Math323

Homework 5

November 23, 2021

Problem 1

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{1}$$

where $ad - bc \neq 0$ be given. Show that for

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{2}$$

then

$$A^{-1}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

Proof:

$$A^{-1}A = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{4}$$

$$A^{-1}A = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{5}$$

$$A^{-1}A = \begin{pmatrix} \frac{ad}{ad-bc} + \frac{-bc}{ad-bc} & \frac{bd}{ad-bc} + \frac{-bd}{ad-bc} \\ \frac{-ac}{ad-bc} + \frac{ac}{ad-bc} & \frac{-bc}{ad-bc} + \frac{ad}{ad-bc} \end{pmatrix} \tag{6}$$

$$A^{-1}A = \begin{pmatrix} \frac{ad-bc}{ad-bc} & \frac{bd-bd}{ad-bc} \\ \frac{-ac+ac}{ad-bc} & \frac{-bc+ad}{ad-bc} \end{pmatrix} \tag{7}$$

$$A^{-1}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{8}$$

$$q.e.d \tag{9}$$