Math 215 Homework 5

Problem 2 a

Explain why the series
1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + ...
is *not* the Taylor series of f centered at 1

When x = 0,

$$\begin{gathered}1.6 - 0.8(0\text{-}1) + 0.4(0\text{-}1)^2 - 0.1(0\text{-}1)^3 + ... \\ 1.6 + 0.8 + 0.4 + 0.1^3 + ... \\ \approx 2.9\end{gathered}$$

However, according to the graph, $f(0) \approx 0.5$

Problem 2 b

Explain why the series
2.8 + 0.5(x-2) + 1.5(x-2)^2 - 0.1(x-2)^3 + ...
is *not* the Taylor series of f centered at 2

When x = 0,

$$\begin{gathered}2.8 + 0.5(0\text{-}2) + 1.5(0\text{-}2)^2 - 0.1(0\text{-}2)^3 + ... \\ 2.8 - 1 + 6 + 0.8 + ... \\ \approx 8.6\end{gathered}$$

However, according to the graph, $f(0) \approx 0.5$

Problem 11

Find the Taylor series for f(x) centered at the given value of a.

f(x) = x^4 - 3x^2 + 1, a = 1

f(x) = x^4 - 3x^2 + 1
f(a) = (1)^4 - 3(1)^2 + 1 = -1
f'(x) = 4x^3 - 6x
f'(a) = 4(1)^3 - 6(1) = -2
f''(x) = 12x^2 - 6
f''(a) = 12(1)^2 - 6 = 6
f'''(x) = 24x
f'''(a) = 24(1) = 24

f(x) = f(a) + f'(a)(x-a) + $\frac{f''(a)(x-a)^2}{2!}$ + $\frac{f'''(a)(x-a)^3}{3!}$ + ...

f(x) = -1 + -2(x-1) + $\frac{6(x-1)^2}{2}$ + $\frac{24(x-1)^3}{6}$ + ...

f(x) = -1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + ...

Problem 12

Find the Taylor series for f(x) centered at the given value of a.

f(x) = x - x^3, a = -2

f(x) = x - x^3
f(a) = (-2) - (-2)^3 = 6
f'(x) = 1 - 3x^2
f'(a) = 1 - 3(-2)^2 = -11
f''(x) = -6x
f''(a) = -6(-2) = 12
f'''(x) = -6
f'''(a) = -6

f(x) = f(a) + f'(a)(x-a) + $\frac{f''(a)(x-a)^2}{2!}$ + $\frac{f'''(a)(x-a)^3}{3!}$ + ...

f(x) = 6 + -11(x+2) + $\frac{12(x+2)^2}{2}$ + $\frac{-6(x+2)^3}{6}$ + ...

f(x) = 6 - 11(x+2) + 6(x+2)^2 - (x+2)^3 + ...

Problem 13

Find the Taylor series for f(x) centered at the given value of a.

f(x) = ln(x), a = 2

f(x) = ln(x)

f(a) = ln(2)

$f'(x) = \frac{1}{x}$

$f'(a) = \frac{1}{2}$

$f''(x) = -\frac{1}{x^2}$

$f''(a) = -\frac{1}{2^2} = -\frac{1}{4}$

$f'''(x) = \frac{2}{x^3}$

$f'''(a) = \frac{2}{2^3} = \frac{1}{4}$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \ldots$

$f(x) = \ln(2) + \frac{1}{2}(x-2) + \frac{-\frac{1}{4}(x-2)^2}{2} + \frac{\frac{1}{4}(x-2)^3}{6} + \ldots$

$f(x) = \ln(2) + \frac{(x-2)}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24} + \ldots$

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Homework 5, Problem 2

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Taylor series centered at a:

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

8.7 Problem 14

$f(x) = \frac{1}{x}$	$f'(x) = \frac{-1}{x^2}$	$f''(x) = \frac{2}{x^3}$	$f'''(x) = \frac{-6}{x^4}$	$\cdots$
$f(-3) = \frac{-1}{3}$	$f'(-3) = \frac{-1}{9}$	$f''(-3) = \frac{-2}{27}$	$f'''(-3) = \frac{-2}{27}$	$\cdots$

$$f(-3) = \frac{-1}{3} + \frac{-1}{9}\frac{1}{1!}(x+3) + \frac{-2}{27}\frac{1}{2!}(x+3)^2 + \frac{-2}{27}\frac{1}{3!}(x+3)^3 + ...$$

$$= \frac{-1}{3}\left(1 + \frac{1}{3}(x+3) + \frac{1}{9}(x+3)^2 + \frac{1}{27}(x+3)^3 + ...\right)$$

$$= \frac{-1}{3}\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^n}$$

8.7 Problem 15

$f(x) = e^{2x}$	$f^{'}(x) = 2e^{2x}$	$f^{''}(x) = 4e^{2x}$	$f^{'''}(x) = 8e^{2x}$	$\cdots$
$f(3) = e^6$	$f^{'}(3) = 2e^6$	$f^{''}(3) = 4e^6$	$f^{'''}(3) = 8e^6$	$\cdots$

$$f(3) = e^6 + 2e^6(x-a) + \frac{4e^6}{2!}(x-a)^2 + \frac{8e^6}{3!}(x-a)^3 + ...$$

$$= e^6 \sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n$$

8.7 Problem 16

$f(x) = sin(x)$	$f^{'}(x) = cos(x)$	$f^{''}(x) = -sin(x)$	$f^{'''}(x) = -cos(x)$	$f^{(4)}(x) = sin(x)$	$\cdots$
$f(\frac{\pi}{2}) = sin(\frac{\pi}{2}) = 1$	$f^{'}(\frac{\pi}{2}) = cos(\frac{\pi}{2}) = 0$	$f^{''}(\frac{\pi}{2}) = -sin(\frac{\pi}{2}) = -1$	$f^{'''}(\frac{\pi}{2}) = -cos(\frac{\pi}{2}) = 0$	$f^{(4)}(\frac{\pi}{2}) = sin(\frac{\pi}{2}) = 1$	$\cdots$

$$f(\frac{\pi}{2}) = sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})(x-a) + \frac{-sin(\frac{\pi}{2})}{2!}(x-a)^2 + \frac{-cos(\frac{\pi}{2})}{3!}(x-a)^3 + \frac{sin(\frac{\pi}{2})}{4!}(x-\frac{\pi}{2})^4 + ...$$

$$= 1 + 0 - \frac{(x-\frac{\pi}{2})^2}{2!} + 0 + \frac{(x-\frac{\pi}{2})^4}{4!} + ...$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(x - \frac{\pi}{2}\right)^{2n}$$

1 Math 215 Homwork 5 Question 3

Evaluate as an infinite series

$$\int \frac{e^x}{x} dx \tag{1}$$

using a Taylor Expansion on e^x:

$$\int \frac{1}{x}(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} ....)dx \tag{2}$$

$$= \int (\frac{1}{x} + 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} ....)dx \tag{3}$$

$$= \int \frac{1}{x} dx + \int dx + \int \frac{x}{2!} dx + \int \frac{x^2}{3!} dx + \int \frac{x^3}{4!} dx.... \tag{4}$$

$$= C + ln|x| + \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} + \frac{x^4}{4 \cdot 4!} .... \tag{5}$$

$$= C + ln|x| + \sum \frac{x^n}{n!} \tag{6}$$

Q.E.D