Math-215: Calculus III Analytic Geometry

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

Exercises 96/100 (12 points each)

1) Section 12.1, problems: 21 - 26.
2) Section 12.1, problems: 29 & 35; do both of these.
3) Section 12.2, problems: 1 - 10.
4) Section 12.2, problems: 43 - 46.
5) Section 12.3, problems: 1 - 4; do all four.
6) Section 12.3, problems: 5 & 6; do both.
7) Section 12.3, problems: 7 - 12
8) Section 12.3, problems: 13 - 19; do as many as possible.

More difficult problems 4/100 (2 points each). *Please submit these on a separate sheet*.

9) Section 12.3, problem: # 30.
10) ...numerical methods for approximating double integrals can be troublesome. The **Monte Carlo method** makes clever use of probability theory to approximate $\iint_R f(x,y)\,\mathrm{d}A$ for a bounded region R. Suppose, for example, that R is contained within the rectangle $0 \le x \le 1$, $0 \le y \le 1$. Generate two random numbers a and b from the uniform distribution on $[0,1]$; this means that every number between 0 and 1 is in some sense equally likely. Determine whether or not the point (a,b) is in the region R and then repeat the process a large number of times. If, for example, 64 out of 100 points generated were within R, explain why a reasonable estimate of the area of R is 0.64 times the area of the rectangle $0 \le x \le 1$, $0 \le y \le 1$. For each point (a,b) that is within R, compute $f(a,b)$. If the average of all of these function values it 13.6, explain why a reasonable estimate of $\iint_R f(x,y)\,\mathrm{d}A$ is $(0.64)\cdot(13.6) = 8.704$. Use the Monte Carlo method to estimate

$$\int_1^2 \int_{\ln x}^{\sqrt{x}} \sin(xy)\;\mathrm{d}y\,\mathrm{d}x.$$

(Hint: Show that y is between $\ln 1 = 0$ and $\sqrt{2} < 2$.)