Due date: Wednesday, November 4^{th} Problem Set # 11

Math-215: Calculus III Analytic Geometry

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

1) Show that for any scalar function $f(x, y, z) = f$ we have $\nabla \times (\nabla f) = \vec{0}$; does this result makes sense? Explain.

Verify Stoke's Theorem

$$\oint_C \vec{F} \cdot \mathrm{d}\vec{r} = \int_S \left(\nabla \times \vec{F}\right) \cdot \mathrm{d}\vec{A}$$

for all of the vector fields below in the *particular case*[1] where C is the directed curve given by the perimeter of the unit square on the first quadrant with one vertex at the origin traversed in the counter-clockwise direction and S is the unit square bounded by C.[2]

Exercises 96/100 (12 points each)

2) $\vec{F} = \langle 7, \pi, e \rangle$
3) $\vec{F} = \langle x, 0, 0 \rangle$
4) $\vec{F} = \langle x, y, z \rangle$
5) $\vec{F} = \langle x^2, y, z+1 \rangle$
6) $\vec{F} = \langle x, y, x \rangle$
7) $\vec{F} = \langle y, x, xy \rangle$
8) $\vec{F} = \langle xy, x, xyz+1 \rangle$

More difficult problems 4/100 (4 points each). *Please submit these on a separate sheet*.

9) Let $\nabla^2 := \nabla \cdot \nabla = \partial_x^2 + \partial_y^2 + \partial_z^2$, show that

$$\nabla \times \left(\nabla \times \vec{F}\right) = \nabla \left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F}.$$

[1] Stoke's Theorem is true for arbitrary shapes of C and S.
[2] This is the unit square discussed in class.