Romeo Capozziello HW 1

Problem 1:
Using direct proof, prove: If n is any even interger, then $(-1)^n = 1$

If n is even then n = 2k
Then $(-1^n) = (-1^{2k}) = (-1^k) * (-1^k)$
If $(-1^k) = 1$, then $(-1^k) * (-1^k) = 1 * 1 = 1$
If $(-1^k) = -1$, then $(-1^k) * (-1^k) = -1 * -1 = 1$
Then $(-1^k) * (-1^k) = 1 \rightarrow (-1^{2k}) = 1 \rightarrow (-1^n) = 1$

Problem 2:
Using induction proof, prove for integer $n \geq 5$, $4n < 2^n$

Base Case: n = 5, $4(5) < 2^5 \rightarrow 20 < 32$
Assume true for n = k, $4k < 2^k$
$4(k+1) = 4k + 4)$
$2^{k+1} = 2^k * 2 = 2^k + 2^k$

$4k + 4 < 2^k + 2k$
$4k < 2^k + 2^k - 4$
$4k < 2^k$ by assumption
Then $4k < 2^k < 2^k + 2^k - 4$
Then $4k < 2^k + 2^k - 4 \rightarrow 4k < 2^{k+1} - 4 \rightarrow 4k + 4 < 2^{k+1}$
Then $4(k+1) < 2^{k+1}$

Problem 3:
Prove by induction that $(11^n - 6)$ is divisible by 5 for every possible integer n.

Base Case: n = 0, 5 | $(11^0 - 6) \rightarrow$ 5 | (1-6) $\rightarrow$ 5 | -5
Assume true for n = k, 5 | $(11^k - 6)$

$5 \mid (11^{k+1} - 6)$
$11^{k+1} - 6 = 11 * 11^k - 6$
$5 \mid (11^k - 6)$ by assumption, thus $11^k = 5x + 6$
$11^{k+1} - 6 = 11 * 11^k - 6 = 11 * (5x + 6) - 6 = 55x + 66 - 6 = 55x + 60$
$55x + 60 = 5 * (11x + 12)$, $5 \mid 5(11x + 12)$
$5 \mid (11^{k+1} - 6)$

Problem 4:
Prove the following statement by Contradiction
The sum of a rational number and an irrational number is irrational.

Assume a the sum of a rational and irrational number was rational
Let x be an irrational number

Then $\frac{a}{b} + x = \frac{c}{d}$
Then x = $\frac{c}{d} - \frac{a}{b}$
Then x = $\frac{bc-da}{bd}$

Since $\frac{a}{b}$ and $\frac{c}{d}$ are rational, then their difference, $\frac{bc-da}{bd}$ is also rational
And since x = $\frac{bc-da}{bd}$, then x is rational
But x is irrational by definition