Math-215 Calculus III Analytic Geometry

Please box your answers for each of the exercises below. Also, be mindful of your presentation, I will deduct 10 points for disorganized or unintelligible answers.

Exercises (12 points each)

The parabola

1) Find the equation, and graph the parabola for the points below:
   - $F = (0, -3)$ and $V = (0, 0)$.
   - $F = (-3, 4)$ and $D : y = 2$.
   - $V = (4, -2)$ and $F = (6, -2)$.
   - $V = (0, 0)$, axis of symmetry: $x = 0$, and containing the point $P = (2, 3)$.
2) Find the *vertex* (V), *focus* (F), and *directrix* (D) of each parabola for the polynomials below
   - $(x + 4)^2 = 16(y + 2)$
   - $y^2 + 2y - x = 0$
   - $x^2 - 4x = 2y$
   - $x^2 + 6x - 4y + 1 = 0$
3) Show that the graph of an equation of the form
$$Cy^2 + Dx + Ey + F = 0,\ C \neq 0$$
   - Is a parabola if $D \neq 0$.
   - Is a horizontal line if $D = 0$ and $E^2 - 4CF = 0$.
   - Is two horizontal lines if $D = 0$ and $E^2 - 4CF > 0$.
   - Contains no points if $D = 0$ and $E^2 - 4CF < 0$.

The Ellipse

4) For the polynomials below, discuss each equation, that is, find the *center*, *foci*, and *vertices* of each ellipse. The ellipse section of Purple Math (link below) explains how to calculate the *foci* and *vertices*.[1]
   - $(x + 5)^2 + 4(y - 4)^2 = 1$
   - $4x^2 + y^2 + 4y = 0$
   - $9x^2 + y^2 - 18x = 0$
   - $x^2 + 3y^2 - 12y + 9 = 0$
5) Graph each function below
   - $f(x) = \sqrt{16 - 4x^2}$
   - $f(x) = \sqrt{9 - 9x^2}$
   - $f(x) = -\sqrt{64 - 16x^2}$
   - $f(x) = -\sqrt{4 - 4x^2}$

The Hyperbola

[1] The link to the ellipse section of purple math is: https://www.purplemath.com/modules/ellipse.htm

6) Find the equations, and graph the hyperbola described below

- Center at $(0, 0)$; focus at $(3, 0)$; and vertex at $(1, 0)$.
- Center at $(0, 0)$; focus at $(0, -6)$; and vertex at $(0, 4)$.
- Foci at $(-5, 0)$ and $(5, 0)$; vertex at $(3, 0)$.

7) For the polynomials below, find the center, transverse axis, vertices, foci, and asymptotes.

- $(y-2)^2 - 4(x+2)^2 = 4$
- $(y-3)^2 - (x+2)^2 = 4$
- $y^2 - 4x^2 - 16x - 2y - 19 = 0$
- $4x^2 - y^2 - 24x - 4y + 16 = 0$

Conic Sections

8) Determine which conic section is associated with each of the polynomials below.

- $x^2 - 4x = y + 4$
- $9x^2 + y^2 - 18x = 0$
- $y = \pm\sqrt{4 - 4x^2}$
- $x^2 + 6x - 46 + 1 = 0$
- $y^2 - x^2 - 4y + 4x - 1 = 0$
- $y = \pm\sqrt{4 - x^2}$
- $y = \pm\sqrt{16 + 4x^2}$

More difficult problems (2 points each)
If the x- and y-axes are rotated through an angle θ, the coordinates (x, y) of a point P relative to the xy-plane and the coordinates (x', y') of the same point relative to the new x'- and y'-axes are related by the formulas

$$x = x' \cos\theta - y' \sin\theta$$
$$y = x' \sin\theta + y' \cos\theta.$$

Applying the rotation formulas above to the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

one can obtain the polynomial in the rotated $x'y'$-axes

$$A'x'^2 + B'x'y' + C'y'^2 + D'x' + E'y' + F' = 0.$$

9) Show that $A + C = A' + C'$, and thus show that $A + C$ is *invariant*; that is, its value does not change under a rotation of axes.

10) Show that $B^2 - 4AC$ is invariant.