

Problem 1:

Find the upper bound of sum  $\sum_{i=1}^n \frac{1}{i}$  (Recall that  $\ln 0$  is undefined)

$$\int_0^n \frac{1}{x} dx$$

$$\ln(x)|_0^n = \ln(n) - \ln(0), \text{ However } \ln(0) \text{ is undefined}$$

$$\text{Upper bound area from } 0 \rightarrow 1 = 1, \text{ so } \int_0^1 \frac{1}{x} dx = 1$$

$$\int_0^n \frac{1}{x} dx = \int_0^1 \frac{1}{x} dx + \int_1^n \frac{1}{x} dx$$

$$= 1 + \int_1^n \frac{1}{x} dx$$

$$= 1 + \ln(x)|_1^n$$

$$= 1 + \ln(n) - \ln(1)$$

$$= 1 + \ln(n) - 0$$

$$\text{Upper bound} = 1 + \ln(n)$$

Problem 2:

(a) Find the lower bound of sum  $\sum_{i=1}^n i^5$  using an integral

$$\int_0^n x^5 dx = \frac{1}{6}x^6|_0^n = \frac{1}{6}n^6 - 0 = \frac{1}{6}n^6$$

(b) manually compute  $\sum_{i=1}^n i^5$  for  $n = 8$ , and compare to the lower bound formula in part (a)

$$\begin{aligned} \sum_{i=1}^8 i^5 &= 1^5 + 2^5 + 3^5 + 4^5 + 5^5 + 6^5 + 7^5 + 8^5 \\ &= 1 + 32 + 243 + 1024 + 3125 + 7776 + 16807 + 32768 = 61776 \\ (n^5 &= d_0d_1d_2\dots d_k n \text{ ? Ex: } \underline{3}^5 = 24\underline{3}, \underline{7}^5 = 1680\underline{7}, \text{ etc.}) \end{aligned}$$

$$\text{Lower bound} = \frac{1}{6}8^6 = \frac{1}{6}262144 \approx 43691 < 61776$$

Problem 3:

Find the limit  $\lim_{n \rightarrow \infty} \frac{n2^n}{e^n}$  (recall  $(2^n)' = 2^n * \ln 2$ )

$$\lim_{n \rightarrow \infty} \frac{n2^n}{e^n} = \lim_{n \rightarrow \infty} \frac{2^n + (2^n \ln 2)n}{e^n} = \lim_{n \rightarrow \infty} \frac{2^n \ln 2 + (2^n + (2^n \ln 2)n) \ln 2}{e^n}$$

L'Hopital's Rule does not simplify to  $\frac{1}{n}$  or  $\frac{1}{n}$ , repeats infinitely

$$\frac{n2^n}{e^n} = n\left(\frac{2}{e}\right)^n$$

Unsure where to go from here :(

Problem 4:

(a) Discuss why for  $f(n) = n * \lg(n^k)$  and  $g(n) = n * \lg(n^m)$  are  $\Theta$  of each other

$$n\lg(n^k) = kn\lg(n), n\lg(n^m) = mn\lg(n)$$

Since k and m are constants, then  $kn\lg(n)$  and  $mn\lg(n)$  are proportional to each other

Thus they are  $\Theta$  of each other

(b) Solve  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n\lg(n^k)}{n\lg(n^m)} = \lim_{n \rightarrow \infty} \frac{kn\lg(n)}{mn\lg(n)} = \lim_{n \rightarrow \infty} \frac{k}{m} = \frac{k}{m}$$