

COSC 483 Final Exam

Things that are allowed in the final exam

- + Group submission: max. 2-3 students/group
- + Notes, books, extra materials, and Internet references.

Things that are NOT allowed during the final exam

- Discussion outside of the group (e.g., Chegg.com – its answers are always incorrect and easy to detect).
- Straight copy from the Internet. You must cite the source and highlight your modification from the source.
- **Late submission (will result in Zero points).**

Submission guidelines

Submit a single PDF file per group containing all the answers. For each question, answer

- (1) thought process and approach
- (2) pseudo-code to solve the problem
- (3) proof of correctness or optimality
- (4) time and space complexity analysis in big-O notation
- (5 – optional) possible improvements.

Questions 1, 4 and 5 are required. Do either question 2 or 3.

Question 1 [20] Given an array $A[]$ of n integers, find the *longest non-decreasing subsequence* such that for every two adjacent elements the bigger number is at least twice the smaller one.

Question 2 [20] Professor X drives a Tesla to travel from city A to city B. The distance between cities A and B is N miles. The Tesla can go up to T miles before it needs a charge and there are many charging locations along the way including those in cities A and B. *Each station requires a different amount of time to fully recharge the car battery, and once stopped the Tesla must be recharged as required.* Professor X wants to minimize *the total battery charging time* so he can get to the destination the soonest. Assume that at any charging location there is at least another charging location within the T -mile radius.

Input: A map of charging stations and required battery recharge time. For example, for $N = 10$, $T = 4$ and the following charging locations (underlined). The battery recharge time is formatted as “(X)” where X is the charging hours

A 1(0) 2 3(2) 4(3) 5 6(1) 7 8(4) 9(2) 10 11(0) **B**

Present an approach for this problem. Does your approach provide an optimal solution? Prove that your solution is optimal or show a counter example? Write a pseudo-code for your procedure. Show its complexity.

Question 3 [20] Amazon.com is renting local trucks to deliver packages in the early morning and afternoon shifts. Each city zone needs several trucks, and those numbers are estimated and provided to the manager at 11:59pm daily. The manager will then rent enough trucks for the early morning shift and reuse them for the afternoon shift. Note that each zone must be completely covered in either shift.

For example, if the city has 4 zones and their truck requirements, in this order, are 1, 6, 11 and 5, the manager could rent 16 trucks to deliver packages in zones #3 (11 trucks) and #4 (5 trucks) in the morning shift and reuse 7 trucks to cover zones #1 (1 truck) and #2 (6 trucks) in the afternoon shift. However, he would leave 9 truck unused in the afternoon shift. Or, the manager could have rented 12 trucks to cover zones #1 (1 truck) and #3

(11 trucks) in the morning and reuse 11 trucks to deliver packages in zones #2 (6 trucks) and #4 (5 trucks). In this plan, he would only have 1 truck left unused. Note that 12 is also the minimum number of trucks needed.

You are interviewed to help the manager *to rent the minimum numbers of trucks for daily delivery*. Of course, you want to give it your best try and get hired. What would your approach be?

Question 4 [20]: Solve the following LP using the Simplex Method. Show all the steps

$$\begin{aligned} \text{Maximize:} \quad & Z = 3x + 5y + 2z \\ \text{Subject to} \quad & x + 2y + z \leq 10 \\ & 3x + 2y + 4z \leq 24 \\ & 2x + 5y + 3z \leq 30 \\ & x, y, z \geq 0 \end{aligned}$$

Question 5 [20] In a graph G , an edge (u, v) is good if all other vertices of G are adjacent to both u and v , i.e., all other vertices are connected to both u and v via different edges. Assume that G is a clique of size n (i.e., a complete graph of n vertices), *what is the least number of edges to be removed so that G does not have any good edges left after the removal. Is this a problem in NP?*