Math-215: Calculus III Analytic Geometry

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

For all of the vector fields below calculate the following integrals and answer the questions below

- $\oint\limits_C \vec{F} \cdot \mathrm{d}\vec{r}$
- $\oint\limits_S \vec{F} \cdot \mathrm{d}\vec{A}$
- $\int\limits_Q \left(\nabla \cdot \vec{F}\right) \mathrm{d}V$
- Does $\vec{F}$ behaves as a sink/source at any point? Explain.
- Verify the divergence theorem for the last two integrals above.

where C is the directed unit square on the first quadrant with one vertex at the origin traversed in the counter-clockwise direction, S is the unit cube on the first octant with one vertex at the origin, and Q is the solid bounded by S.[1]

Exercises 96/100 (12 points each)

1) $\vec{F} = \langle 1, 2, 3 \rangle$
2) $\vec{F} = \langle 7, \pi, e \rangle$
3) $\vec{F} = \langle x, 0, 0 \rangle$
4) $\vec{F} = \langle x, y, z \rangle$
5) $\vec{F} = \langle x^2, y, z+1 \rangle$
6) $\vec{F} = \langle x, y, 0 \rangle$
7) $\vec{F} = \langle y, x, 0 \rangle$
8) $\vec{F} = \langle xy, x, 0 \rangle$

More difficult problems 4/100 (2 points each). *Please submit these on a separate sheet*.

9) Verify the Divergence Theorem for $\vec{F} = \langle x, y, z \rangle$ where S is the unit sphere centered at the origin.
10) Verify the Divergence Theorem for $\vec{F} = \alpha \frac{\hat{r}}{4\pi ||\vec{r}||^2}$ where $\vec{r} = \langle x, y, z \rangle$, S is the unit sphere centered at the origin, and $\nabla \cdot \left(\frac{\hat{r}}{4\pi ||\vec{r}||^2} \right) = \delta^3(x, y, z)$ is the delta function in three dimensions; $\frac{\alpha}{4\pi} = kq_1q_2$ for the Coulomb Force and $\frac{\alpha}{4\pi} = -Gm_1m_2$ for the gravitational force.

[1] These are the unit square and unit cube we have discussed in class.