

Math215

Homework 3

November 24, 2021

Problem 8

Show that $\vec{A} \cdot (\vec{A} \times \vec{B}) = \vec{0}$ and $\vec{B} \cdot (\vec{A} \times \vec{B}) = \vec{0}$ for all $\vec{A} = \langle a_1, a_2, a_3 \rangle$ and $\vec{B} = \langle b_1, b_2, b_3 \rangle$

Proof:

$$\vec{A} \cdot (\vec{A} \times \vec{B}) = \quad (1)$$

$$\langle a_1, a_2, a_3 \rangle \cdot (\langle a_1, a_2, a_3 \rangle \times \langle b_1, b_2, b_3 \rangle) = \quad (2)$$

$$\langle a_1, a_2, a_3 \rangle \cdot (\langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle) = \quad (3)$$

$$a_1(a_2b_3 - a_3b_2) + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_2 - a_2b_1) = \quad (4)$$

$$a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_2a_1b_3 + a_3a_1b_2 - a_3a_2b_1 = \quad (5)$$

$$a_1a_2b_3 - a_1a_2b_3 + a_2a_3b_1 - a_2a_3b_1 + a_1a_3b_2 - a_1a_3b_2 = \quad (6)$$

$$0 \quad (7)$$

$$\vec{B} \cdot (\vec{A} \times \vec{B}) = \quad (8)$$

$$\langle b_1, b_2, b_3 \rangle \cdot (\langle a_1, a_2, a_3 \rangle \times \langle b_1, b_2, b_3 \rangle) = \quad (9)$$

$$\langle b_1, b_2, b_3 \rangle \cdot (\langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle) = \quad (10)$$

$$b_1(a_2b_3 - a_3b_2) + b_2(a_3b_1 - a_1b_3) + b_3(a_1b_2 - a_2b_1) = \quad (11)$$

$$b_1a_2b_3 - b_1a_3b_2 + b_2a_3b_1 - b_2a_1b_3 + b_3a_1b_2 - b_3a_2b_1 = \quad (12)$$

$$b_1a_2b_3 - b_1a_2b_3 + b_2a_3b_1 - b_2a_3b_1 + b_3a_1b_2 - b_3a_1b_2 = \quad (13)$$

$$0 \quad (14)$$