Numerical Methods I

Assignment Sheet 3. Due: February 26, 2020, 11:15 sharp

—

Exercise 11 [5 x 3 Points]: Let $f(x) = x^3 - 3x^2 + 2x - 6$ and consider starting points $x_0 = 2$ and $x_1 = 5$.

  a) Find the roots of the function f analytically.

  b) Check whether Bisection, Newton, and Secant method can be applied.

  c) Apply two steps of the Bisection, Newton, and Secant method. (For the Newton method start with x_1).

  d) Compare the errors of the results computed in c).

  e) Which of the methods can be expected to converge? Explain your answers.

—

Exercise 12 [15 Points]: Given a function $f \in C^2[a, b]$, where $[a, b]$ is a neighborhood of the root r of f with $f'(r) \neq 0$. Find an example of a function f and starting points x_0 and x_1, such that the Secant method does not converge, while the Bisection method converges.

—

Exercise 13 [not graded, w/o Points]: Write down algorithms in pseudo code for the bisection method, Newton's method and the Secant method.

—

Exercise 14 [not graded, w/o Points]: Starting with $(1, 1)$, apply two iteration steps of the Newton method to solve the following system of non-linear equations

$$x^2 + y^2 + xy = 3,$$
$$e^x - 2xy = 3.$$

—

Exercise 15 [not graded, w/o Points]: The so called *Steffensen's method* considers the iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)}$$

for approximating roots of f. Show that, under suitable assumptions, this iteration is quadratically convergent.