Math 215 Homework 5

Problem 2 a

Explain why the series
1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + ...
is *not* the Taylor series of f centered at 1

When x = 0,

$$\begin{gathered} 1.6 - 0.8(0-1) + 0.4(0-1)^2 - 0.1(0-1)^3 + ... \\ 1.6 + 0.8 + 0.4 + 0.1^3 + ... \\ \approx 2.9 \end{gathered}$$

However, according to the graph, $f(0) \approx 0.5$

Problem 2 b

Explain why the series
2.8 + 0.5(x-2) + 1.5(x-2)^2 - 0.1(x-2)^3 + ...
is *not* the Taylor series of f centered at 2

When x = 0,

$$\begin{gathered} 2.8 + 0.5(0-2) + 1.5(0-2)^2 - 0.1(0-2)^3 + ... \\ 2.8 - 1 + 6 + 0.8 + ... \\ \approx 8.6 \end{gathered}$$

However, according to the graph, $f(0) \approx 0.5$

Problem 11

Find the Taylor series for f(x) centered at the given value of a.

f(x) = x^4 - 3x^2 + 1, a = 1

f(x) = x^4 - 3x^2 + 1
f(a) = (1)^4 - 3(1)^2 + 1 = -1
f'(x) = 4x^3 - 6x
f'(a) = 4(1)^3 - 6(1) = -2
f''(x) = 12x^2 - 6
f''(a) = 12(1)^2 - 6 = 6
f'''(x) = 24x
f'''(a) = 24(1) = 24

f(x) = f(a) + f'(a)(x-a) + $\frac{f''(a)(x-a)^2}{2!}$ + $\frac{f'''(a)(x-a)^3}{3!}$ + ...

f(x) = -1 + -2(x-1) + $\frac{6(x-1)^2}{2}$ + $\frac{24(x-1)^3}{6}$ + ...

f(x) = -1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + ...

Problem 12

Find the Taylor series for f(x) centered at the given value of a.

f(x) = x - x^3, a = -2

f(x) = x - x^3
f(a) = (-2) - (-2)^3 = 6
f'(x) = 1 - 3x^2
f'(a) = 1 - 3(-2)^2 = -11
f''(x) = -6x
f''(a) = -6(-2) = 12
f'''(x) = -6
f'''(a) = -6

f(x) = f(a) + f'(a)(x-a) + $\frac{f''(a)(x-a)^2}{2!}$ + $\frac{f'''(a)(x-a)^3}{3!}$ + ...

f(x) = 6 + -11(x+2) + $\frac{12(x+2)^2}{2}$ + $\frac{-6(x+2)^3}{6}$ + ...

f(x) = 6 - 11(x+2) + 6(x+2)^2 - (x+2)^3 + ...

Problem 13

Find the Taylor series for f(x) centered at the given value of a.

f(x) = ln(x), a = 2

$f(\mathrm{x}) = \ln(\mathrm{x})$

$f(\mathrm{a}) = \ln(2)$

$f'(\mathrm{x}) = \frac{1}{x}$

$f'(\mathrm{a}) = \frac{1}{2}$

$f''(\mathrm{x}) = -\frac{1}{x^2}$

$f''(\mathrm{a}) = -\frac{1}{2^2} = -\frac{1}{4}$

$f'''(\mathrm{x}) = \frac{2}{x^3}$

$f'''(\mathrm{a}) = \frac{2}{2^3} = \frac{1}{4}$

$f(\mathrm{x}) = f(\mathrm{a}) + f'(\mathrm{a})(\mathrm{x}-\mathrm{a}) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \ldots$

$f(\mathrm{x}) = \ln(2) + \frac{1}{2}(\mathrm{x}-2) + \frac{-\frac{1}{4}(x-2)^2}{2} + \frac{\frac{1}{4}(x-2)^3}{6} + \ldots$

$f(\mathrm{x}) = \ln(2) + \frac{(x-2)}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24} + \ldots$