

Math 215 Homework 4

Problem 25

Find an equation of the plane that passes through the points $(0,1,1)$, $(1,0,1)$, and $(1,1,0)$

Finding vector normal to the plane

$$\begin{aligned}\text{Let } p &= (0,1,1), q = (1,0,1), r = (1,1,0) \\ \bar{A} = \vec{pq} &= \langle 1-0, 0-1, 1-1 \rangle = \langle 1, -1, 0 \rangle \\ \bar{B} = \vec{pr} &= \langle 1-0, 1-1, 0-1 \rangle = \langle 1, 0, -1 \rangle\end{aligned}$$

$$\text{Normal Vector} = \bar{A} \times \bar{B}$$

$$\begin{aligned}\bar{Y} = \bar{A} \times \bar{B} &= \langle (-1)(-1) - (0)(0), (0)(0) - (1)(-1), (1)(0) - (1)(-1) \rangle \\ \bar{Y} &= \langle 1, 1, 1 \rangle\end{aligned}$$

Finding equation of the plane

$$\begin{aligned}\bar{Y} = \langle a, b, c \rangle &= \langle 1, 1, 1 \rangle \\ p &= (x_0, y_0, z_0) = (0, 1, 1) \\ a(x-x_0) + b(y-y_0) + c(z-z_0) &= 0 \\ 1(x-0) + 1(y-1) + 1(z-1) & \\ x + (y-1) + (z-1) & \\ x + y + z - 2 & \\ x + y + z &= 2\end{aligned}$$

Problem 26

Find an equation of the plane that passes through the origin and the points (2,-4,6) and (5,1,3)

Finding vector normal to the plane

$$\begin{aligned}\text{Let } p &= (0,0,0), q = (2,-4,6), r = (5,1,3) \\ \vec{A} &= \vec{pq} = \langle 2-0, -4-0, 6-0 \rangle = \langle 2, -4, 6 \rangle \\ \vec{B} &= \vec{pr} = \langle 5-0, 1-0, 3-0 \rangle = \langle 5, 1, 3 \rangle\end{aligned}$$

$$\text{Normal Vector} = \vec{A} \times \vec{B}$$

$$\begin{aligned}\vec{Y} &= \vec{A} \times \vec{B} = \langle (-4)(3) - (6)(1), (6)(5) - (2)(3), (2)(1) - (-4)(5) \rangle \\ \vec{Y} &= \langle -18, 24, 22 \rangle\end{aligned}$$

Finding equation of the plane

$$\begin{aligned}\vec{Y} &= \langle a, b, c \rangle = \langle -18, 24, 22 \rangle \\ q &= (x_0, y_0, z_0) = (2, -4, 6) \\ a(x-x_0) + b(y-y_0) + c(z-z_0) &= 0 \\ 2(x-(-18)) + -4(y-24) + 6(z-22) & \\ 2(x+18) - 4(y-24) + 6(z-22) & \\ 2x - 4y + 6z - 36 + 116 - 66 &= 0 \\ 2x + 4y + 6z &= 14\end{aligned}$$

Math215

Homework 4, Problem 3

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10.5 Pr31

Find an equation of the plane that passes through the point (1,5,1) and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$

Finding vector normal to the plane

$$\langle a_1, a_2, a_3 \rangle \times \langle b_1, b_2, b_3 \rangle = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle \quad (1)$$

$$\langle 2, 1, -2 \rangle \times \langle 1, 0, 3 \rangle = \langle (1)(3) - (-2)(0), (-2)(1) - (2)(3), (2)(0) - (1)(1) \rangle = \langle 3, -8, -1 \rangle$$

Finding normal vector through the point

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \quad (2)$$

$$3(x - 1) - 8(y - 5) - (z - 1) = 0$$

$$3x - 3 - 8y + 40 - z + 1 = 0$$

$$3x - 8y - z = -38$$

1 Math 215 Homework 4 Question 7

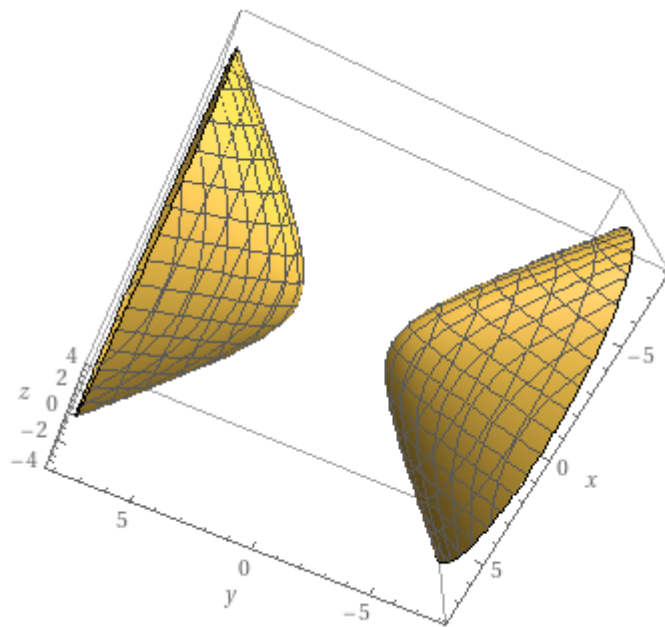
Reduce to Standard Form and State the shape

24: $y^2 = x^2 + 4z^2 + 4$

$$y^2 - x^2 - 4z^2 = 4$$

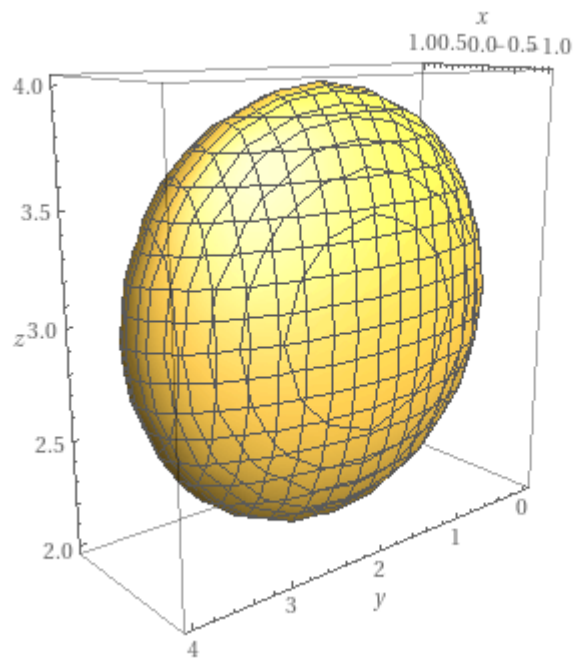
$$\frac{y^2}{4} - \frac{x^2}{4} - z^2 = 1$$

shape is a hyperboloid of two sheets



$$\begin{aligned}
25: & 4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0 \\
& 4x^2 + y^2 + 4(z^2 - 6z + 9) = 0 \\
& 4x^2 + y^2 - 4y + 4(z - 3)^2 = 0 \\
& 4x^2 + y^2 - 4y + 4 + 4(z - 3)^2 = 4 \\
& 4x^2 + (y - 2)^2 + 4(z - 3)^2 = 4 \\
& x^2 + \frac{(y-2)^2}{4} + (z - 3)^2 = 1
\end{aligned}$$

shape is an ellipsoid



$$\begin{aligned}
26: & 4y^2 + z^2 - x - 16y - 4z + 20 = 0 \\
& 4y^2 - 16y + 16 + z^2 - 4z + 4 = x \\
& 4(y^2 - 4y + 4) + z^2 - 4z + 4 = x \\
& 4(y - 2)^2 + (z - 2)^2 = x \\
& (y - 2)^2 + \frac{(z-2)^2}{4} = \frac{x}{4}
\end{aligned}$$

shape is an elliptic parabaloid