Romeo Capozziello HW 2

Problem 1:

A binary search of a sorted array with 3 unique elements where the search is always successful can be viewed as an array with each element having a probability of being selected = 1/3 and the number of comparisonsa binary search requires to find the element is shown inside the array.

2	1	2

Then the averagenumber of comparisons would be A(3) = (1/3)[2 + 1 + 2] = 5/3
Find the A(7) for an array with 7 elements for the same binary search.

(a) Draw the 7-element array showing the number of comparisons needed to find each element.

3	2	3	1	3	2	3

(b) Determine A(7)

A(7) = (1/7)[3 + 2 + 3 + 1 + 3 + 2 + 3] = (1/7)(17) = $\frac{17}{7}$ = $2\frac{3}{7}$

(c) Using what you learned from parts (a) and (b), find the summation expression for A(n). To simplify your work, assume that n = 2^k–1. Use k in your summation. Leave your answer in closed form in terms of n.

A(n) = $\frac{\sum_{i=1}^{k} i2^{i-1}}{2^i-1} = \frac{(k-1)2^i+1}{2^i-1} = \frac{(lg(n+1)-1)(n+1)+1}{n}$

(d) Simplify your a(n) for large values of n.

$\frac{(lg(n+1)-1)(n+1)+1}{n} \approx \frac{(lg(n))(n)}{n} \approx$ lg(n) for large values of n

---

Problem 2:

Consider the algorithm below
Precondition: n is a non-negative integer

```
function f(n)
{
    temp = 0
    if (n != 0)
    {
        for (i = 1; i ¡= 3; i++)
            temp = temp + n * f(n-1)
        return temp
    }
    else
        return 1;
}
```

Solve for the closed form by repeated substitution

T(0) = 1
T(n) = 3nT(n-1)
T(n-1) = 3(n-1)T(n-2)
T(n-2) = 3(n-2)T(n-3)
T(n) = 3nT(n-1)
= 3n[3(n-1)T(n-2)]

$= 3n[3(n-1)[3(n-2)T(n-3)]]$
$= 3^3 n(n-1)(n-2)T(n-3)$

$T(n) = 3^n (n-1)(n-2) \ldots (n-(k-2))(n-(k-1))T(n-k)$

Problem 3:

The recurrence relation is given as
$a_n = 2a_{n-1} + 3a_{n-2}$
Use the method linear homogeneous characteristic roots to solve for the closed form, with given initial conditions
$a_0 = 2$ and $a_1 = 4$

(a) Find the general solution

$a_n = r_n$
$r^n = 2r^{n-1} + 3r^{n-2}$
$r^2 = 2r + 3$
$r^2 - 2r - 3 = 0$
(r - 3)(r + 1) = 0
r = 3, -1
$a_n = \alpha(3^n) + \beta(-1^n)$

(b) Find the Specific solution

$a_0 = 2 = \alpha(3^0) + \beta(-1^0)$
$= \alpha + \beta$
$a_1 = 4 = \alpha(3^1) + \beta(-1^1)$
$= \alpha(3) + \beta(-1)$

$2 = \alpha + \beta$
$4 = 3\alpha - \beta$
$6 = 4\alpha,\ \frac{3}{2} = \alpha$

$2 = (\frac{3}{2}) + \beta$
$2 - \frac{3}{2} = \beta,\ \frac{1}{2} = \beta$

$a_n = \frac{3}{2}(3)^n + \frac{1}{2}(-1)^n$

(c) Use your closed form result to find a_5

$a_5 = \frac{3}{2}(3)^5 + \frac{1}{2}(-1)^5$
$a_5 = \frac{3}{2}(243) - \frac{1}{2}$
$a_5 = 364.5 - 0.5 = 364$

Problem 4:

Two average complexities are given below.
Variable p is probability value in the interval of [0.0, 1.0]. Variable n is the problem size.

$A_1 = p(n^2 + 1) + (1 - p)3n$
$A_2 = (1 - p)(3n^2 + n) + p(2n)$

(a) Determine the condition for which A_1 is faster than A_2.

$A_1 < A_2$
$p(n^2+1)+(1-p)3n < (1-p)(3n^2+n)+p(2n)$
$p(n^2)+p+3n-3pn < 3n^2+n-3p(n^2)-pn+2pn$
$p(n^2)+p-3pn+3p(n^2)+pn-2pn < 3n^2+n-3n$
$4p(n^2)-4pn+p < 3n^2-2n$
$p(4(n^2)-4n+1) < 3n^2-2n$
$p < \frac{3n^2-2n}{4n^2-4n+1}$

(b) Approximate the range of p values for which A_1 is faster than A_2 for large values of n.

$\lim_{p\to\infty}[\frac{3n^2-2n}{4n^2-4n+1}] = \frac{3}{4}$

(c) Given n=5,find the range of values for p where A_2 is faster than A_1.

$A_1 > A_2$
$p > \frac{3n^2-2n}{4n^2-4n+1}$

n = 5

$p > \frac{3(5^2)-2(5)}{4(5^2)-4(5)+1}$
$p > \frac{75-10}{100-20+1}$
$p > \frac{65}{81}$

(d) Can you determine the problem sizen where A_1 is always faster? Why or why not?

No because A_1 and A_2 both depend on p which varies independantly from the problem size n
So given a fixed problem size n, A_1 can still be fast or slower than A_2 depending on the value of p