

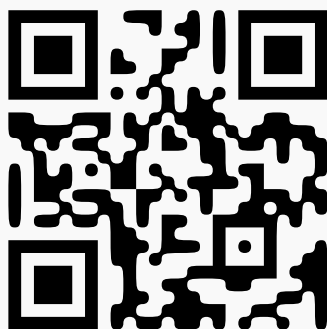
# Probabilistic Pose Estimation from Image Features

Feeding the ~~beast~~ Kalman filter with well-calibrated estimates

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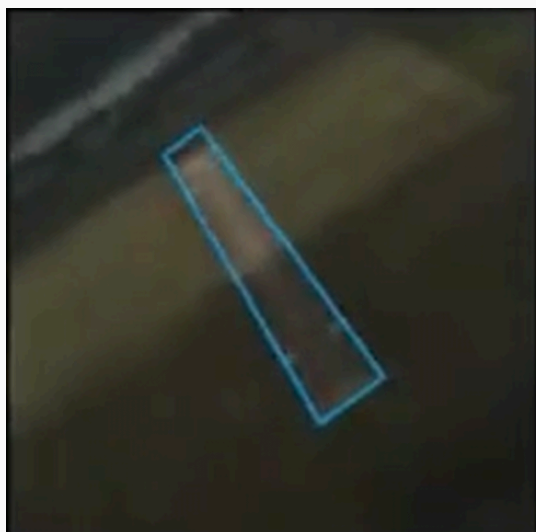
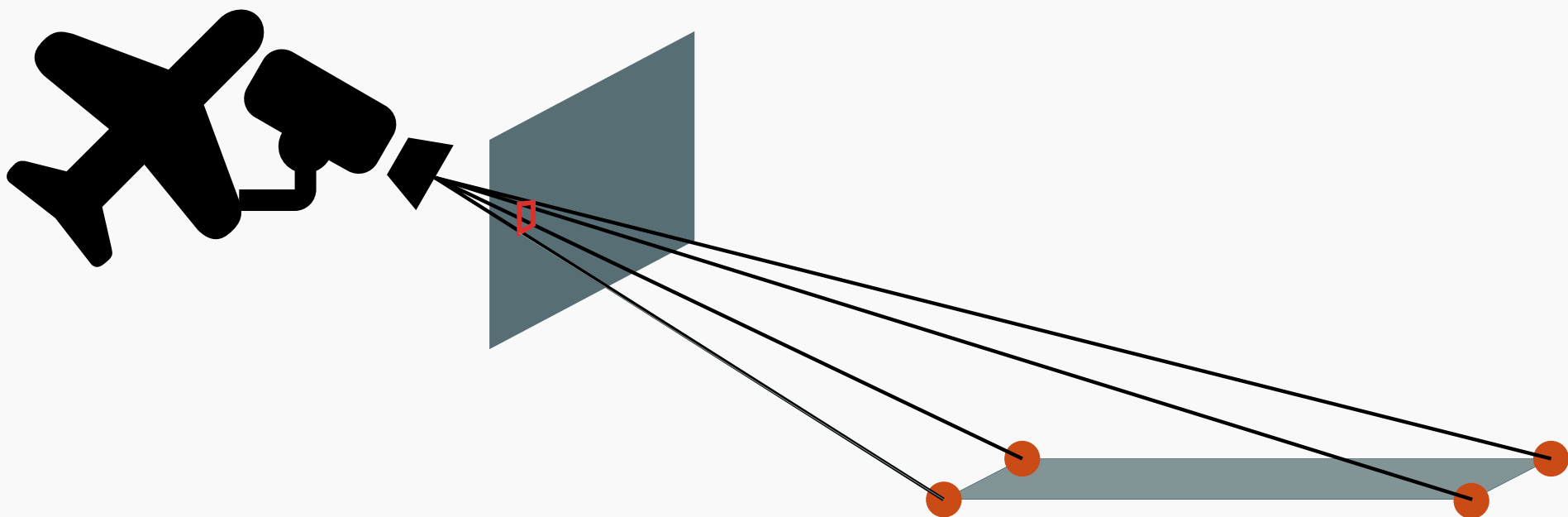
Romeo Valentin (romeov@stanford.edu)

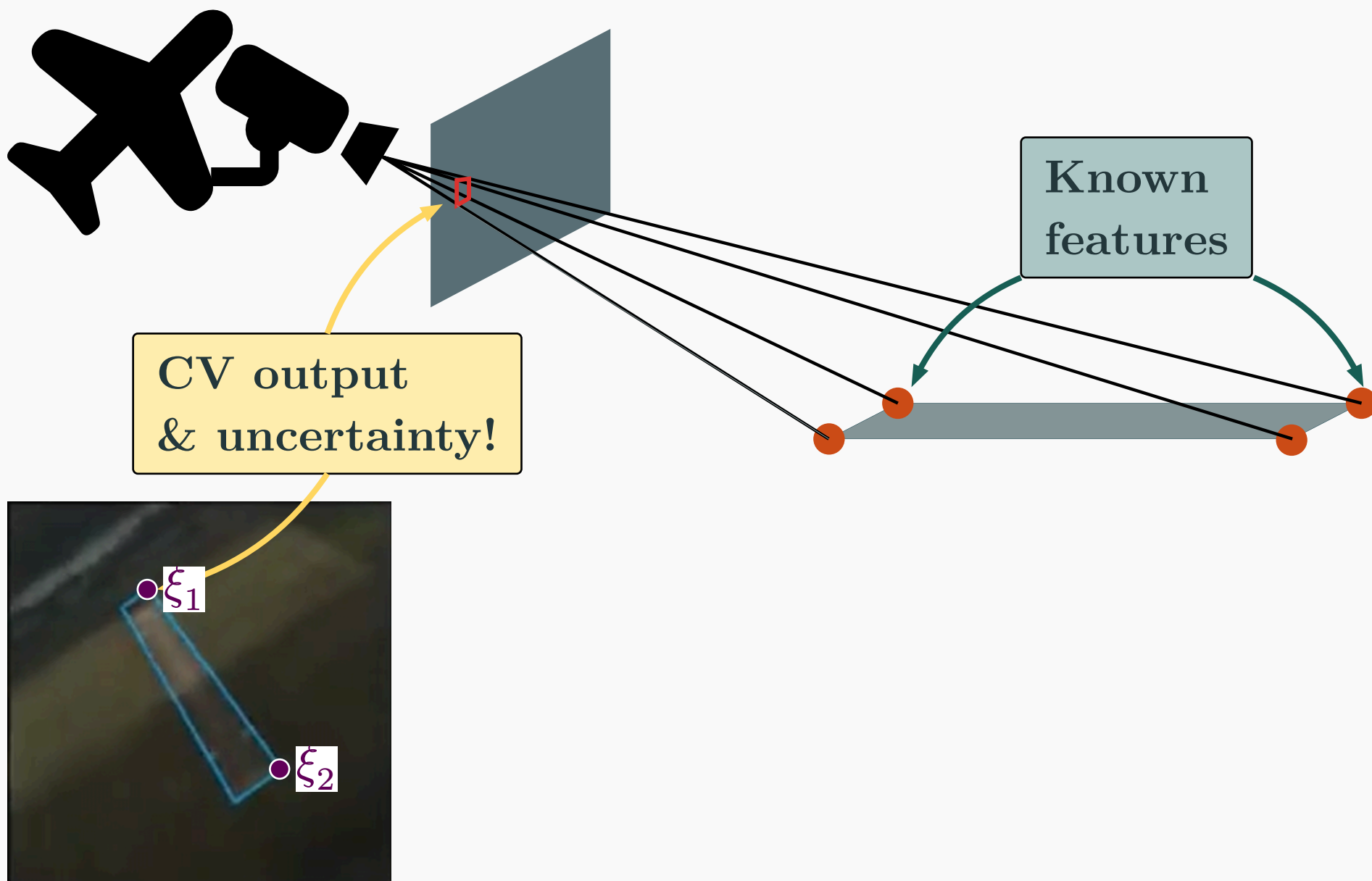
September 2024 • DASC

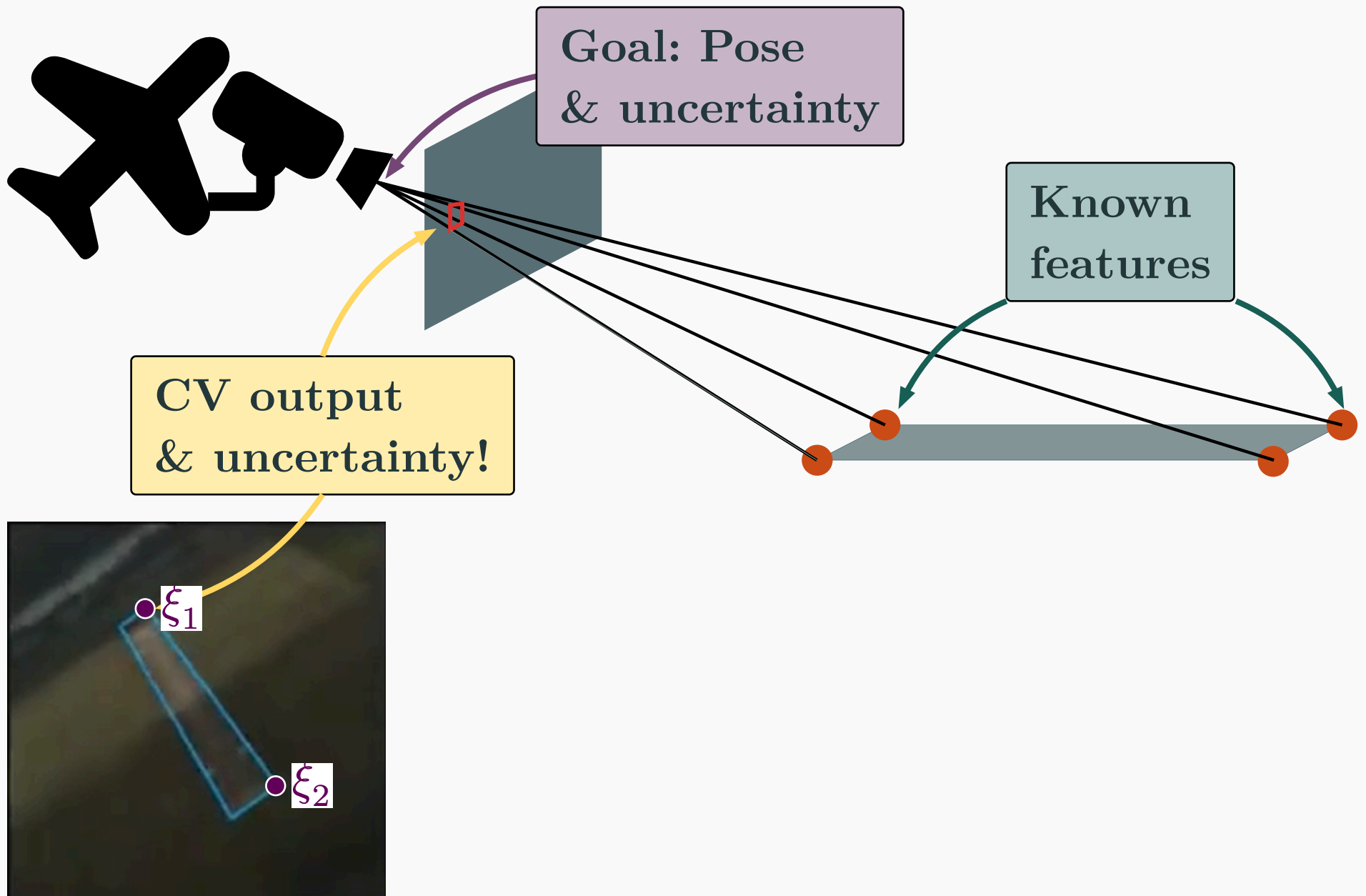


Collaborators:

- Sydney Katz
- Joon Lee
- Mykel Kochenderfer
- Don Walker
- Matthew Sorgenfrei



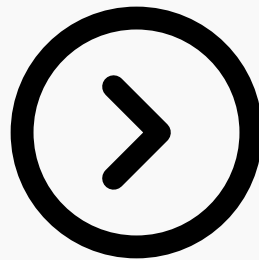




GPS:

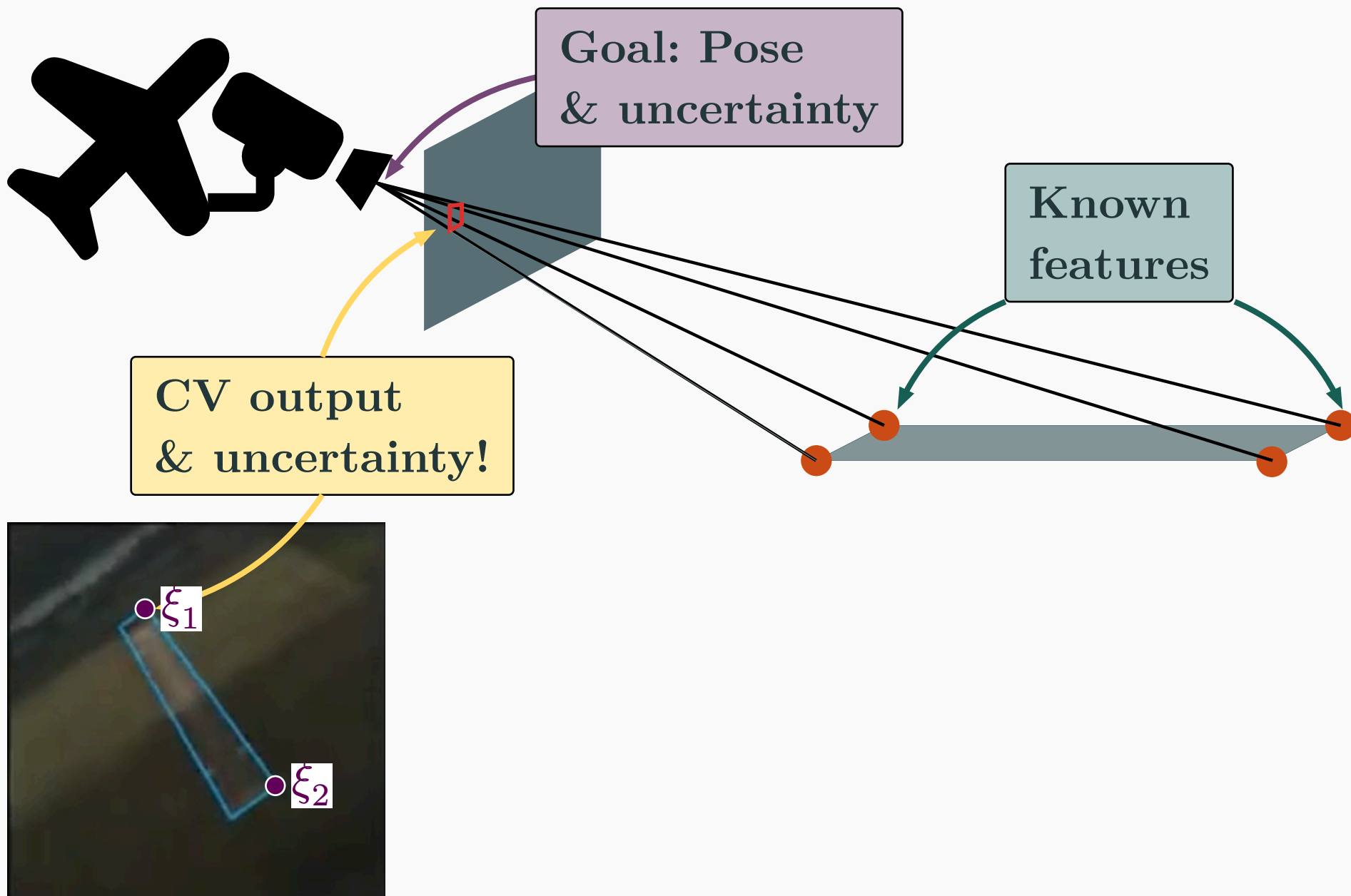


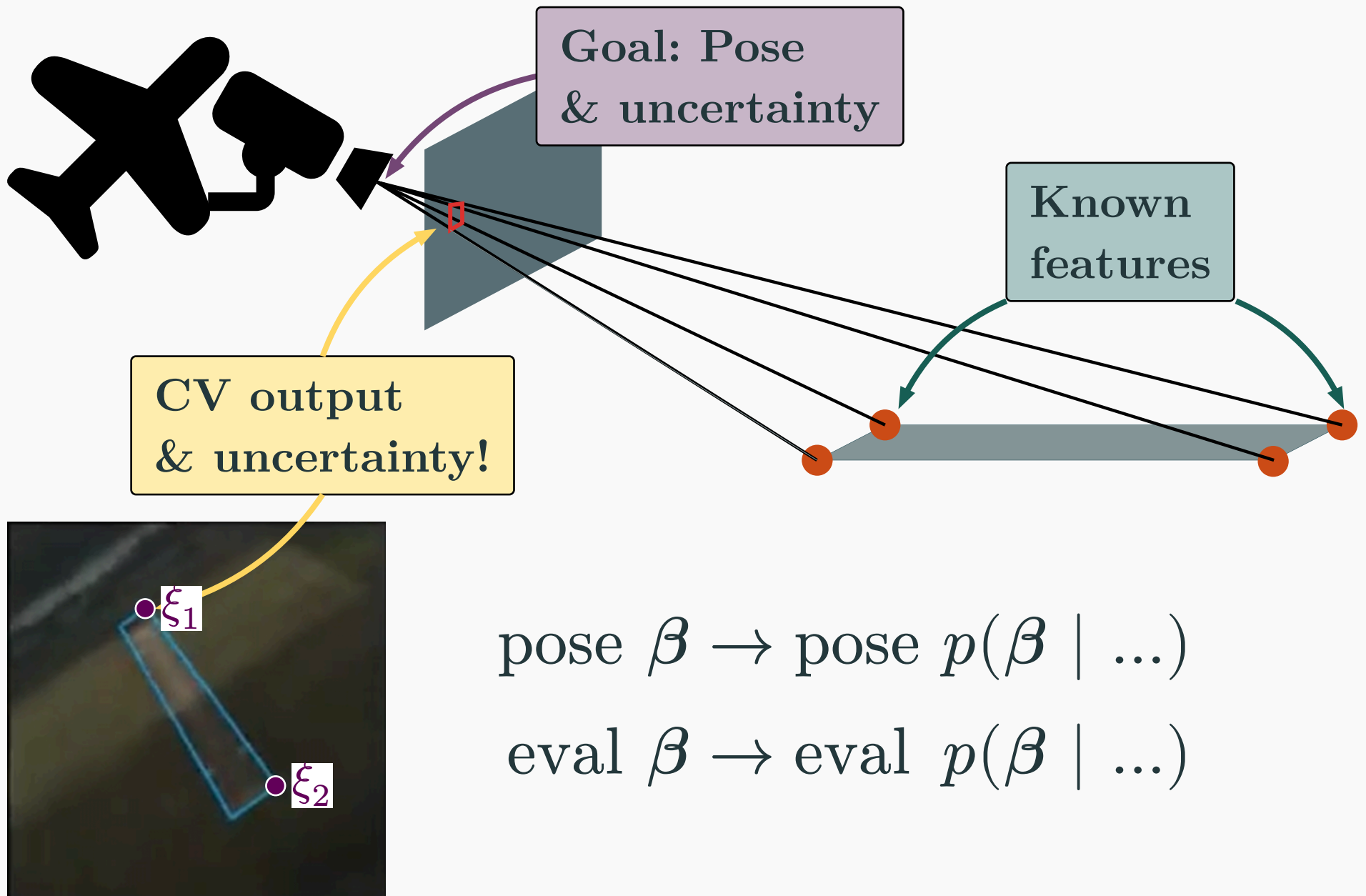
IMU:



Camera?



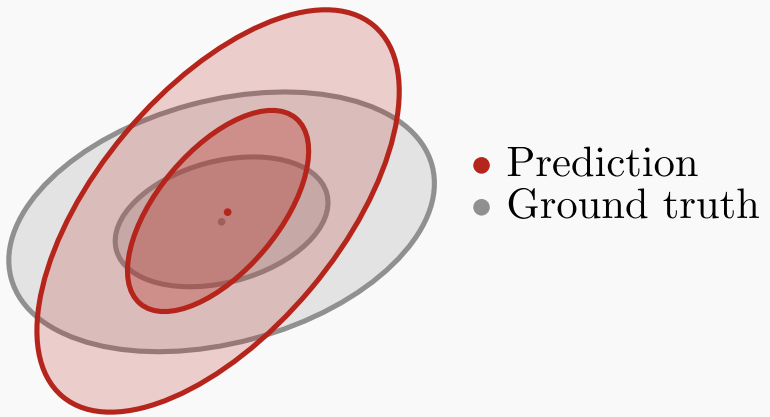




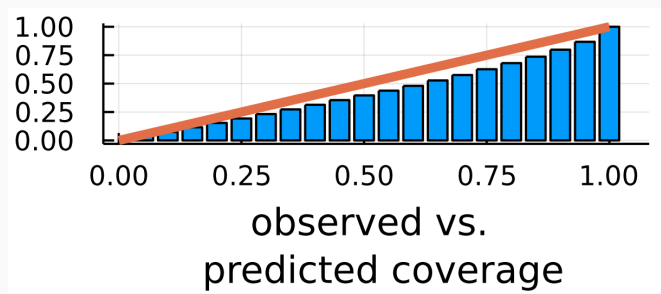
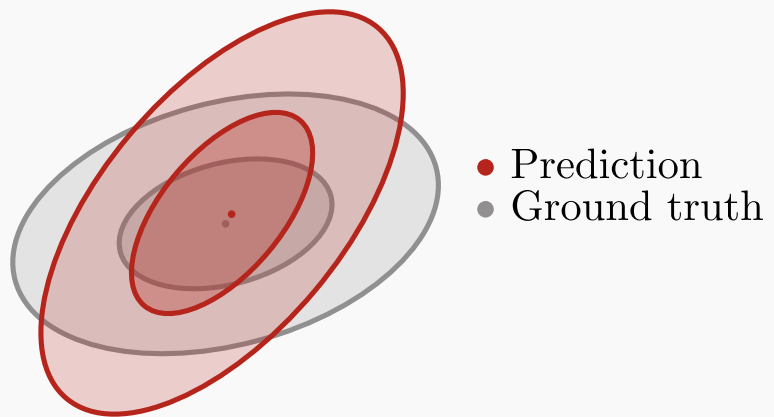
What makes a good probability  
prediction?



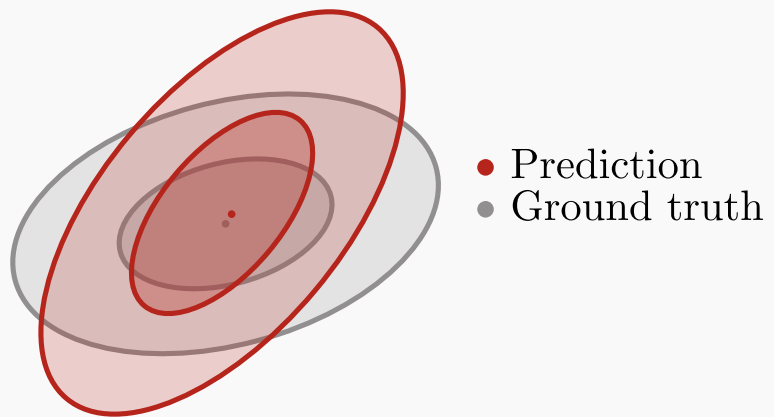
Calibration curve:



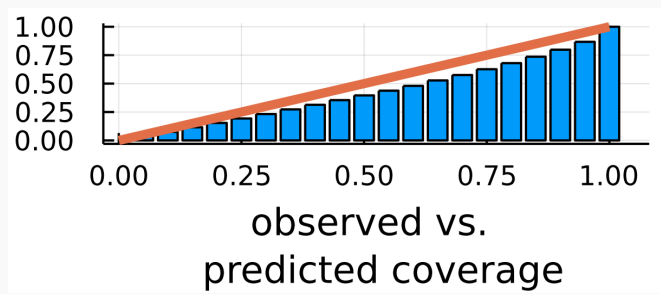
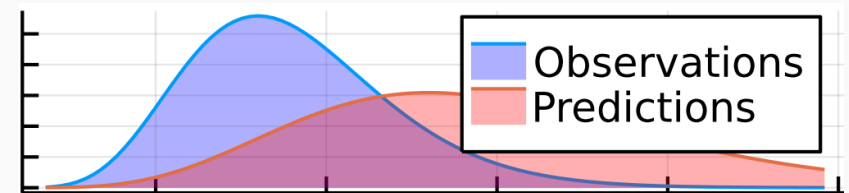
Calibration curve:



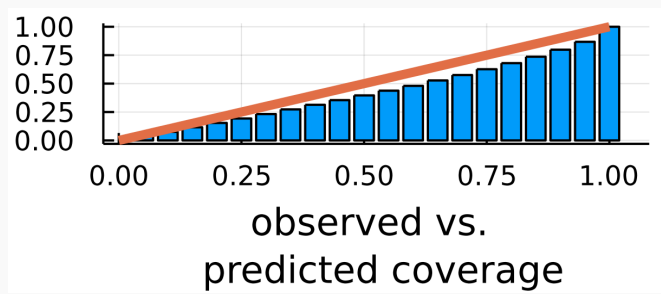
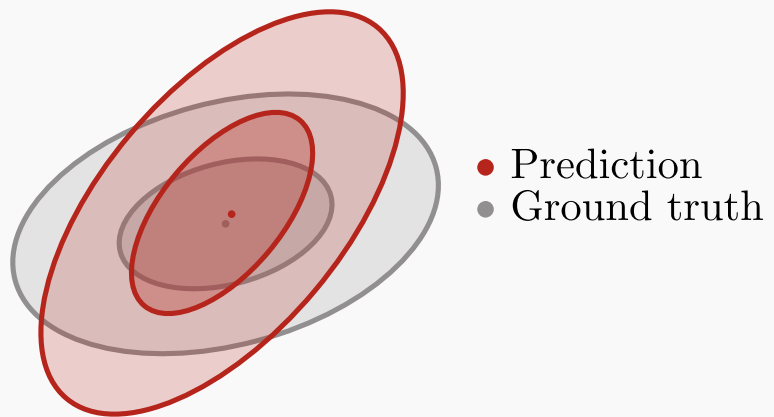
## Calibration curve:



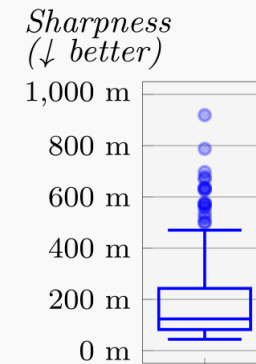
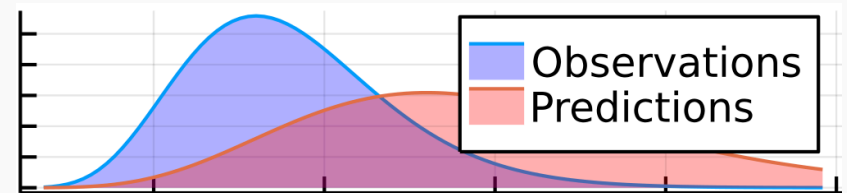
## Sharpness:



## Calibration curve:



## Sharpness:



# Three Probabilistic Estimators

## LSQEstimator

Sample noise, repeatedly solve least squares problem

$$\beta = \arg \min_{\theta} \sum w_i \cdot (\text{proj}_{\beta}(x_i) - y_i)^2.$$

---

## LinearApproxEstimator

Assume everything Gaussian & linear

$$\mu_{\beta} = \arg \min_{\beta} \mathbf{r}(\beta)^T \mathbf{r}(\beta)$$
$$\Sigma_{\beta} = (J^T \Sigma_{\varepsilon}^{-1} J)^{-1} J^T \Sigma_{\varepsilon}^{-1} J (J^T \Sigma_{\varepsilon}^{-1} J)^{-1}.$$

---

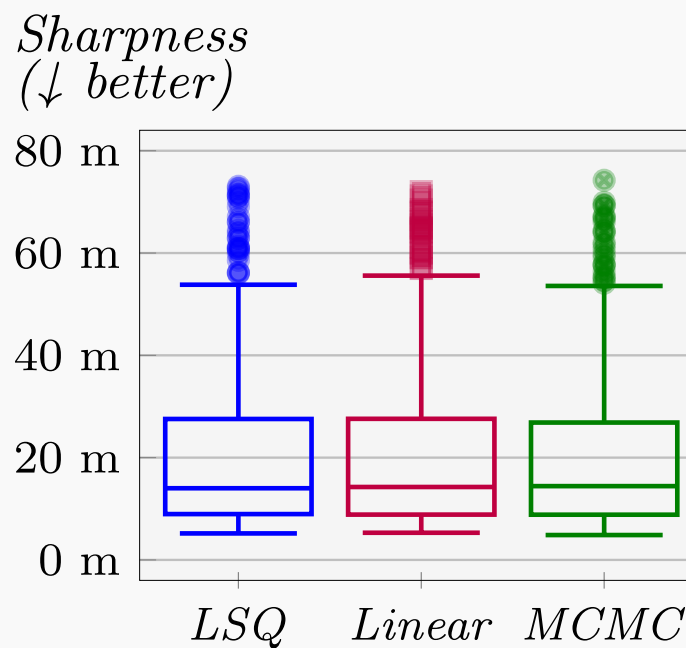
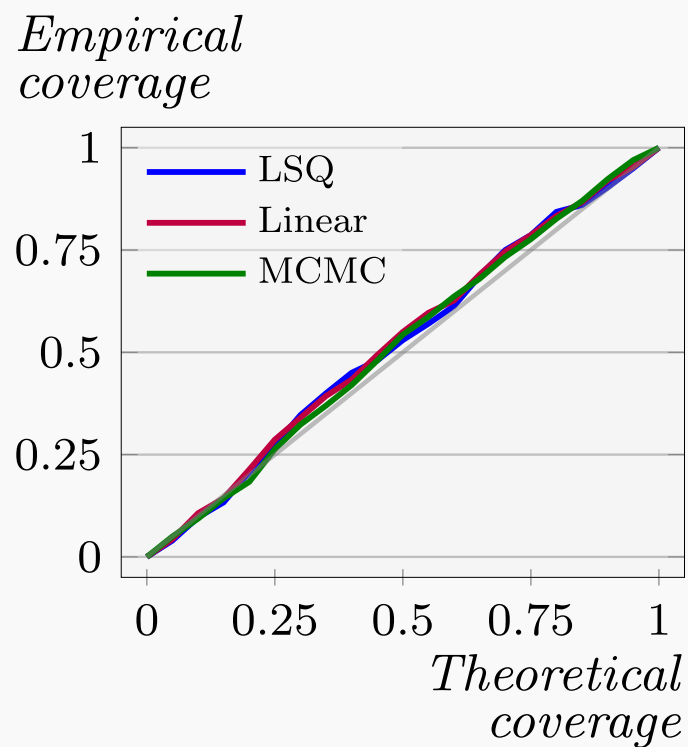
## MCMCEstimator

Automagically sample directly from pose distribution

$$\beta \sim p(\beta \mid \{\text{proj}_{\beta}(x_i), y_i\}_i).$$

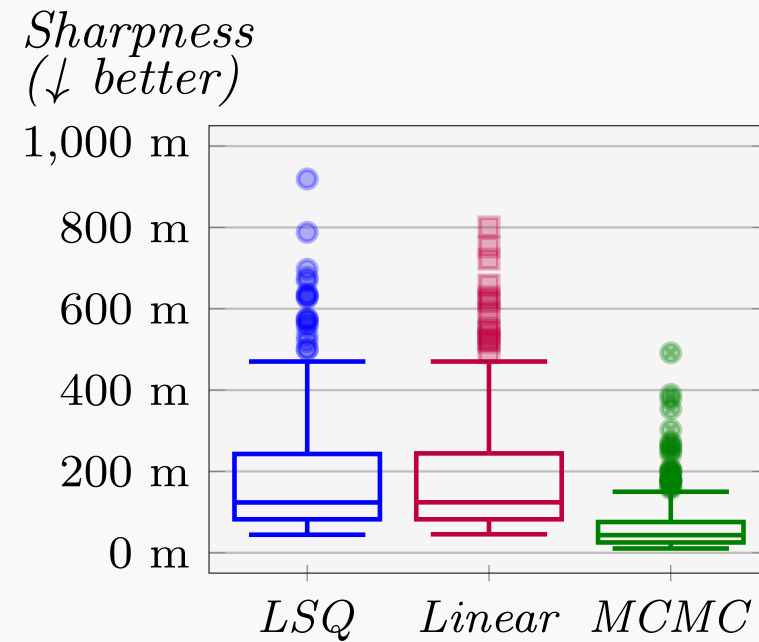
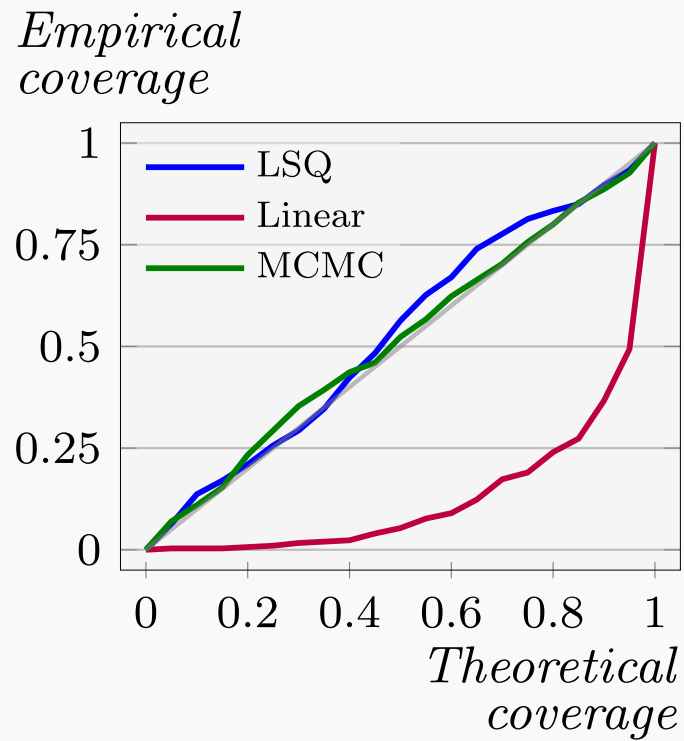
Putting it together!

# Uncorrelated normal noise:



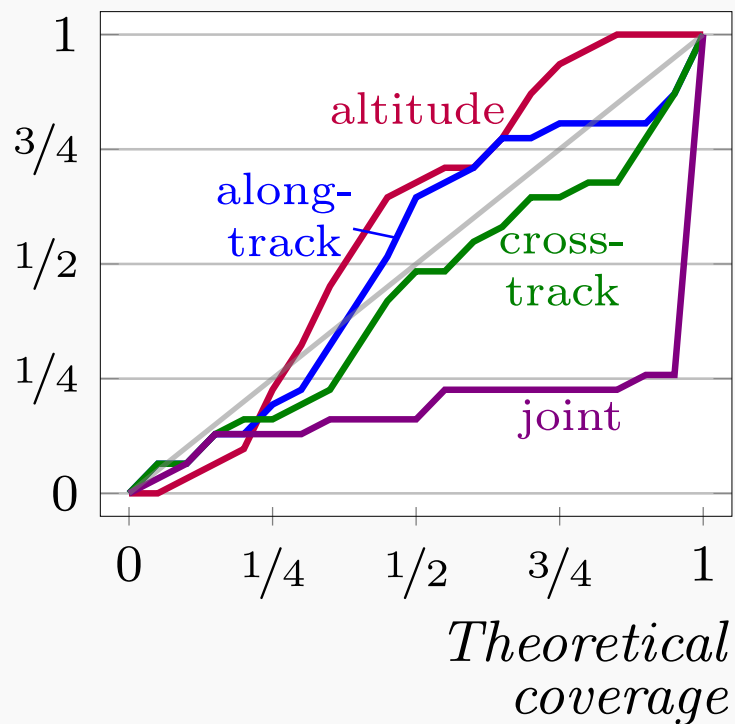


# Long tail noise:

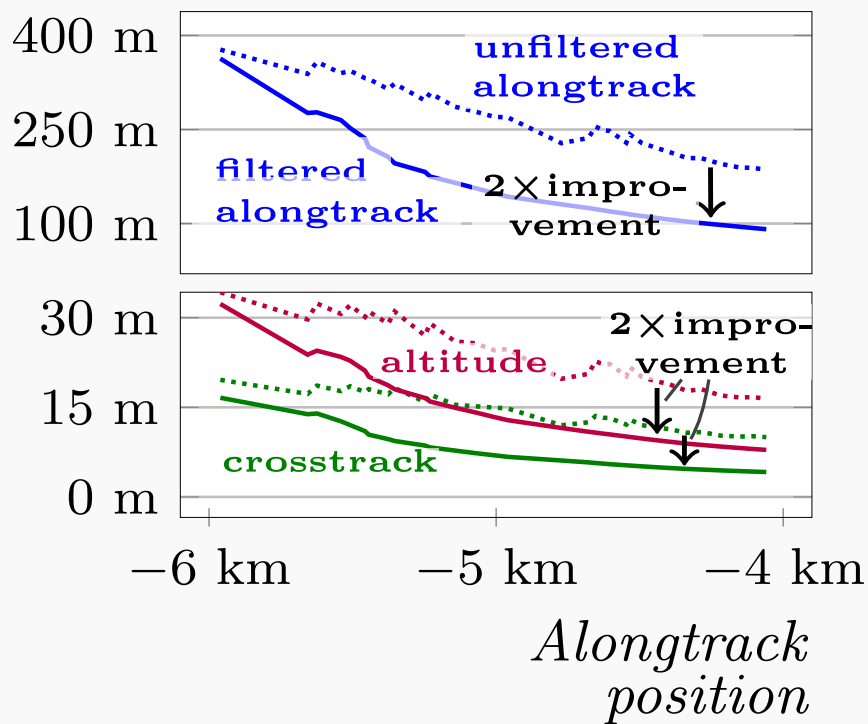


# Results after Kalman filter:

*Empirical coverage*



*Sharpness*  
(↓ better)



## Runtime costs:

---

Noise sampling	311 ms
Lin. Approx	0.4 ms
MCMC	183 ms

---

# ProbabilisticParameterEstimators.jl

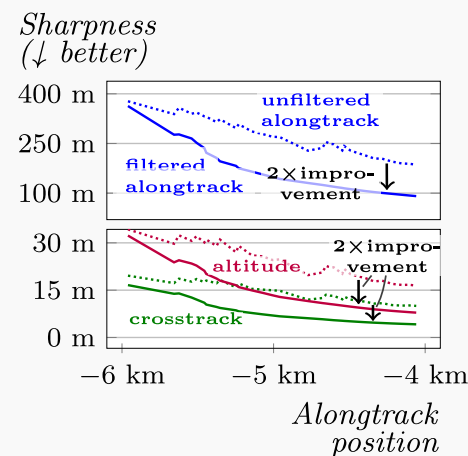
```
using ProbabilisticParameterEstimators, RunwayLib, GeodesyXYZExt
runway_corners = [
    XYZ(0.0m, -25m, 0m), XYZ(0.0m, 25m, 0m),
    XYZ(3000.0m, -25m, 0), XYZ(3000.0m, 25m, 0m),
]
projection_measurements = mymodel(...)
paramprior = MvNormal(...); noisemodel=UncorrGaussianNoiseModel(...)

pose_dist = predictdist(LSQEstimator(), projection_fn,
    runway_corners, projection_measurements, paramprior, noisemodel)
```

Implements three estimators:

- LSQEstimator
- LinearApproxEstimator
- MCMCEstimator

→ read more in the paper.



# Experimental C library

```
#include <stdio.h>
#include <stdlib.h>

// Julia headers (for initialization and gc commands)
#include "julia_init.h"
#include "runwaynpnsolve.h"

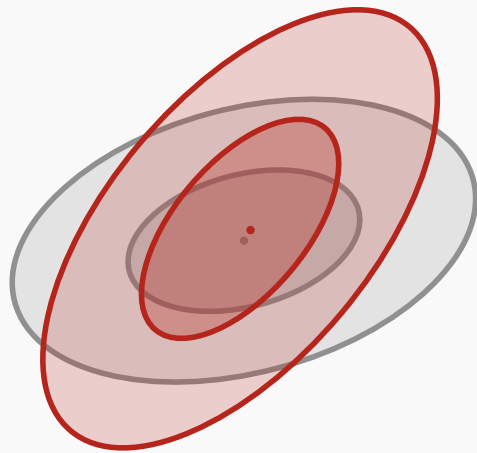
int main(int argc, char *argv[])
{
    init_julia(argc, argv);

    int ret;
    double dst_pos[3] = {0., 0., 0.};
    double dst_cov[9] = {0., 0., 0., 0., 0., 0., 0., 0., 0.};

    double rwylength = 3500.0; // m
    double rwywidth = 61.0; // m
    double rwycorners[3*4] = { /* insert runway geometry */ };

    double measuredprojs[2*4] = { /* get from sensor */ };

    int n_rwycorners = 4;
    ret = predict_pose_c_interface(dst_pos, dst_cov,
                                   rwycorners, n_rwycorners,
                                   measuredprojs);
}
```



- Prediction
- Ground truth

How can we quantify “quality” of predictions?

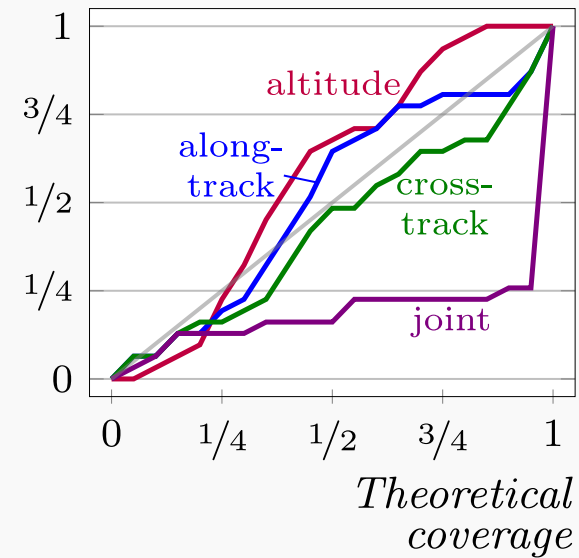
→ read more in the paper.

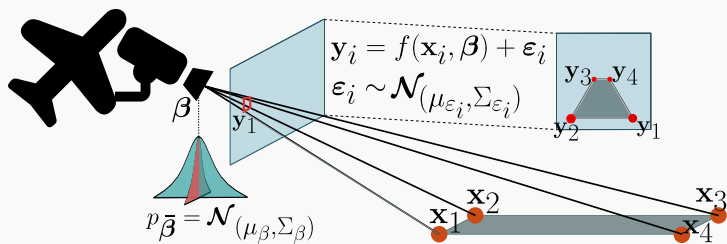
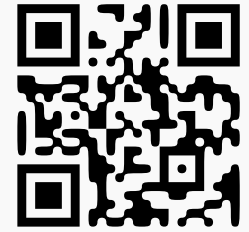
## Usage Example (Julia):

```
using MvNormalCalibration
preds = [predictdist(...) for t in timesteps]
truevals = # ...

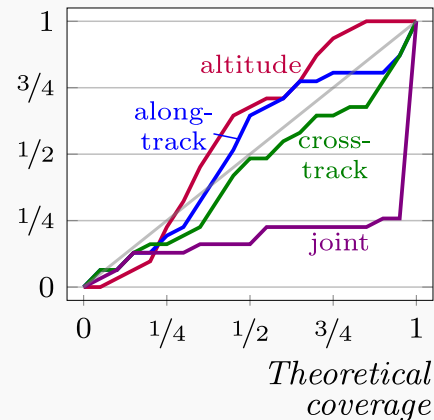
est = computecalibration(preds, truevals)
plot(est...)
```

*Empirical coverage*

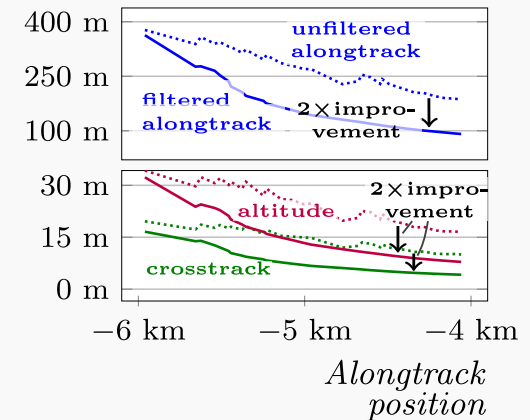




Empirical coverage



Sharpness (↓ better)



**Goal:** Estimate  $p(\beta \mid \dots)$  from probabilistic measurements.

### Three estimators:

- i) LSQEstimator
- ii) LinearApproxEstimator
- iii) MCMCEstimator

& different **noise models**.

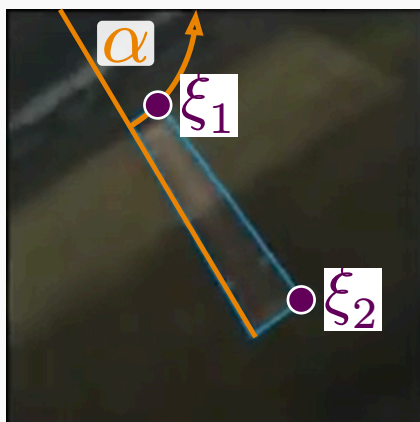
### Calibration & filtering:

- New calibration metric for  $\mathcal{N}(\mu, \Sigma)$
- Demonstrate calibration after filtering.
- Filtering despite bias (WIP)

# What is the new approach?

Consider **probabilistic inputs / outputs**:

$\alpha \in \mathbb{R}$	$\longrightarrow$	$\alpha \in \mathbb{P}(\mathbb{R})$	e.g., $\mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$
$\xi \in \mathbb{R}^2$	$\longrightarrow$	$\xi \in \mathbb{P}(\mathbb{R}^2)$	e.g., $\mathcal{N}(\mu_\xi, \Sigma_\xi)$
$\beta \in \mathbb{R}^6$	$\longrightarrow$	$\beta \in \mathbb{P}(\mathbb{R}^6)$	
$\arg \min_{\beta} \sum_i (\cdot)_i^2$	$\longrightarrow$	$p(\beta \mid \alpha, \xi_1, \xi_2, \dots)$	e.g., $\mathcal{N}(\mu_\beta, \Sigma_\beta)$
$\text{metric} := \ \cdot\ _2$	$\longrightarrow$	$\text{metric} := \begin{pmatrix} \text{calibration} \\ \& \text{sharpness} \end{pmatrix}$	



## Three estimators:

- i) Noise Sampling + least squares
- ii) Linear Approximation for  $\Sigma_\beta$
- iii) Markov chain Monte Carlo



# The RunwayPNPSolve.jl ecosystem

## **Paper:**

See resources – soon [arxiv](#), then DASC.

## **Registered:**

`ProbabilisticPoseEstimators.jl`

`MvNormalCalibration.jl`

## **Unregistered:**

`GeodesyXYZExt.jl`

`RunwayLib.jl`

`RunwayPNPSolve.jl`

`RunwayPNPSolveBinary.jl` (unpublished)

Experimental feature: strongly\_typed unitful type restrictions.

## Usage Example (Julia):

```
using GeodesyXYZExt
camera_pos = XYZ(-2500.0m, 20m, 300m)
camera_rot = RotXYZ(roll=10.0°, pitch=3°, yaw=-1.5°)

function project(pos::XYZ{<:WithDims(m)}, rot::Rot3, world_pt::LLA)
    # ...
    return ImgProj(...)
end
```

Restrict:

- Coordinate system
- Unit

But flexible with:

- Number type, such as  
Float64 or Dual{Float64}

## RunwayLib.jl

Some utility functions, such as point and line projections.

### Usage Example (Julia):

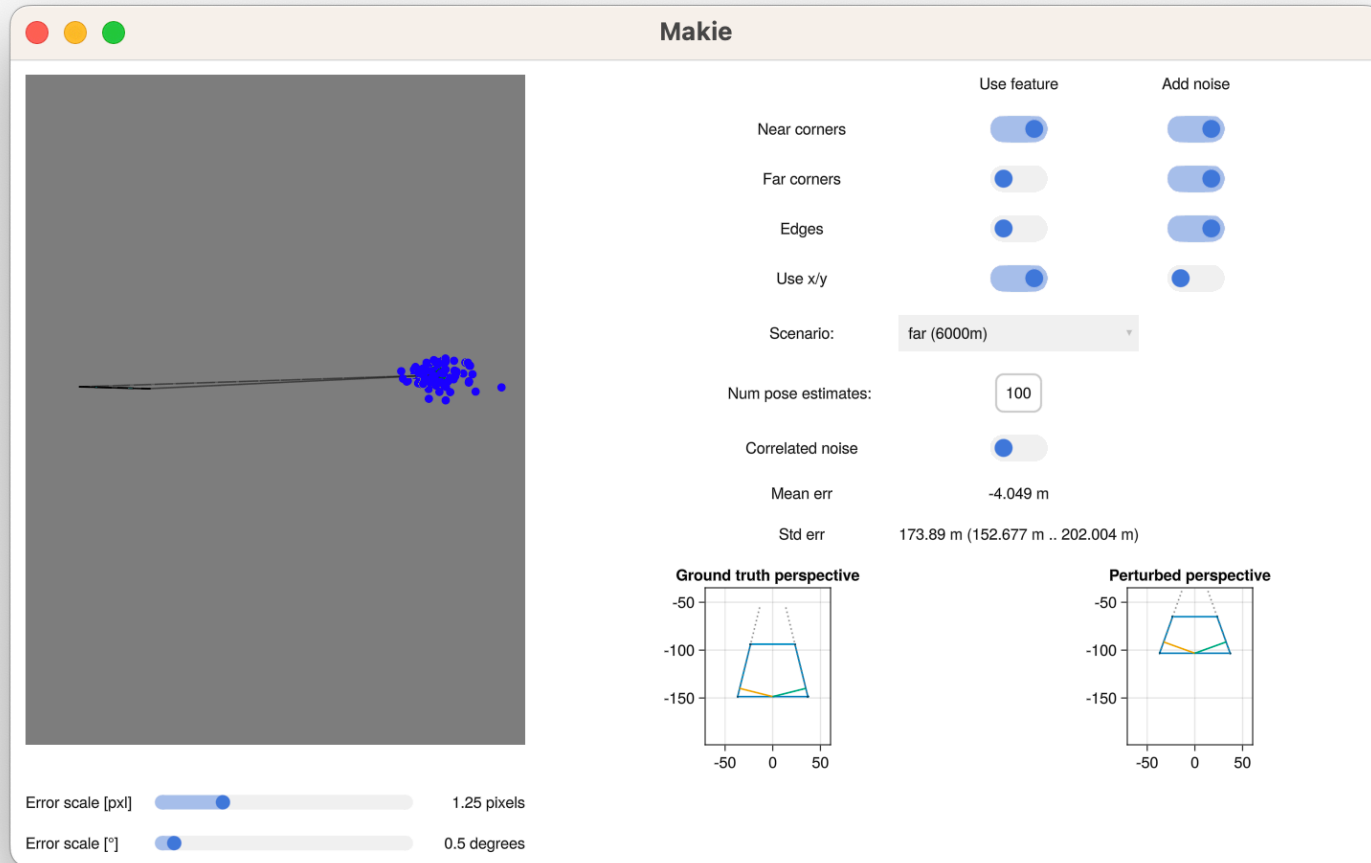
```
cam_pose = CamTransform(camera_rot, cam_pos)
world_pt = XYZ(300.0m, 20.0m, 0.0m)

projection::ImgProj = project(cam_pose, world_pt)
rho_theta = project_line(camera_pose, ImgProj(0.0pxl, 0.0pxl),
                        (world_pt, world_pt + XYZ(100.0m, 0m, 0m)))
```

## RunwayPNPSolveBinary.jl (unpublished)

- binary generation using PackageCompiler.jl
- can run without Julia installation, e.g. onboard
- 0.5ms to 250ms for one parameter estimate
  - fast enough for real-time!

# Experimental visualizations with Makie.jl



# The RunwayPNPSolve.jl ecosystem.

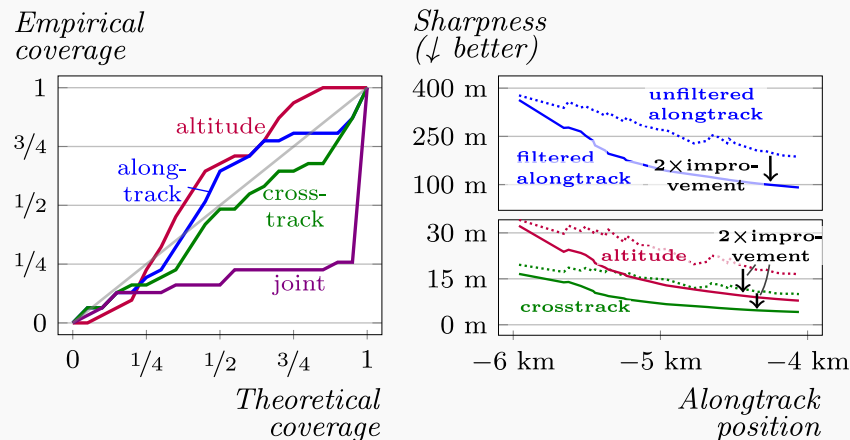
## Packages:

- ProbabilisticPoseEstimators.jl
- MvNormalCalibration.jl
- GeodesyXYZExt.jl
- RunwayLib.jl
- RunwayPNPSolveBinary.jl

## Acknowledgements:

- NonlinearSolve.jl and the SciML ecosystem
- Makie.jl ecosystem
- CoordinateTransformations.jl, Rotations.jl, and Geodesy.jl
- Unitful.jl and UnitfulAngles.jl

→ Check out the paper [in the talk resources](#) to learn more about these figures!



... and my co-authors

- Sydney Katz, Joon Lee, Mykel Kochenderfer
- Don Walker, Matt Sorgenfrei ( $A^3$  by Airbus)

Also, come see my poster on ThreadedDenseSparseMul.jl at 7pm in Method (1.5)!

# Why camera based?

We already have

- GPS
- IMU / INS
- radio-based (VOR)

so why **camera-based**?

⇒ **independent failure mode**