

Math 215 Homework 4

Problem 25

Find an equation of the plane that passes through the points $(0,1,1)$, $(1,0,1)$, and $(1,1,0)$

Finding vector normal to the plane

$$\begin{aligned}\text{Let } p &= (0,1,1), q = (1,0,1), r = (1,1,0) \\ \bar{A} = \vec{pq} &= \langle 1-0, 0-1, 1-1 \rangle = \langle 1, -1, 0 \rangle \\ \bar{B} = \vec{pr} &= \langle 1-0, 1-1, 0-1 \rangle = \langle 1, 0, -1 \rangle\end{aligned}$$

$$\text{Normal Vector} = \bar{A} \times \bar{B}$$

$$\begin{aligned}\bar{Y} = \bar{A} \times \bar{B} &= \langle (-1)(-1) - (0)(0), (0)(0) - (1)(-1), (1)(0) - (1)(-1) \rangle \\ \bar{Y} &= \langle 1, 1, 1 \rangle\end{aligned}$$

Finding equation of the plane

$$\begin{aligned}\bar{Y} = \langle a, b, c \rangle &= \langle 1, 1, 1 \rangle \\ p &= (x_0, y_0, z_0) = (0, 1, 1) \\ a(x-x_0) + b(y-y_0) + c(z-z_0) &= 0 \\ 1(x-0) + 1(y-1) + 1(z-1) & \\ x + (y-1) + (z-1) & \\ x + y + z - 2 & \\ x + y + z = 2 &\end{aligned}$$

Problem 26

Find an equation of the plane that passes through the origin and the points (2,-4,6) and (5,1,3)

Finding vector normal to the plane

$$\begin{aligned}\text{Let } p &= (0,0,0), q = (2,-4,6), r = (5,1,3) \\ \bar{A} &= \bar{p}q = \langle 2-0, -4-0, 6-0 \rangle = \langle 2, -4, 6 \rangle \\ \bar{B} &= \bar{p}r = \langle 5-0, 1-0, 3-0 \rangle = \langle 5, 1, 3 \rangle\end{aligned}$$

$$\text{Normal Vector} = \bar{A} \times \bar{B}$$

$$\begin{aligned}\bar{Y} &= \bar{A} \times \bar{B} = \langle (-4)(3) - (6)(1), (6)(5) - (2)(3), (2)(1) - (-4)(5) \rangle \\ \bar{Y} &= \langle -18, 24, 22 \rangle\end{aligned}$$

Finding equation of the plane

$$\begin{aligned}\bar{Y} &= \langle a, b, c \rangle = \langle -18, 24, 22 \rangle \\ q &= (x_0, y_0, z_0) = (2, -4, 6) \\ a(x-x_0) + b(y-y_0) + c(z-z_0) &= 0 \\ 2(x-(-18)) + -4(y-24) + 6(z-22) & \\ 2(x+18) - 4(y-24) + 6(z-22) & \\ 2x - 4y + 6z - 36 + 116 - 66 &= 0 \\ 2x + 4y + 6z &= 14\end{aligned}$$