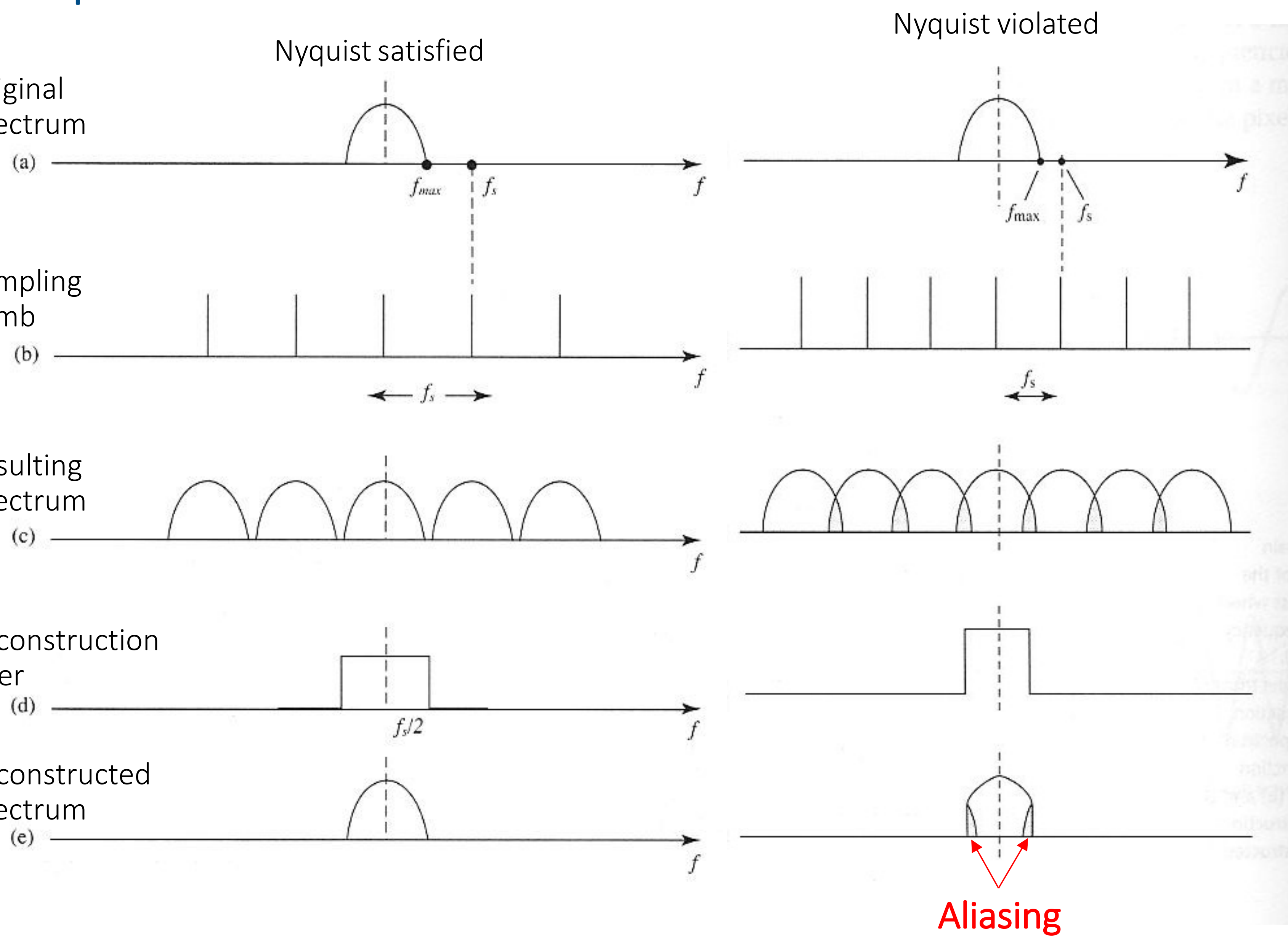
Lecture 10:

Anti-Aliasing and Super-Sampling

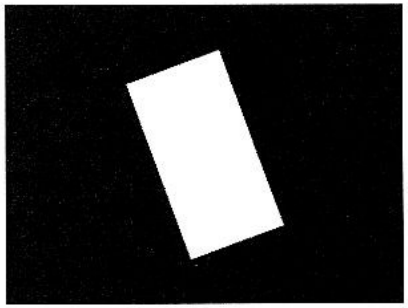
Contents

In Fourier space

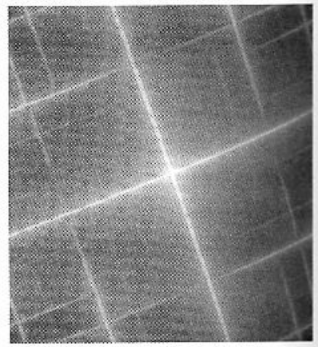
- Original spectrum
- Sampling comb
- Resulting spectrum
- Reconstruction filter
- Reconstructed spectrum



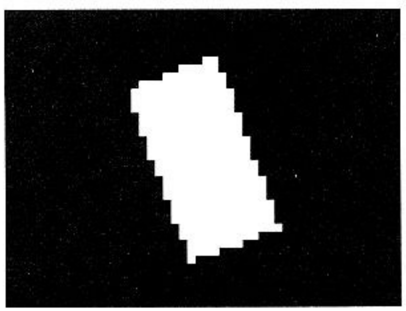
Motivation: Aliasing



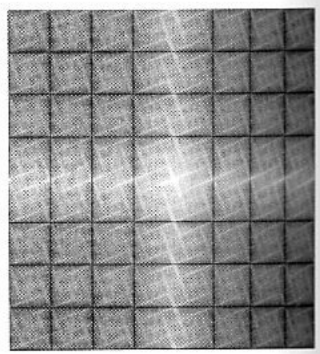
(a) Simulation of a perfect line



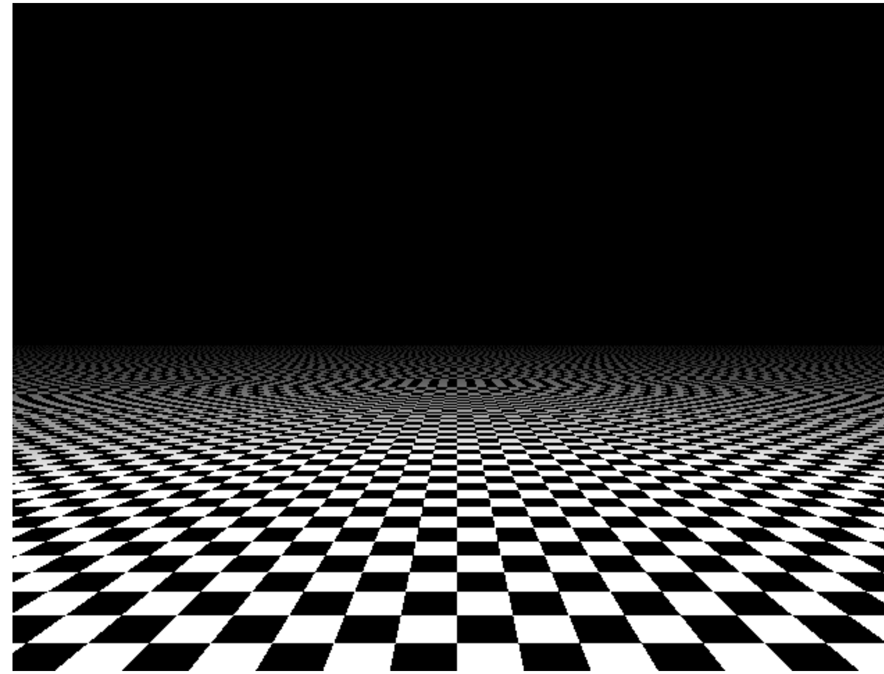
(b) Fourier transform of (a)

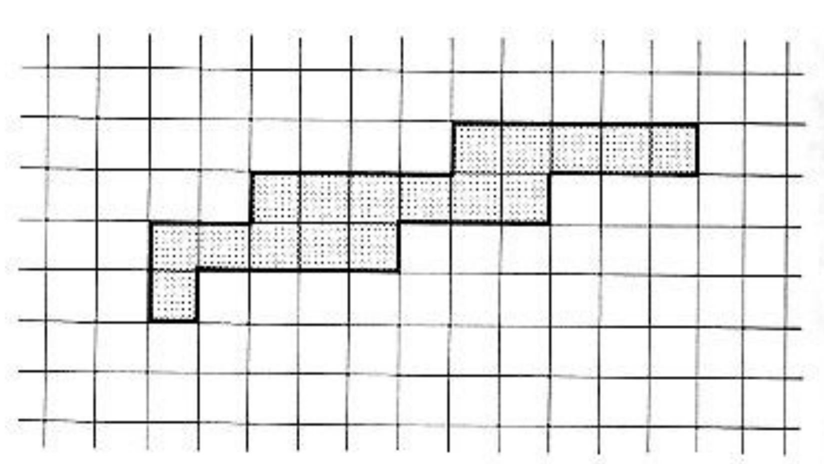


(c) Simulation of a jagged line



(d) Fourier transform of (c)





Spatial aliasing:

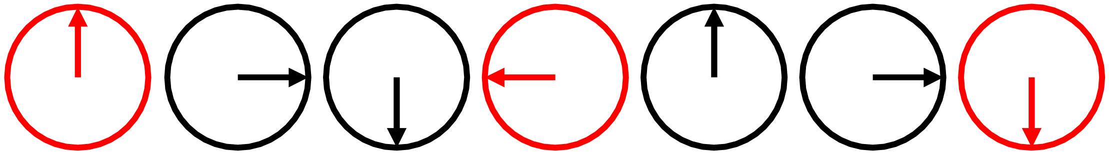
- Stair cases, Moiré patterns, *etc*.

Solutions:

- Increasing the sampling rate
  - Ok, but infinite frequencies at sharp edges
- Post-filtering (after reconstruction)
  - Does not work - only leads to blurred stair cases
- Pre-filtering (Blurring) of sharp geometry features
  - Slowly make geometry transparent at the edges
  - Correct solution in principal
  - Analytic low-pass filtering hard to implement
  - Super-sampling

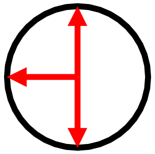
Temporal Aliasing:

- Car wheels, ...



Solutions

- Increasing the frame rate
  - OK
- Pre-filtering (Motion Blur)
  - Yes, possible for simple geometry (*e.g.*, Cartoons)
  - Problems with texture, *etc*.
- Post-filtering (Averaging several frames)
  - Does not work – only multiple detail



Important

- Distinction between **aliasing errors** and **reconstruction errors**

It all comes from sampling at discrete points

- Multiplied with comb function, no smoothly weighted filters
- Comb function: repeats frequency spectrum

Or, from using non band limited primitives

- Hard edges ⇒ infinitely high frequencies

In reality, integration over finite region necessary

- *E.g.*, finite CCD pixel size

Computer: Analytic integration often not possible

- No analytic description of radiance or visible geometry available

Only way: numerical integration

- Estimate integral by taking multiple point samples, average
  - Leads to aliasing
- Computationally expensive
- Approximate

Geometry

- Edges, vertices, sharp boundaries
- Silhouettes (view dependent)
- ...

Texture

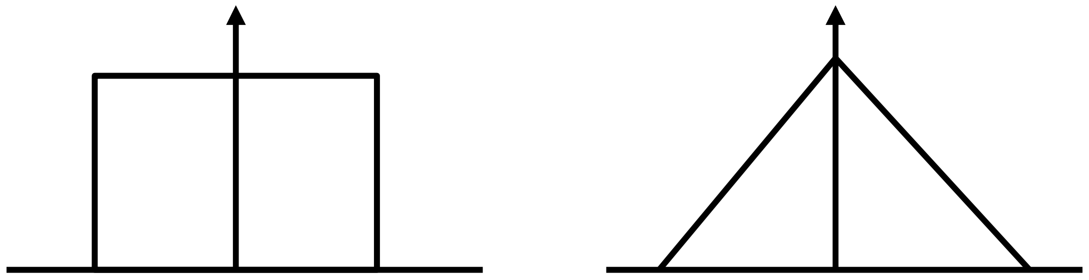
- *E.g.*, checkerboard pattern, other discontinuities, ...

Illumination

- Shadows, lighting effects, projections, ...

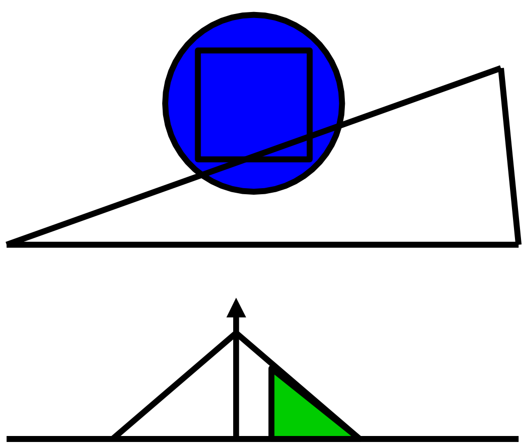
⇒ Analytic filtering almost impossible

- Even with the most simple filters



Analytic low-pass filtering

- Ideally eliminates aliasing completely
- Hard to implement
    - Only works for polygon edges with constant color
    - Weighted or unweighted area sampling
    - Compute distance from pixel to a line
    - Filter values can be stored in look-up tables
        - Possibly taking into account slope
        - Distance correction
        - Non rotationally symmetric filters
    - Does not work for corners



Over-/Super-sampling

- Very easy to implement
- Does not eliminate aliasing completely
    - Sharp edges contain infinitely high frequencies

Filtering before sampling

- Band-limiting signal
- Analog / analytic or
- Reduce Nyquist frequency for chosen sampling-rate

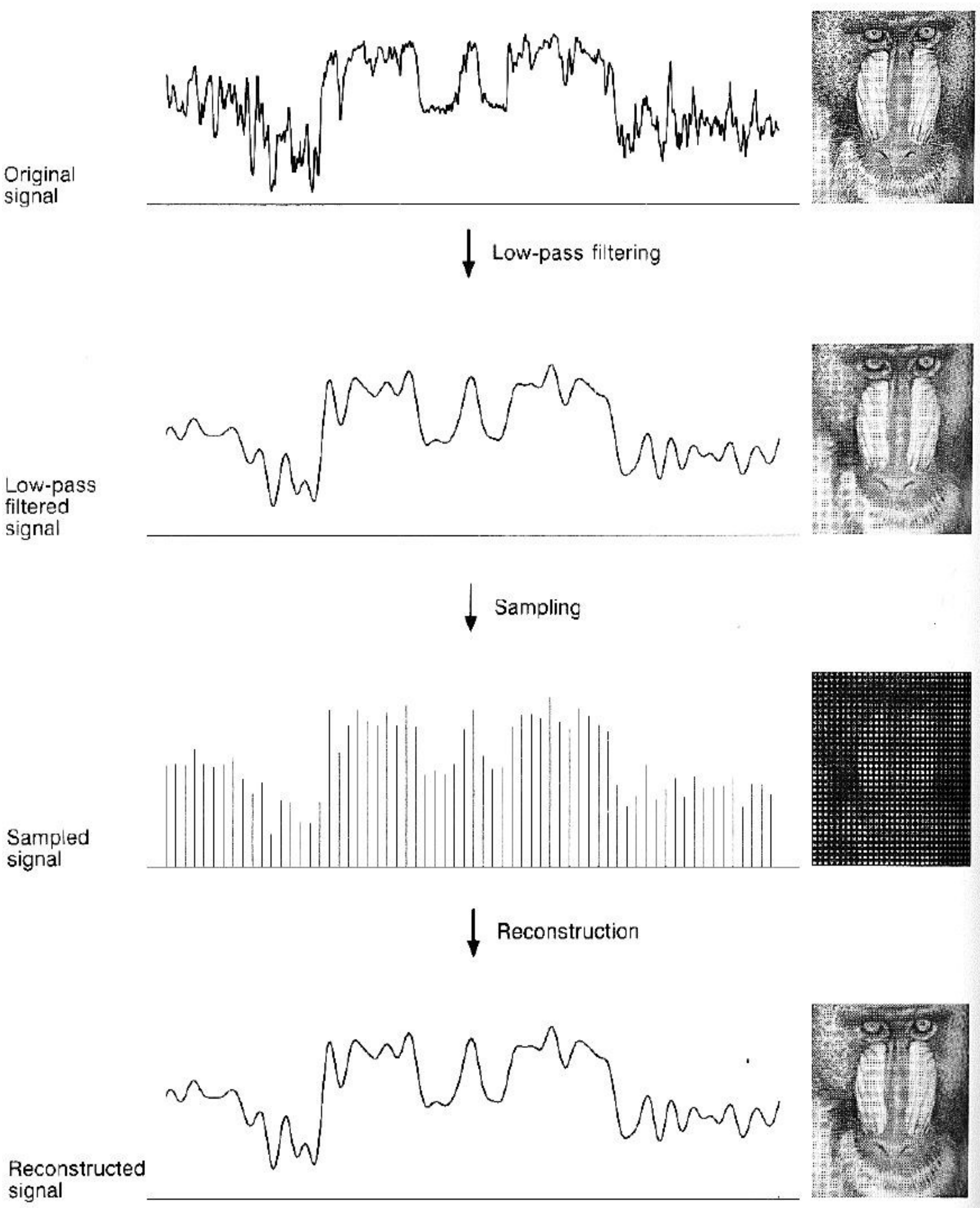
Ideal reconstruction

- Convolution with $sinc$

Practical reconstruction

- Convolution with
  - Box filter, Bartlett (Tent)

⇒ Reconstruction error

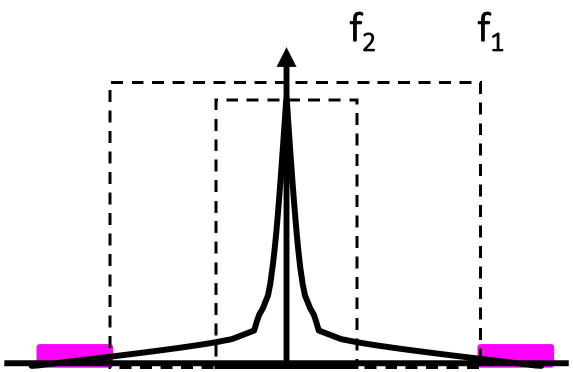


Assumption

- Energy in high frequencies decreases quickly
- Reduced aliasing by intermediate sampling with higher frequencies

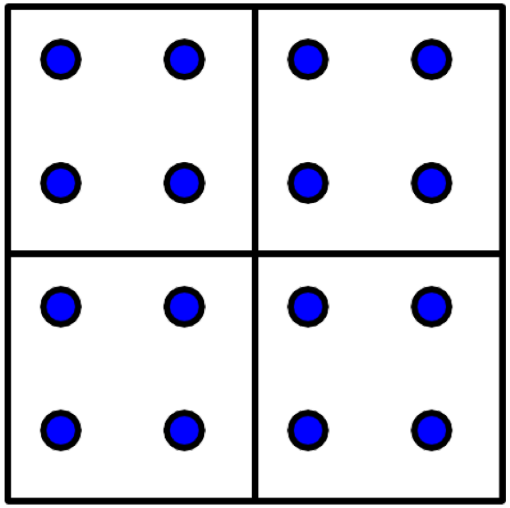
Algorithm

- Super-sampling
  - Sample continuous signal with boundary frequency $\mathbf{f_1}$
  - Aliasing with energy beyond $\mathbf{f_1}$ (assumed to be small)
- Reconstruction of signal
  - Filtering with $g_1(x)$: *e.g.* convolution with $sinc(\mathrm{f}_1)$
- Analytic low-pass filtering of signal
  - Filtering with filter $g_2(x)$ with $\mathbf{f_2} << \mathbf{f_1}$
  - Signal is now band limited w.r.t. $\mathbf{f_2}$
- Re-sampling with a sampling frequency that is compatible with $\mathbf{f_2}$
  - No additional aliasing
- Filters $g_1(x)$ and $g_2(x)$ can be combined
- Hardware support (OpenGL multisampling extension)



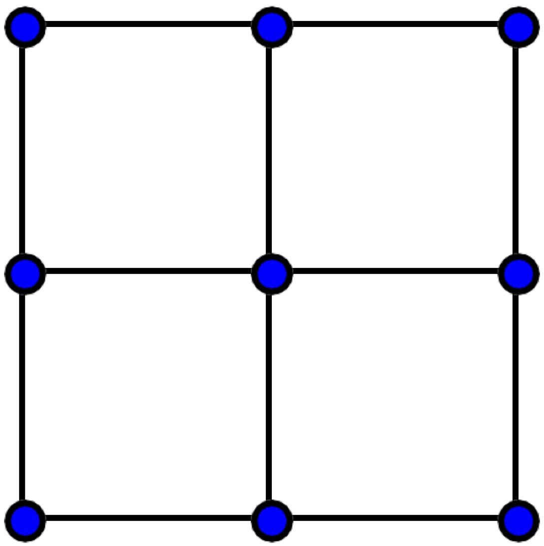
Regular super-sampling

- Averaging of N samples per pixel on a grid
- N :
  - 4 quite good
  - 16 almost always sufficient
- Samples
  - Rays, z-buffer, reflection, motion, ...
- Averaging
  - Box filter
  - Others: Pyramid (Bartlett), B-spline, Hexagonal, ...
- Regular super-sampling
  - Nyquist frequency for aliasing only shifted
    - ⇒ Irregular sampling patterns



Popular mistake

- Sampling at the corners of every pixel
- Pixel color by averaging
- Free super-sampling ???



Problem

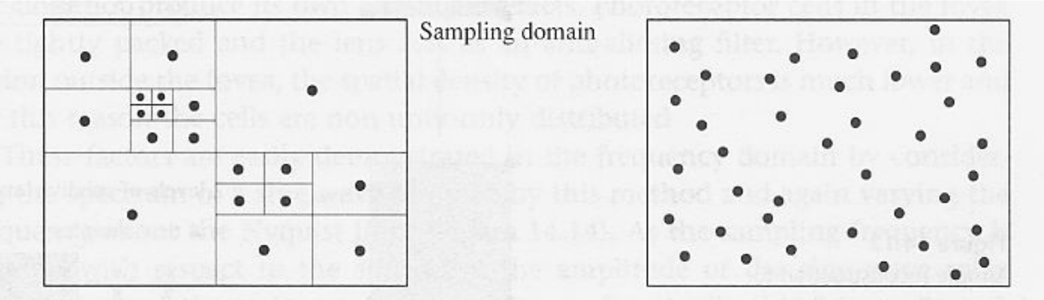
- Wrong reconstruction filter!!!
- Same sampling frequency, but post-filtering with a hat function
- Blurring: Loss of information

Post-Reconstruction Blur

1x1 Sampling, 3x3 Blur

1x1 Sampling, 7x7 Blur

⇒ „Super-sampling“ does not come for free

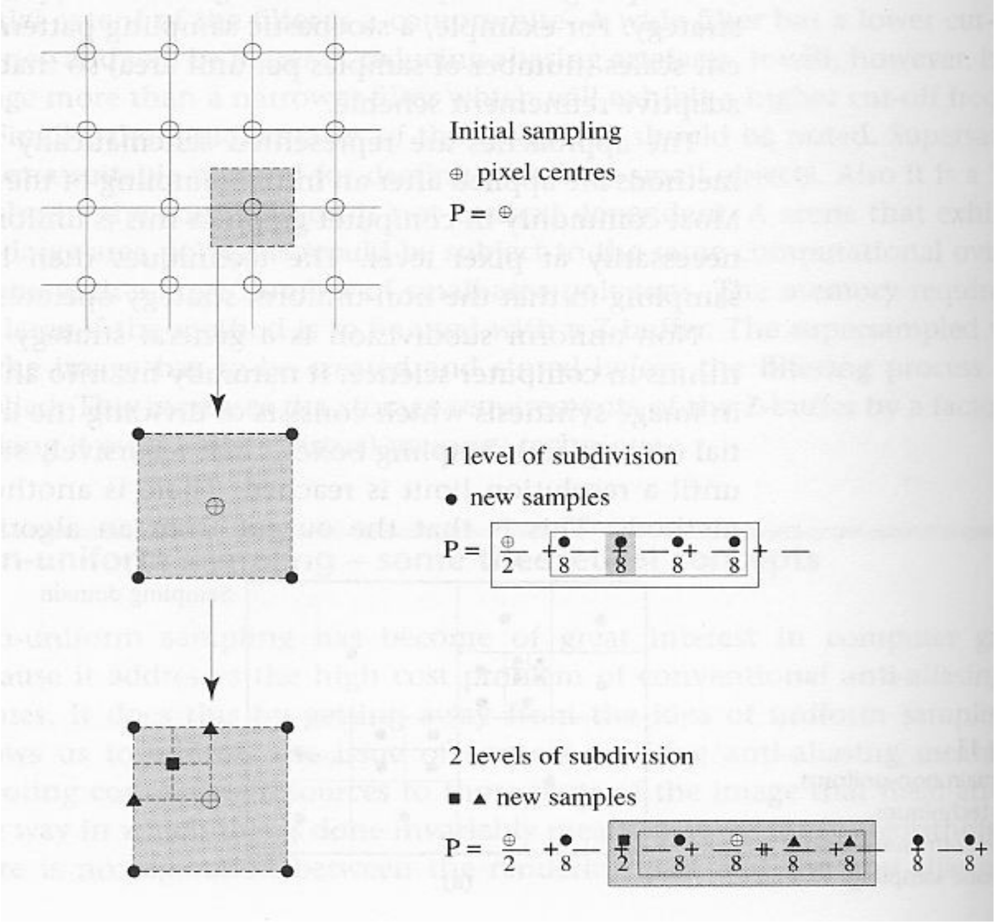


Adaptive super-sampling

- Idea: locally adapt sampling density
    - Slowly varying signal:

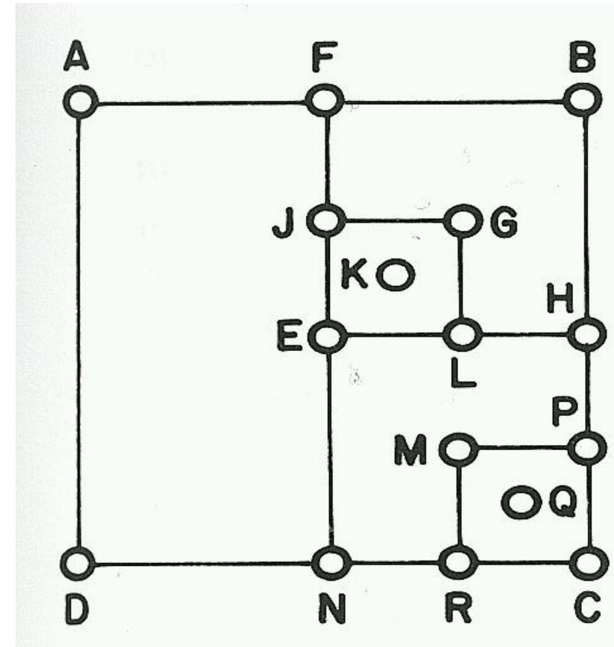
      low sampling rate
    - Strong changes:

      high sampling rate
- Decide sampling density locally
- Decision criterion needed
    - Differences of pixel values
    - Contrast (relative difference)
        - $|A-B| / |A|+|B|$

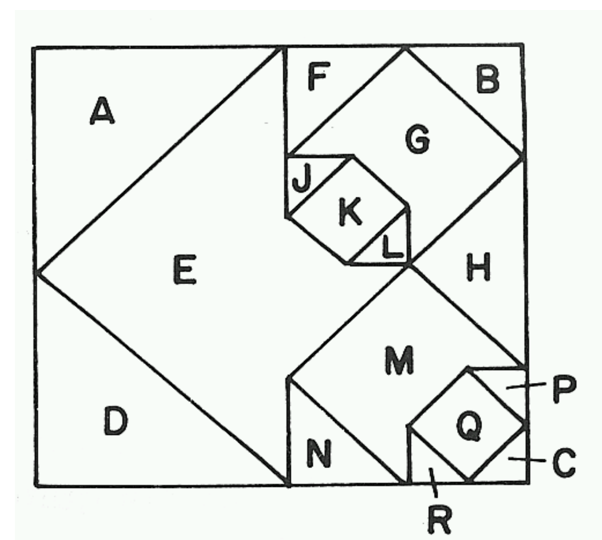


Algorithm

- Sampling at corners and mid points
- Recursive subdivision of each quadrant
- Decision criterion
  - Differences, contrast, object-IDs, ray trees, ...
- Filtering with weighted averaging
  - ¼ from each quadrant
  - Quadrant: ½ (midpoint + corner)
    - Recursion



$$\frac{1}{4}\left(\frac{A+E}{2}+\frac{D+E}{2}+\frac{1}{4}\left[\frac{F+G}{2}+\frac{B+G}{2}+\frac{H+G}{2}+\frac{1}{4}\left\{\frac{J+K}{2}+\frac{G+K}{2}+\frac{L+K}{2}+\frac{E+K}{2}\right\}\right]\right.$$
$$\left.+\frac{1}{4}\left[\frac{E+M}{2}+\frac{H+M}{2}+\frac{N+M}{2}+\frac{1}{4}\left\{\frac{M+Q}{2}+\frac{P+Q}{2}+\frac{C+Q}{2}+\frac{R+Q}{2}\right\}\right]\right)$$



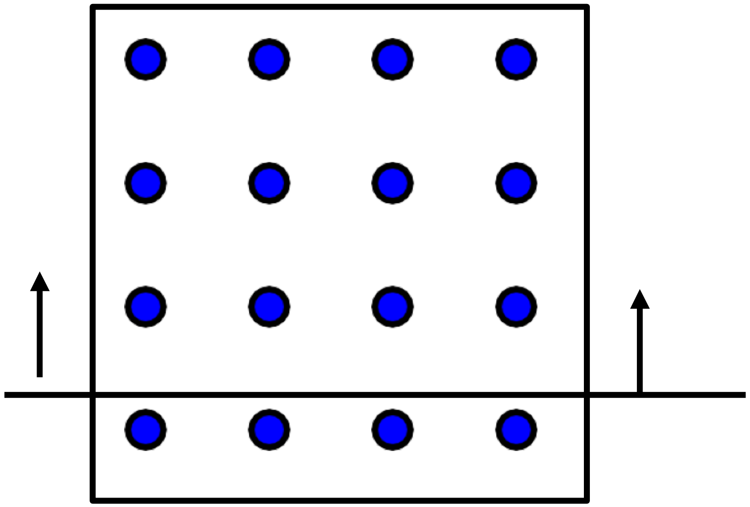
Extension

- Jittering of sample points

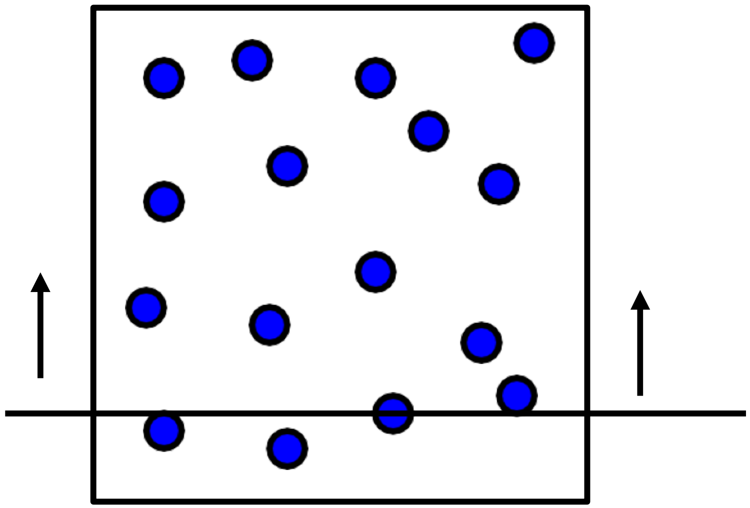
Problems with regular super-sampling

- Expensive: 4-fold to 16-fold effort
- Non-adaptive: Same effort everywhere
- Too regular: Reduced number of levels

Introduce irregular sampling pattern



0 → 4/16 → 8/16 → 12/16 → 16/16



Better, but noisy

Stochastic super-sampling

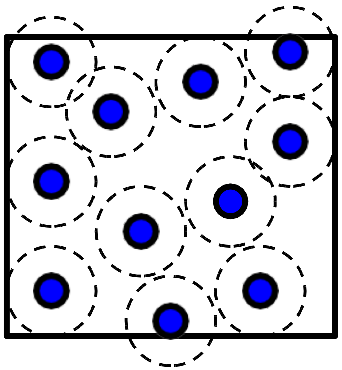
- Or analytic computation of pixel coverage and pixel mask

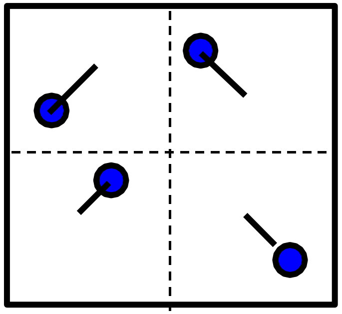
Requirements

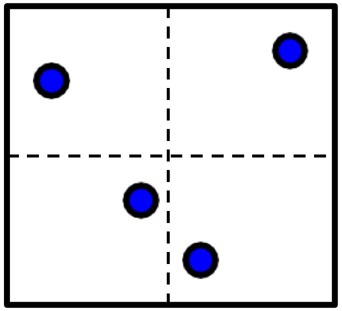
- Even distribution
- Little correlation between samples
- Incremental generation

Generation of samples

- Poisson-disk sampling
  - Fixes a minimum distance between samples
  - Random generation of samples
    - Rejection, if too close to other samples
- Jittered sampling
  - Random perturbation from regular positions
- Stratified Sampling
  - Subdivision into areas with one random sample each
- Quasi-random numbers (Quasi-Monte Carlo)
  - *E.g.* Halton Sequence
  - Advanced feature



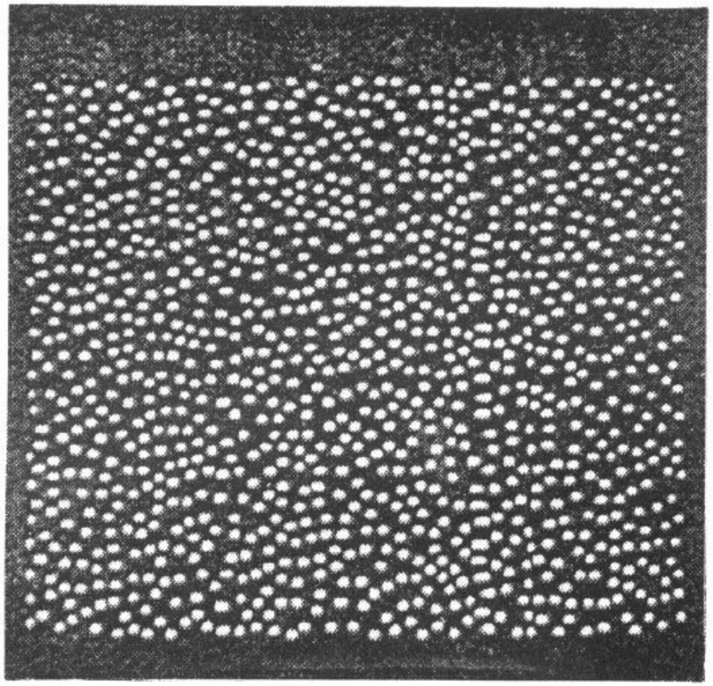




Motivation

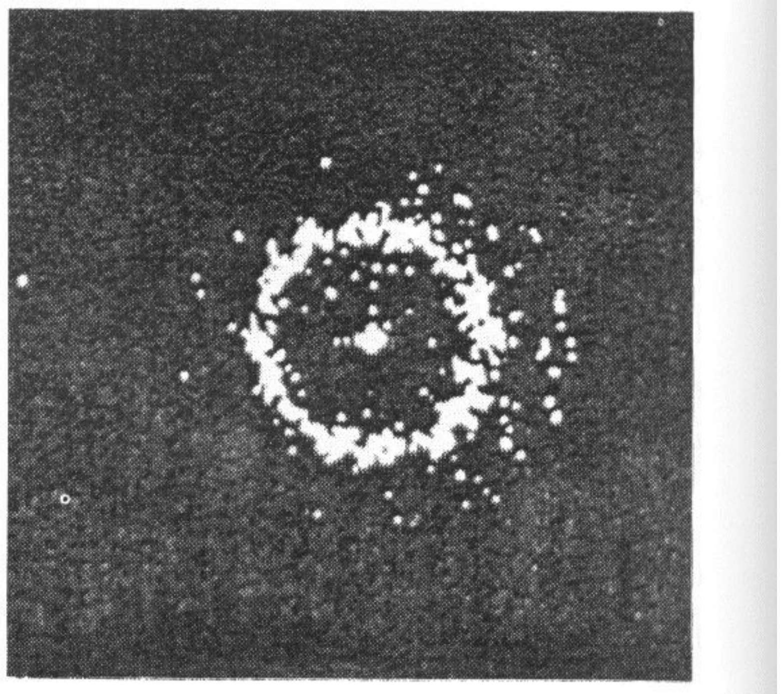
- Distribution of the optical receptors on the retina (here: ape)

(a)



Distribution of the receptors

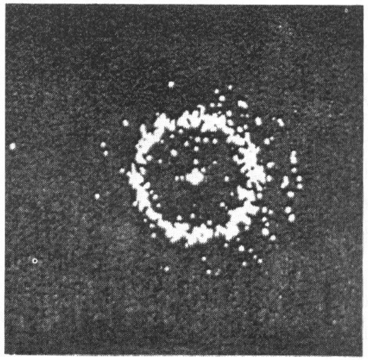
(b)

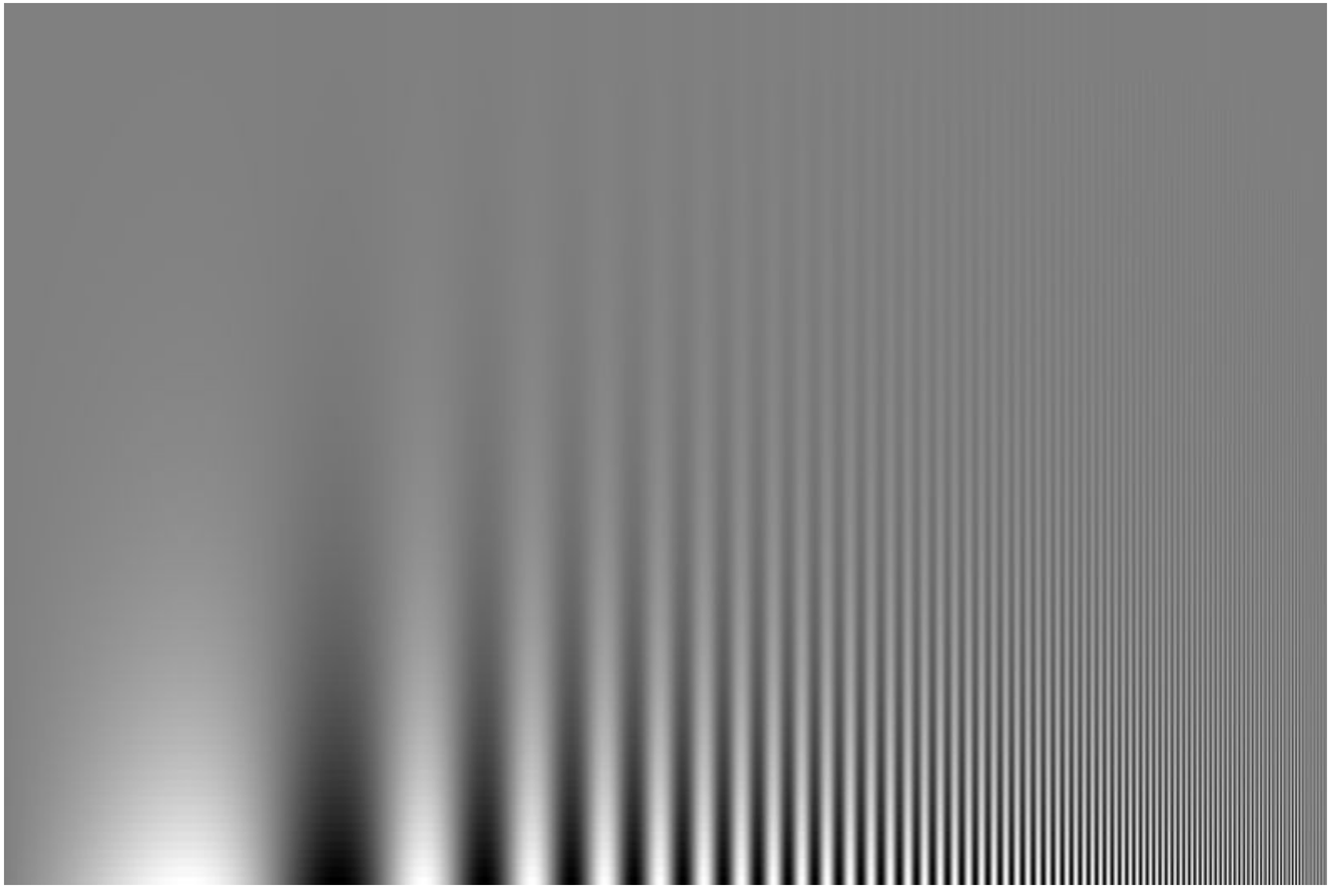


Fourier analysis

Human perception

- Very sensitive to regular structures
- Insensitive against (high frequency) noise

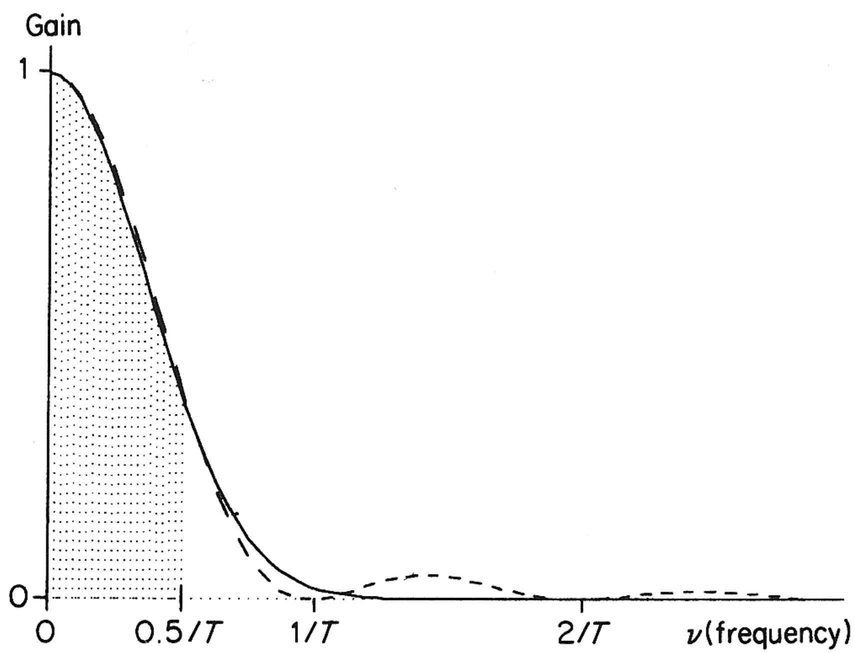




Campbell-Robson contrast sensitivity chart

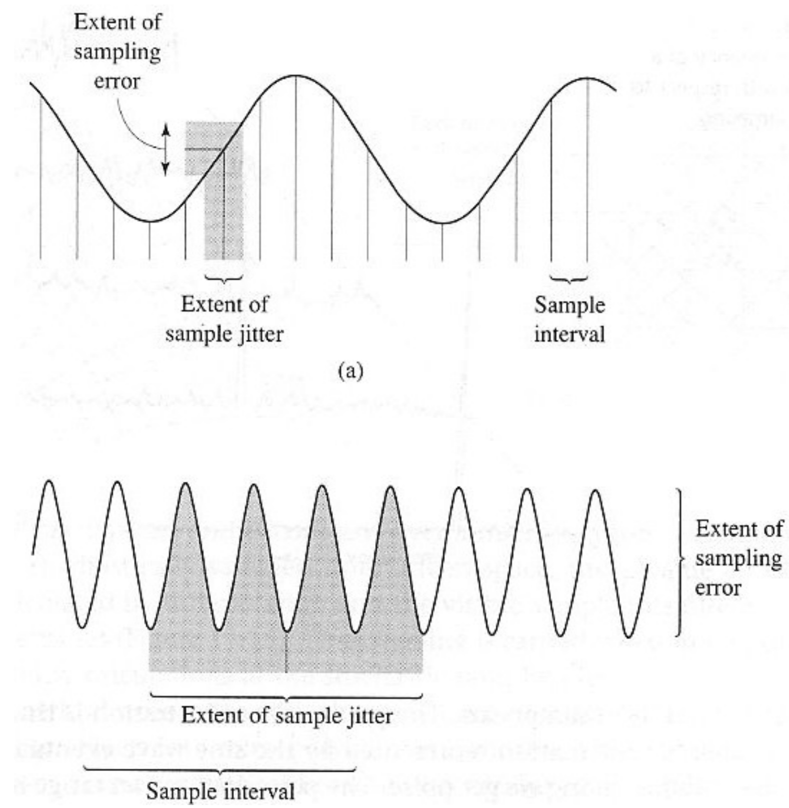
Stochastic Sampling

- Transforms energy in high frequency bands into noise



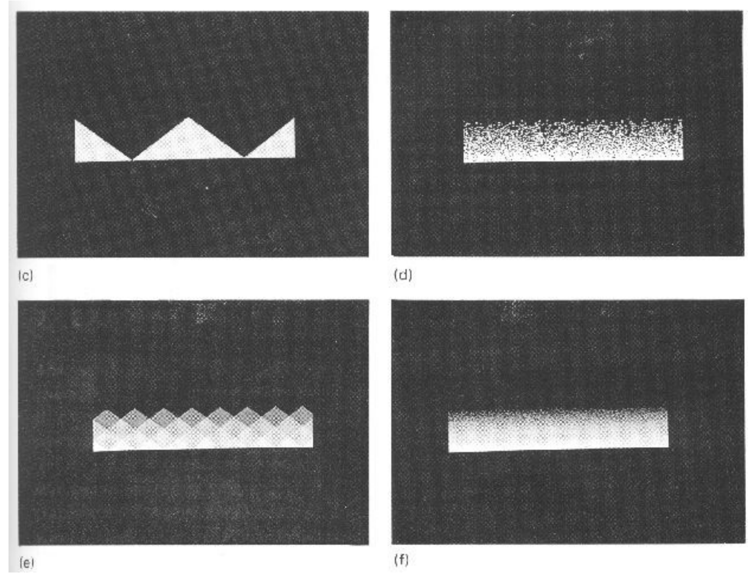
Gauss Jittering

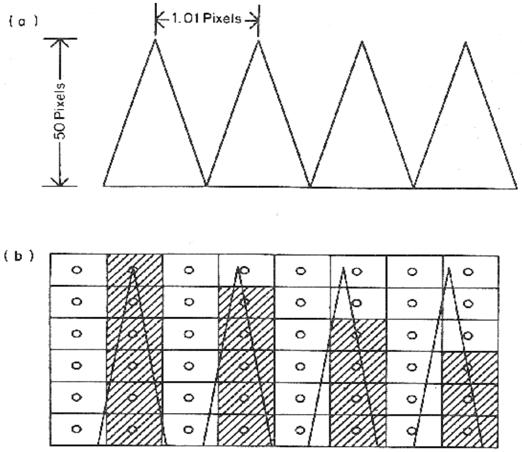
White-Noise Jittering



Triangle comb
(Width: 1.01 pix, Heigth: 50 pix):

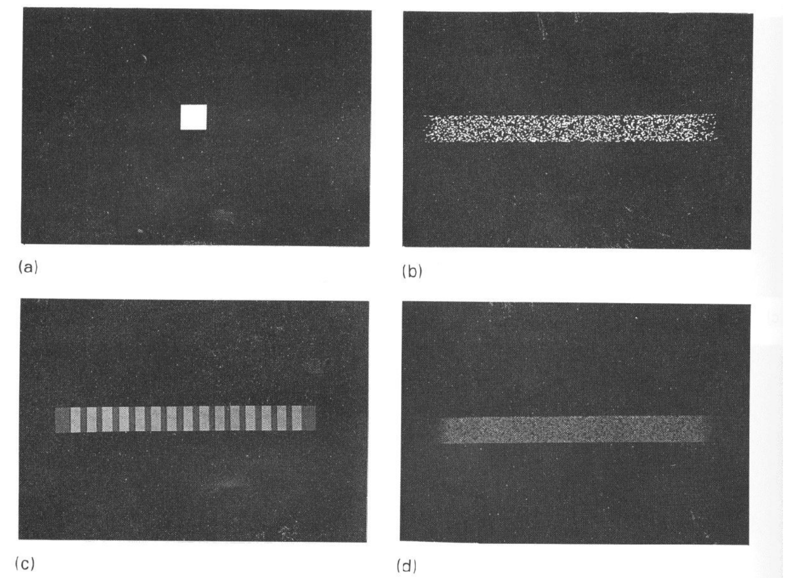
- 1 sample, no jittering
- 1 sample, jittering
- 16 samples, no jittering
- 16 samples, jittering





Motion Blur:

- 1 sample, no jittering
- 1 sample, jittering
- 16 samples, no jittering
- 16 samples, jittering



Comparison

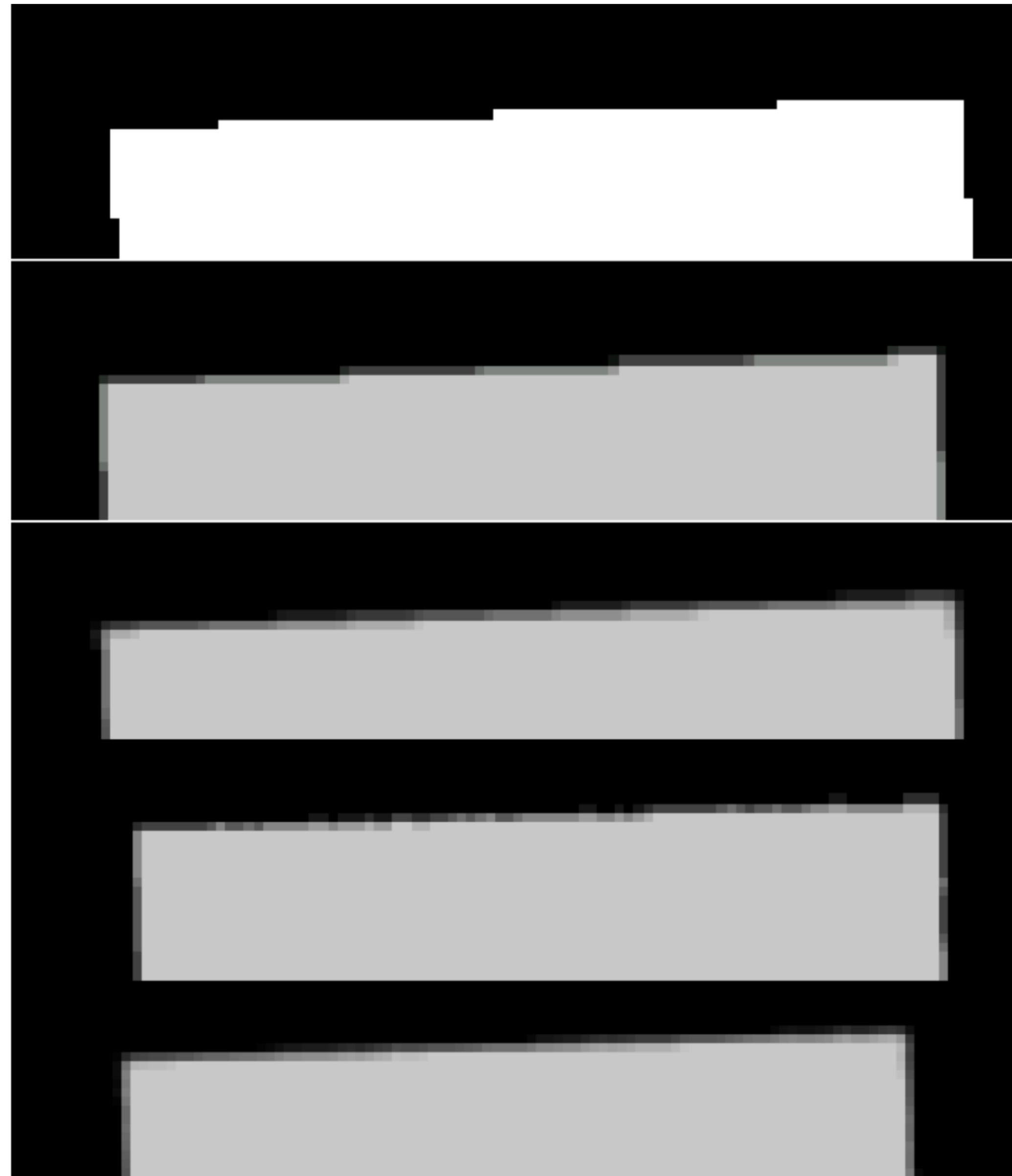
Regular, 1x1

Regular 3x3

Regular, 7x7

Jittered, 3x3

Jittered, 7x7



Assignment 3 (Theoretical part)

Submission deadline: Friday, 18. October 2019 9:45 (before the lecture)
Written solutions have to be submitted in the lecture room before the lecture. Every assignment sheets counts 100 points (theory and practice)

3.* Ray Quadric Intersection (30 Points) (voluntary / bonus points)

Given a ray $\vec{r}(t) = \vec{o} + t \cdot \vec{d}$ and quadric
$ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + iz + j = 0$.

a) Compute the values t for which the ray intersects the quadric.
b) Derive the ray-sphere intersection formula from it, as a special case.

This is a pure theoretical exercise which has not to be implemented, but you can of course if you like.