Nest Predation Problems

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Contents

1   Introduction

Infanticide is a behavior of killing an infant, typically killing a newborn, shortly after birth, which has occurred throughout history and across various cultures. Historically, infanticide has been associated with a range of factors, including poverty, societal norms, cultural beliefs, and the desire to control population size. In some cases, infants were killed due to physical deformities, perceived disabilities, or because they were unwanted for various reasons. Infanticide has also been linked to issues such as gender discrimination, where female infants were more likely to be victims in societies that preferred male offspring. Additionally, in many societies, especially in the past, infanticide was usually used as a way to control and regulate the population. Therefore, it particularly affected female children, since having fewer women meant having a lower rate of reproduction (fewer children).

As for infanticide among animals, although the detrimental outcome for the infant is fairly constant, individuals responsible for infanticide may benefit. Sources of increased benefits from infanticide include:

- Exploitation of the infant as a resource

- Elimination of a competitor for resources

- Increased maternal survival or lifetime reproductive success for either mother or father by elimination of an ill-timed, handicapped, or supernumerary infant

- Increased access for individuals of one sex for reproductive investment by the other sex at the expense of same-sex competitors [3].

In general, infanticide of animals does improve the survival of the fittest and also maintains resource conditions for the remaining animals in the population.

Egg-tossing is a kind of infanticide actions for birds, which is caused by social conflict between birds and restrict of limited resource. Tossing of eggs is non-accidental; the individual rolls the egg to the edge of the nest by repeatedly flicking it with its beak [1]. During co-nesting, before a bird starts laying its own eggs, it will toss out eggs laid previously by other females [8]. As a result, the last egg-layers may contribute more eggs to the common nest [8], and this will increase the chances that newly laid eggs bearing the genetic material of that female will have a better chance of survival [5, 7]. In some species, egg-tossing is a strategy of clutch coordination; eggs are tossed until all birds in the common nest are ready to proceed with brooding [5]. This helps to prevent early egg-layers from dominating reproduction [5, 8].

The gray-capped weaver is one of the birds species that takes this egg-tossing action. Some members of this species may toss eggs in adverse situations, such as to make space for new eggs when a nest is full or when the egg is unfertilized. Recently, a new event was discovered to trigger egg toss: when predation levels are high and clutch size becomes too small, some gray-capped social weaver parents may toss their eggs as a strategy to maximize its reproductive success, which is alleged to be a new feature in social birds, and is called parental tossing. Through tossing their eggs, birds are actually removing the main resources of their eggs' predators, keeping its populational sizes under control and possibly enhancing the survival probability of the future clutch[2].

2 Assumption

In the context of the Population-Egg-Bird (PEB) mathematical models, certain assumptions are made to simplify and frame the analysis of infanticide, particularly egg-tossing behavior in grey-capped weavers. These assumptions serve as foundational premises upon which the mathematical models are constructed. It is crucial to acknowledge that these assumptions, while aiding in the clarity of the models, may not capture all the intricacies of real-world scenarios. The assumptions include:

1. The models assume a homogeneous environment, implying that the factors influencing the population dynamics of predators, eggs, and birds remain constant throughout the study period. This assumption helps in establishing a baseline scenario for analysis and eliminates the possibility of any disastrous catastrophe, like tsunami or earthquake.

2. The predators do not intend to eat weaver birds' eggs. And the total energy of the system is sufficient enough. That is one of our key assumptions that there are various kinds of predators (Eagles and snakes) in the system and enormous numbers of preys (apart from our research subject grey-capped weavers). Alternatively speaking, rather than choose weavers' eggs as a main source of food, all predators would have sufficient other better choices for starving issue. It also explains reasons why we choose to disregard any interaction between predators and eggs in our simulation.

3. The collision rates between birds and predators, as well as between birds and eggs, are assumed to be stationary over time. This assumption implies that the frequency of interactions leading to predation or egg-tossing remains relatively constant during the study period.

4. The parameters involved in the models, such as the rate of collision, natural death rates, and reproduction rates, are considered deterministic (positive constant) and do not exhibit stochastic variations. This assumption facilitates the application of differential equations for modeling.

5. The models assume interactions primarily between two species: birds (grey-capped weavers) and predators. Interactions with other potential external factors are not explicitly considered in this simplified model since we just wonder the interactions involving our research subject. But such external factors can not be ruled out in a general picture.

6. The probability of egg-tossing (P_{toss}) is assumed to be independent of external factors not explicitly considered in the model. This independence simplifies the analysis by isolating the impact of egg-tossing within the context of the defined parameters, for example, wind.

3 Methods

In this section, we will discuss PEB mathematical model with and without infanticide, the logic here is to build up model systems and methodology used to determine the stability of ordinary differentiable time-invariant (TI) system.

3.1   PEB mathematical model without infanticide

This model consists of three ordinary differential equations, describing the evolution of the population of predators, eggs, and birds concerning time:

$$\frac{dP}{dt} = \mu BP - kP$$
$$\frac{dE}{dt} = cEB - \frac{\lambda EB}{1 + E} \tag{1}$$
$$\frac{dB}{dt} = \frac{\lambda EB}{1 + E} - \phi BP - \gamma B$$

where the uppercase English letters (P, E, B) represent the number of predators, eggs, and birds accordingly. All the other parameters (a, μ, k, c, λ, ϕ, γ) are positive so as to have a rational meaning in our model. Specifically, all the parameters are defined as follows:

- μ denotes the rate of collision, that is the probability of two species (birds and predators) colliding with each other and resulting in predation (one species is eaten by another one).

- k denotes the natural death rate of predators.

- c denotes the probability of eggs being produced by one female Grey-capped weaver in one period after the collision between weavers and eggs.

- λ denotes the optimal number of eggs being fully incubated by one Grey-capped weaver and transferred into fledglings.

- ϕ is the rate of collision, which resembles , but is different on the basis that a single predation would not directly contribute to the increase in the number of predators.

- γ denotes the death rate of birds which excludes the death caused by predation.

The number of predators grows by predation μ BP because of the presence of birds (as food of predators) and it decreases caused by the natural death kP.

As for the growth rate of birds $\frac{dB}{dt}$, it increases due to hatching from eggs $\frac{\lambda BE}{1+E}$ and it decreases due to predation ϕ BP and other causes of death γ B. The growth rate of eggs $\frac{dE}{dt}$ increases depending on cBE the total number of eggs produced by all female Gray-capped weavers in one period after the collision between weavers and eggs and decreases depending on $\frac{\lambda BE}{1+E}$ the total number of eggs being fully incubated and transferred into fledglings after the collision between weavers and eggs.

3.2   PEB mathematical model with infanticide (particularly egg-tossing)

$$\frac{dP}{dt} = \mu BP - kP$$
$$\frac{dE}{dt} = cEB(1 - \frac{2E}{1 + E^2}) - \frac{\lambda EB}{1 + E} - gEB \tag{2}$$
$$\frac{dB}{dt} = \frac{\lambda EB}{1 + E} - \phi BP - \gamma B$$

 Compared with the previous model without infanticide, there are two extra terms and one extra parameter g, whose representation is the following:

- g is the product of two parameters: g_1 and g_2. g_1 denotes the collision rate of two species: birds and eggs, and g_2 denotes the percentage of social conflict. Social conflict is defined as interspecies conflict which may occur when birds fight for limited resources, for example, shortage of capacity in the nest and insufficient resources to feed up a fledgling. Cheng, Dustin R. and Shen's paper have suggested that social conflict is an important factor that causes a particular egg-tossing behaviour in the parent grey-capped weavers in Kenya [2].

In the PEB mathematical model with infanticide (particularly egg-tossing), we take extra consideration the decline of the growth rate of eggs $\frac{dE}{dt}$ caused by egg-tossing. The term $\frac{2E}{1+E^2}$ is the probability of the egg-tossing and the constant 2 there is to constrain the function to be valued within [0,1] with E being positive.

Additionally, The work of Cheng, Dustin R. and Shen also suggests that the probability of egg-tossing decreases as the number of eggs decreases [2]. That is to say, fewer numbers of eggs in the hatch may incentivize the related parents to discard the remaining eggs to balance the expected fitness of lives in the long run. After several experiments, one can find that if there is only one egg remaining in the hatch, then parents would infanticide their eggs without doubt. The following picture:
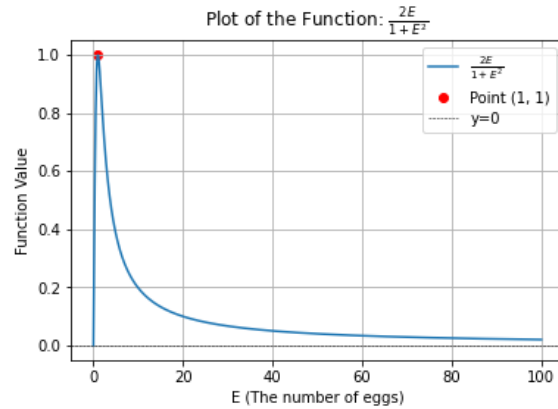

Figure 1: Plot of the Function $2E/(1+E^2)$

The term gBE is representing the amount of eggs that is discarded on account of social conflict issue.

3.3   Routh-Hurwitz criteria

The Routh-Hurwitz criteria is a mathematical method used to determine the stability of a linear time-invariant (LTI) system by analyzing the coefficients of its characteristic equation [6]. This criterion is named after its developers, Edward John Routh and Adolf Hurwitz. It is widely used in control systems analysis to determine the stability of a system without explicitly calculating the roots of the characteristic equation. In this paper, a more practical theorem is applied as following rather than the generalized Routh-Hurwitz criteria which requires (n-1) matrices to obtain the results:

Theorem 1. For a cubic polynomials of variable x:

$$x^3 + a_1 x^2 + a_2 x + a_3 = 0$$

where a_3, a_2, a_1, a_0 are all complex numbers (real numbers), in particular, a_3 should be nontrivial to demonstrate the polynomial of degree three. And if the following criteria are satiated, then all the real parts of the complex root to this cubic polynomials are negative:

1. a_1 is positive

2. a_3 is positive

3. $a_1 \times a_2$ is larger than a_3

This theorem is the only criteria that we applied in the report, considering the fact that we are dealing with 3×3 matrices (or cubic polynomials) in both *PEB* models (with or without egg-tossing).

Theorem 2. Now, if you're interested in finding the real pa rt of the complex root, you can use the fact that the real part of a complex number z is given by the assumption $\text{Re}(z) = \frac{z + \bar{z}}{2}$, where $\bar{z}$ is the complex conjugate of z. So, if z is a complex root obtained from the cubic formula, the real part would be:

$$\text{Re}(z) = \frac{1}{2}\left(\sqrt[3]{-\frac{d}{a} + \sqrt{(\frac{c}{d})^3 + (\frac{d}{a})^2}} + \sqrt[3]{-\frac{d}{a} - \sqrt{(\frac{c}{d})^3 + (\frac{d}{a})^2}}\right) + \frac{1}{2}$$

(complex conjugate of the above expression).

4   Discussion

In this section, we will have numerical analyze about the models without tossing strategies and those with tossing strategies, explore whether there exist equilibrium points, and discuss their stability in depth. Additionally, we will discuss the limitations and examine the biological implications of models based on prior assumptions. Them methodology we applied here is to do stability analysis firstly under the criteria of functional analysis (without input data) for both models. Then we substitute data with all the parameters (constants) to explore results discretely under real scenarios.

4.1   Models without tossing strategies

Let's first consider non-tossing strategies. When seeking equilibrium solutions to the system of equations (1), we equalize all the right-hand side terms of the equations in the system to 0. Therefore we divide our discussions into three categories:

1. One patent solution to the system is the original point: (P, E, B) = (0, 0, 0)

2. Apart from the original point, we want to know whether there exists any other points that characterized by at least either one of (P,E,B) is zero.

- if $B = 0$, in the first equation of (1), P must equal 0 and $E \in \mathbb{R}$
- if $P = 0$, then $B = 0$ (from the first equation) and E can be arbitrary as well (from the second equation); the other case when $B \neq 0$ is impossible to be true if analyzing the combination of the second and third equation.
- if $E = 0$, then B must equal 0 and P must equal 0

3. The last condition we have is non-zero solutions, in this case, we have $(P^*, E^*, B^*) =$

$$\left(\frac{\lambda - c - \gamma}{\phi}, \frac{\lambda}{c} - 1, \frac{k}{\mu} \right)$$

With the help of Jacobian matrix, the qualitative behaviors (especially stability or instability) of the above equilibrium points can be evaluated separately. The Jacobian matrix in the general form is:

$$J_{(P,E,B)} = \begin{bmatrix} \mu B - k & 0 & \mu P \\ 0 & \frac{\lambda B E}{(1+E)^2} + Bc - \frac{B\lambda}{E+1} & Ec - \frac{\lambda E}{E+1} \\ -\phi B & \frac{\lambda B}{1+E} - \frac{BE\lambda}{(1+E)^2} & \frac{E\lambda}{1+E} - \phi P - \gamma \end{bmatrix}$$

The above three equilibrium points are substituted into the Jacobian matrix respectively, and it can be judged as follows:

- For J(0,0,0) = $\begin{bmatrix} -k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma \end{bmatrix}$, the trace of the matrix is $\text{Tr}(J) = -k - \gamma < 0$, which implies a stability. The determinant of the matrix $\det(J) = 0$.
  Clearly it conforms to non-isolated points (The biological intrepretation of these is futile to find).

- For $J(P^*, E^*, B^*) = \begin{bmatrix} 0 & 0 & \mu P* \\ 0 & \frac{\lambda B*E*}{(1+E*)^2} & 0 \\ -\phi B* & \frac{\lambda B*}{(1+E*)^2} & 0 \end{bmatrix}$
  The generalized characteristic equation of a cubic polynomial with real coefficients is:

$$\lambda^3 - tr(J*)\lambda^2 + \lambda \sum minors - det(J*) = 0 \tag{3}$$

Here, $\sum minors$ denotes the sum of the terms on the triangular line of trace, for which $\sum minors = PB\mu\phi$, $tr(J*) = \frac{BE\lambda}{(1+E)^2}$, $det(J*) = \frac{P\mu b^2 E\lambda\phi}{(1+E)^2}$. Using **Theorem 2** that any cubic polynomial must have at least one complex root. And we can calculate the real part of the eigenvalues (λ) without input any values to substitute all parameters. By **Theorem 3**, we find out the explict expression of the real part of the root as:

$$\frac{BE\lambda}{E^2 + 2E + 1} = \frac{c(\lambda - c)}{\lambda\mu} \tag{*}$$

Take c as the unknown variable, this leads to the discussion of a polynomial of degree two. Figure 2 below shows a general structure of the curve though it's a handwritten draft. From assumption 4 , we can see that $c > 0$, so around c = λ the curve changes its sign from positive to

negative. Using **Hopf's bifurcation Theorem**, we would have a periodical solution around the point $c = \lambda$. Besides, for c in the range $(0, \lambda)$, we would have a positive sign for the above (*) which conveys an idea that all equilibrium are unstable. Similarly, for $c > \lambda$, stability would be reached.
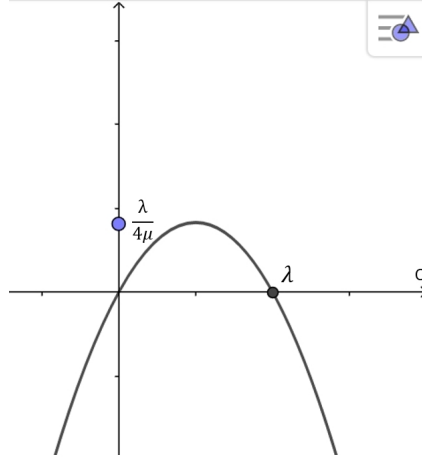

Figure 2: graph of $c(\lambda - c)/(\lambda\mu)$

4.2   Model with tossing strategies

A further discussion of the model using tossing strategies is showed below. When seeking solutions to the system (2), the similar methods are applied here to equal the three equations to zero. Though this system is way too complex to solve, we have to apply assembled procedures here.

1. As well, the origin point (0,0,0) is a steady state.

2. Then we need to analysis whether there are other solutions such that any component in (P, E, B) is zero.

   - if $B = 0$, the (4) equation tells us $P = 0$, (5) and (6) verify the arbitrariness of Eggs.
   - if $P = 0$ and $B = 0$, then $E \in \mathbb{Z}$
   - if $P = 0$, but $B \neq 0$ then the (5) equation gives that $c(1 - \frac{\theta E}{1+E^2}) - \frac{\gamma}{E} - g = 0$ (when $E \neq 0$)

3. For PEB all non-zero, we have (P^*, E^*, B^*) must satisfied the following equation systems:

   $$\mu B - k = 0$$

   $$BEc\left(\frac{2E^2\theta}{(1+E^2)^2} - \frac{\theta}{E^2+1}\right) + \frac{BE\lambda}{(E+1)^2} - \frac{B\lambda}{1+E} + Bc\left(1 - \frac{E\lambda}{E^2+1}\right) - Bg = 0$$

   $$\frac{\lambda E}{1+E} - \phi P - \gamma = 0$$

   After simplification we have that: $B* = \frac{k-a}{\mu}$, $E^2 = \frac{(c-1)\lambda}{2\theta-\lambda}$, the number of predators P takes the form: $\frac{\lambda E}{1+E} - \theta P - \gamma = 0$

The next step is to analyze the Jacobian matrix as before, the generalized Jacobian matrix can be formulated as

$$J_{(P,E,B)} = \begin{bmatrix} \mu B + a - k & 0 & \mu P \\ 0 & \frac{\lambda BE}{(1+E^2)^2} + \frac{BEc(2\theta E^2 - \lambda E^2 - \lambda)}{(E^2+1)^2} & 0 \\ -\phi B & \frac{\lambda B}{(1+E)^2} & \frac{E\lambda}{1+E} - P\phi - \gamma \end{bmatrix}$$

The above equilibrium points are substituted into the Jacobian matrix respectively, and it can be judged as follows:

- For $J(0,0,0) = \begin{bmatrix} -k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma \end{bmatrix}$, its trace is $\text{Tr}(J) = -k - \gamma$ and determinant is $\det(J) = 0$. Clearly it conforms to the behavior of non-isolated equilibrium points. This conformity demonstrates that the strategy of egg-tossing does little impact on the behavior of equilibrium state at the origin.

- In terms of the nontrivial solutions, it's a extreme headache to deal with given its complexity of its essence as a three times three matrix with non-linear terms. Namely, for

$$J(P^*, E^*, B^*) = \begin{bmatrix} 0 & 0 & \mu P^* \\ 0 & \frac{\lambda B^* E^*}{(1+(E^*)^2)^2} + \frac{B^* E^* c(2\theta(E^*)^2 - \lambda(E^*)^2 - \lambda)}{((E^*)^2+1)^2} & 0 \\ -\phi B^* & \frac{\lambda B^*}{(1+E^*)^2} & 0 \end{bmatrix}$$

The characteristic equation conforms to the form as equation (2) (It's a generally accepted theorem, hence we will not provide any proof for this).

Here, $tr(J*) = \frac{BE\lambda}{(1+E)^2}$, $\sum minors = PB\mu\phi det(J*) = \frac{P\mu B^2 E\phi[(2\theta-\lambda)cE^2+(1-c)\lambda]}{(1+E*^2)^2}$. By utilizing **Routh-Hurwitz Criteria** from theorem 1, we find a problem that one condition cannot satisfied: $a_1 \times a_2 > a_3$. The reason is that from our computation, $a_1 \times a_2 = a_3$. Thus, the solution may be periodic, we proceed by investigating the real part of complex roots. Python gives us the real part of the complex roots (Theorem 2) in the cubic polynomials by inputting values like $k = 0.3$, $\mu = 0.02$ (this is given based on the predation rate between predators and preys in the system (Kenya statistics) mutiplied by the effect of marginal return of energy consumption in nature) and we derive:

$$-0.837 \cdot \left(\left(\text{re}(P\phi)^2 + \text{im}(P\phi)^2 \right)^{\frac{1}{4}} \cdot \cos\left(\frac{\arctan(\text{im}(P\phi), \text{re}(P\phi))}{2} \right) \right)$$

The primary analysis of this term is to find out its relationship with 0, notice that there is a negative sign before the coefficient, and the term $\text{re}(P\phi)^2 + \text{im}(P\phi)^{2\frac{1}{4}}$ would be positive anyway. So the sign of cosine and arc triangular function matters. The $cos\theta$ function returns a positive value between $\theta \subset (-\frac{\pi}{2}, \frac{\pi}{2})$ as the below figure, and within such area, the arc triangular returns the tangent value of the line crossing the origin and $(re(P^*\phi), im(P^*\phi))$. Thus we have the below proposition

Proposition2. If $re(P^*) > 0$ (real part of P^* is positive), all the equilibrium points of the non-zero solutions would have negative real parts, if $re(P^*) = 0$ which means P^* is a pure imaginary number. And by Hopf's bifurcation theorem, we would have a periodic solution when some
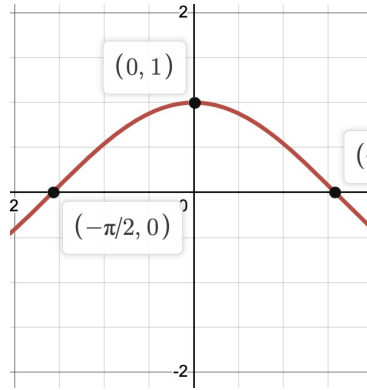
parameters satisfy the equation: $\frac{E\lambda}{1+E} - P\phi - \gamma$, and around that value for small perturbation, it would have a according limit cycle; if $re(P^*) < 0$, all equilibrium solutions would be unstable by similar analysis.

5   Results

Beyond the above functional analysis, we now assign specific values to our parameters to observe any existence of particular pattern and to validate our analysis by spotting the existence of a limit cycle. Generally speaking, the values are shown as follows: $k = 0.3, \mu = 0.02, c = 0.5, \lambda = 100, g = 0.015, \phi = 0.3, \gamma = 0.2$

Linearization We fix the equilibrium points in both system while systematically adjusting the initial points by assigning perturbation to the points - a well-known method named linearization. One can observed that, irrespective of the position of diverse initial points, the trajectories of them consistently converge to the limit cycle for both models. This intriguing finding prompts us to conduct a more detailed examination by generating 2D plots to further explore the impact of infanticide strategy.

2D plot analysis

In figure 4, the horizontal axis represents the number of predators, while the vertical axis represents the number of birds. For the scenario without egg tossing, the x-axis values are set in the range of (331, 333.5), while the y-axis values position between (14.4, 15.6). Comparatively, with the introduction of egg tossing strategy, the x-axis values shift to (315, 350), and the y-axis values to (5, 30). Such differences are novelty to attend to, and the biological interpretation can be that, once involving the infanticide strategy in our PEB system, the limit cycle enlarges to a large scale. That is to say, the oscillation of both number predators and birds are waved more dramatically. Though with increased oscillation, the periodic solutions seems to be apparent as well. We can reach a conclusion at this point: the introduce of infanticide strategy would not impair the stability of the PEB system. This really conforms to our understanding of the mother nature.

Take a closer look at the figure 4 and figure 5 . Firstly, the curve oscillates with time, which implies a periodic behavior within the system. This showcases the co-existences of birds and predators in the system in endless time if neglecting other environmental impacts (assumption 1). It is noted that the

Phase portrait of the nonzero equilibria plus small perturbation (without egg tossing)
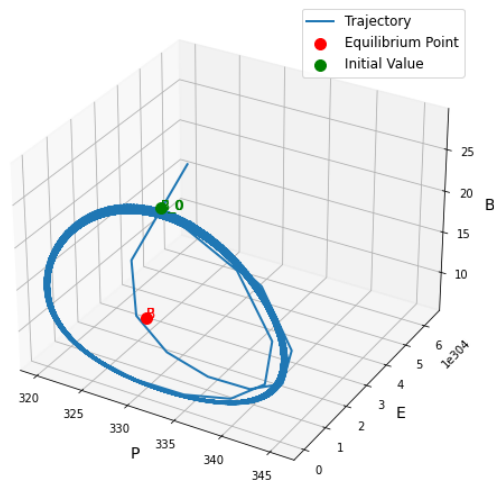
Phase portrait of the nonzero equilibria plus small perturbation (with egg tossing)
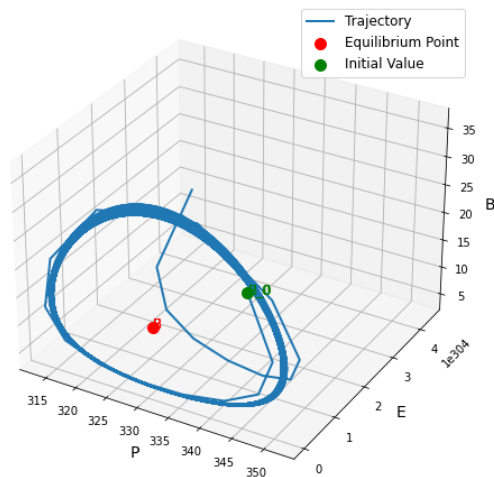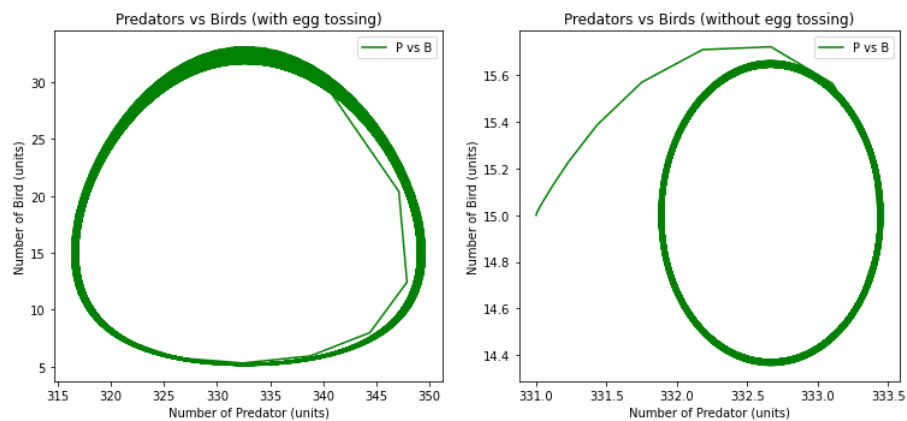

Figure 3: equilibrium points in 3D plots

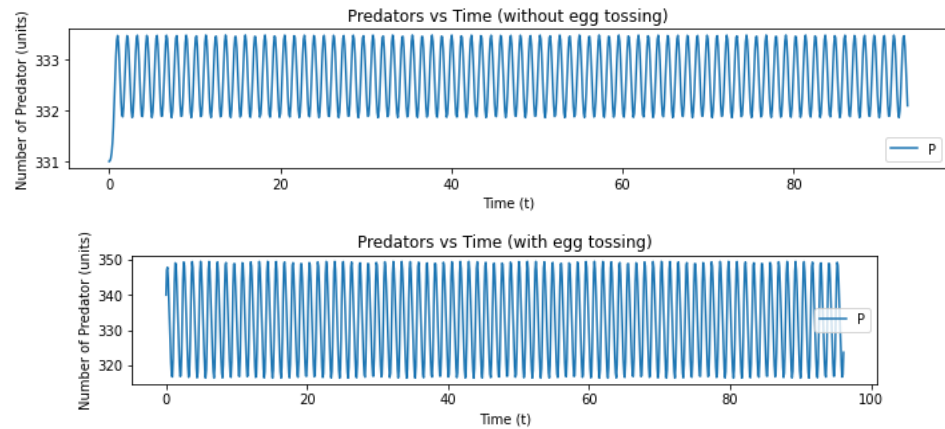Figure 4: equilibrium points in 2D plots
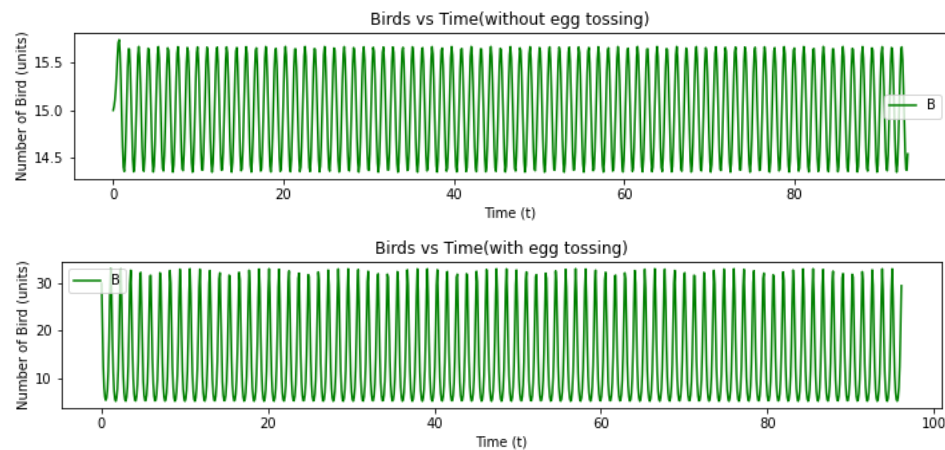
Figure 5: Predators vs Times

Figure 6: Birds vs Times

amplitude of oscillation vs time, for instance, at intervals of figure 6, experiences a significant increase compared to the figure in non egg-tossing. To be more concrete, with the help of infanticide method, the scale of oscillation for predators increased 15 times (discrepancy 30 compared to 2). This is an incredible figure which implies that the discrepancy enlarges following the variance of time. In figure 6, comparative things happen as well. The number of birds is experiencing a two times larger variations in this periodic range in the PEB with tossing.

In summary, the 3D model's resilience to changes in initial values, coupled with the pronounced increase in oscillation amplitude in the 2D representation, suggests that the introduction of egg tossing contributes to heightened stability within the model. This finding holds implications for understanding and potentially managing ecological systems, providing valuable insights for future research and practical applications.

5.1 The biological implications

Egg tossing has been interpreted as an adaptive strategy, and its biological implications can be deeply understood as an evolutionary response of gray-top weaves population to environmental changes, which mainly includes the following aspects:

- The PEB system firstly show its periodic behavior in the normal scenario. This implies that predators, eggs and weaver birds can co-exist in the system, and they contribute and confine others to construct a highly stable (proven by perturbation) system where their numbers change within particular regions with the fly of time.

- Egg-tossing strategy is invented to balance the expected fitness of the long run life cycle by sacrificing the current insufficient living condition (mainly conducted by predation) [2]. The differences of two model in figure 4, 5, 6 illustrate that such introduction of infanticide approach verifies such expected fitness in the long run because it prolongs and enlarges the variation in the system for all kinds (P,E,B). However, the periodic solution remains existent. This guarantees that our system can sustain external brutal forces to some content. To put it another way, if something happens and leads to the deficiency of some resources in the system that may contribute to parental egg-tossing behavior. Then the status quo of birds family would variate but still preserve a periodicity and recuperate to a better condition for hatching and incubating in the long term.

- It is seen as a resource management strategy to conserve energy and resources and improve the chances of reproductive success. This can be interpreted as an adaptive behavior under resource scarcity or environmental stress.

- This strategy reduces investment in the current reproductive cycle and increases the success of future reproductive cycles. This reflects a long-term reproductive strategy that maximizes the chances of reproductive success by allocating resources adaptively.

- It may be an adaptive response to environmental stress, especially when predation pressure is high or nest capacity is limited. This behavior helps adapt to changing environmental conditions and improves the chances of survival and reproduction.

The model seeks to understand the long-term effects of tossing strategies on overall population dynamics

- Adjustment of ecological balance scale: The introduction of tossing eggs may lead to long-term adjustment of population dynamics. Through mathematical modeling, the effects of tossing strategies on the equilibrium points and the stability of these equilibrium points were studied. This provides insight into how populations of gray-top weaves adapt tossing strategies to maintain ecological balance.

- Likelihood of periodic behavior: Models suggest that tossing strategies may lead to periodic fluctuations in population size. This suggests that populations may undergo periodic changes over a certain time range, which may be related to environmental conditions or other intrinsic dynamic factors.

- Adaptive evolution of populations: The effects of tossing strategies are considered as part of the adaptive evolution of populations. Through the mathematical model, it can be inferred that the tossing behavior may affect the adaptive evolution of the population of gray top weaver finch, so that it can adapt to the current ecological environment.

6   Conclusion

In-depth mathematical modeling of the population dynamics of grey-capped weavers has led to a number of key conclusions that have helped deepen our understanding of this ecosystem. We first analyze a mathematical model of behavior different from tossing eggs, under which we observe equilibrium points, some of which appear as non-isolated points. This suggests that in the absence of egg dropping, population dynamics may take on relatively simple characteristics.

We then introduced a mathematical model of egg tossing behavior, taking into account its potential impact on population dynamics. In this model, we observe more complex equilibrium points and periodic solutions, suggesting that egg tossing may alter the ecological dynamics of the population to some extent. Our model further reveals that egg tossing behavior may be an adaptive strategy, especially when resources are scarce or environmental stress is high. By reducing investment in the current breeding cycle and increasing the success of future breeding cycles, gray-top weaverbird populations may be better adapted to complex ecological environments.

Our study also highlights the limitations of the model, chief among which is the simplified treatment of seasonal variations and environmental dynamics. Seasonal changes and environmental fluctuations present in the actual ecosystem can have an important impact on the accuracy of the model. Therefore, these factors need to be considered more carefully when applying the model to real-world scenarios.

Finally, our study provides a useful mathematical framework for the field of ecology, which lays the foundation for further studies of population dynamics and adaptive behavior of grain-top weavers and other ecosystems. We encourage future research to pay more attention to the assumptions and limitations of models when using mathematical models for ecological exploration in order to more fully understand complex ecological interactions in nature.

6.1   Limitations

Although our model provides a preliminary understanding of the population dynamics of gray-top weavers, it also has some limitations that need to be taken into account when interpreting and applying the model results.

First, our model may ignore seasonal variations. In fact, ecosystems can experience significant impacts from season to season, such as fluctuations in temperature, food availability, and so on. These factors may be oversimplified in the model, which limits its accurate description of seasonal dynamics. Second, our models may not adequately account for the dynamics of the environment. An ecosystem is a constantly changing system involving changes in habitat, fluctuations in predator populations, and more. If these dynamics are not properly accounted for, the model's predictions can be affected. In support of our assertion of model limitations [4] found in their study that assumptions and simplifications

of models of bird behavior can lead to underestimates or overestimates of population dynamics and ecosystem responses. This study highlights the need for careful handling of assumptions in model interpretation and application to fully understand the scope of application of the model.

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