Romeo Capozziello HW 3

Problem 1:

Find the upper bound of sum $\sum_{i=1}^{n} \frac{1}{i}$ (Recall that ln0 is undefined)

$\int_0^n \frac{1}{x}\, dx$

$\ln(x)|_0^n = \ln(n) - \ln(0)$, However ln(0) is undefined

Upper bound area from $0 \rightarrow 1 = 1$, so $\int_0^1 \frac{1}{x}\, dx = 1$

$\int_0^n \frac{1}{x}\, dx = \int_0^1 \frac{1}{x}\, dx + \int_1^n \frac{1}{x}\, dx$

$= 1 + \int_1^n \frac{1}{x}\, dx$

$= 1 + \ln(x)|_1^n$

$= 1 + \ln(n) - \ln(1)$

$= 1 + \ln(n) - 0$

Upper bound = 1 + ln(n)

Problem 2:

(a) Find the lower bound of sum $\sum_{i=1}^{n} i^5$ using an integral

$\int_0^n x^5\, dx = \frac{1}{6}x^6|_0^n = \frac{1}{6}n^6 - 0 = \frac{1}{6}n^6$

(b) manually compute $\sum_{i=1}^{n} i^5$ for n = 8, and compare to the lower bound formula in part (a)

$\sum_{i=1}^{8} i^5 = 1^5 + 2^5 + 3^5 + 4^5 + 5^5 + 6^5 + 7^5 + 8^5$
$= 1 + 32 + 243 + 1024 + 3125 + 7776 + 16807 + 32768 = 61776$
$(n^5 = d_0 d_1 d_2 ... d_k n$? Ex: $\underline{3}^5 = 24\underline{3}$, $\underline{7}^5 = 1680\underline{7}$, etc.)

Lower bound $= \frac{1}{6}8^6 = \frac{1}{6}262144 \approx 43691 < 61776$

Problem 3:

Find the limit $lim_{n\rightarrow\infty}\ \frac{n2^n}{e^n}$ (recall $(2^n)' = 2^n * ln2$)

$lim_{n\rightarrow\infty}\ \frac{n2^n}{e^n} = lim_{n\rightarrow\infty}\ \frac{2^n + (2^n ln2)n}{e^n} = lim_{n\rightarrow\infty}\ \frac{2^n ln2 + (2^n + (2^n ln2)n) ln2}{e^n}$

L'Hopital's Rule does not simplify to $\frac{1}{n}$ or $\frac{1}{n}$, repeats infinately

$\frac{n2^n}{e^n} = n(\frac{2}{e})^n$

Unsure where to go from here :(

Problem 4:

(a) Discuss why for f(n) = $n * lg(n^k)$ and g(n) = $n * lg(n^m)$ are Θ of each other

$nlg(n^k) = knlg(n)$, $nlg(n^m) = mnlg(n)$
Since k and m are constants, then $knlg(n)$ and $mnlg(n)$ are proportional to each other
Thus they are Θ of each other

(b) Solve $lim_{n\to\infty} \frac{f(n)}{g(n)}$

$lim_{n\to\infty} \frac{f(n)}{g(n)} = lim_{n\to\infty} \frac{nlg(n^k)}{nlg(n^m)} = lim_{n\to\infty} \frac{knlg(n)}{mnlg(n)} = lim_{n\to\infty} \frac{k}{m} = \frac{k}{m}$