Math215

Homework 5, Problem 2

November 29, 2021

Taylor series centered at a:

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

8.7 Problem 14

$f(x) = \frac{1}{x}$	$f^{'}(x) = \frac{-1}{x^2}$	$f^{''}(x) = \frac{2}{x^3}$	$f^{'''}(x) = \frac{-6}{x^4}$	$\cdots$
$f(-3) = \frac{-1}{3}$	$f^{'}(-3) = \frac{-1}{9}$	$f^{''}(-3) = \frac{-2}{27}$	$f^{'''}(-3) = \frac{-2}{27}$	$\cdots$

$$f(-3) = \frac{-1}{3} + \frac{-1}{9}\frac{1}{1!}(x+3) + \frac{-2}{27}\frac{1}{2!}(x+3)^2 + \frac{-2}{27}\frac{1}{3!}(x+3)^3 + ...$$

$$= \frac{-1}{3}\left(1 + \frac{1}{3}(x+3) + \frac{1}{9}(x+3)^2 + \frac{1}{27}(x+3)^3 + ...\right)$$

$$= \frac{-1}{3}\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^n}$$

8.7 Problem 15

$f(x) = e^{2x}$	$f^{'}(x) = 2e^{2x}$	$f^{''}(x) = 4e^{2x}$	$f^{'''}(x) = 8e^{2x}$	$\cdots$
$f(3) = e^{6}$	$f^{'}(3) = 2e^{6}$	$f^{''}(3) = 4e^{6}$	$f^{'''}(3) = 8e^{6}$	$\cdots$

$$f(3) = e^6 + 2e^6(x-a) + \frac{4e^6}{2!}(x-a)^2 + \frac{8e^6}{3!}(x-a)^3 + ...$$

$$= e^6 \sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n$$

8.7 Problem 16

$f(x) = sin(x)$	$f^{'}(x) = cos(x)$	$f^{''}(x) = -sin(x)$	$f^{'''}(x) = -cos(x)$	$f^{(4)}(x) = sin(x)$	$\cdots$
$f(\frac{\pi}{2}) = sin(\frac{\pi}{2}) = 1$	$f^{'}(\frac{\pi}{2}) = cos(\frac{\pi}{2}) = 0$	$f^{''}(\frac{\pi}{2}) = -sin(\frac{\pi}{2}) = -1$	$f^{'''}(\frac{\pi}{2}) = -cos(\frac{\pi}{2}) = 0$	$f^{(4)}(\frac{\pi}{2}) = sin(\frac{\pi}{2}) = 1$	$\cdots$

$$f(\frac{\pi}{2}) = sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})(x-a) + \frac{-sin(\frac{\pi}{2})}{2!}(x-a)^2 + \frac{-cos(\frac{\pi}{2})}{3!}(x-a)^3 + \frac{sin(\frac{\pi}{2})}{4!}(x-\frac{\pi}{2})^4 + ...$$

$$= 1 + 0 - \frac{(x-\frac{\pi}{2})^2}{2!} + 0 + \frac{(x-\frac{\pi}{2})^4}{4!} + ...$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(x - \frac{\pi}{2}\right)^{2n}$$