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Homework 2

Problem 2.1

Solution:

a) $777_8 + 1_8 = 511_{10} + 1_{10} = 512_{10} = 1000_8$

b) $888_{16} + 1_{16} = 2184_{10} + 1_{10} = 2185_{10} = 889_{16}$

c) $32007_8 + 1_8 = 13319_{10} + 1 = 13320_{10} = 32010_8$

d) $32108_{16} + 1_{16} = 205065_{10} = 32109_{16}$

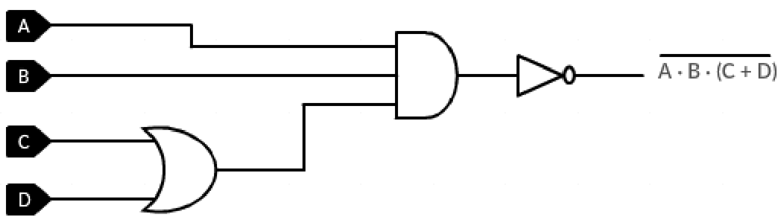
e) $8BFF_{16} + 1_{16} = 35839_{10} + 1 = 35840_{10} = 8C00_{16}$

f) $1219_{16} + 1_{16} = 4634_{10} = 121A_{16}$

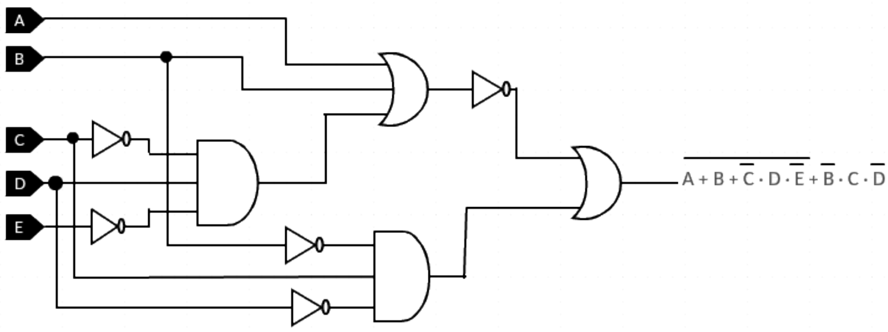
Problem 2.2

Solution:

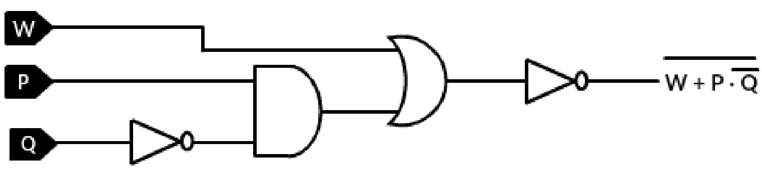
a)



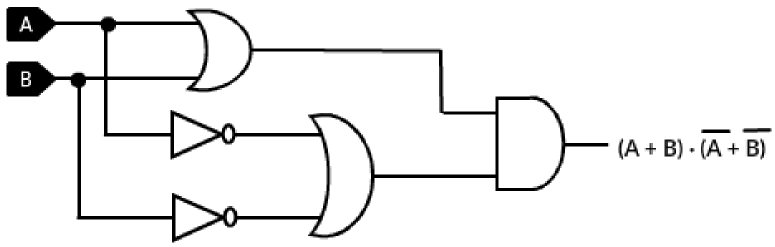
b)



c)



d)



Problem 2.3

Solution:

Truth Table

M	N	Q	x
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

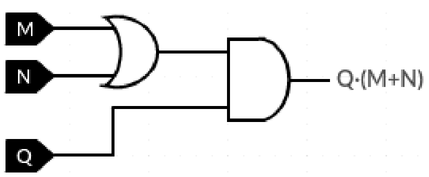
The sum of products expression, considering the gate model is:

$$
\begin{aligned}
x &= \overline{\overline{MNQ} \cdot \overline{M\overline{N}Q} \cdot \overline{\overline{M}NQ}} && \\
&= \overline{\overline{MNQ + M\overline{N}Q + \overline{M}NQ}} && \text{R5} \\
&= MNQ + \overline{M}NQ + M\overline{N}Q && \text{R6} \\
&= MNQ + \overline{M}NQ + MNQ + M\overline{N}Q && \text{R2, R4} \\
&= NQ(M + \overline{M}) + MQ(N + \overline{N}) && \text{R1} \\
&= NQ + MQ = Q(M + N) && \text{R3, R1}
\end{aligned}
$$

Where:

R1 - Distributivity: $XY + YZ = Y(X + Z)$
R2 - Commutativity: $X + Y = Y + X$
R3 - Complement: $\overline{X} + X = 1$
R4 - Indermpotent: $X = X + X$
R5 - De Morgan: $\overline{X}\,\overline{Y} = \overline{X + Y}$
R6 - Involution: $\overline{\overline{X}} = X$

Therefore, the new simplified circuit is:



Problem 2.4

Solution:

a) $X + \overline{X} \cdot Y = X + Y$

X	Y	$\overline{X}$	$\overline{X} \cdot Y$	$X + \overline{X} \cdot Y$	$X + Y$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

Considering the two last columns, the statement is proved.

b) $\overline{X} + X \cdot Y = \overline{X} + Y$

X	Y	$\overline{X}$	$X \cdot Y$	$\overline{X} + X \cdot Y$	$\overline{X} + Y$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

Considering the two last columns of the table, we say that the expression is proved to be true.

Problem 2.5

Solution:

a) $A + 1 = 1$
b) $A \cdot A = A$
c) $B \cdot \overline{B} = 0$
d) $C + C = C$
e) $x \cdot 0 = 0$
f) $D \cdot 1 = D$
g) $D + 0 = D$
h) $C + \overline{C} = 1$
i) $G + G \cdot F = G(1 + F) = G$
j) $y + \overline{w} \cdot y = y(1 + \overline{w}) = y$

Problem 2.6

Solution:

According to De Morgan's first theorem, we have: $\overline{X + Y} = \overline{X} \cdot \overline{Y}$. The corresponding truth table would be:

X	Y	$\overline{X}$	$\overline{Y}$	X+Y	$\overline{X + Y}$	$\overline{X} \cdot \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

Therefore, considering the two last columns, we can see that theorem is proved to be true.

De Morgan's second theorem: $\overline{X \cdot Y} = \overline{X} + \overline{Y}$. The corresponding truth table would be:

X	Y	$\overline{X}$	$\overline{Y}$	$X \cdot Y$	$\overline{X \cdot Y}$	$\overline{X} + \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

Considering the two last columns, we say that the second theorem is also proved to be true.

Problem 2.7

Solution:

Sum-of-products expression:

$$
\begin{aligned}
&\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + ABCD \\
&= \bar{A}\bar{B}D(\bar{C}+C) + \bar{A}BD(\bar{C}+C) + ABD(\bar{C}+C) + A\bar{B}\bar{C}\bar{D} && R1 \\
&= \bar{A}\bar{B}D + \bar{A}BD + ABD + A\bar{B}\bar{C}\bar{D} && R2 \\
&= \bar{A}\bar{B}D + \bar{A}BD + \bar{A}BD + ABD + A\bar{B}\bar{C}\bar{D} && R3 \\
&= \bar{A}D(\bar{B}+B) + BD(\bar{A}+A) + A\bar{B}\bar{C}\bar{D} && R1 \\
&= \bar{A}D + BD + A\bar{B}\bar{C}\bar{D} && R2
\end{aligned}
$$

Where we know that:

R1 - Distributivity: $XY + YZ = Y(X+Z)$

R2 - Complement: $\overline{X} + X = 1$

R3 - Indermpotent: $X = X + X$

Problem 2.8

Solution:

The map is as follows:

	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	0_1	1_2	1_3	0_4
$\overline{A}B$	0_5	1_6	1_7	0_8
AB	0_9	1_{10}	1_{11}	0_{12}
$A\overline{B}$	1_{13}	0_{14}	0_{15}	0_{16}

We 'identify' each cell with its specific index(cell number). Considering the table above, we have one quad consisting of cells 2,3,6,7. This quad is looped since there is no isolated pair. Another quad can be formed by cells 6,7,10,11. Meanwhile, we have a 1 that isn't adjacent to any other 1, so cell 13 is also looped. As a result, we get the following expression, which is identical to the analytical one, derived from Problem 2.7:

$$x = \bar{A}D + BD + A\bar{B}\bar{C}\bar{D}$$