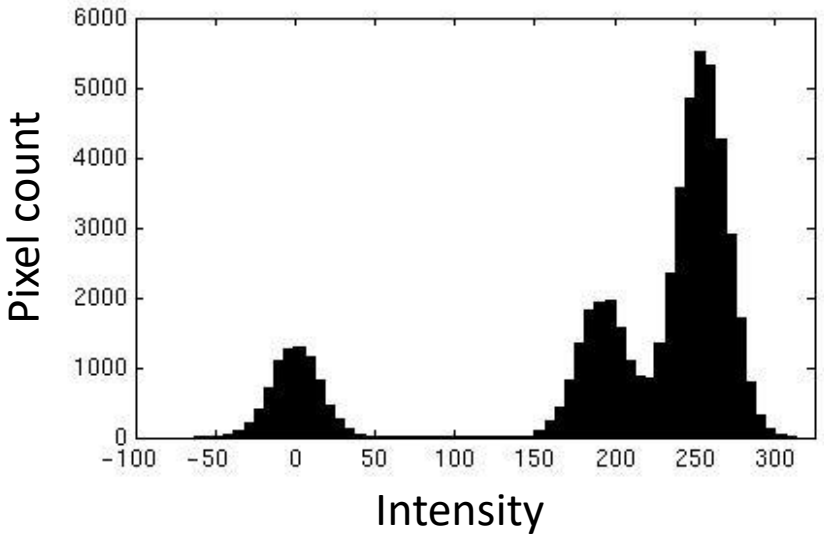


Input image

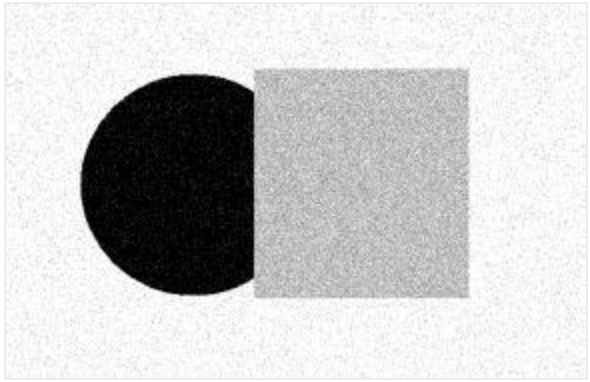




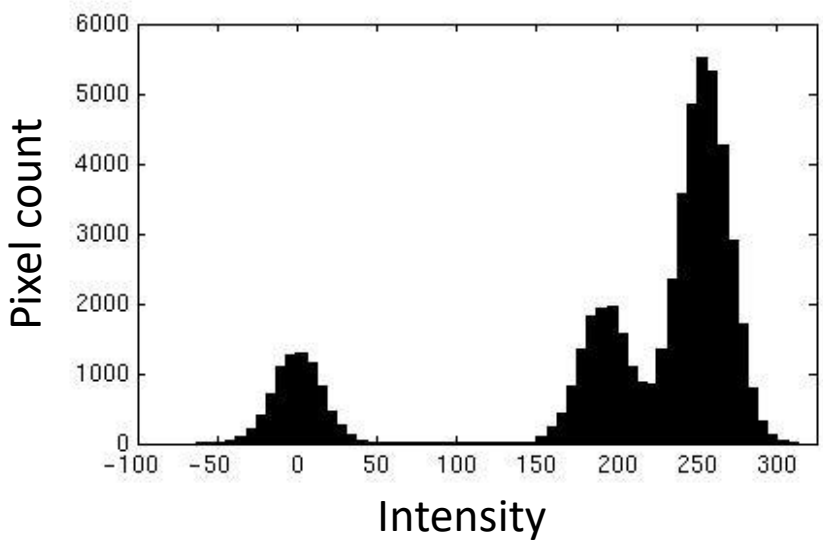
Input image



Slide credit: Kristen Grauman

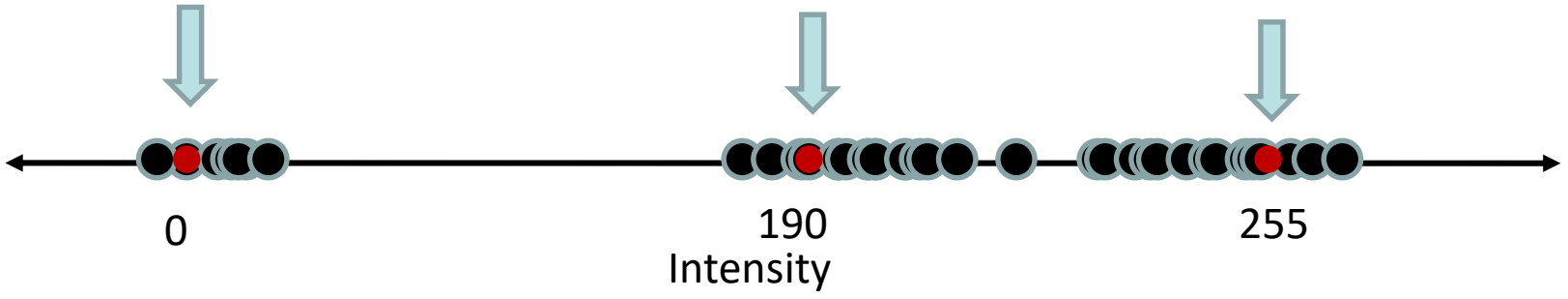


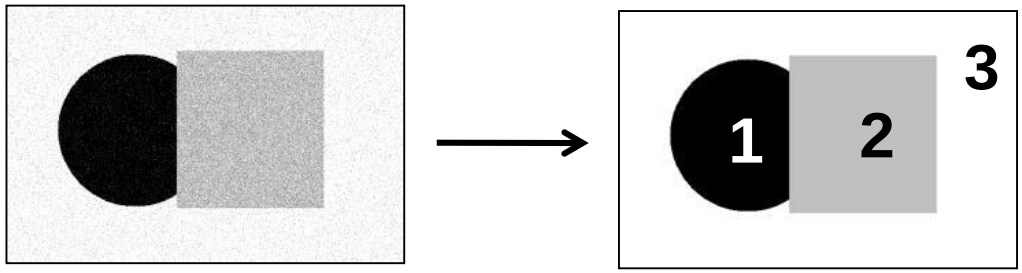
Input image



- Now how to determine the three main intensities that define our groups?
- We need to cluster.

Slide credit: Kristen Grauman





- Goal: choose three "centers" as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize Sum of Square Distance (SSD) between all points and their nearest cluster center c_i:

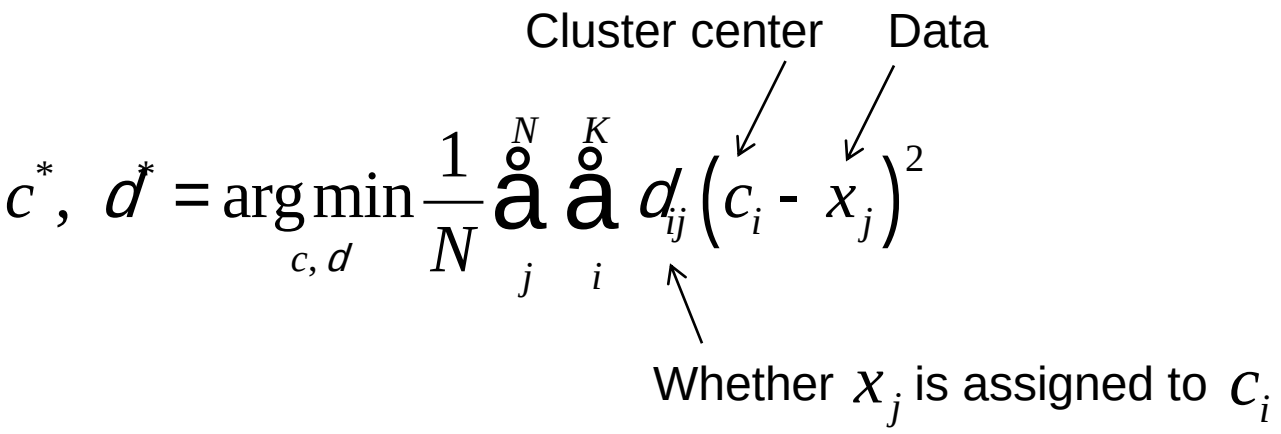
$$SSD = \sum_{cluster\,i} \sum_{x \in cluster\,i} (x - c_i)^2$$

Slide credit: Kristen Grauman

Clustering for Summarization

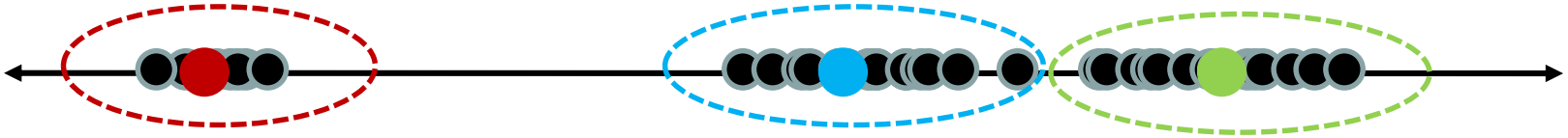
Goal: cluster to minimize variance in data given clusters

– Preserve information

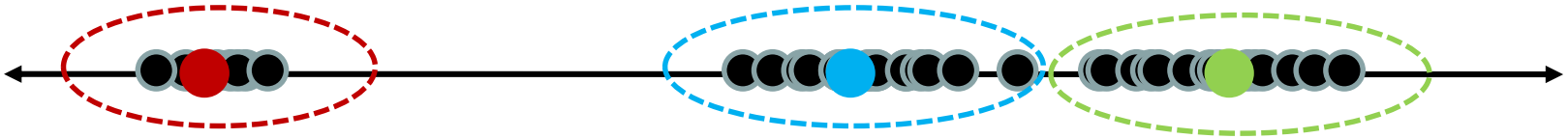


Clustering

- With this objective, it is a "chicken and egg" problem:
  - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.



  - If we knew the *group memberships*, we could get the centers by computing the mean per group.



Slide credit: Kristen Grauman

K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$

2. Compute d^t: assign each point to the closest center
   - d^t denotes the set of assignment for each x_j to cluster c_i at iteration t

$$d^t = \arg\min_d \frac{1}{N} \sum_j^N \sum_i^K d_{ij}^{t-1} \left(c_i^{t-1} - x_j \right)^2$$

1. Computer c^t: update cluster centers as the mean of the points

$$c^t = \arg\min_c \frac{1}{N} \sum_j^N \sum_i^K d_{ij}^t \left(c_i^{t-1} - x_j \right)^2$$

1. Update $t = t + 1$, Repeat Step 2-3 till stopped

K-means clustering

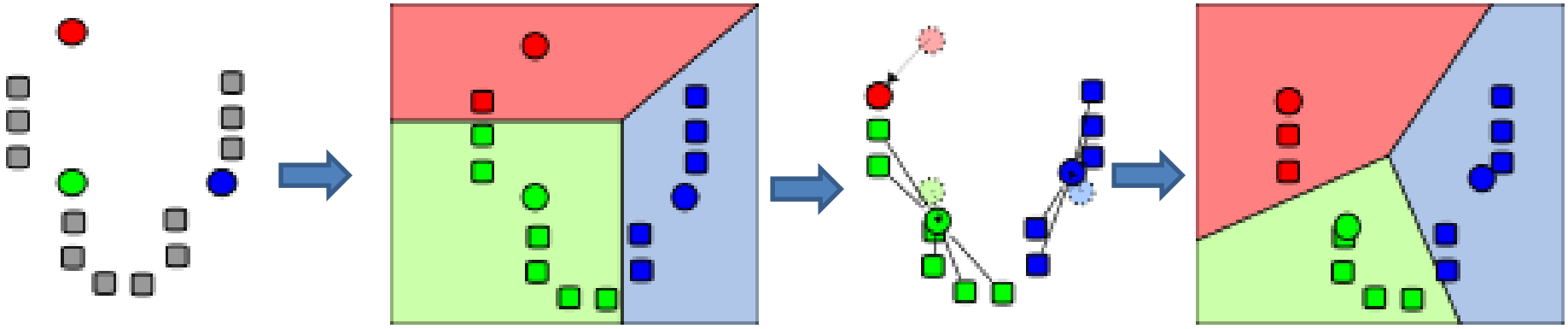
1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$
   - Commonly used: random initialization
   - Or greedily choose K to minimize residual
2. Compute d^t: assign each point to the closest center
   - Typical distance measure:
     - Euclidean $sim(x, x') = x^T x'$
     - Cosine $sim(x, x') = x^T x' / (\|x\| \times \|x'\|)$
     - Others

1. Computer c^t: update cluster centers as the mean of the points

$$c^t = \arg\min_c \frac{1}{N} \sum_j^N \sum_i^K d_{ij}^t \left(c_i^{t-1} - x_j \right)^2$$

2. Update $t = t + 1$, Repeat Step 2-3 till stopped
   - c^t doesn't change anymore.

K-means clustering



1. Initialize Cluster Centers

2. Assign Points to Clusters

3. Re-compute Means

Repeat (2) and (3)

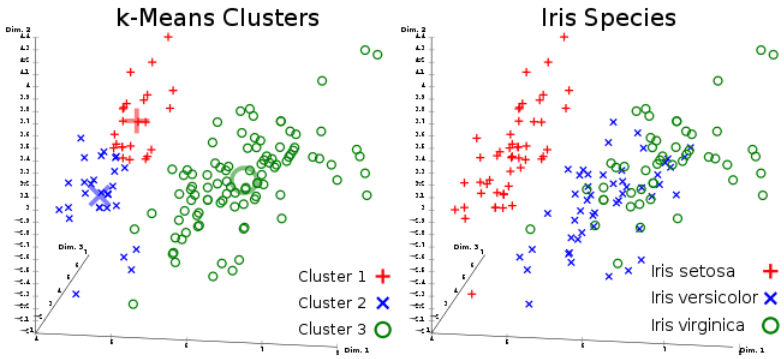
http://stanford.edu/class/ee103/visualizations/kmeans/kmeans.html

Illustration Source: wikipedia

K-means clustering

- Converges to a *local minimum* solution
  - Initialize multiple runs

- Better fit for spherical data

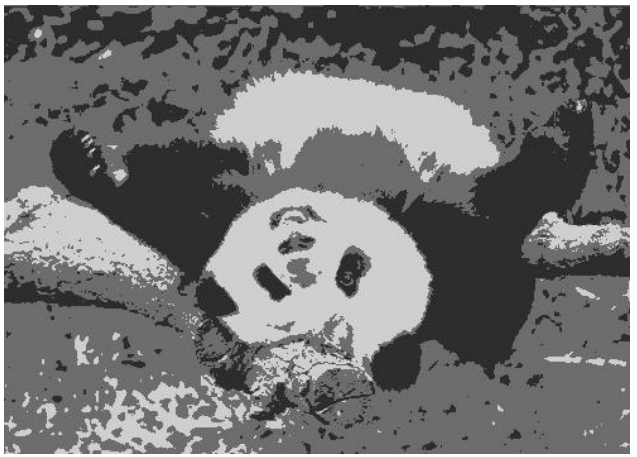


- Need to pick K (# of clusters)

Segmentation as Clustering

Original image

2 clusters



3 clusters

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

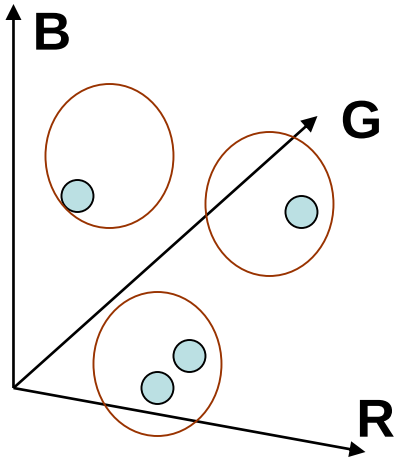
- Grouping pixels based on intensity similarity

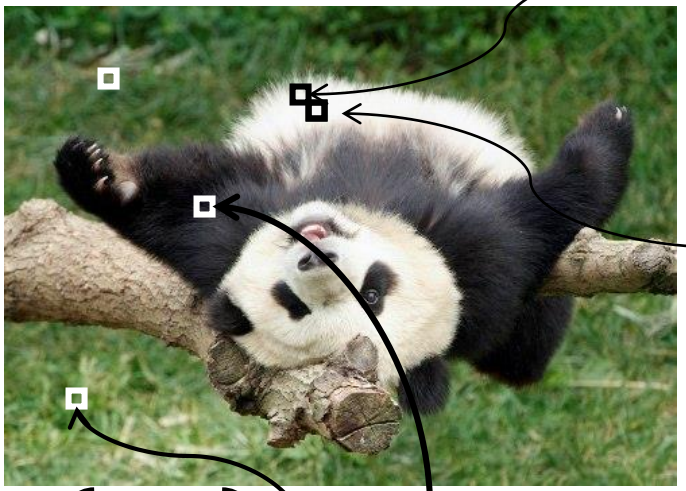
- Feature space: intensity value (1D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on color similarity



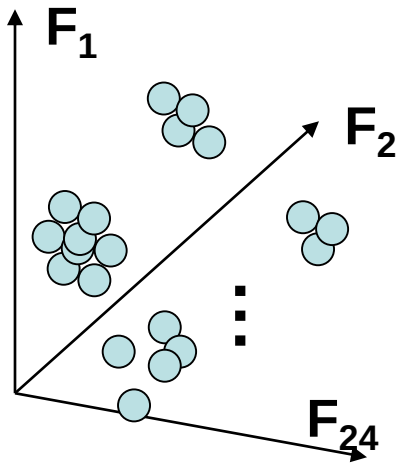


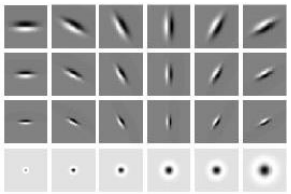
- Feature space: color value (3D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on texture similarity





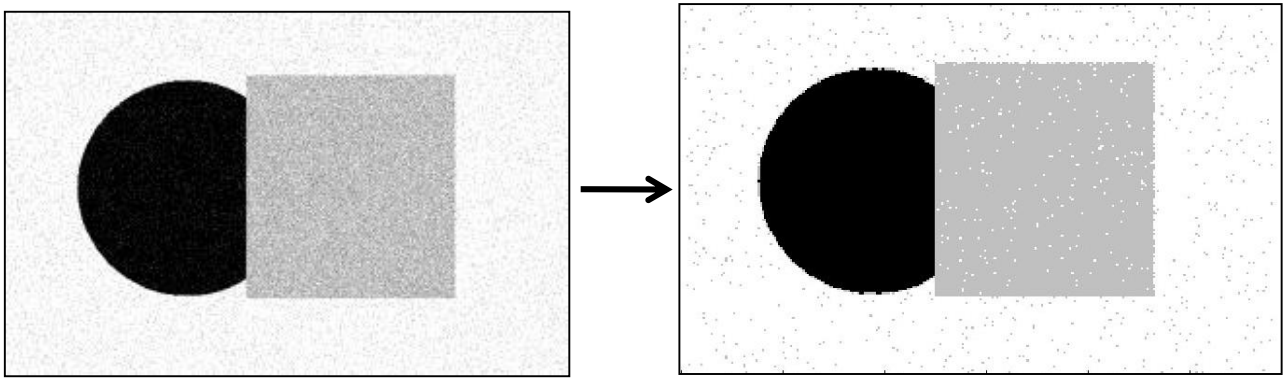
Filter bank of 24 filters

- Feature space: filter bank responses (e.g., 24D)

Slide credit: Kristen Grauman

Smoothing Out Cluster Assignments

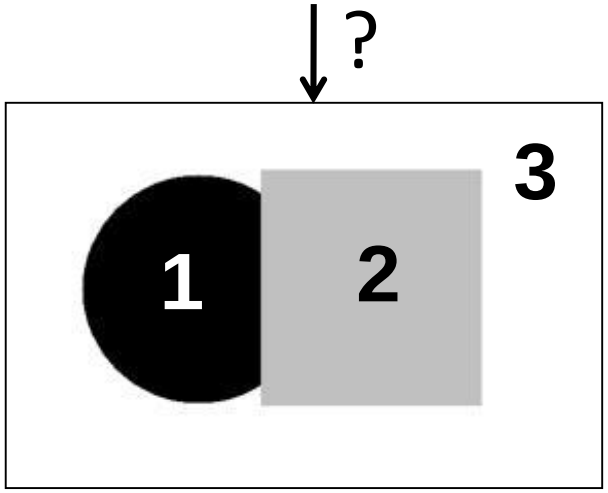
- Assigning a cluster label per pixel may yield outliers:



Original

Labeled by cluster center's intensity

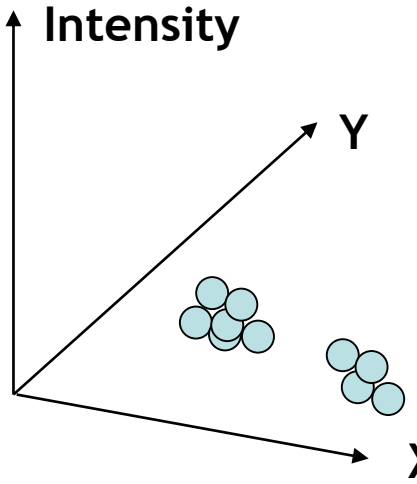
- How can we ensure they are spatially smooth?

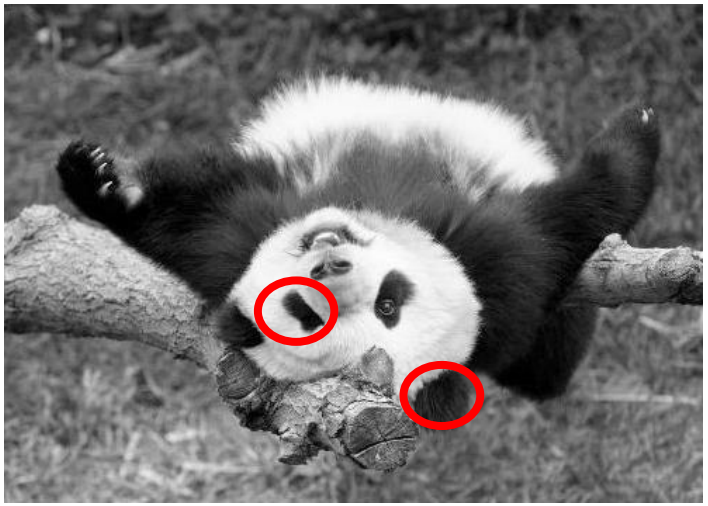


Slide credit: Kristen Grauman

Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on *intensity+position* similarity





$\Rightarrow$ Way to encode both *similarity* and *proximity.*

Slide credit: Kristen Grauman

K-Means Clustering Results

- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent

Image

Intensity-based clusters

Color-based clusters

Image source: Forsyth & Ponce

K-Means Clustering Results

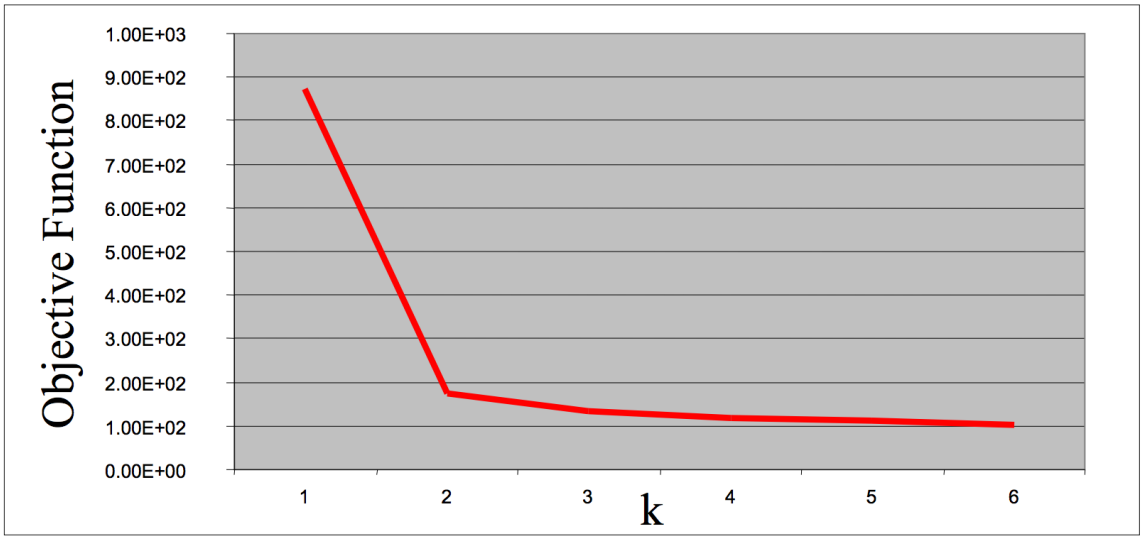
- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent
- Clustering based on (r,g,b,x,y) values enforces more spatial coherence

How to choose the number of clusters?

Try different numbers of clusters in a validation set and look at performance.

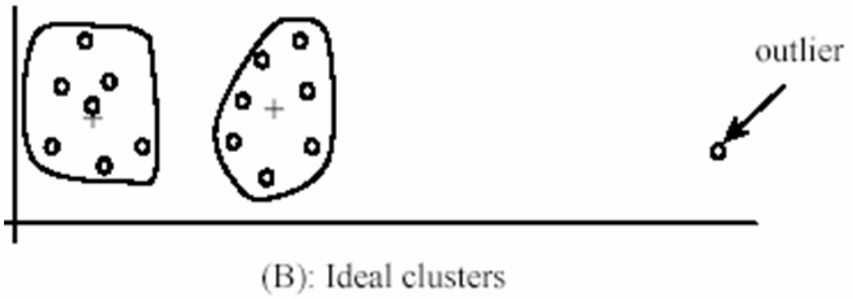
We can plot the objective function values for k equals 1 to 6…

The abrupt change at k = 2, is highly suggestive of two clusters in the data. This technique for determining the number of clusters is known as "knee finding" or "elbow finding".

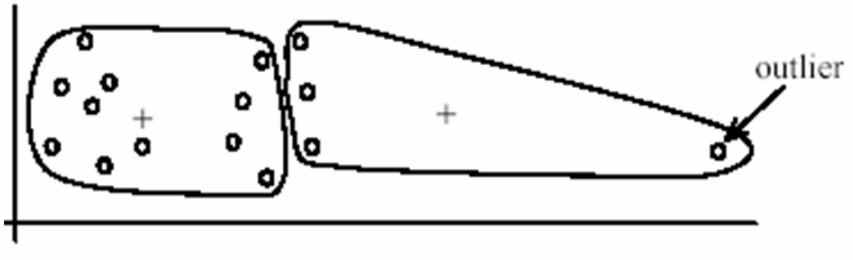


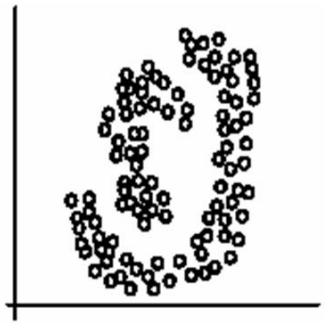
K-Means pros and cons

- Pros
  - Finds cluster centers that minimize conditional variance (good representation of data)
  - Simple and fast, Easy to implement
- Cons
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - *Can be slow: each iteration is O(KNd) for N d-dimensional points
- Usage
  - Unsupervised clustering
  - Rarely used for pixel segmentation

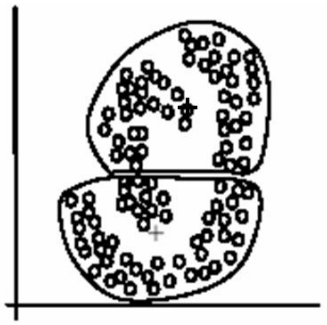


(B): Ideal clusters





(A): Two natural clusters



(B): *k*-means clusters

What will we learn today?

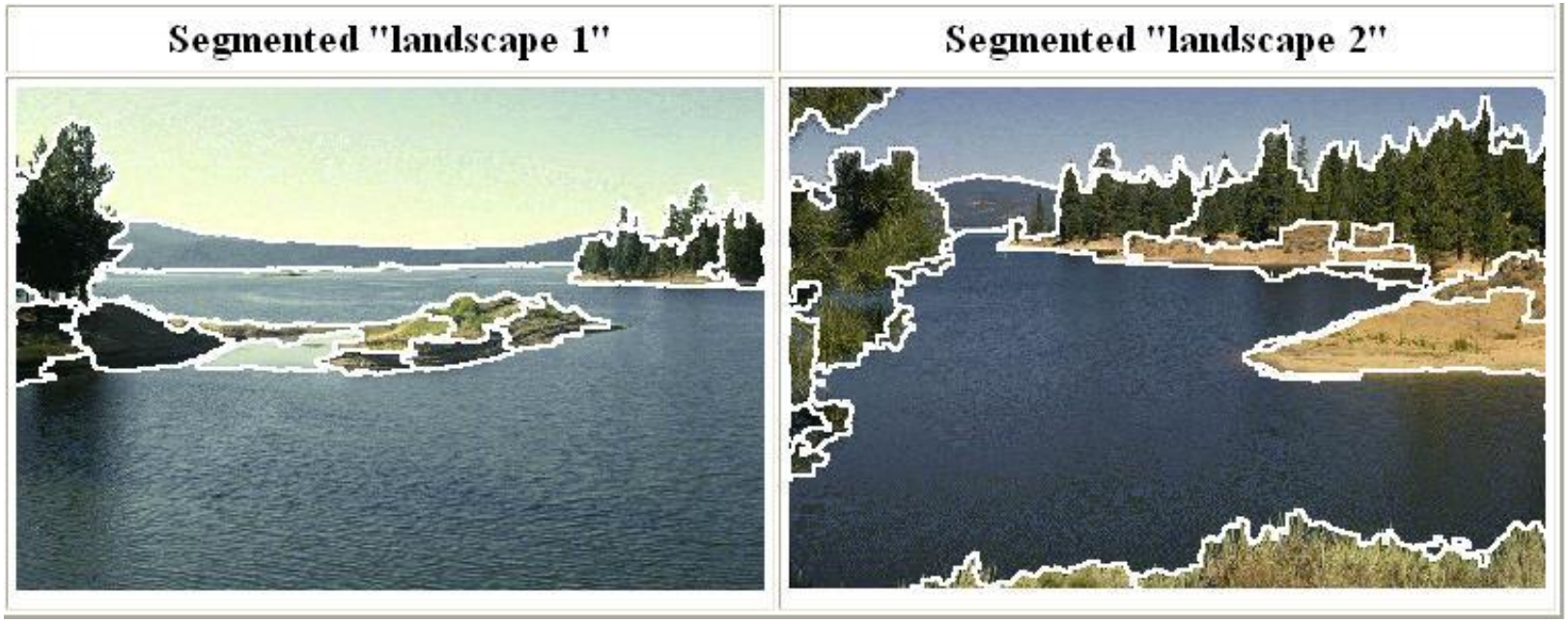
- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation

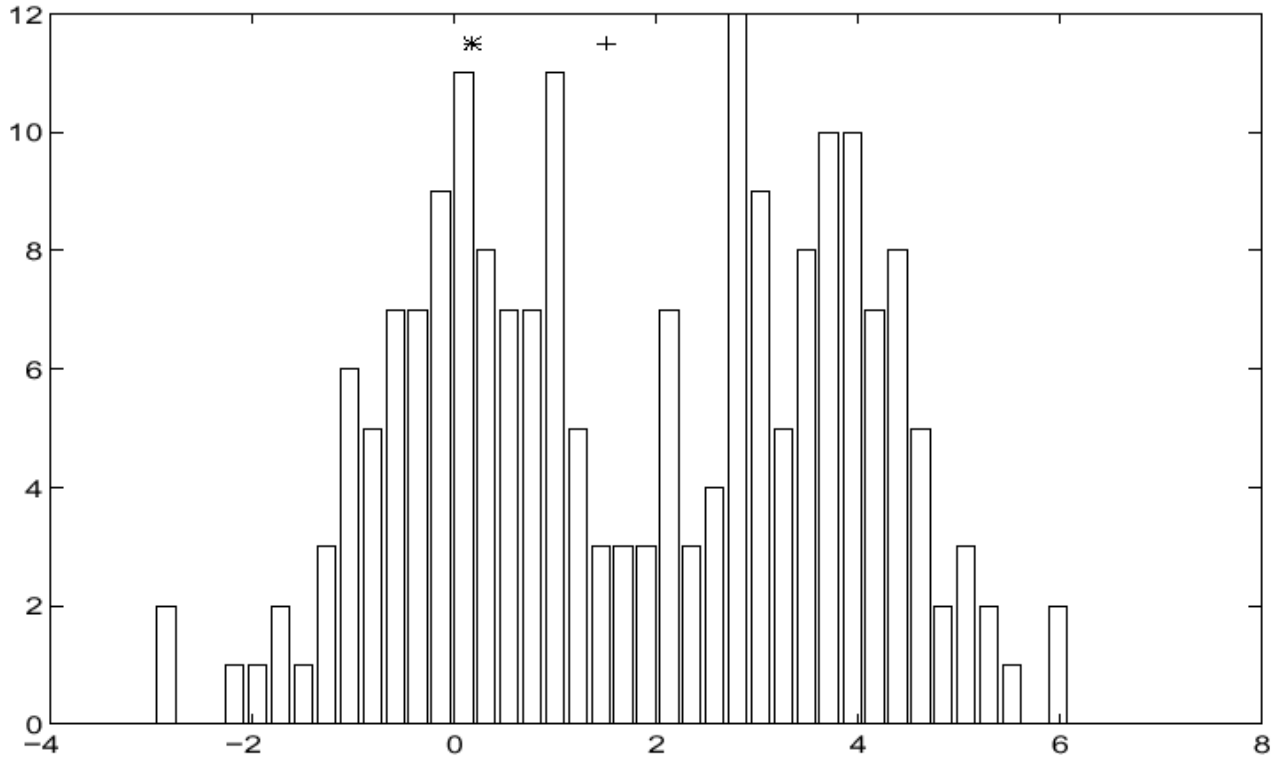


http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Slide credit: Svetlana Lazebnik

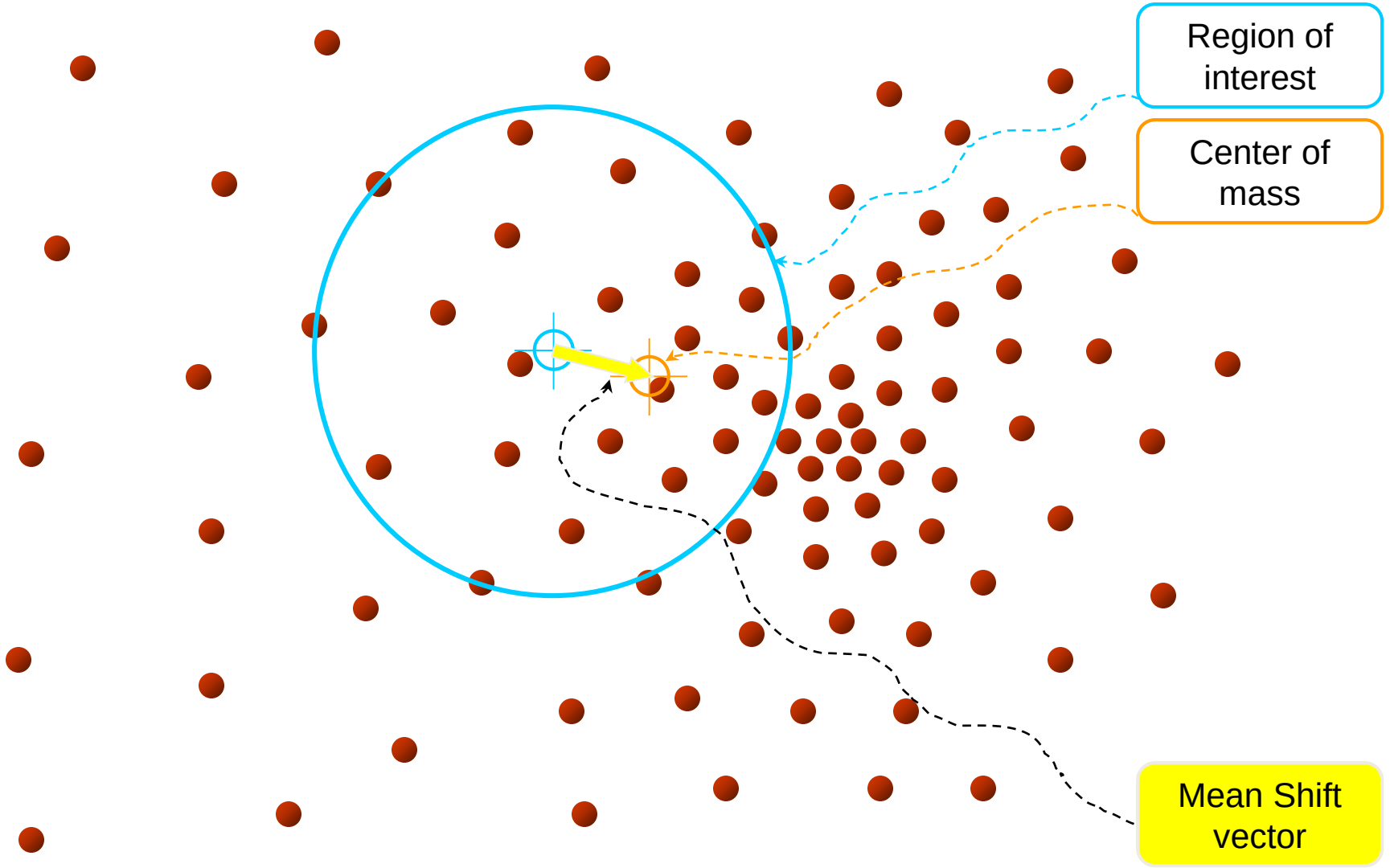
Mean-Shift Algorithm



- Iterative Mode Search
  1. Initialize random seed, and window W
  2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} xH(x)$
  3. Shift the search window to the mean
  4. Repeat Step 2 until convergence

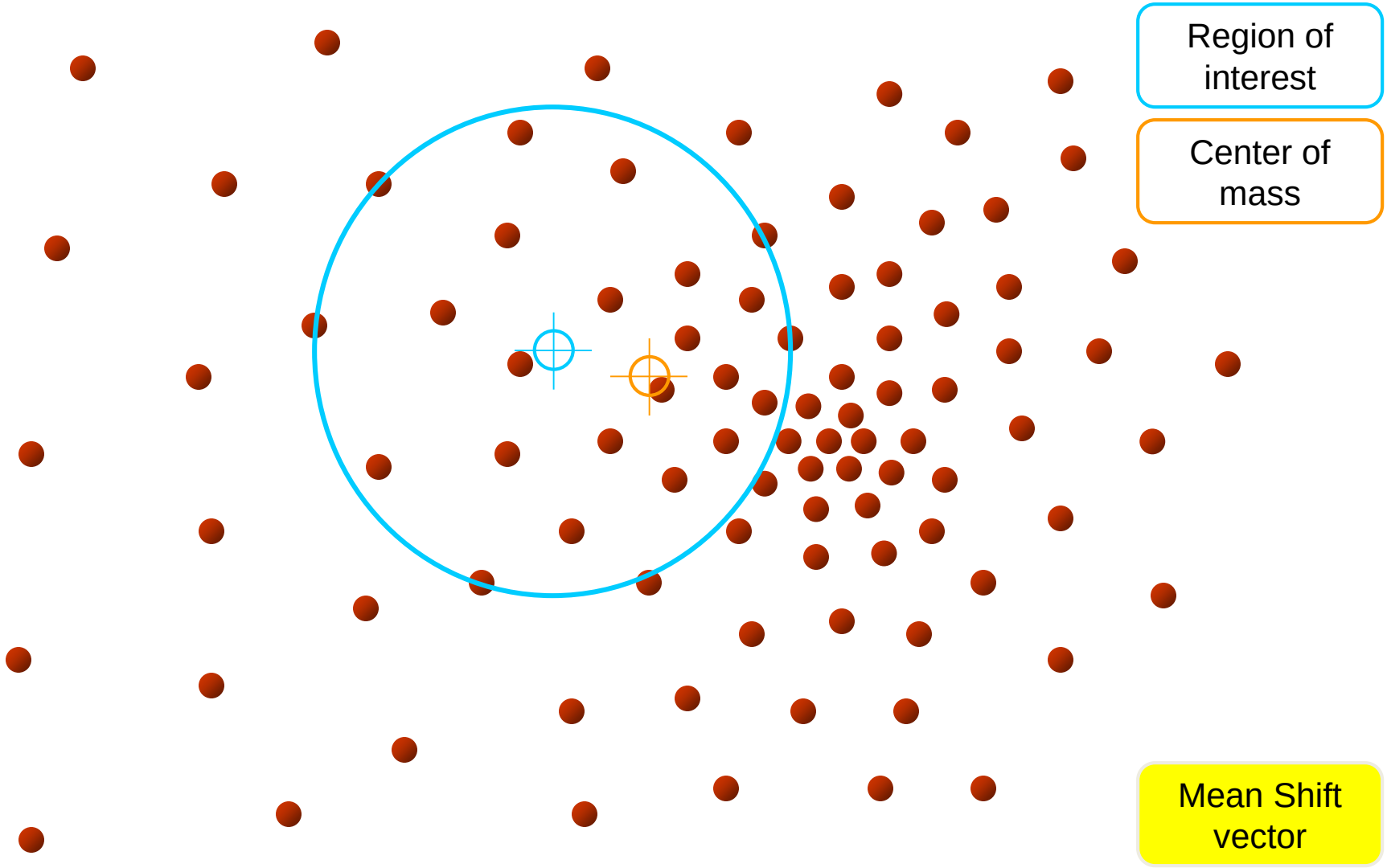
Slide credit: Steve Seitz

Mean-Shift



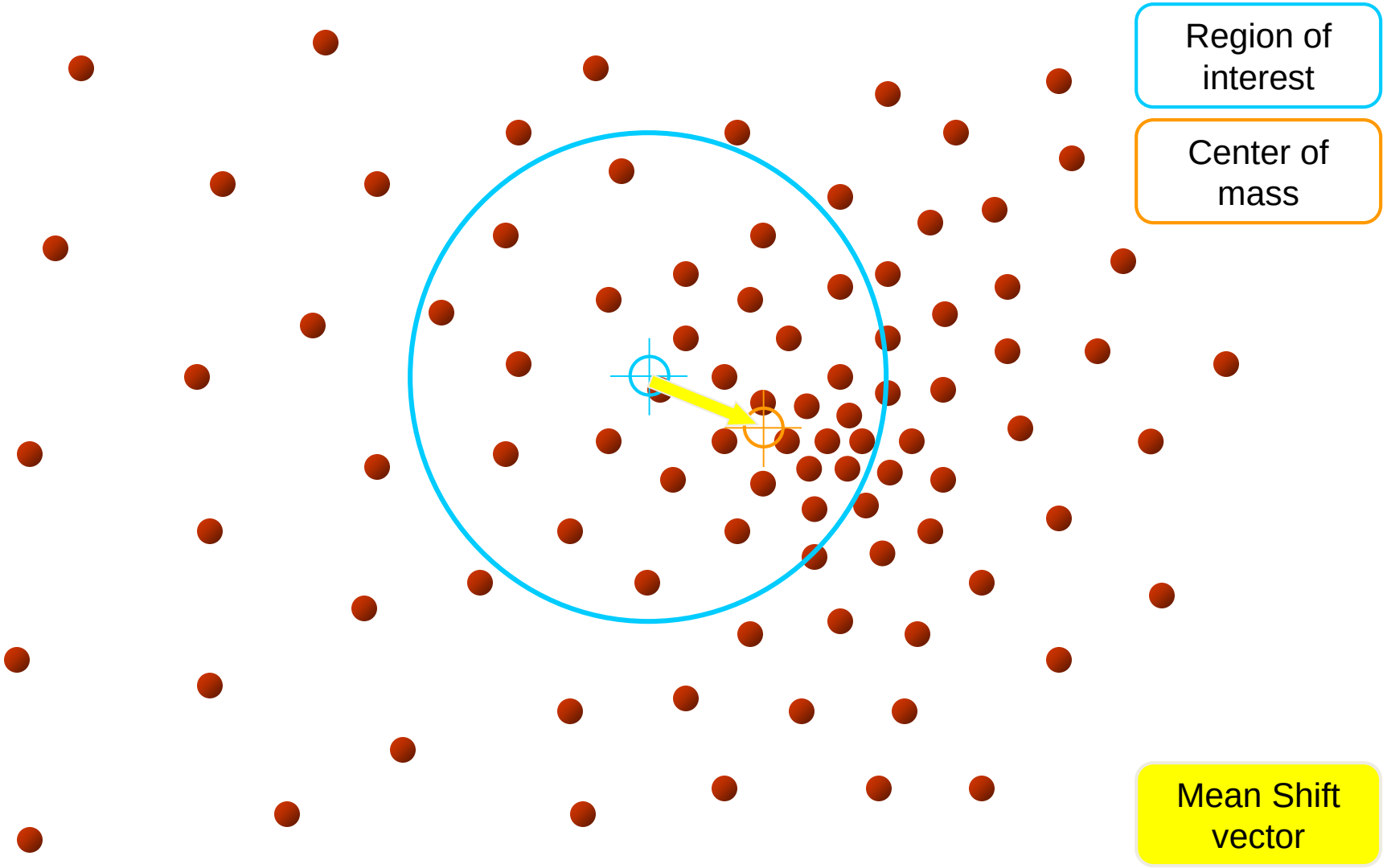
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



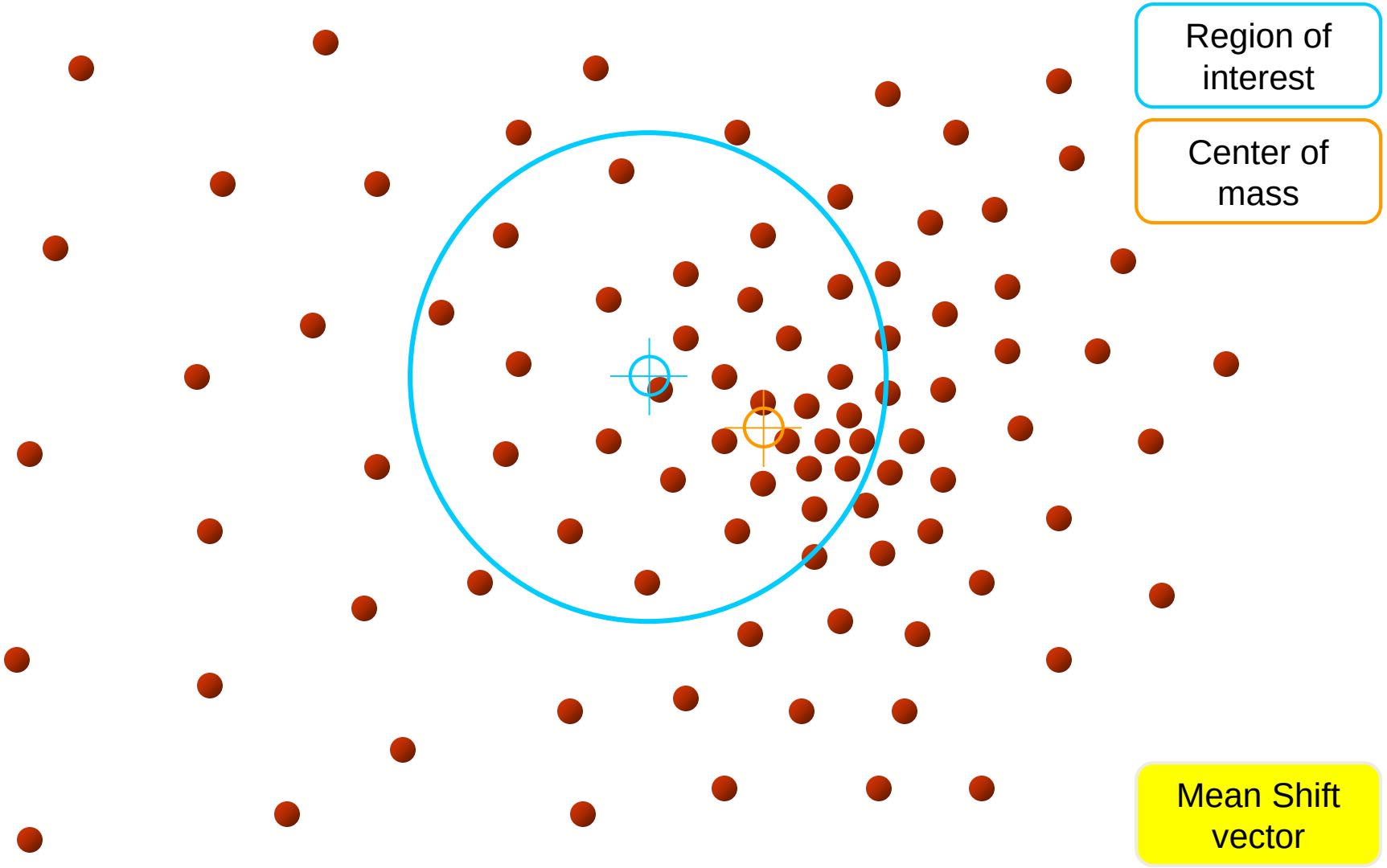
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



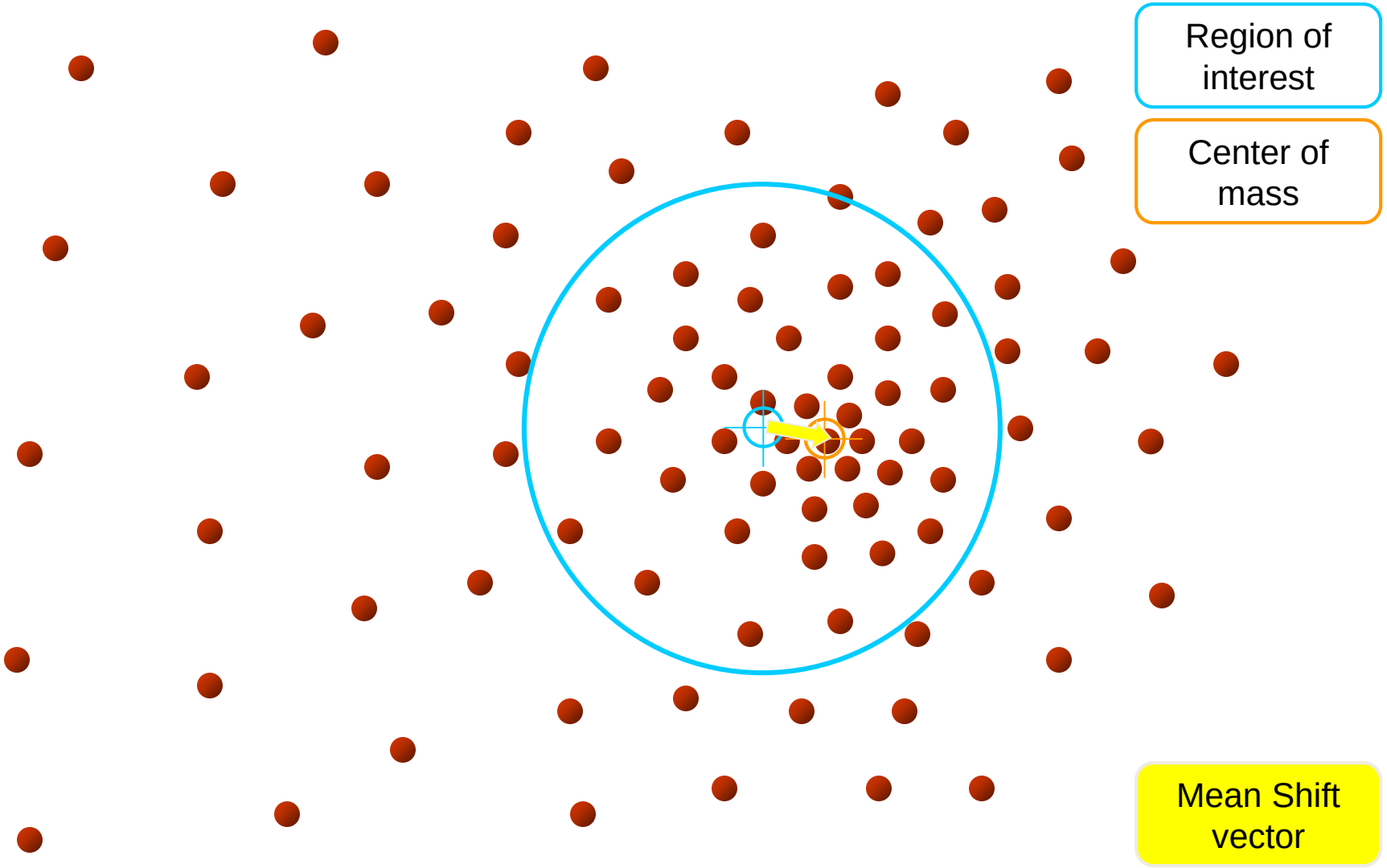
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



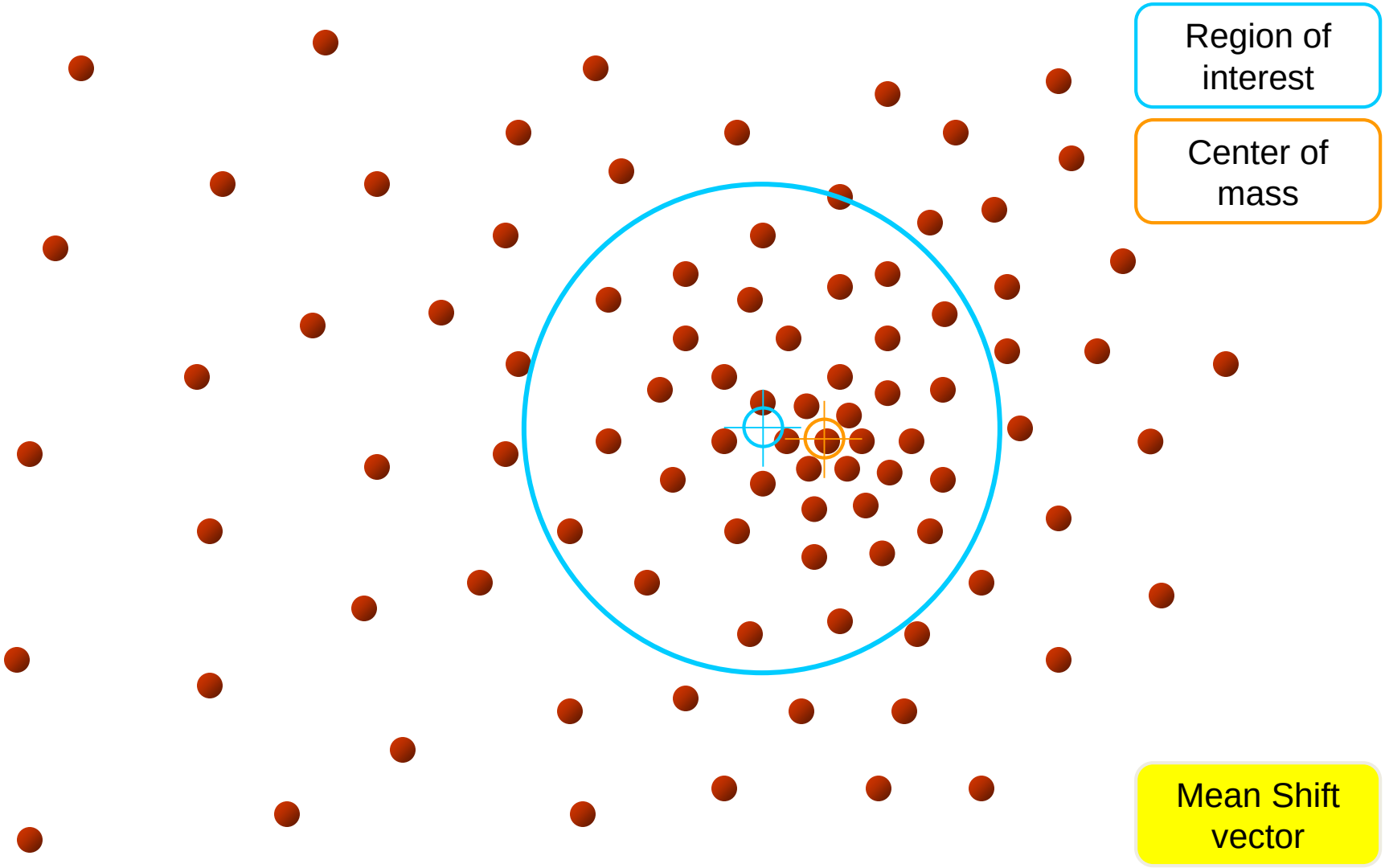
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



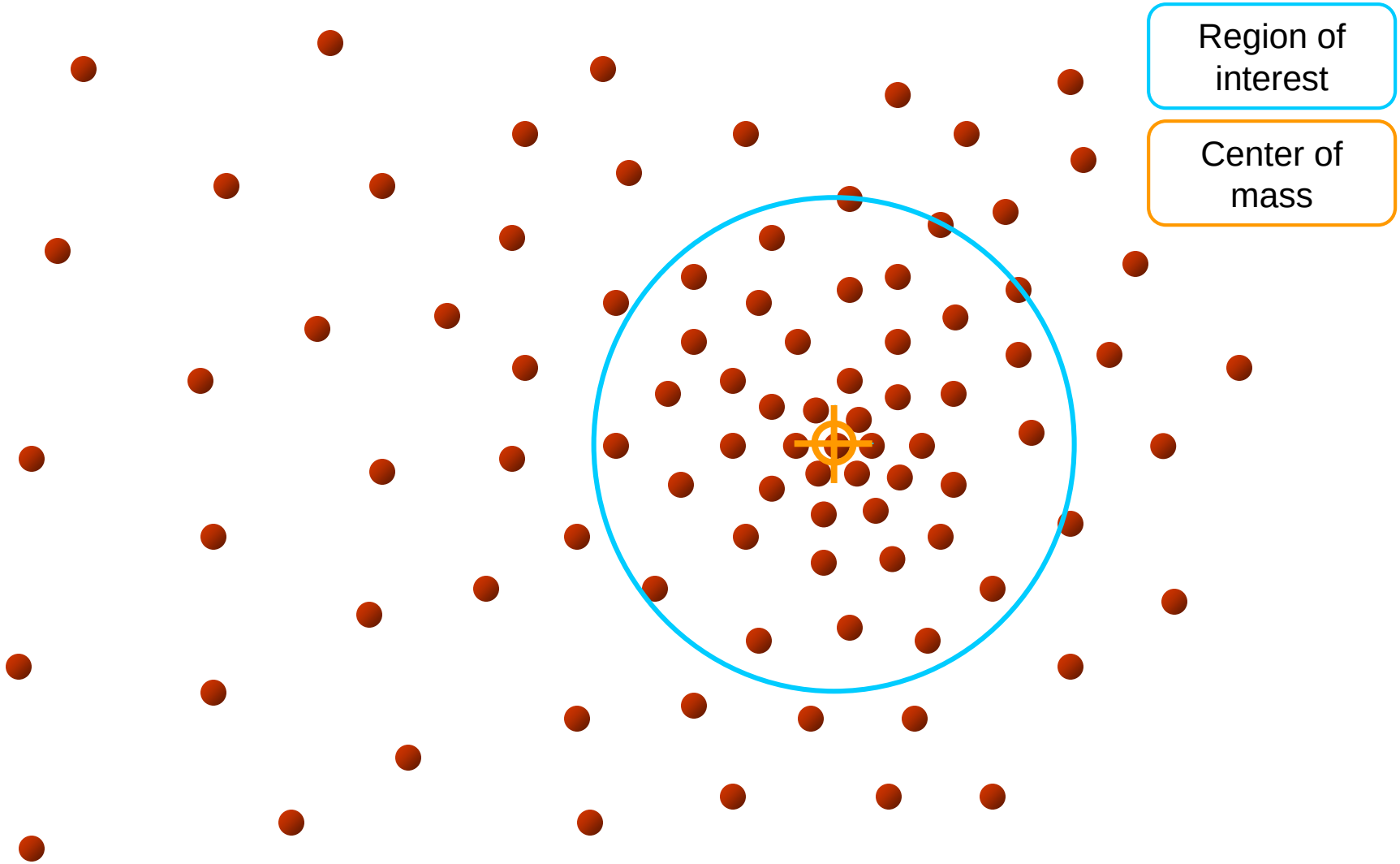
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



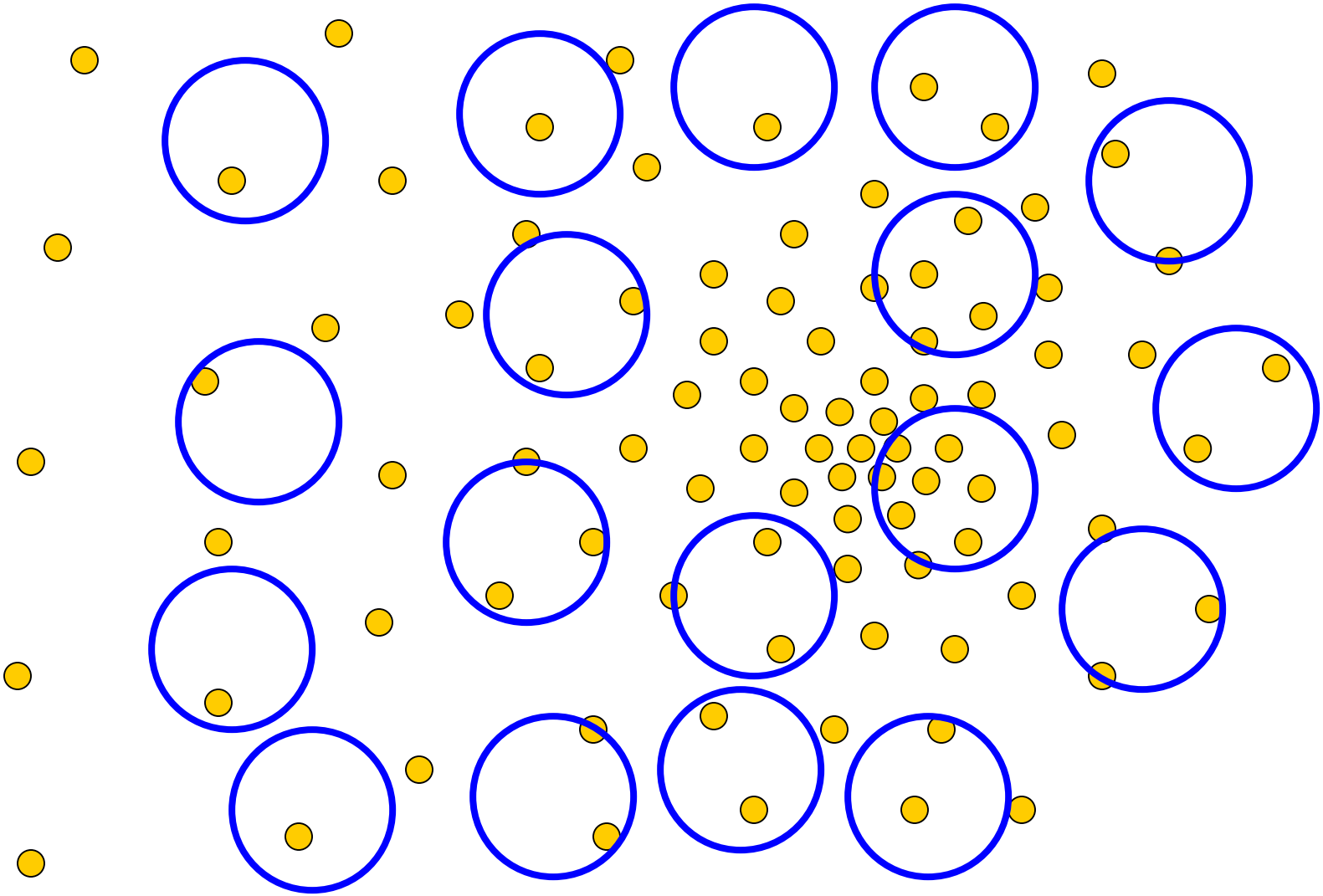
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

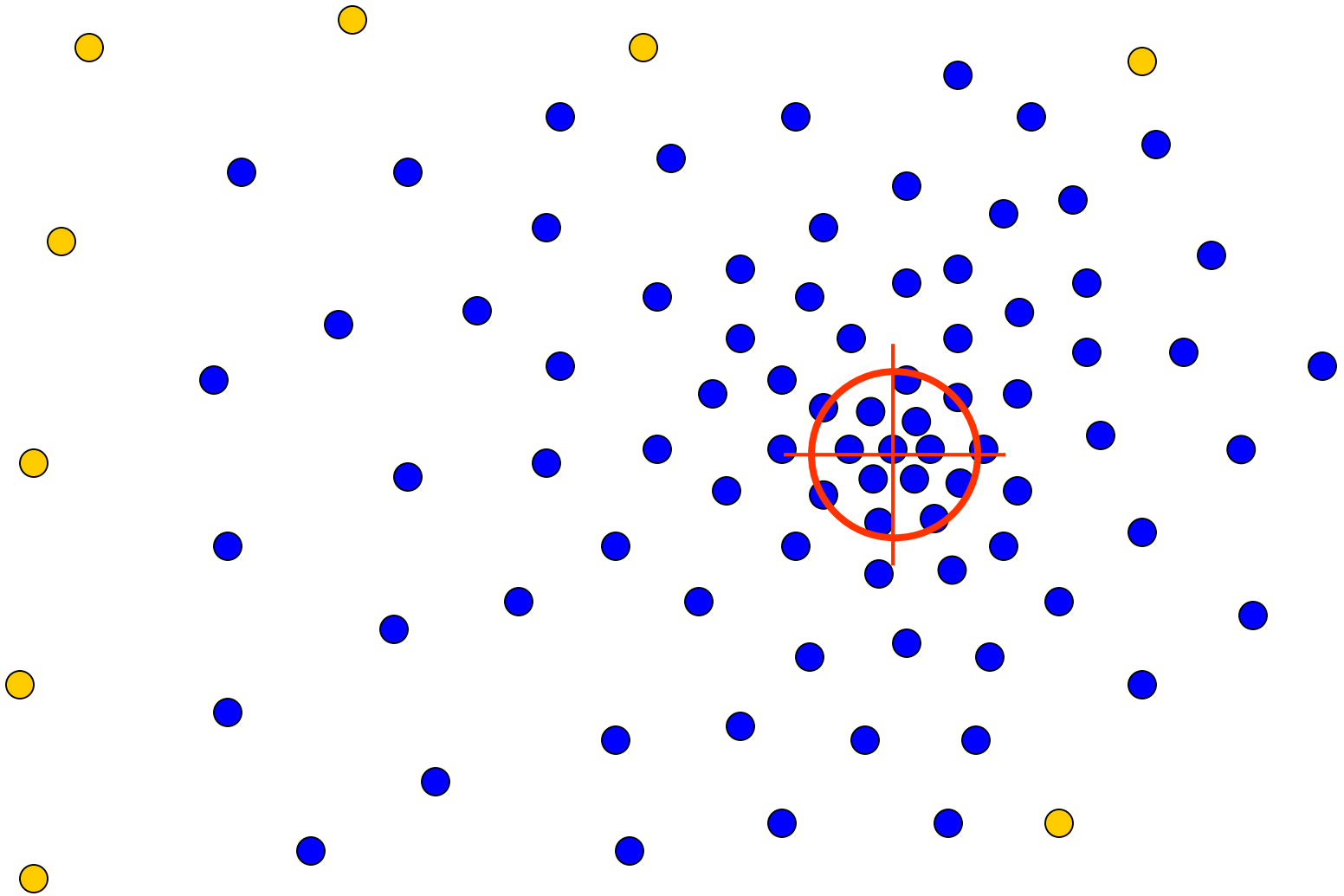


Tessellate the space with windows

Run the procedure in parallel

Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

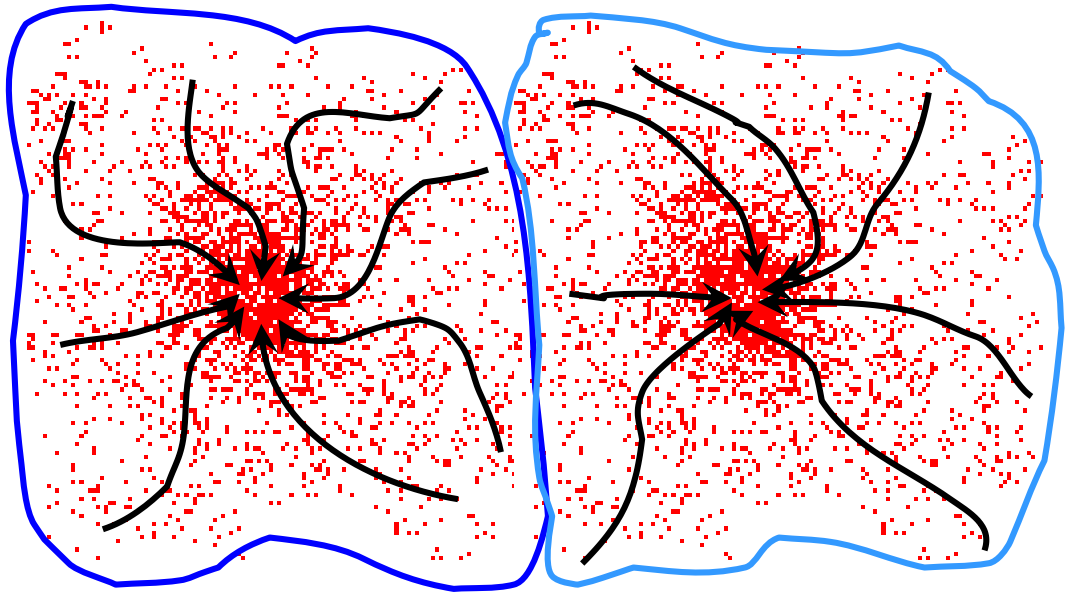


The blue data points were traversed by the windows towards the mode.

Slide by Y. Ukrainitz & B. Sarel

Mean-Shift Clustering

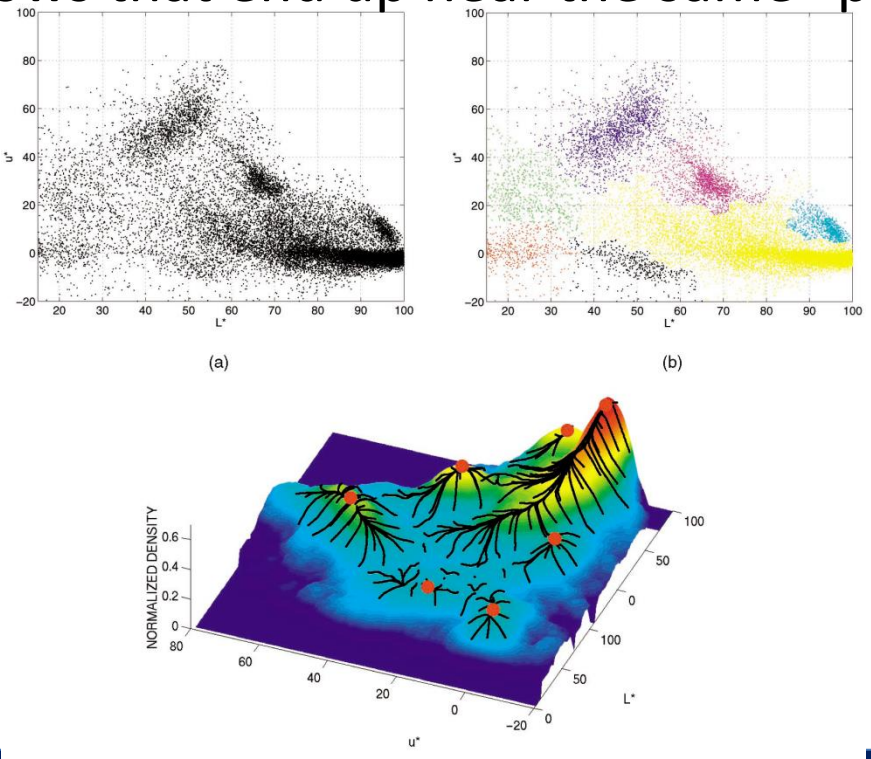
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



Slide by Y. Ukrainitz & B. Sarel

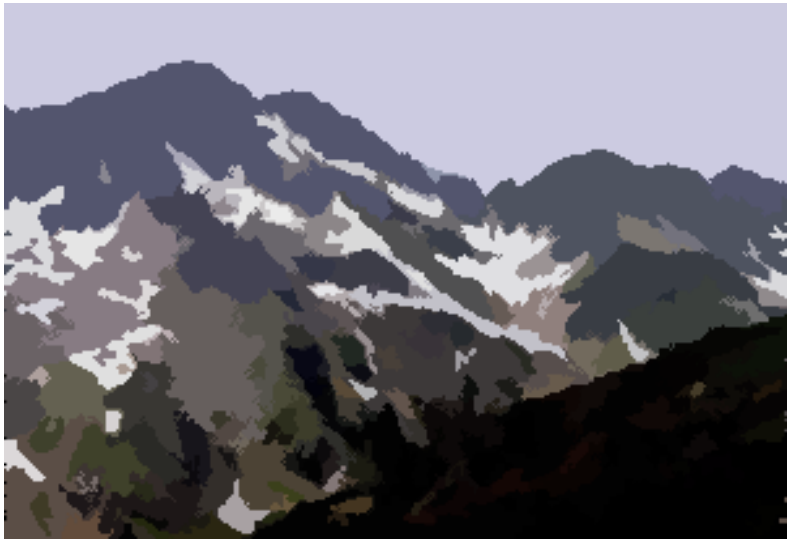
Mean-Shift Clustering/Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



Slide credit: Svetlana Lazebnik

Mean-Shift Segmentation Results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

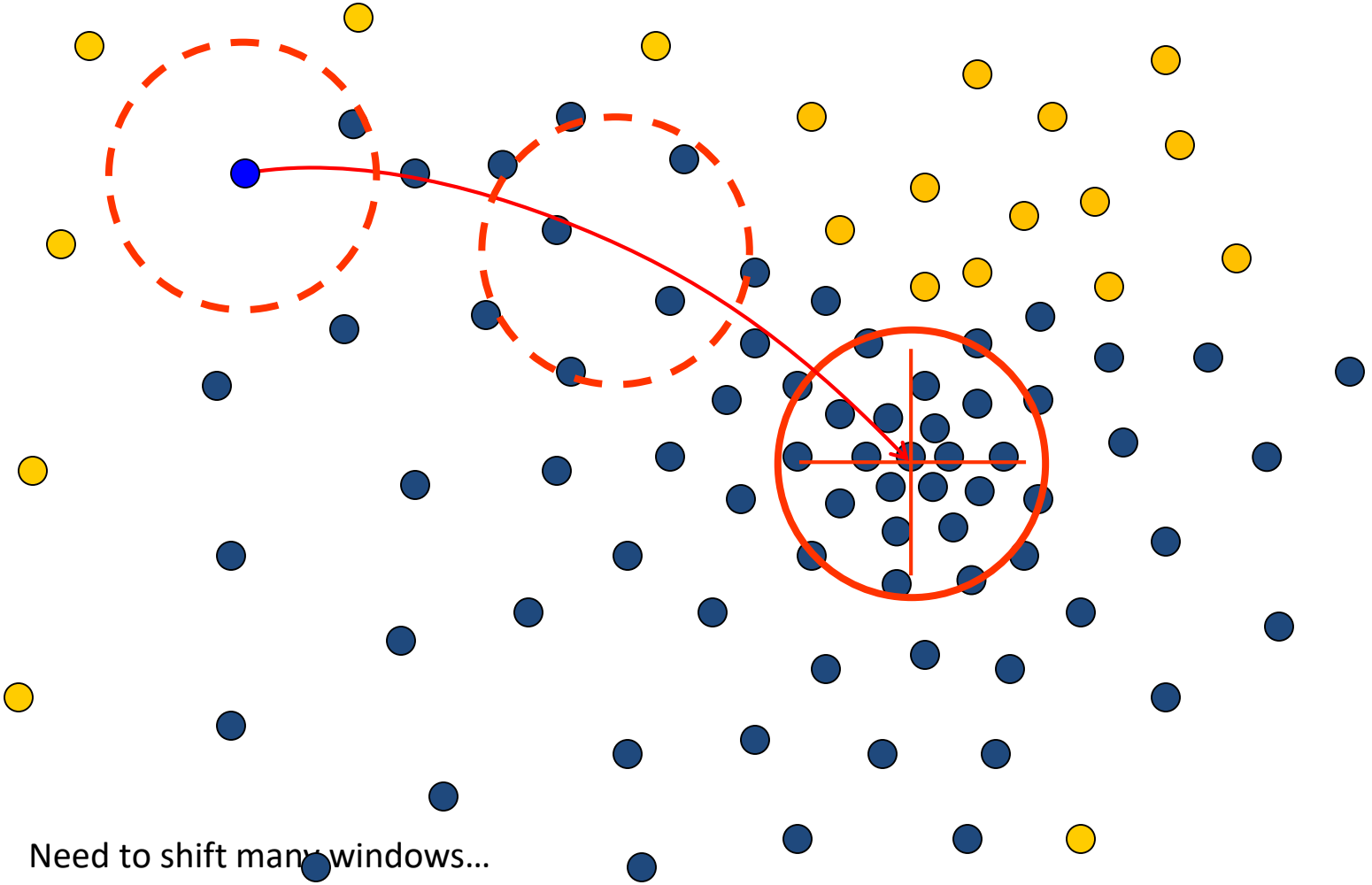
Slide credit: Svetlana Lazebnik

More Results

Slide credit: Svetlana Lazebnik

More Results

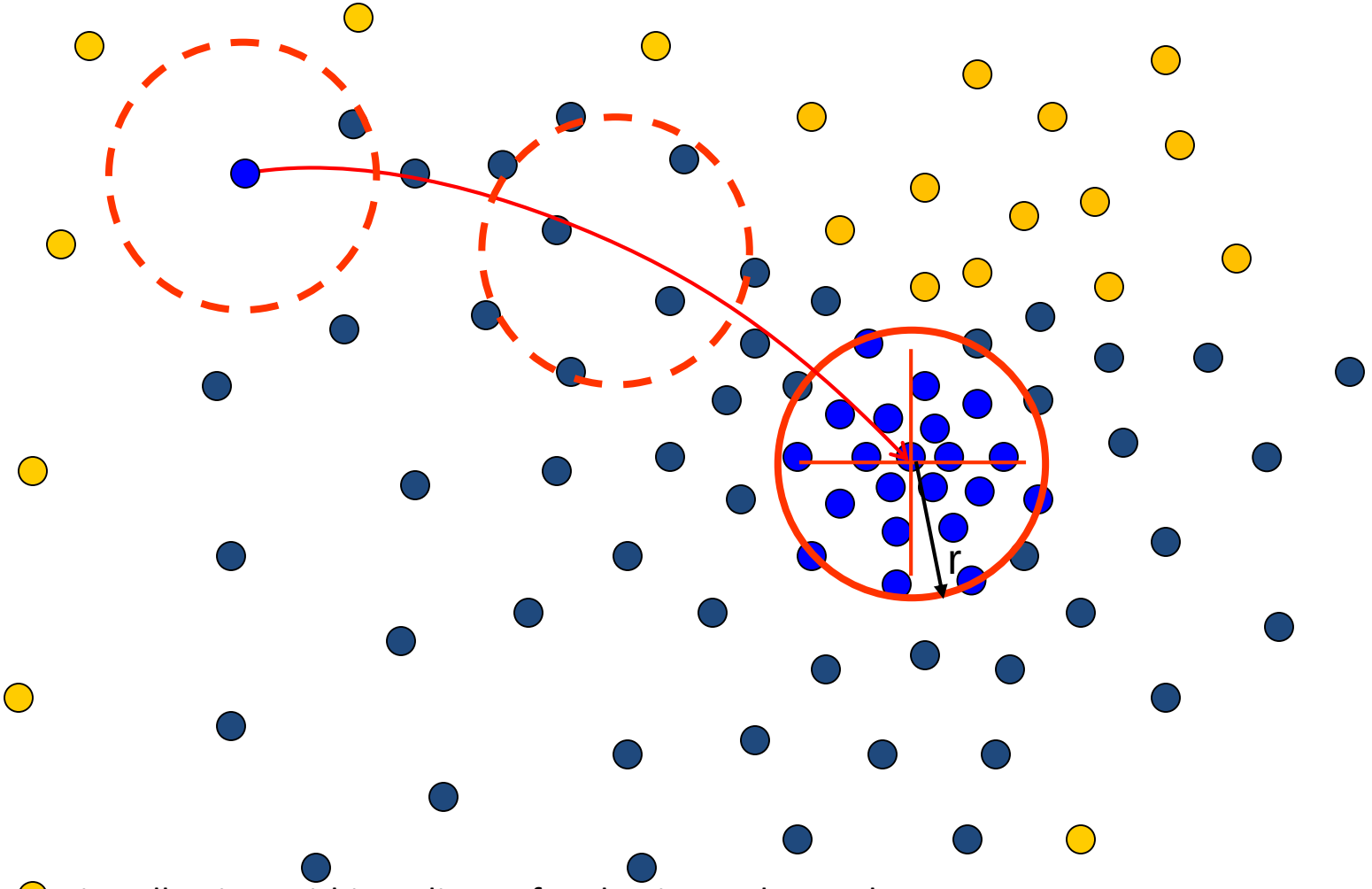
Problem: Computational Complexity



- Need to shift many windows…
- Many computations will be redundant.

Slide credit: Bastian Leibe

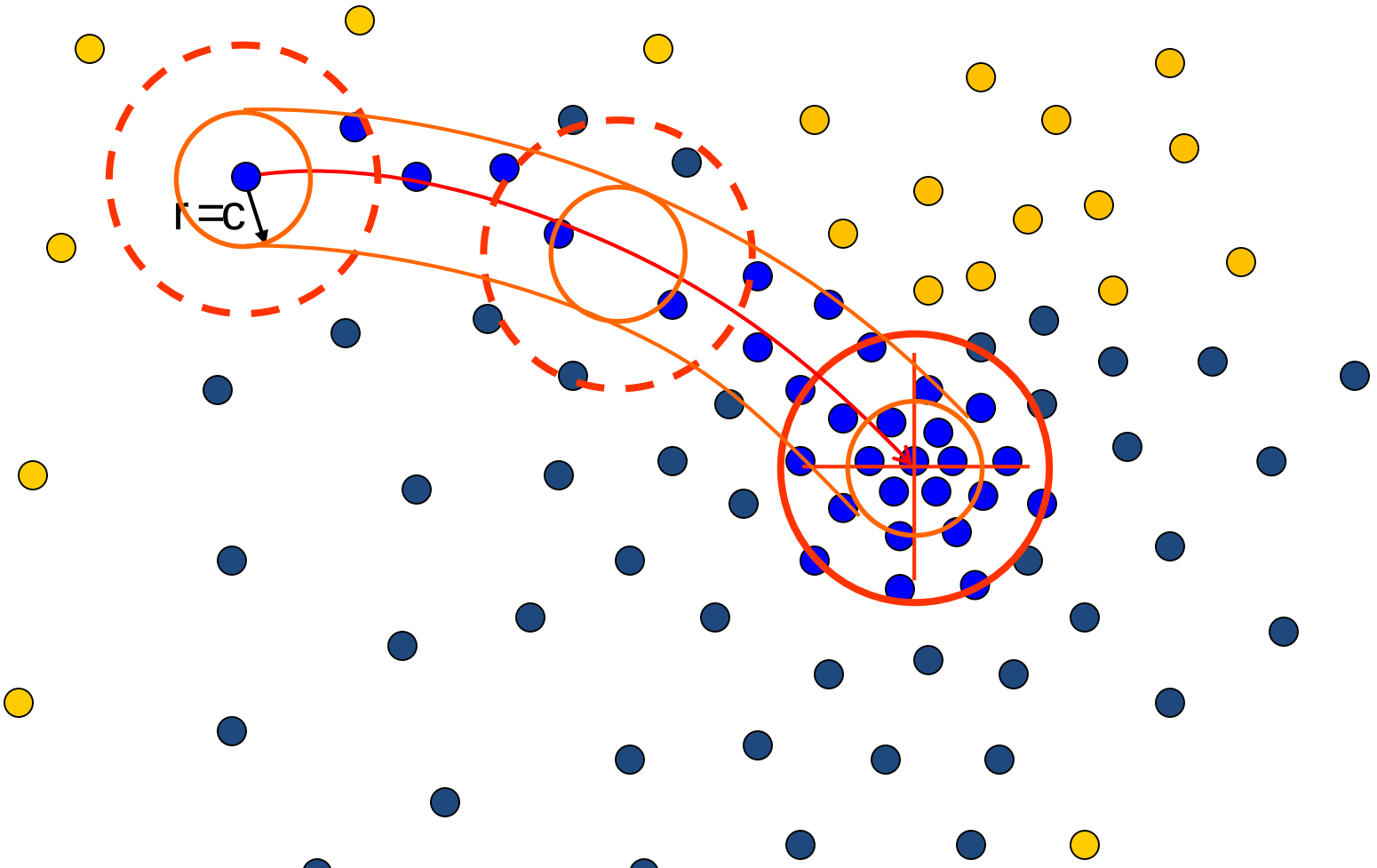
Speedups: Basin of Attraction



1. Assign all points within radius r of end point to the mode.

Slide credit: Bastian Leibe

Speedups



2. Assign all points within radius r/c of the search path to the mode -> reduce the number of data points to search.

Slide credit: Bastian Leibe

Technical Details

Given n data points $\mathbf{x}_i \in \mathbb{R}^d$, the multivariate kernel density estimate using a radially symmetric kernel[1] (e.g., Epanechnikov and Gaussian kernels), $K(\mathbf{x})$, is given by,

$$\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right), \tag{1}$$

where h (termed the *bandwidth* parameter) defines the radius of kernel. The radially symmetric kernel is defined as,

$$K(\mathbf{x}) = c_k k(\|\mathbf{x}\|^2), \tag{2}$$

where c_k represents a normalization constant.

Comaniciu & Meer, 2002

Technical Details

$$\nabla \hat{f}(\mathbf{x}) = \underbrace{\frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \right]}_{\text{term 1}} \underbrace{\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \right]}_{\text{term 2}}, \qquad (3)$$

where $g(x) = -k'(x)$ denotes the derivative of the selected kernel profile.

- Term1: this is proportional to the density estimate at x (similar to equation 1 from the previous slide).
- Term2: this is the mean-shift vector that points towards the direction of maximum density.

Comaniciu & Meer, 2002

Technical Details

Finally, the mean shift procedure from a given point x_t is:

1. Computer the mean shirt vector m:

$$\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \right]$$

2. Translate the density window:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{m}(\mathbf{x}_i^t).$$

3. Iterate steps 1 and 2 until convergence.

$$\nabla f(\mathbf{x}_i) = 0.$$

Comaniciu & Meer, 2002

Summary Mean-Shift

- Pros
  - General, application-independent tool
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
  - Just a single parameter (window size h)
    - h has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Window size (bandwidth) selection is not trivial
  - Computationally (relatively) expensive (~2s/image)
  - Does not scale well with dimension of feature space

Slide credit: Svetlana Lazebnik