Math-215 Fall 2019

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

Exercises 96/100 (12 points each)

1) Simplify each expression. Express your answer so that only positive exponents occur. Assume that any variables are positive.

$$\sqrt[8]{x^4} \qquad \sqrt[6]{x^3}$$

$$x^{3/2}x^{-1/2} \qquad x^{5/4}x^{-1/4} \qquad \sqrt{x^3}\sqrt[4]{x} \qquad \sqrt[3]{x^2}\sqrt{x}$$

$$(x^2y)^{1/3}(xy^2)^{2/3} \qquad (xy)^{1/4}(x^2y^2)^{1/2} \qquad (x^3y^6)^{2/3} \qquad (x^4y^8)^{5/4}$$

$$\left(\frac{x^{2/5}y^{-1/5}}{x^{-1/3}}\right)^{15} \qquad \left(\frac{x^{1/2}}{y^2}\right)^4\left(\frac{y^{1/3}}{x^{-2/3}}\right)^3 \qquad (16x^2y^{-1/3})^{3/4} \qquad (4x^{-1}y^{1/3})^{3/2}$$

2) Write each expression as a sum and/or difference of logarithms. Express powers as factors.

$$\ln(x^2\sqrt{1-x}) \qquad \ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{1/3} \qquad \ln\frac{5x\sqrt{1-3x}}{(x-4)^5}$$

$$\ln(x\sqrt{1+x^2}) \qquad \ln\left[\frac{(x-4)^2}{x^2-1}\right]^{2/3} \qquad \ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right]$$

3) Write each expression as a single logarithm. When possible, simplify each expression by factoring polynomials.

$$3\log_5 u + 4\log_5 v \qquad \ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln(x^2-1) \qquad \log\left(\frac{x^2+2x-3}{x-4}\right) - \log\left(\frac{x^2+7x+6}{x+2}\right)$$

$$\log_3 u^2 - \log_3 v \qquad \log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right)$$

4) Calculate the derivatives $y' = \frac{\mathrm{d}y}{\mathrm{d}x}$ for the functions below:

$$y = (3x+7)^{10} \qquad y = \sqrt{x^2+1}$$

$$y = (5x^2+11x)^{20} \qquad y = e^{\sqrt{x}}$$

5) Calculate the derivatives $x' = \frac{\mathrm{d}x}{\mathrm{d}t}$ for the functions below:

$$x = \tan\left(5t^2\right) \qquad x = \sec\left(e^t\right)$$

$$x = \sin\left(\frac{t}{4}\right) \qquad x = \left(\frac{3t}{4t+2}\right)^2$$

6) Calculate the derivatives $f'(z) = \frac{\mathrm{d}f}{\mathrm{d}z}$ for the functions below:

$$f(z) = \left[(z+2)(3z^3+3z)\right]^4 \qquad f(z) = e^{-z^2}$$

7) Calculate the derivatives $f' = \frac{\mathrm{d}f}{\mathrm{d}u}$ for the functions below:

$$f = ue^u \qquad f = ue^{2u}$$

$$f = u^2e^u \qquad f = ue^{u^2}$$

8) Calculate the integrals below

$$\int te^t \, \mathrm{d}t \qquad \int se^{-2s} \, \mathrm{d}s \qquad \int e^{2\theta} \sin 6\theta \, \mathrm{d}\theta$$

$$\int 2xe^{3x} \, \mathrm{d}x \qquad \int te^{-st} \, \mathrm{d}t, \;\; s \in \mathbb{R} \qquad \int t^3 e^{-t} \, \mathrm{d}t$$

9) Calculate the integrals below

$$\int \sin 2x \, \mathrm{d}x \qquad \int \sin 3z \cos 7z \, \mathrm{d}z \qquad \int u^2 e^u \, \mathrm{d}u$$

$$\int \sin^2 t \, \mathrm{d}t \qquad \int \sin 3y \sin 2y \, \mathrm{d}y \qquad \int u^2 e^{-u} \, \mathrm{d}u$$

More difficult problems (4 points). *Please submit these on a separate sheet*.

10) Use integration by parts to show that for $m \neq -1$,

$$\int x^m \ln x \, \mathrm{d}x = \frac{x^{m+1}}{m+1} \left(\ln x - \frac{1}{m+1} \right) + C$$

and for $m = -1$,

$$\int \frac{\ln x}{x} \, \mathrm{d}x = \frac{1}{2} \ln^2 x + C.$$