Math 215

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

Exercises

1) Let (x_n) and (y_n) be given such that (x_n) converges and $\forall n > n_0 \ y_n = x_n$. Prove that (y_n) converges.

2) Let $c \in \mathbb{R}$, $X := (x_n)$, and $Y := (y_n)$ be given such that $\lim(x_n)$ and $\lim(y_n)$ exist. Prove that

   - $\lim(x_n - y_n) = \lim(x_n) - \lim(y_n)$, and
   - $\lim(cx_n) = c\lim(x_n)$.

3) Prove that if $0 < c < 1$, then $\lim(c^{1/n}) = 1$.

4) Let $b \in \mathbb{R}$: $0 < b < 1$. Show that $\lim(nb^n) = 0$.

5) Show that if $\lim(x_n) = x > 0$, then $\exists \ K \in \mathbb{N} : \ \forall n \geq K \ \frac{1}{2}x < x_n < 2x$.

6) If (b_n) is a bounded sequence and $\lim(a_n) = 0$, show that $\lim(a_n b_n) = 0$. Explain why you are not allowed to use any of the theorems we derived in class.

7) Give an example of two divergent sequences X and Y such that:

   - $X + Y$ converges.
   - XY converges.

8) Prove that if $(x_n) \searrow$ is bounded below then $\lim(x_n)$ exists.

9) Textbook problem TBA.