Computer Architecture and Programming Languages
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Homework 1

Problem 1.1

Solution:

a) We perform the multiplication of each of the binary digits with 2 to the power of the specific position of each digit, as shown below:

$\underset{43210}{10100}_2 \rightarrow 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 16 + 4 = 20_{10}$

b)

$\underset{76543210}{11011011}_2 \rightarrow 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 128 + 64 + 16 + 8 + 2 + 1 = 219_{10}$

c)

$001001001_2 \rightarrow 0 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 64 + 8 + 1 = 73_{10}$

d)

$111111111111_2 \rightarrow 1 \cdot 2^{11} + 1 \cdot 2^{10} + 1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 4095_{10}$

e) We follow the same idea as before: the digits of the octal will be multiplied with 8 to the power of the position of the digit.

$\underset{43210}{75077}_8 \rightarrow 7 \cdot 8^4 + 5 \cdot 8^3 + 0 \cdot 8^2 + 7 \cdot 8^1 + 7 \cdot 8^0 = 28672 + 2560 + 56 + 7 = 31295_{10}$

f) Same logic: the digits of the ternary number will be multiplied with 3 to the power of the position of the digit.

$\underset{43210}{12101}_3 \rightarrow 1 \cdot 3^4 + 2 \cdot 3^3 + 1 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 = 81 + 54 + 9 + 1 = 145$

g)

$\underset{43210}{26601}_7 \rightarrow 2 \cdot 7^4 + 6 \cdot 7^3 + 6 \cdot 7^2 + 0 \cdot 7^1 + 1 \cdot 7^0 = 4802 + 2058 + 294 + 1 = 7155$

h)

$\underset{543210}{431021}_5 \rightarrow 4 \cdot 5^5 + 3 \cdot 5^4 + 1 \cdot 5^3 + 0 \cdot 5^2 + 2 \cdot 5^1 + 1 \cdot 5^0 = 12500 + 1875 + 125 + 10 + 1 = 14511$

Problem 1.2

Solution:

a) There are many ways to perform the conversion into binary. One of them is getting the remainders of the repeated division by 2.

4272_{10}

$4272/2 = 2136 \rightarrow 0$
$2136/2 = 1068 \rightarrow 0$
$1068/2 = 534 \rightarrow 0$
$534/2 = 267 \rightarrow 0$
$267/2 = 133 \rightarrow 1$
$133/2 = 66 \rightarrow 1$
$66/2 = 33 \rightarrow 0$

$33/2 = 16 \rightarrow 1$
$16/2 = 8 \rightarrow 0$
$8/2 = 4 \rightarrow 0$
$4/2 = 2 \rightarrow 0$
$2/2 = 1 \rightarrow 0$
$1/2 = 0 \rightarrow 1$

The resulting binary number is: 1000010110000_2

b) We use the table from the lecture to convert the hexadecimal digits to a set of 4 digits binary representation.
Our number is CBA_{16}. Looking from the table in the slides.

C $\rightarrow$ 1100
B $\rightarrow$ 1011
A $\rightarrow$ 1010
As a result, the binary representation is: 1100 1011 1010_2

c) Using the same approach as in point b, considering the table from the lecture, we have:

$B \rightarrow 11\,;C \rightarrow 12$

$B8C_{16} \rightarrow 11 \cdot 16^2 + 8 \cdot 16^1 + 12 \cdot 16^0 = 2816 + 128 + 1 = 2956_{10}$

d) For $29D8_{16}$ the value for D is 13.

$29D8_{16} \rightarrow 2 \cdot 16^3 + 9 \cdot 16^2 + 13 \cdot 16^1 + 8 \cdot 16^0 = 8192 + 2304 + 208 + 8 = 10712_{10}$

e) Using the table from the lectures, we get:

$8CE_{16} \rightarrow 8CF_{16} \rightarrow 8D1_{16} \rightarrow 8D2_{16} \rightarrow 8D3_{16} \rightarrow 8D4_{16}$

Problem 1.3

Solution:

a) Using BCD we can convert 732_{10} simply using the table from the lecture for each of the digits.

$732_{10}: \quad 7 \rightarrow 0111_2 \quad 3 \rightarrow 0011_2 \quad \rightarrow 0010_2$

The result of this conversion is: 011100110010_2

b) Binary Coded Decimal numbers range from 0 to 9. Therefore, codes who represent values bigger then 9_{10} are invaid BCD codes. Specifically, we have:

$$\begin{array}{ccc} \text{Decimal} & & \text{Binary} \\ 10 & & 1010 \\ 11 & & 1011 \\ 12 & \Leftrightarrow & 1100 \\ 13 & & 1101 \\ 14 & & 1110 \\ 15 & & 1111 \end{array} \tag{1}$$

c) In order to convert from BCD to decimal, we divide the digits into groups of 4, each representing a digit in the decimal representation.

$$\begin{array}{ccc} & 1001\ 0101\ 0110_{BCD} & \\ \swarrow & \downarrow & \searrow \\ 1001 \rightarrow 9_{10} & 0101 \rightarrow 5_{10} & 0110 \rightarrow 6_{10} \end{array}$$

The result of this conversion is : 956_{10}

d) Since M represents a value of 77 in the ASCII code, using same method as in probelm 2, its binary value is:

$77/2 = 38 \rightarrow 1$
$38/2 = 19 \rightarrow 0$
$19/2 = 9 \rightarrow 1$
$9/2 = 4 \rightarrow 1$
$4/2 = 2 \rightarrow 0$
$2/2 = 1 \rightarrow 0$
$1/2 = 0 \rightarrow 1$

$$\begin{array}{cc} \multicolumn{2}{c}{0100\ 1101_2} \\ \swarrow & \searrow \\ 0100 \rightarrow 4 & 1101 \rightarrow D \end{array}$$

The binary value is: 01001101_2
The hexadecimal value is: $4D_{16}$

e) Using same approach as before to convert 109_{10} into binary:

$109/2 = 54 \rightarrow 1$
$54/2 = 27 \rightarrow 0$
$27/2 = 13 \rightarrow 1$
$13/2 = 6 \rightarrow 1$
$6/2 = 3 \rightarrow 0$
$3/2 = 1 \rightarrow 1$
$1/2 = 0 \rightarrow 1$

$$0110\ 1101_2$$

$$\swarrow \qquad \searrow$$

$$0110 \rightarrow 6 \qquad 1101 \rightarrow D$$

The binary value is: 01101101_2
The hexadecimal value is: $6D_{16}$

Problem 1.4

Solution:
a) The NOT function does not satisfy this requirement as when a NOT gate's input is low the output is high and vice versa. For the other function, we check the respective truth tables for AND and OR gates:

OR Table

Input	Input	Output
0	0	0
0	1	1
1	0	1
1	1	1

AND Table

Input	Input	Output
0	0	0
0	1	0
1	0	0
1	1	1

As we can see, the AND function has 3 cases where at least 1 of inputs is low the output is still low. On the other side, there is 1 case for the OR function when this requirement is fulfilled. Therefore, both functions can provide a low output in response to one or more low inputs, but the AND function is the one that always provides a low input if one or more of the inputs is low. (correct answer is (iii)AND)

b) Considering the truth tables in the previous question, we can see that both OR or AND functions have 1 respective case when the requirement is met (first rows of the tables). But the OR function is the one that has a low output only when all the inputs are low. (correct answer is (i)OR)

Problem 1.5

Solution:

AND Gate Table

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Problem 1.6

Solution:

OR Gate Table

A	B	C	D	Output
0	0	0	0	0
0	0	0	1	1
0	1	0	0	1
0	1	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	1	0	0	1
1	1	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	1	0	1
1	1	1	1	1