Math-215 Fall 2018

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

Exercises 96/100 (12 points each)

Taylor Series

1) Section 8.7, problems: # 2, 11 - 13.
2) Section 8.7, problems: # 14 - 16.
3) Section Ch. 8 Review, problem: # 51.

Quadric Surfaces

4) Section 10.6, problems: # 21 - 23; only if you did not do it in the previous homework.
5) Section 10.6, problems: # 24 - 25; only if you did not do it in the previous homework.
6) Section 10.6, problems: # 26; only if you did not do it in the previous homework.
7) Section 10.6, problems: # 27; only if you did not do it in the previous homework.
8) Section 10.6, problems: # 28; only if you did not do it in the previous homework.

More difficult problems 4/100 (2 points each). *Please submit these on a separate sheet*

Taylor Series

9) Use the fact that the Taylor series for e^x is absolutely convergent (you can rearrange its terms) to show that

$$e^{i\theta} = \cos\theta + i\sin\theta, \text{ where } i^2 = -1.$$

Note: If we associate with $x + iy$ a vector in V_2 given by $\vec{r} = \langle x, y\rangle$ then the rotation of $\vec{r}$ in the xy-plane can be represented by

$$e^{i\theta}(x+iy) = \underbrace{x\cos\theta - y\sin\theta}_{x'} + i\,\underbrace{x\sin\theta + y\cos\theta}_{y'}$$

where $\vec{r}' = \langle x', y'\rangle$ is the rotated vector. One can visualize $e^{i\theta}$ as the rotation operator in the complex xy-plane.

10) The exponential of an operator $\hat{\mathcal{O}}$ is defined in terms of the Taylor expansion of the exponential function, namely:

$$e^{\hat{\mathcal{O}}} \equiv \sum_{n=0}^{\infty} \frac{\hat{\mathcal{O}}^n}{n!}.$$

Consider a function $f(t)$ with infinite radius of convergence and show that

$$f(t+\Delta t) = \left[e^{\Delta t \frac{\mathrm{d}}{\mathrm{d}t}}\right] f(t).$$

One says that $e^{\Delta t \frac{\mathrm{d}}{\mathrm{d}t}}$ is the time-evolution operator, and $i\frac{\mathrm{d}}{\mathrm{d}t}$ is the generator of the transformation.